COMMODITY FUTURES AND SPOT PRICE DETERMINATION AND HEDGING IN CAPITAL MARKET EQUILIBRIUM

by

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An integral element of most models of futures markets are hedgers typically viewed as involved in the storage or production process and wishing to avoid price risk associated with holdings of the underlying commodity by futures market transactions. Speculators accept the risk and receive compensation the size of which is in considerable dispute. [Keynes (1930), Telser (1958), Cootner (1960), Dusak (1973)]. This "insurance" view of hedging is sometimes expanded to allow for "discretionary" or "selective" hedging which tends to arise when expectations differ across individuals.[1] Narrow models of hedging in the commodities market [Johnson (1960), Heifner (1972), Peck (1975)], in the foreign exchange market [Ethier, 1973] and in the bank loan market [Pyle, 1971] as well as more general models of the determination of spot and futures prices that incorporate hedging [Stein, 1961] have preceded or ignored the theory of equilibrium asset prices [Sharpe (1964), Lintner (1965)]. On the other hand recent models of the valuation of futures contracts in capital market equilibrium have not considered the role of hedgers [Grauer and Litzenberger (1979)]

Traditional explanations of hedging are problematical since they fail to take proper account of the risk spreading opportunities available in the capital market as a whole as opposed to the futures market alone. Thus it can be shown that in a world of perfect capital markets where shares in the production or storage process and futures contracts are freely traded at no cost, there is no need for hedging [Baron (1976), Stoll (1976)].

Such a demonstration is, however, at variance with the observed concern of many firms and individuals with appropriate futures market hedging strategies. The basis for hedging offered in this paper is the inability or reluctance of individuals to trade ownership claims on certain assets or production techniques with which they are endowed. As a result risk is not passed on in the stock market but may (in part) be passed on in the futures market. This rationale is appropriate for commodity futures markets in which many firms are privately held and where there are individual farmers.[2] Alternative rationales for hedging are possible. See for example Anderson and Danthine (1978) for a rather general approach and Dumas (1977) for a rationale based primarily on bankruptcy.

In this paper a simple model of spot and futures prices is developed in context of capital market equilibrium in which, a la Mayers (1972), there are certain non-tradeable assets. The principal problems are the allocation of risk in the capital market, which consists of a stock market, a futures market and a bond market; and the allocation of the amount to be stored. The assumptions underlying the analysis are presented first. In section II the equilibrium futures price, interest rate, stock price, and level of hedging and speculation are determined conditional on a particular distribution of storage. In section III the optimal hedging and storage decision of the individual storer are developed. The equilibrium spot price, basis and distribution of storage is considered in section IV.

I. Assumptions and Notation

For simplicity, and without loss of generality, one security exists in the stock market, in the bond market and in the futures market. Two points in time are assumed. The stock can be thought of as a mutual fund holding the shares of all companies. The price per share today and the random dollar payoff tomorrow per share are denoted by p and $\tilde{v}$ respectively. The aggregate supply of shares is $\bar{x}$. Each bond instrument is assumed to pay one dollar with certainty at time 2. Today's price of a bond is denoted by 1/R where R is one plus the riskless rate of interest, R_f. The aggregate supply of bonds is zero.

The futures market is a market in futures contracts; not a market in the underlying commodity. Those holding long positions promise to take delivery of the underlying commodity and make payment at maturity at the futures price of the contract, and those holding short positions promise to make delivery of the underlying commodity and receive payment at maturity at the futures price of the contract. Since longs equal shorts, the aggregate supply of futures contracts is zero. One contract calls for delivery of one bushel, and the futures price is denoted by F. Since positions acquired through delivery can be liquidated at the time 2 spot price, $\tilde{S}$, a random variable today, the payoff to a long position in one futures contract is $(\tilde{S}-F)$. The contract is a pure bet, and the futures price is determined so that neither party to the contract pays the other party any amount.[3] With two points in time, this means the value of the futures contract is zero in period one.

Speculators are defined as having no position in the underlying commodity; they may hold futures contracts. Hedgers are distinguished from speculators by their ownership of a production process which in its simplest form is pure storage of the underlying commodity. A key assumption is that shares in this process are not traded in the stock market. A broader model might take as endogenous this lack of tradeability but the scope of this paper is more limited.[4] The union of owernship and management functions of commodity storage firms is taken as a datum, and the paper analyses futures and spot price determination in this context. The future dollar payoff to individual i from storing Q_i bushels is

$$\tilde{\pi}_i Q_i = \tilde{S} Q_i - C_i(Q_i) - \tilde{u}_i Q_i \tag{1}$$

where

$\tilde{\pi}_i$ = per bushel payoff (a function of Q_i)

$C_i(Q_i)$ = deterministic portion of the total cost of storage function.

$$\frac{\partial\, C_i(Q_i)}{\partial\, Q_i} = C_i'(Q_i) > 0; \qquad C_i''(Q_i) < 0.$$

$\tilde{u}_i$ = random variable that represents uncertainties in per bushel storage costs. The distribution of $\tilde{u}_i$ is normal with $E(\tilde{u}_i) = 0$, and it is independent of Q_i

Payment of storage costs is made at time 2. The cost function, $C_i(Q_i)$, reflects out-of-pocket storage costs (i.e., rent, insurance, but not interest) and a convenience yield that declines with the quantity stored. This traditional convenience yield of Kaldor (1939) and Brennan (1958) recognizes that a commodity will be stored even when the return to storage does not cover out-of-pocket costs because it is "convenient" to have the commodity on hand to maintain a production process or because there is an explicit monetary yield (as when foreign exchange is stored in the form of interest bearing instruments or when cattle that increase in weight are stored). A pure storer would not realize a non-pecuniary convenience yield. The storage industry is small relative to other industries, and payments to its factors of production are assumed not to affect capital market equilibrium or the aggregate time pattern of consumption.[5]

The aggregate amount of the commodity to be stored is given, and the spot price is determined so that the available bushels are willingly stored. In this paper the allocation between current consumption and storage is not considered.[6] Storers are not individually endowed with quantities to be stored, and each stores according to his abilities as reflected in his cost function; in other words there is tradeability among storers in the underlying commodity. What is not tradeable is claims on the production-storage process. The number of storers is exogenously fixed.

