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LEASING, BORROWING AND FINANCIAL RISK

by

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Introduction

Much ink and even some invective have been spilled in the debate over the merits of long-term financial lease contracts. But in sharp contrast to some of the more polemical discussions that have accompanied the development of modern financial theory, the frank exchange of views engendered by the problem of evaluating financial leases has not been a sterile exercise in product differentiation. A direct and important result of the debate has been a clarification of far-reaching importance of the principles which underlie the valuation of financing alternatives in a world of uncertanty. The relevant literature is voluminous. See, for example, Bower [1] Gordon [2], Lewellen et al. [4], and Myers et al. [6].

One important, and perhaps the most important, result of the debate has been the identification of the central (and often overlooked) problem of lease evaluation, that is, the need to neutralize financial risk in the evaluation of lease and other financial alternatives. The crucial importance of neutralizing the risk differential stems directly from the lease evaluation equation presented in Myers, Dill, and Bautista [6]. Using this equation, we present, in this paper, a relatively simple and straightforward, and therefore operational, solution to the practical problem of neutralizing the risk differential induced by lease contracts.[1]

Defining the Cash Flow

Although the valuation of the desirability of a leasing arrangement appears, on the surface at least, to be straightforward and perhaps even simple, nothing could be further from the truth. The frustration felt by many financial specialists when confronted with the received doctrine of leasing has been expressed recently by Myron Gordon:

"At various times over the last twenty years I have presented classes in finance with cases involving the choice between buying or leasing in acquiring a capital asset, and invariably I have been unhappy with the solutions proposed in the solution manuals and literature as well as the ones presented by the student ," [2, p. 245].

Some of the difficulty in evaluating a lease proposal reflects an underlying ambiguity regarding the relevant alternative that should be used as the benchmark for comparison with the lease. Should the lease be compared with buying or with borrowing? Obviously, a lease or borrow" comparison also implies a "lease or buy" comparison, so to avoid possible confusion we shall denote by "lease or buy" the comparison with a purchase that is financed by the firm's standard debt-equity mix.

On the surface, it would appear that the choice between the two alternatives -- lease or buy --is relatively simple. Assuming that both options have positive net present values, it might be argued that the firm should follow the alternative with the higher NPV. If the net present value of the lease, NPV(L), is greater than the net present value of the purchase option, NPV(P), the machine should be leased (and conversely in the case in which NPV(P) exceeds NPV(L)), But despite the apparent plausibility of this approach it can readily be shown that such a solution is incorrect.

Comparing the net present values of the buy or lease alternatives involves us in a comparison of apples and oranges, because the two cash flows differ in a fundamental sense. The lease arrangement is like borrowing in that it commits the firm to a series of fixed rental payments. Thus, even if the lease alternative has a greater NPV, it may also expose the firm's shareholders to greater risk.

The differential risk can be identified by carefully specifying the cash flows of the two alternatives. Let us first turn to the problem of determining the cash flow engendered by the lease. For computational convenience only, we shall assume that the lease is defined over the duration of the asset's life and that there is no residual value.

Lease Cash Flow

Assume that the firm leases a machine for n years and pays a rent of L_t in year t, and that the firm earns revenue from the sale of the machine's output equal to S_t in year t. The production cost (labor, raw materials, electricity, and the like) associated with this output is C_t. The firm incurs no depreciation cost, since it does not actually own the machine. Hence, the net cash flow engendered by the lease can be written as follows:

$$(1-T)\ (S_t - C_t - L_t) = (1-T)\ (S_t - C_t) - (1-T)\ L_t \quad (1)$$

where T denotes the appropriate corporate tax-rate.

Now if we assume that the riskiness of the net receipts from this investment, $(S_t - C_t)$, does not deviate significantly from the frim's standard risk, and that the appropriate after-tax discount rate is equal to k_τ the latter rate should be used to calculate the present value of the first component of the cash flow, $(1-T)\ (S_t - C_t)$. On the other hand, the after-tax lease payments $(1-T)L_t$, like the payments on a bond, constitute a fixed charge and therefore should be discounted using the interest rate, r.[2] Thus the net present value of the lease, NPV(L), is given by,

$$NPV(L) = \sum_{t=1}^{n} \frac{(1-T)\ (S_t - C_t)}{(1+k_\tau)^t} \quad \sum_{t=1}^{n} \frac{(1-T)L_t}{(1+r)^t} \ . \tag{2}$$