Notation for time 2 dollar payoffs per unit of financial asset or commodity, price today per unit, number of units held by individual i, and aggregate supply of units is summarized as follows:

	Price today per unit	Payoff tomorrow per unit	Number of units held by individual i	Aggregate Supply of units
Stock	p	$\tilde{v}$	x_i	$\bar{x}$
Bond	$\frac{1}{R}$	1	b_i	0
Futures	0	$\tilde{S}-F$	Z_i	0
Commodity	S_o	$\tilde{\pi}_i$	Q_i	$\bar{Q}$

The tilde denotes a random variable. For each individual the joint distribution of payoffs at time 2 is assumed to be multi-variate normal. There are homogenous expectations about payoffs to financial assets that are freely traded, but payoff to holding the underlying commodity may vary across individual storers because of differences in either the deterministic or random component of the storage cost function. There are no transactions costs or taxes. Short sales of the bond, stock and futures contract are permitted, but individuals are sufficiently risk averse so that borrowing is not excessive and the probability of bankruptcy may be assumed to be zero.

II. Hedgers, Speculators and Capital Market Equilibrium

A. Basic Result

Each individual maximizes the utility of lifetime consumption. Consumption in period 2 of individual i is

$$\tilde{W}_i = x_i\tilde{v} + b_i + Z_i(\tilde{S}-F) + Q_i\tilde{\pi}_i, \tag{2}$$

where $\tilde{\pi}_i$ is given by (1). Hedgers are defined by $Q_i>0$; speculators, by $Q_i=0$. When the distinction between hedgers and speculators needs to be made more explicitly the subscripts "h" for hedgers and "s" for speculators are utilized. The expected value, E_i, and variance, σ_i^2, of future consumption are

$$E_i = x_iE(\tilde{v}) + b_i + Z_i[E(\tilde{S})-F] + Q_iE(\tilde{\pi}_i) \tag{3}$$

$$\sigma_i^2 = x_i^2\sigma^2(\tilde{v}) + Z_i^2\sigma^2(\tilde{S}) + Q_i^2\sigma^2(\tilde{\pi}_i) + 2x_iZ_i\text{cov}(\tilde{S},\tilde{v})$$
$$+ 2x_iQ_i\text{cov}(\tilde{\pi}_i,v) + 2Z_iQ_i\text{cov}(\tilde{S},\tilde{\pi}_i) \tag{4}$$

Since the distribution of $\tilde{W}_i$ is normal by assumption, one can justify a preference function, $G_i(c_{1i},E_i,\sigma_i^2)$, which depends on consumption at time one and only on the expected value and variance of future consumption.

It is to be maximized subject to the budget constraint, $W_{oi} = c_{1i} + x_i p + \frac{b_i}{R} + S_o Q_i$, where W_{oi} = individual i's current wealth. Consider first the problem of the optimal selection of financial assets conditional on S_o and Q_i. The relevant Lagrangian is

$$L_i = G_i(c_{1i}, E_i, \sigma_i^2) + \lambda_i[W_{oi} - c_{1i} - x_i p - \frac{b_i}{R} - S_o Q_i] \tag{5}$$

where λ_i is the Lagrangian multiplier. Taking derivatives with respect to c_{1i}, b_i, x_i and Z_i yields the optimality conditions (for convenience tildes are henceforth omitted):

$$\frac{\partial G_i}{\partial c_{1i}} - \lambda_i = 0 \tag{6}$$

$$\frac{\partial G_i}{\partial E_i} - \lambda_i \frac{1}{R} = 0 \tag{7}$$

$$\frac{\partial G_i}{\partial E_i} E(v) + \frac{\partial G_i}{\partial \sigma_i^2} [2x_i \sigma^2(v) + 2Z_i \operatorname{cov}(S,v) + 2Q_i \operatorname{cov}(\pi_i, v)] - \lambda_i p = 0 \tag{8}$$

$$\frac{\partial G_i}{\partial E_i}[E(S)-F] + \frac{\partial G_i}{\partial \sigma_i^2} [2Z_i \sigma^2(S) + 2x_i \operatorname{cov}(S,v) + 2Q_i \operatorname{cov}(S,\pi_i)] = 0 \tag{9}$$

The first 2 conditions show that each individual borrows or lends to equate the marginal rate of substitution of current consumption for expected future consumption with minus one plus the risk free rate:

$$-\frac{\partial G_i/\partial c_{1i}}{\partial G_i/\partial E_i} = \frac{dE_i}{dc_{1i}} = -R \tag{10}$$

Equilibrium in the futures market is determined by the requirement that the demand for futures contracts by speculators equals the supply of futures contracts by hedgers. In addition the requirement that all shares are held by the m individuals in the economy must be met: $\sum_{i=1}^{m} x_i = \bar{x}$; as must the condition that the bushels of the commodity available for storage must be stored: $\sum_{i=1}^{m} Q_i = \bar{Q}$. Before aggregating divide through (9) by $\partial G_i/\partial C_{1i}$ and use (10) to get

$$\frac{\Theta_{hi}}{2}\left[\frac{E(S)-F}{R}\right] = Z_{hi}\sigma^2(S) + X_{hi}\text{cov}(S,v) + Q_i\ \text{cov}(\pi_i,S) = \text{cov}(S,W_{hi}) \tag{11}$$

for hedger i and

$$\frac{\Theta_{sj}}{2}\left[\frac{E(S)-F}{R}\right] = Z_{sj}\sigma^2(S) + X_{sj}\text{cov}(S,v) = \text{cov}(S,W_{sj}) \tag{12}$$

for speculator j, where

$$\Theta_i = -\frac{\partial G_i/\partial c_{1i}}{\partial G_i/\partial \sigma_i^2} = \frac{d\sigma_i^2}{dc_{1i}},$$

the marginal rate of substitution of current consumption for variance of future consumption, and where W_{hi} and W_{sj} are the terminal wealth of hedger i and speculator j respectively. Now summing over m_h hedgers and m_s speculators respectively yields

$$\frac{\Theta_h}{2}\left[\frac{E(S)-F}{R}\right] = Z_h\sigma^2(S) + x_h \operatorname{cov}(S,v) + \sum_{i=1}^{m_h} Q_i \operatorname{cov}(\pi_i,S)$$

$$= \sum_{i=1}^{m_h} \operatorname{cov}(S,W_{hi}) \tag{13}$$

$$\frac{\Theta_s}{2}\left[\frac{E(S)-F}{R}\right] = Z_s\sigma^2(S) + x_s \operatorname{cov}(S,v) = \sum_{j=1}^{m_s} \operatorname{cov}(S,W_{sj}) \tag{14}$$

where $Z_k = \sum_{i=1}^{m_k} Z_{ki}$; $x_k = \sum_{i=1}^{m_k} x_{ki}$; $\Theta_k = \sum_{i=1}^{m_k} \Theta_{ki}$; $k=h,s$.

Making use of the fact [from (1)] that

$$\operatorname{cov}(\pi_i,S) = \sigma^2(S) - \operatorname{cov}(u_i,S)$$

and rearranging terms yields

HH: $$F_h = E(S) - \frac{2R}{\Theta_h} \sum_{i=1}^{m_h} \operatorname{cov}(S,W_{hi}) = E(S) - \frac{2R}{\Theta_h} x_h \operatorname{cov}(S,v)$$

$$- \frac{2R}{\Theta_h}\left[\bar{Q}\sigma^2(S) - \sum_{i=1}^{m_h} Q_i \operatorname{cov}(S,u_i)\right] - \frac{2R}{\Theta_h} Z_h\, \sigma^2(S) \tag{13.a}$$

SS: $$F_s = E(S) - \frac{2R}{\Theta_s} \sum_{j=1}^{m_s} \operatorname{cov}(S,W_{si})$$

$$= E(S) - \frac{2R}{\Theta_s} x_s \operatorname{cov}(S,v) - \frac{2R}{\Theta_s} Z_s\sigma^2(S) \tag{14.a}$$

These demand equations are plotted in Figure 1 in a graph that is in the sprit of Cootner but with a more precise interpretation. The equilibrium futures price is found where $Z_s = - Z_h$:

$$F^* = E(S) - \frac{2R}{\Theta} [\sum_{i=1}^{m_h} \text{cov}(S,W_{hi}) + \sum_{i=1}^{m_s} \text{cov}(S,W_{si})]$$

$$= E(S) - \frac{2R}{\Theta} [\bar{x} \text{ cov}(S,v) + \bar{Q}\sigma^2(S) - \sum_{i=1}^{m_h} Q_i \text{cov}(S,u_i)] \qquad (15)$$

where $\Theta = \sum_{i=1}^{m} \Theta_i = \Theta_h + \Theta_s$

When there are homogenous expectations the critical element giving rise to futures trading is the holding by storers of an asset not tradeable in the stock market. This causes the displacement of the SS and HH schedules. If claims on the storage activity could be sold in the stock market, hedgers and speculators would hold identical portfolios except for the proportion invested in risk free asset. In that case the term involving Q in the HH equation is zero (because the return to holding ownership shares in Q is subsumed in v. Furthermore when all assets are tradeable one can show that holdings of any risky asset differ across individuals only by a scale factor which is Θ_i in this case.[7] This means that $X_s/\Theta_s = X_h/\Theta_h$ and $Z_s/\Theta_s = Z_s/\Theta_h$. Inspection of 13.a and 14.a show that these conditions can be met only if $Z_s = - Z_h = 0$. The SS and HH schedule would collapse to a single point, and future trading would not need to exist. This is so <u>irrespective of tastes</u> which are reflected in Θ_s and Θ_h. Of course a rationale for futures markets would arise if individuals disagree about the future spot price distribution, but that rationale alone does not convey the usual distinction between speculators and hedgers. In this model the inability to sell shares in the storage process gives rise to hedging behavior in the traditional sense of Keynes (1930) and Cootner (1960). Hedgers buy insurance from speculators in exactly the same way that individuals whose human wealth is not tradeable buy life insurance. The price of insurance in the aggregate is the risk premium $E(S)-F$ defined by (15). When $E(S)-F>0$ normal backwardation is said to obtain [Keynes, 1930].

Although expectations are homogeneous, hedgers with different underlying storage or production processes have different relative needs for hedging because of differences in $\mathrm{cov}(S,u_i)$. As a result different hedgers are willing to pay different amounts for the same insurance. Values of $\mathrm{cov}(S,u_i)$ may differ because technology of production or storage differs or because certain other hedging actions are taken. For example, forward contracting of rental payments and labor payments may reduce the need for hedging in the futures market.

B. Normal Backwardation and the Telser-Cootner Debate

The dispute over normal backwardation has been fought primarily in terms of the shape and location of the HH and SS schedules of Figure 1. The SS schedule is drawn to reflect the normal backwardation of Keynes (1930) and Cootner (1960).[8] Telser (1958) has drawn the SS schedule perfectly elastic (horizontal) and passing through $E(S)$ so that $F=E(S)$ and no price is paid for insurance. In terms of this model, normal backwardation exists if the last term of (15) is negative. If the term is positive, contango [where $F>E(S)$] is implied. Since $R,\theta,\bar{x},Q$ are positive the issue depends on the signs and magnitudes of cov(S,v), $\sigma^2(S)$ and $\sum_i^{m_h} Q_i\ \text{cov}(S,u_i)$. Since the covariance terms can be positive or negative, anything is possible. But empirical work by Dusak (1973) suggests cov(S,v) = 0. One would suppose $\sigma^2(S) > \text{cov}(S,u_i)$ because per bushel commodity price fluctuations exceed per bushel storage cost fluctuations. In that case normal backwardation would exist even with zero covariance between commodity prices and stock prices. Should there be negative covariance between commodity prices and stock prices, contango could result.[10]

C. Commodity Futures Returns and the Capital Asset Pricing Model

The equilibrium stock price, p, is found by aggregating (8) over all m individuals and invoking the market clearing conditions. Using (10) the result is

$$\frac{\Theta}{2}\left[\frac{E(v)}{R} - p\right] = \bar{x}\sigma^2(v) + \sum_{i=1}^{m_h} Q_i \text{cov}(\pi_i, v) \tag{16}$$

This may be written as

$$\frac{2}{\Theta} = \frac{E(Rm) - R_f}{R[P\sigma^2(R_m) + \sum_{i=1}^{m} Q_i \text{cov}(\pi_i, R_m)]} \tag{17}$$

where $P = \bar{x}\, p$, $R_m = \frac{v}{p} - 1$, $R_f = R-1$

The equilibrium futures price in term of stock market variables can be written by using (17) to eliminate $2/\Theta$ from (15). Writing the result in rate of return dimensions yields.

$$\frac{E(S)-F}{F} = \frac{E(R_m)-R_f}{\sigma^2(R_m) + \sum_{i=1}^{m_h} \frac{FQ_i}{P} \operatorname{cov}(\frac{\pi_i}{F},R_m)} [\operatorname{cov}(R_s,R_m) + \frac{F\bar{Q}}{P} \sigma^2(R_s)$$

$$- \sum_{i=1}^{m_h} \frac{FQ_i}{P} \operatorname{cov}(R_s, \frac{u_i}{F})] \qquad (18)$$

when $R_s = \frac{S}{F} - 1$.