Purchase Cash Flow

Suppose now that the firm decides to buy rather than lease the machine. Assuming a purchase price of I dollars, and an annual depreciation expense of D_t, the relevant cash flow of the purchase option in year t is given by

$$(1 - T)\ (S_t - C_t - M_t - D_t) + D_t \ . \tag{3}$$

We first subtract the depreciation expense, D_t, in order to calculate the corporate tax liability, but then add it back because depreciation is not a cash outflow. We also deduct M_t, which denotes any additional maintenance, insurance, or other costs engendered by the decision to buy rather than lease the machine. However, with no loss of generality we shall assume, for simplicity, that the sum of all such costs is zero. Hence the net cash flow of the purchase option in year t reduces to

$$(1 - T)\ (S_t - C_t) + TD_t \ . \tag{4}$$

The net present value of the purchase option, NPV(P), is given by the formula,

$$NPV(P) = \sum_{t=1}^{n} \frac{(1-T)\ (S_t - C_t)}{(1+k_\tau)^t} + \Sigma \frac{TD_t}{(1+r)^t}\, t + \frac{S'_n}{(1+r_2)^n} - I \tag{5}$$

where I denotes the initial investment outlay, $S_n^{'}$ is the estimated after-tax salvage value of the equipment, and r_1 and r_2 denote the appropriate discount factors for the tax shield and the salvage value. The reader should note that the tax shelter, TD_t, is for all practical purposes almost completely certain since it can be obtained against the income of other projects should the project in question fail to generate any taxable income. However, even in the case in which the firm suffers an overall loss, the tax claim can still be carried forward (or backward) and applied against the firm's future (or past) years' taxable income. Thus the discount rate (r_1) which is used to calculate the present value of the tax shield is only slightly higher than the riskless interest rate. The second discount rate, r_2, reflects the riskiness of realizing the salvage value; in general, the greater the specificity of the equipment, the higher the discount rate.

Abstracting from the slight riskiness of the tax shield and the existence of any terminal (salvage) value, a more easily managable formula can be derived:

$$NPV(P) = \sum_{t=1}^{n} \frac{(1-T)\ (S_t - C_t)}{(1+k_\tau)^t} + \frac{TD_t}{(1+r)^t} - I. \qquad (5a)$$

Because the tax shield from the depreciation is considered to be certain, this component of the cash flow is discounted using the interest rate, r, rather than the discount rate, k_τ as the latter reflects the firm's overall risk level. The logic underlying the use of the interest rate, r, as the discount rate for calculating the present value of the tax shield from depreciation is given in Levy and Arditti [3].

Comparing Alternatives

The differential cash flow which is engendered by the decision to buy can now be derived by subtracting the annual lease cash flow from the annual buy cash flow:

$$CF(P) - CF(L) = (1 - T)L_t + TD_t \tag{6}$$

where CF(P) and CF(L) denote the cash flow of the purchase and lease options, respectively. The lease commits the firm to a series of annual fixed after-tax rentals, $(1-T)L_t$, the purchase option of course involves an initial investment outlay of I but adds with certainty the annual tax shield from depreciation, TD_t. Before we can make a meaningful comparison of the two alternatives, we must first neutralize any additional financial risk inherent in the lease i.e., we must hold the risk constant when comparing the two alternatives.

Thus, the fundamental difficulty in comparing the buy or lease alternatives relates to risk. Only if the risk incurred in both of these alternatives is identical can the difference in net present values be used as a guide to action:

$$NPV(P) - NPV(L) = \sum_{t=1}^{n} \frac{TD_t}{(1+r)^t} + \sum_{t=1}^{n} \frac{(1-T)L_t}{(1+r)^t} - I. \tag{7}$$

In such a case, a positive differential indicates that the purchase option is preferable, and, conversely, a negative result indicates a preference for the lease. Equivalently, the critical maximum lease payment, L_t^*, which leaves the firm indifferent between the two alternative methods of acquiring the services of the asset can be derived from the following formula:

$$\sum_{t=1}^{n} \frac{TD_t + (1-T)L_t^*}{(1+r)^t} = I.$$

If we recall that the discount rate k_τ in Equations (2) and (5) above is a weighted cost of capital which inter alia reflects the borrowing power and debt shelter conferred by the cash flows being discounted, this implies some standard level of borrowing against the flows from the asset purchases. Equations (7) and (8) implicitly assume that obligating the firm under the lease, and giving up the depreciation shelter, engenders no sacrifice in the firm's overall borrowing power.