This is the Mayers (1972) result with a different interpretation. The expected dollar return on a futures contract, relative to the futures price, depends on the market price of risk (the fraction outside the brackets) and the risk of the futures contract which is measured by the covariance of the commodity return with the return of the other assets. Using F as a base for expressing a futures market return is arbitrary, but proper so long as the relevant risk terms are expressed in the same way. There is no investment of F or other amount and thus no minimum risk free return is required[10].

As Mayers notes, the market price of risk, usually estimated as $\frac{E(R_m)-R_f}{\sigma^2(R_m)}$, is overstated when there are non tradeable assets that covary with the tradeable assets. The discrepancy depends (roughly) on the value of the amount stored ($\sum_{i=1}^{m_h} FQ_i$) relative to the value of the stock.[11] The risk of the futures contract depends on the usual covariance of its return with the return of the market. But it depends also on the covariance with the other assets in the economy--the crop in storage $[\sigma^2(R_s)]$ and the storage process itself $[\mathrm{cov}(R_s, \frac{u_i}{F})]$.

There are two risks associated with the commodity: (1) the price risk ($\tilde{S}$) arising from future supply and demand uncertainties and (2) the risk of storage ($\tilde{u}$) occuring from uncertainties of storage costs. The futures market permits only the price risk to be passed on. If shares in the storage process were traded in the stock market, both the price risk and basis risk would be passed on, in fixed proportions in the stock market. In that case the Sharpe-Lintner (S-L) CAPM pricing equation would result:

$$\frac{E(S)-F}{F} = \frac{E(R_m^{'})-R_f}{\sigma^2(R_m^{'})} \mathrm{cov}(R_s, R_m^{'}), \tag{19}$$

where $R_m^{'}$ now refers to the return on a market portfolio including shares in the storage firms. Reliance on the futures market rather than the stock market may depend on factors such as the small magnitude of the storage risk (and the ability to hedge that risk by forward contracting storage costs), the costs associated with public ownership (annual reports, meetings, etc.), and the greater of flexibility in risk sharing through the futures market.

Dusak finds $\mathrm{cov}(R_s, R_m) = 0$ for 3 commodities (wheat, corn, soybeans). Since this standard measure of risk under the Sharpe-Linter capital asset pricing model (S-L CAPM) is zero, she concludes that it is not surprising to find (as she does) that average realized profits on futures contracts are also zero. However according to (18) even if $\mathrm{cov}(R_s, R_m) = 0$, the expected return on futures contracts is not zero. To the extent that the market value of commodities is small relative to market value of all shares of stock, P, the Dusak approximation may be appropriate. This is an empirical question.

III. The Individual Storer-Hedger

Assume the individual storer-hedger operates in a competitive industry and that the current price of the commodity, S_o, and the aggregate quantity to be stored, $\bar{Q}$, are given. Furthermore assume he takes the futures price and stock price as given. He has a portfolio-production problem in which he chooses simultaneously the holdings of securities, x_i, Z_i, b_i and his production of storage, Q_i. Except for b_i these decisions are not independent. In particular actions in the capital market are not independent of production.

A. The optimal hedge

A frequently asked question is whether storers are fully hedged in the sense that the volume of short sales in the futures market matches inventory holdings of the physical commodity. It is for the moment assumed that Q_i is not a decision variable, that the amount to be stored is exogenously given to the individual. From (9) and (10)

$$Z_i = \frac{\Theta_i}{2R\sigma^2(S)}[E(S)-F] - x_i \frac{\text{cov}(S,v)}{\sigma^2(S)} - Q_i[1- \frac{\text{cov}(S,u_i)}{\sigma^2(S)}] \tag{20}$$

Using (8), (10) and (1) to eliminate x_i in (20) gives

$$Z_i(1-r_{Sv}^2) = \frac{\Theta_i}{2R\sigma^2(S)}\{E(S)-F-[E(v)-Rp] \frac{\text{cov}(S,v)}{\sigma^2(v)}\}$$

$$+ Q_i[\frac{\text{cov}^2(S,v)-\text{cov}(u_i,v)\text{cov}(S,v)}{\sigma^2(S)\sigma^2(v)}] - Q_i[1- \frac{\text{cov}(S,u_i)}{\sigma^2(S)}] \tag{21}$$

where $$r_{Sv}^2 = \frac{\text{cov}^2(S,v)}{\sigma^2(S)\sigma^2(v)}$$

Simplifying (21) gives the optimal hedge

$$Z_i = \Theta_i\left[\frac{E(S)-F-[E(v)-Rp]\beta_{Sv}}{2R\sigma^2(S)(1-r_{Sv}^2)}\right] - Q_i\left[1-\frac{cov(S,u_i)-cov(u_i,v)\beta_{Sv}}{\sigma^2(S)(1-r_{Sv}^2)}\right] \tag{22}$$

where $\beta_{Sv} = \frac{cov(S,v)}{\sigma^2(v)}$

Whether $Z_i = -Q_i$ depends on the cost of hedging reflected in the numerator of the first term and on the degree of "natural" hedge existing in the storage business represented by the second term. The first term will tend to exceed zero since the analysis of (15) indicated that normal backwardation would exceed that implied by the S-L CAPM. The second term is likely to be less negative than $-Q_i$ since Dusak's empirical evidence suggests $\beta_{Sv} = 0$ and since one would suppose $cov(S,u) > 0$. As a result storers will tend to underhedge somewhat.

As a practical matter it may be appropriate to accept the S-L CAPM for the reasons given at the end of the preceding section. The first term in (22) is then zero. If in addition Dusak is correct that commodities have no portfolio risk and $\beta_{Sv} = 0$, (22) becomes

$$Z_i = -Q_i\left[1-\frac{cov(S,u_i)}{\sigma^2(S)}\right] \tag{23}$$

The formula for the optimal hedge becomes quite simple and identical to the formula that would result if a storage firm engaged in futures market hedging so as to minimize variance.[12] This is an understandable result since other dependencies have been assumed away. A positive value of $cov(S,u_i)$ is a good thing because it implies the existence of a natural hedge in the storage process in that revenues and costs are positively correlated. The larger $cov(S,u_i)$ the less the need for short hedging in the futures market. Finally if $cov(S, u_i) = 0$ (naturally or because storers forward contract all storage costs), full hedging is necessary to eliminate the commodity price risk, the only risk remaining to be hedged.

B. Optimal Production

The owner of the storage process must determine the optimal amount of storage to produce. This turns out to be a complicated problem because the financial portfolio decision and the production decision are interdependent. Taking the partial derivative of (5) with respect to Q_i and using (10) gives

$$\frac{\Theta_i}{2}\left[\frac{E(S)-C_i'(Q_i)}{R} - S_o\right] = x_i cov(\pi_i,v) + Z_i cov(\pi_i,S)$$

$$+ Q_i\sigma^2(\pi_i) = cov(\pi_i,W_{hi}) \qquad (24)$$

Following the same procedure for the stock and the futures contract results in 3 equations in the three unknowns, x_i, Z_i, Q_i. The solution for Q_i is represented as follows

$$F-S_o = C_i'(Q_i) + R_f\, S_o + H_i, \qquad (25)$$

where H_i is a complicated risk premium term given in the footnote.[13] The risk premium required by the i^{th} storer depends on his attitude toward risk; his assessment of the risk which depends on Q_i, $\sigma^2(u_i)$ and the covariance of u_i with the return on the other assets held by the storer; and on the prices of the other risks in the economy.

The storage decision is represented in Figure 2 in which the basis, $F-S_o$, and expected storage costs are plotted. The basis is the market determined price of storage, and Q_1 represents the level of storage at which expected marginal physical storage cost, $C_i'(Q_i)$, plus the marginal interest cost of holding the commodity, $R_f S_o$, are just covered.

The risk term, H_i, can be positive or negative and therefore optimal storage may be the right or left of Q_1. An argument can be made that $H_i < 0$. Relying again on the Dusak result that $\beta_{Sv} = 0$ greatly simplifies (25) as written in footnote 13 to give

$$F-S_o = C_i'(Q_i) + R_f S_o - [E(S)-F]\beta_{u_i S} - [E(v)-Rp]\beta_{u_i v}$$

$$+ \frac{2RQ_i}{\Theta_i} [\sigma^2(u_i)-\sigma^2(v)\beta^2_{u_i v} - \sigma^2(S)\beta^2_{u_i S}]$$

If $\beta_{u_i S}>0$, $\beta_{u_i v}>0$ and $\sigma^2(u_i)$ is not too large, the risk term is negative. In this case the storer would produce at a point like Q_2 at which expected storage costs exceed the basis. The distance y is a new type of convenience yield that, contrary to the traditional view of Kaldor (1939) and Brennan (1958), arises even for pure storers. The reason is that in a portfolio context the storage business can be risk reducing if storage costs are positively correlated with returns on the other assets (stock and commodity futures). Note that the convenience yield may be different for each storer depending on the characteristics of his own storage process.

If storage costs are uncorrelated with other asset returns so that $\beta_{u_i S} = \beta_{u_i v} = 0$ and if one maintains assumption that $\beta_{vS} = 0$, a more traditional answer results:

$$F\text{-}S_o = C_i'(Q_i) + R_f S_o + \frac{2RQ_i}{\Theta_i}\sigma^2(u_i) \tag{27}$$

The basis exceeds expected storage costs by a storage risk premium that in this case depends only on the variance of per unit storage costs, the storer's attitude toward risk as reflected in Θ_i, the amount stored, Q_i, and the interest rate, R_f. In this case output is to the left of Q_1. Finally if the storer forward contracts all storage costs, $\sigma^2(u_i) = 0$, output is at Q_1, and profits are certain. Although forward contracting of rent etc. may be possible, it is not always optimal. If there is a convenience yield of the kind indirected above, forward contracting of storage costs would be undesirable and risk increasing. Even if there is not a convenience yield as in (27), the price concessions necessary to arrange forward contracts must also be considered.by the individual storer.[14]

IV. Equilibrium Storage, Spot Price and Basis

The equilibrium spot price is one such that the available supply of the commodity, $\bar{Q}$, is willingly held by storers. It is found by summing (24) over all m_h storers:

$$S_o = \frac{1}{R}[E(S) - \sum_{i=1}^{m_h} \frac{\Theta_i}{\Theta_h} C_i'(Q_i^*) - \frac{2R}{\Theta_h} \sum_{i=1}^{m_h} \text{cov}(\pi_i^*, W_{hi}^*)], \tag{28}$$

where the asterisk (*) indicates individuals' optimal holding of assets.[15] The spot price is the discounted value of the expected spot price adjusted for the aggregate expected marginal storage costs and a risk term that depends on the sum of covariances between individual payoffs to storage and terminal wealth as well as on hedger attitudes toward risk. Storers differ in their storage abilities (as reflected in $C_i(Q_i^*)$, their attitude toward risk (as reflected in Θ_i) and their assessment of risk (as reflected in cov (π_i^*, W_{hi}^*).[16] The distribution of storage among the m_h storers depends on these factors. For example even if all storers have indentical cost functions, quantities stored will differ according to risk attitudes.