However, in general, the riskiness of the two alternatives will not be identical. The lease option, as we have already noted, commits the firm to a stream of rental payments, fixed in advance. Hence, in order to neutralize the risk differential, the analysis of the purchase option must be made on the explicit assumption that the purchase is partially financed by a loan which commmits the firm to a stream of fixed payments (of principal and interest), thereby equating the cash flows of the purchase and lease alternatives; therefore the two cash flows will have the same risk. The crucial question is the amount that should be borrowed.

Clearly, when the firm borrows, thereby incurring an interest payment of say R dollars in year t, the resulting tax shield, TR_t, must be taken into account if the differential riskiness of the "lease and "purchase" alternatives is to be neutralized. And since we have assumed that the annual net receipts generated by the asset, $(1-T)(S_t - C_t)$, are identical in both financing alternatives, it follows from Equation (6) and the tax deductibility of interest payments that the payments stream on the loan (principal repayment plus pre-tax interest actually paid) which is required to neutralize the differential risk between the lease and the purchase options is given by:

$$(TD + (1-T)L_t + TR_t) \tag{9}$$

where TR denotes the interest tax shield in year t.

A fully equivalent and a more readily applicable formulation sets out the critical risk-equating payment stream on the loan in terms of principal repayment and the <u>after-tax</u> interest payment (<u>i.e.</u>, $(1-T)R_t$):

$$(TD_t + (1-T)L_t). \tag{10}$$

The equivalence of the pre-tax and post-tax formulations of the loan payments stream can be easily demonstrated. Denoting by B_t the balance of the loan outstanding at the end of period t, the amount of principal repaid in period t is equal to $B_{t-1} - B_t$. Hence, the after-tax payment stream (10) can be rewritten as

$$TD_t + (1-T)L_t = (B_{t-1} - B_t) + (1-T)rB_{t-1} \tag{11}$$

where r denotes the interest rate. Recalling that by definition $R_t = rB_{t-1}$, we obtain

$$TD_t + (1-T)L_t = (B_{t-1} - B_t) + (1-T)R_t \tag{12}$$

which reduces to

$$TD_t + (1-T)L_t + TR_t = (B_{t-1} - B_t) + R_t.$$

Thus the pre-tax and after-tax formulations (9) and (10) can be used interchangeably to equalize the risk of the lease and purchase options.[3] The former is set out in terms of the interest actually received by the bank (creditor), and therefore constitutes a pre-tax payment from the firm's viewpoint; the latter formulation incorporates the firm's after tax interest payment, $(1-T)R_t$.

Since the post-tax formulation is somewhat easier to apply, Equation (14) is used to calculate the critical loan payment schedules in the numerical examples which follow (See Exhibit 1).

If the risk-neutralizing payments stream (appropriately defined) permits us to borrow an amount of money that exceeds the purchase price of the equipment, I, the equipment should be purchased rather than leased.

In this instance the firm can finance the purchase out of the proceeds from the loan and still have some money left over.

A practical, but fully equivalent, way to reach the optimal solution is to find the critical lease payment that leaves the firm indifferent between the lease.

and buy alternatives. The critical value is the maximum lease payment, L_t^*, that the firm can pay before the purchase option becomes preferable and is given by

$$\sum_{t=1}^{n} \frac{TD_t + (1-T)L_t^* + TR_t}{(1+r)^t} = I. \qquad (13)$$

This formula has an obvious disadvantage; the tax shield on the borrowed funds is difficult to calculate since it diminishes over time. Fortunately the tax shield can be eliminated from the numerator by discounting the payments stream using the after-tax interest rate (1-T)r. This transformation yields the more operational formula:[4]

$$\sum_{t=1}^{n} \frac{TD_t + (1-T)L_t^*}{[1+(1-T)r]^t} = I. \qquad (14)$$

Numerical Example

The underlying logic of the above solution to the thorny lease or buy decision can be made even more transparent by considering a specific numerical example. Assume that the firm has already decided to acquire a particular machine, but is considering whether it should be purchased or leased. The price of the machine, I, is $10,000, and its economic (and accounting) life, n, is ten years. The interest rate, r, is 10%; the corporate tax rate, T, is 50%; accelerated depreciation is calculated using the sum of the years' digits (SYD) method. (The depreciation schedule is given in column (4) of Exhibit 1.). As before, (S-C) denotes the net annual revenue earned from the operation of the machine, which for simplicity only we assume to be constant.

Should the firm buy or lease the machine? The answer depends on the magnitude and timing of the lease payments confronting the firm; however, if we assume constant annual lease payments, the critical value of the lease payment can be found by solving Equation (14) for L^*. If the proposed lease payments are less than L^*, the machine should be leased; for lease payments greater than L^*, the purchase option is preferable.