The covariance term in (28) depends on both commodity risk and storage risk (because $\tilde{\pi}_i$ depends on both $\tilde{S}$ and $\tilde{u}$). Both risks need not be borne by storers. By selling futures they can largely eliminate commodity price risk and act on the basis, $F-S_o$, rather than on the relation between the expected and current spot price. The equilibrium basis is given by the difference between the equilibrium futures price, (15), and the equilibrium spot price, (28):

$$F-S_o = R_f S_o + \sum_{i=1}^{m_h} \frac{\Theta_i}{\Theta_h} C_i'(Q_i^*) - \frac{2R}{\Theta_h} \sum_{i=1}^{m_h} \mathrm{cov}(u_i, W_{hi}^*) \tag{29}$$

$$+ \frac{2R}{\Theta}[-\mathrm{cov}(S,W_s^*) + \frac{\Theta_s}{\Theta_h} \mathrm{cov}(S,W_h^*)]$$

where $W_h = \sum_{i=1}^{m_h} W_{hi}$, $W_s = \sum_{i=1}^{m_s} W_{sj}$.

The basis depends on

1) The interest cost of holding the commodity.

2) The weighted average expected marginal storage cost for the m_h storers, where the weights depend on risk attitudes.

3) An aggregate basis risk which is the sum across storers of storage cost risk. Note that in (28) the comparable term was $\mathrm{cov}(\pi_i^*, W_{hi}^*)$ whereas in (29) $\mathrm{cov}(u_i, W_{hi}^*)$ is relevant. If the sum of these covariance terms is positive, the storage business is risk reducing in a portfolio context; and the equilibrium basis may be less than expected storage costs.

4) The last term in (29) reflects the netting out of most of the commodity price risk between hedgers and speculators. If the covariance terms were proportional to $\frac{\Theta_s}{\Theta_h}$, the last term would be zero. This need not be the case. The sign depends on the extent of the natural hedge provided storers when $\mathrm{cov}(S,u_i) > 0$ and the size for hedgers relative to speculators of the commodity price risk as measured by the covariance with the value of marketable assets.[17]

Note that equilibrium basis may not cover expected marginal storage costs thereby reflecting new type of covenience yield discussed earlier. This convenience yield is caused by the existence of a natural hedge in the storage business and can arise at any scale of storage, unlike the traditional convenience yield of Kaldor.

V. Conclusions

In this paper a Keynes-Cootner insurance justification of hedging is for the first time developed in the framework of capital market equilibrium under homogenous expectations. The critical assumption is that shares in the storage process owned by certain individuals are not tradeable. As a result the futures market is used to pass on some of the risk associated with holding the physical commodity.

Equilibrium hedging and speculation and the equilibrium futures price are derived conditional on a particular allocation of the harvest between current consumption and storage. The model assumes positive storage, and in that environment normal backwardation (futures price less than expected spot price) is the natural result. However contango (futures price greater than expected spot price) is possible, even with positive storage, if the covariance between commodity and stock prices is sufficiently negative.

The optimal hedge and optimal output of the individual storer are dervived. One finds that in general the storer will underhedge somewhat his holdings of the physical commodity. The assumptions necessary to yield the simple decision rule, "Hedge so as to minimize the variance of the storage firm's terminal cash throwoff," are specified. The optimal output decision involves solving simultaneously the portfolio and production problem of the individual storer. In equilibrium, storage is supplied until the marginal cost of storage less a risk adjustment term equals the basis. However there is reason to argue that the storage process is risk reducing in a portfolio context. In that case the basis would not cover out-of-pocket storage costs, thereby resulting in a new type of convenience yield.

Finally the paper considers determination of the equilibrium spot price and basis. The spot price is the discounted value of the expected spot price adjusted to reflect expected marginal storage costs and storer attitudes toward and assessments of risk.

FOOTNOTES

[1]See the discussion in Working [1953]. According to Hirshleifer [1975], heterogenous expectations are a necessary condition for futures markets and hedging.

[2]Major commodity storage and processing firms that are privately held are Continental Grain, Cargill, Dreyfus.

[3]See Black [1976] for a discussion of the valuation of futures contracts and for this point.

[4]For example the interesting paper by Jensen and Meckling [1976] implies private ownership is optimal since it avoids certain mangerial dis-incentives and monitoring costs which arise when outside equity financing is sought.

[5]This does not necessarily imply that the amount stored is insignificant, only that the cost of providing storage facilities is a small fraction of national income.

[6]It is the standard assumption in the capital asset pricig literature that the supplies of assets on exogenous. However see Black (1972) for a verbal discussion of an iterative procedure to determine the allocation between consumption and storage as well as prices of assets.

[7] To see this solve (8) and (9) for portfolio holdings, X_i, Z_i. One can show that the ratio X_i/Θ_i depends only on exogenous variables which are the same for all individuals so long as all assets are traded.

[8]In the absence of transactions costs Cootner would have the SS schedule pass through E(S). This is only the case if the speculation holds no other assets ($X_s = 0$) or if cov $(S_1V) = 0$.

[9]There would be contango while at the same time speculators were long. Speculators would willingly incur losses because commodity futures contracts reduced risk. This explanation for contango is different from the traditional one which requires hedgers to be long and speculators to be short. Long hedging to meet future needs for the commodity forces up the futures price relative to the expected spot price. The traditional explanation ultimately depends on the desire to hedge future consumption needs, and the specification of a complete model is quite complex--more complex than the symetry argument put forward by Stein (1961), e.g.

[10]Margin to guarantee performance of the futures contract may be posted in the form of interest earning assets.

[11]Making use of (1) the market price of risk in (18) is

$$\frac{E(R_m) - R_f}{\sigma^2(R_m) + \frac{F\bar{Q}}{P}\,\mathrm{cov}(R_s,R_m) - \sum_{i=1}^{m_h} \frac{FQ_i}{P}\,\mathrm{cov}(\frac{u_i}{F}, R_m)}$$

Since the covariance terms could have either sign the direction of the discrepancy is not necessarily clear. The same reasoning applies here as applied in the discussion of normal backwardation.

[12]Define the firm as the last 2 terms of (2) and use (1):

$$\tilde{T}_i = Q_i\tilde{S} - C_i(Q_i) - \tilde{u}_iQ_i + Z_i(\tilde{S}-F)$$

where $\tilde{T}_i$ is the terminal value of the firm i. The variance of $\tilde{T}_i$ is

$$\sigma_T^2 = (Q_i+Z_i)^2\sigma^2(S) + Q_i^2\sigma^2(\tilde{u}_i) - 2\,Q_i(Q_i+Z_i)\,\mathrm{cov}(u_i,S)$$

First order conditions for a minimum σ_T^2 give (24).

[13]The exact solution for Q_i is

$$F-S_o = C_i'(Q_i) + R_fS_o - [E(S)-F]\,\frac{\beta_{uS}-\beta_{uv}\beta_{vS}}{1-r_{Sv}^2} - [E(v)-Rp]\,\frac{\beta_{uv}-\beta_{Sv}\beta_{uS}}{1-r_{Sv}^2}$$

$$+ \frac{2RQ_i}{\Theta_i}\quad\sigma^2(u_i) - \frac{\sigma^2(v)}{1-r_{Sv}^2}\,[\beta_{uv}^2 + \beta_{Sv}^2 + \beta_{Sv}\beta_{uv} + \beta_{uv}\beta_{Sv}\beta_{uS}]$$

$$- \frac{\sigma^2(S)}{1-r_{Sv}^2}\,[\beta_{uS}^2 + \beta_{uv}\beta_{vS}-\beta_{uS}\beta_{uv}\beta_{vS} - \beta_{uS}\beta_{Sv}\beta_{vS}]$$

where $\beta_{xy} = \frac{\mathrm{cov}(x,y)}{\sigma^2(y)}$, $\beta_{yx} = \frac{\mathrm{cov}(x,y)}{\sigma^2(x)}$

$$r_{xy}^2 = \beta_{xy}\beta_{yx}$$

and terms involving u are understood to be specific to individual i.

[14]When $\tilde{u}_i = \tilde{u}_j$ for all i,j there is homogenous endowment of storage abilities. Then a single forward market in storage costs is sufficient to make markets complete and allow all risk to be passed on. In that case a storer-hedger could eliminate all risk in the storage business and rely on his financial portfolio to give him the optimal risk-return combination. In other words the dependency of the financial portfolio decision and output decision would disappear. But when individual differ in their storage functions, the costs of forward contracting may be favorable to some and unfavorable to others with the result that there is not indifference as to whether forward contracting is undertaken.

[15]At this equilibrium the following constraints are satisfied:

$$\sum_{i=1}^{m_h} Q_i = \bar{Q}; \quad \sum_{i=1}^{m} x_i = \bar{x}; \quad \sum_{i=1}^{m} Z_i = 0$$

[16]The differences in the assessment of risk referred to in this sentence arise not from heterogeneous expectations but from the fact that storers hold different efficient portfolios therefore causing the risk of assets (which is measured by covariance with the efficient portfolio) to differ. Furthermore $\tilde{u}_i$, a component of π_i, may differ across individuals.

[17]More detailed versions of (29) are possible by using the definition of terminal wealth, (2), and writing out the covariance terms. One version is

$$F-S_o = R_f S_o + \sum_{i=1}^{m_h} C_i'(Q_i^*) - \frac{2R}{\Theta_i} \sum_{i=1}^{m_h} \mathrm{cov}(u_i, M_{hi}^*)$$

$$- \frac{2R}{\Theta_h} \sum_{i=1}^{m_h} Q_i^*[\mathrm{cov}(u_i,S)-\sigma^2(u_i)] + \frac{2R}{\Theta}[\frac{\Theta_s}{\Theta_h} \mathrm{cov}(S,M^*)-\mathrm{cov}(S,W_S^*)]$$

$$+ \frac{2R}{\Theta} \sum_{i=1}^{m_h} Q_i^*[\sigma^2(S)-\mathrm{cov}(S,u_i)] \qquad (29.a)$$

where $M_{hi} = x_{hi}v + Z_{hi}(\tilde{S}-F)$

$$M = \sum_{i=1}^{m_h} M_{hi}$$

The last term of (29) is negative if the sum of the last two terms of (29.a) is negative. Further manipulation yields

$$F-S_o = R_f S_o + \sum_{i=1}^{m_h} \frac{\Theta_i}{\Theta_h} C_i'(Q_i^*) - \frac{2R}{\Theta_h} \sum_{i=1}^{m_h} x_{hi}^* \text{cov}(u_i,v) + \frac{2R}{\Theta_h} \sum_{i=1}^{m_h} Q_i^* \sigma^2(u_i)$$

$$+ \frac{2R}{\Theta} \sigma^2(S)[\bar{Q} + Z_h^*(\frac{\Theta_s}{\Theta_h} + 1)] + \frac{2R}{\Theta} \text{cov}(S,v) [\frac{\Theta_s}{\Theta_h} x_h^* - x_s^*] \quad (29.b)$$

$$- \frac{2R}{\Theta} \sum_{i=1}^{m_h} [Q_i^* \text{cov}(u_i,S) + \frac{\Theta}{\Theta_h} Q_i^* \text{cov}(u_i,S) + \frac{\Theta}{\Theta_h} Z_{hi}^* \text{cov}(u_i,S)]$$