Solving Equation (14) explicitly for L^* we obtain,

$$L^* = \frac{I - \sum_{t=1}^{n} TD_t / (1 + (1-T)r)^t}{\sum_{t=1}^{n} (1-T) / (1 + (1-T)r)^t} . \tag{15}$$

Using the data of our hypothetical numerical example and the SYD depreciation assumption, the critical annual lease payment, L^*, is equal to \$1,517.50

Before going on to a demonstration that for an annual lease payment of \$1,517.50 the firm will indeed be indifferent between the lease and buy alternatives, let us first verify that the computational procedure has actually equated the riskiness of the two financing strategies. (The neutralization process is shown for the first year only but the interested reader can verify the risk neutralization in all of the ten years under consideration by examining Exhibit 1.)

If the firm leases the machine the annual cash flow is given by

$$(1-T)(S-C) - (1-T)L = (1-T)(S-C) - 0.50 \times \$1,517.50 =$$
$$(1-T)(S-C) - \$758.75.$$

If the firm buys the machine, the first year's cash flow becomes

$$(1-T)(S-C) + TD_1 = (1-T)(S-C) + 0.50 \times \$1,818.30 =$$
$$(1-T)(S-C) + \$909.15.$$

(\$1,818.30 represents the first year's depreciation allowance using the SYD method (see Exhibit 1, Column 4.)

But, in addition, if the firm buys the machine, it must also make an initial outlay of \$10,000 (the purchase price of the machine). What is the size of the loan required to <u>neutralize</u> the leverage effect? Recalling our previous analysis, we can find the annual payments of interest and principal required to equalize the annual cash flows of <u>both</u> alternatives

Using Equation (10), the after-tax payment required to neutralize the risk in the first year is equal to:

$$TD_1 + (1-T)L^* = \$909.15 + \$758.75 = \$1,667.90 \doteq \$1,668.$$

Thus, if the machine is purchased, it should be financed by a loan, which engenders a payment in the first year (principal repayment plus after-tax interest) of \$1,667.90. Plugging this loan payment into the first year purchase cash flow exactly equates the adjusted annual cash flows of the purchase and lease options:

$$(1-T)\ (S-C) + TD - \$1,667.90 =$$

$$(1-T)\ (S-C) + \$909.15 - \$1,667.90 = (1-T)\ (S-C) - \$758.75.$$

Similarly, by construction, the adjusted annual purchase cash flow of all the other years is now identical to the cash flow of the lease option and therefore must also have the same risk. And since the same discount rates, k_T and r, are used to capitalize the components of the payments stream in both alternatives, their net present values must also be identical. Hence, the buy or lease decision depends strictly on the amount we are able to borrow, B, in return for the annual payments of principal and interest required to neutralize the leverage, $(1-T)L_t + TD_t$, and the size of the initial investment outlay required to purchase the machine, I. If the loan, B, is greater than the required initial outlay, I, buying is better than leasing, because if B > I, we can borrow the amount B, invest the amount I, and still have some cash left over. Conversely, if B is less than I, the machine should be leased.

An alternative, but equivalent, procedure for analyzing the lease or buy alternatives may prove helpful. Since the firm incurs no initial outlay should it decide to lease, let us assume that it deposits I dollars of its own capital in the bank at a post-tax interest rate (1-T)r. Each year the firm reduces its deposit by the amount $(1-T)\ L + TD_t$ and adds this amount to the annual cash flow of the lease. This procedure is similar to the one described in the text with one distinction. Instead of paying interest and principal, the firm simply incorporates this sum in the annual cash flow of the lease. Once again this results in identical cash flows for both the lease and buy options. If the firm has to deposit in the bank more than I in order to ensure a receipt in year t of $(1-T)\ L + TD_t$, buying the machine is preferable. If a deposit of less than I is sufficient, leasing is the better alternative.

Now let us turn to Exhibit 1 and examine the logic of the case in which the firm will be indifferent between the two alternatives. Given the assumed annual lease payment of $1,517.50, the firm will be indifferent between the lease and buy option if the required loan, B, is exactly equal to the initial outlay, I (in this example $10,000). The numerical proof of this statement is given in Exhibit 1.