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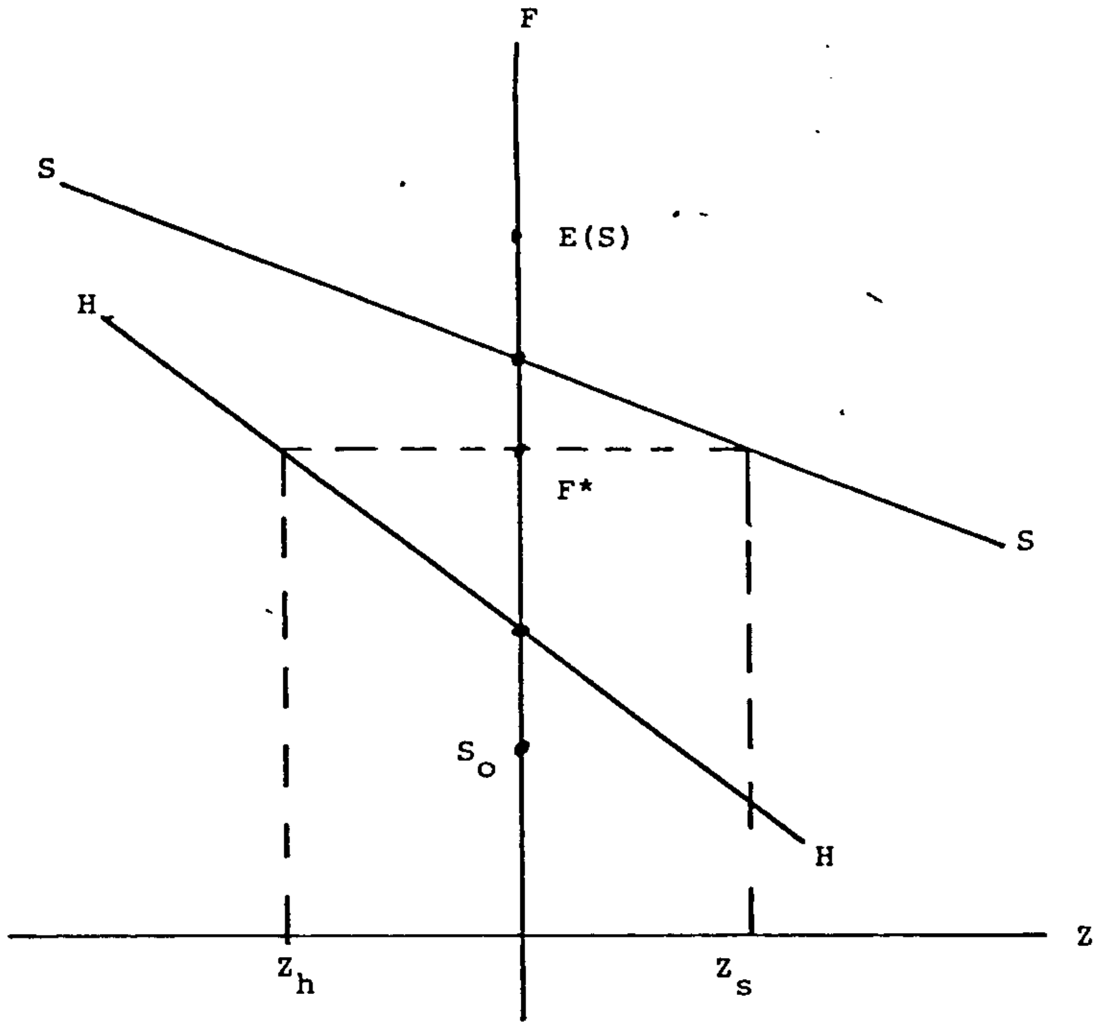
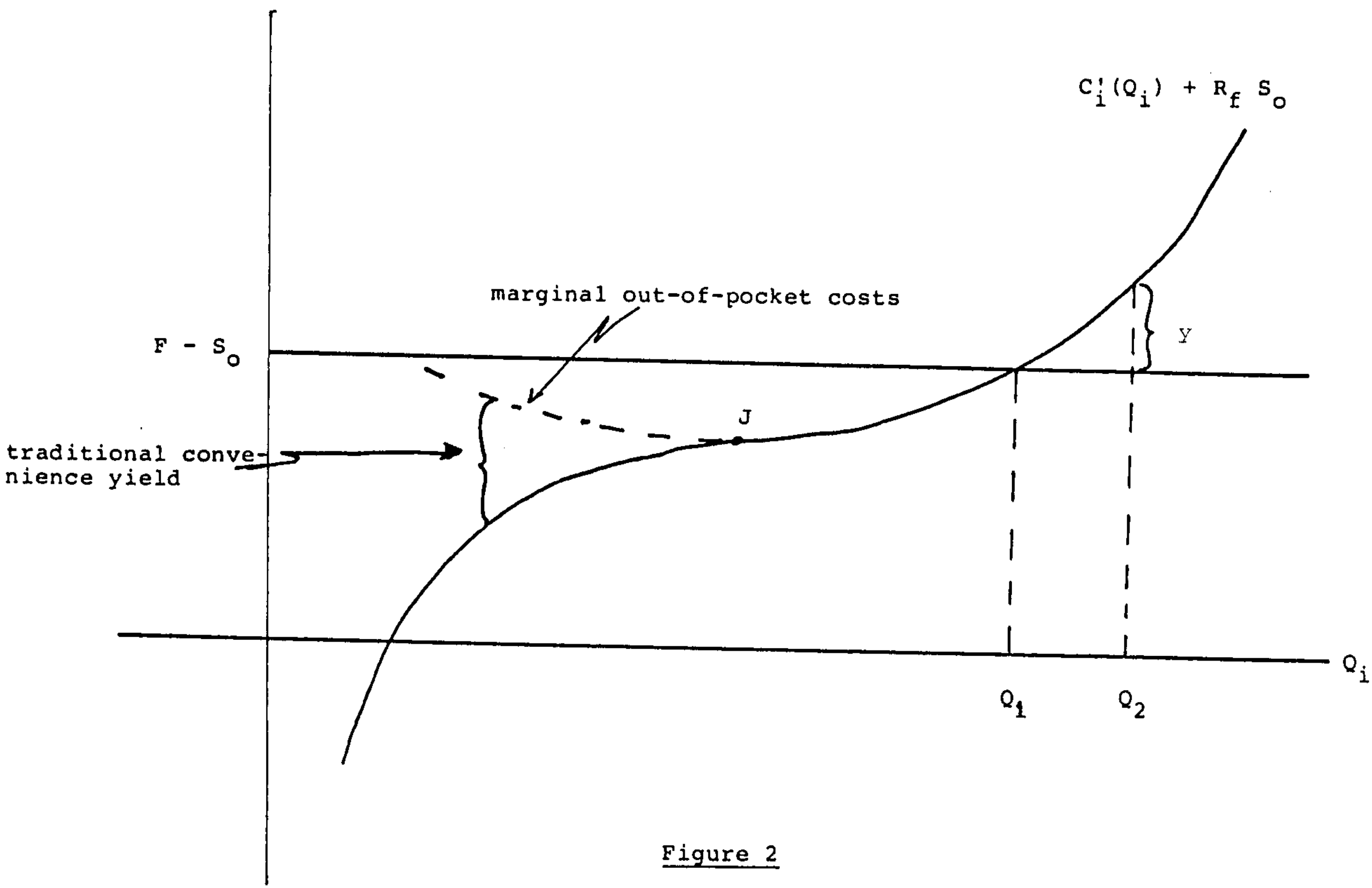


Figure 1



C'i(Qi) + Rf So
marginal out-of-pocket costs
F - So
Y
J
traditional convenience yield
Q1
Q2
Qi

Figure 2