If the firm borrows $10,000, it must pay $500 in after-tax interest at the end of the first year, (1-T)10% × $10,000 = $500. Since the total after-tax payment in the first year can be as high as $1,668, an additional $1,168 can be repaid at the end of the first year on principal. The latter payment, of course, is not tax deductible. Thus, the outstanding loan at the beginning of the second year is $10,000 - $1,168 = $8,832 (see column 9 of Exhibit 1). In the second year, after-tax interest of $441.60, (1-T)10% × $8,832 = $461.60, plus $1,135 on , is paid to the bank. Continuing this procedure for ten years is just sufficient to pay the interest bill and repay the loan. Thus, if the firm borrows $10,000 at 10% interest and services the loan by paying $(1-T)L^* + TD_t$ in year t (see column 6 of Exhibit 1) on principal and interest, it will be indifferent between the lease and buy alternatives. In effect there is no net initial outlay, since the purchase is completely financed by the $10,000 loan, and the loan repayment schedule creates an annual cash flow which is identical to that of the lease.

The above example was constructed in such a way that at the assumed annual rental of $1,517.50 the firm is just indifferent between the lease and buy alternatives. Now let us assume that the annual rental is greater than $1,517.50 but that there are no changes in the other parameters. In such a case, the firm can borrow more than $10,000, say $11,000, since the annual payments of principal and interest required to neutralize the leverage, which are equal to $TD_t + (1-T)L$, exceed the amounts given in column 6 of Exhibit 1. The firm can still buy the machine for $10,000 and create identical cash flows for the buy and lease alternatives, but with one distinction. If the firm borrows and buys the machine it will have an additional \$1,000 since the machine costs less than the loan required to equalize the riskiness of the two alternatives. Thus, in this instance buying is preferable to leasing. Conversely, if the lease payment is less than \$1,517.50, the annual charges on the loan required to equalize the cash flows will be less than that of Exhibit 1, so the firm must borrow less than \$10,000, say \$9,000, if the leverage is to be neutralized. This amount is insufficient to purchase the machine and an additional \$1,000 is required. Thus, in this case leasing is preferable to the purchase option.

In the preceding discussion the logic underlying the correct appraisal of a lease or buy decision has been illustrated by numerical examples. Fortunately there is no need, in practice, to carry out all of the cumbersome arithmetic which was used to illustrate the problem. As we have already indicated, a direct solution to the lease or buy problem can be obtained by solving for the critical value of L^* [see Equation (15)] and comparing the critical value with the proposed lease payments. This confirms the numerical analysis given above which indicated that for a lease payment of \$1,517.50 the firm would be indifferent between the lease and buy options. For $L >$ \$1,517.50 the firm should buy the machine, and for $L <$ \$1,517.50 leasing is the better financing strategy. This is shown in Exhibit 2, which gives the results of calculations similar to those of Exhibit 1 for alternative levels of lease payments. As we have just noted, the firm is indifferent between the lease or buy option for a lease payment of \$1,517.50. For L = \$1,000, a loss of \$1,998 is incurred if the machine is bought rather than leased, and for L = \$2,500 a gain of \$3,792 results should the firm decide to buy rather than rent the machine.

Summary

This essay has attempted to analyze a problem that has been growing in importance as more and more types of assets become available for long-term rental. Although at first glance the lease or buy question appears relatively simple, a correct decision requires an appraisal of almost all the facets of financial decision-making. The lease or buy decision is a type of capital budgeting problem requiring the application of present value techniques. It also has tax implications, and the relevant after-tax cash flows of the two alternatives must be set out with great care. The choice of discount rates requires a decomposition of the cash flows into their risky and riskless components. And finally, the correct solution requires a neutralization of the differential financial risk implicit in the lease vs. purchase comparison.

References

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FOOTNOTES

1. A simple "break-even" formula for evaluating the desirability of a proposed lease, once the risk differential has been neutralized, is derived in the Appendix.

This formula can also be obtained from the valuation formula of Myers, Dill, and Bautista [6] by setting the net present value of the lease equal to zero and solving for the equilibrium lease payment. However, the derivation of the MDB result presented in this paper is more general: no debt constraint need be defined, and it is not required that firms borrow 100% of their interest and depreciation tax shields. All that our proof requires (regarding debt capacity) is that the firm keep its financial risk constant, whether or not it is optimal.

over the duration of the asset's life and that there is no residual value.

2. For ease of presentation we shall assume that the interest rate, r, is riskless. This is tantamount to assuming the absence of bankruptcy risk. The introduction of bankruptcy risk complicates the analysis but does not alter the conclusions; in such a case the appropriate interest rate would be the marginal borrowing rate.

3. It has been noted that if the corporate tax rate (T) is uncertain or systematically varies with the economy, or if income cannot always be found to exploit the tax shelter, risk is not neutralized by the formula in the text, as there are different amounts of shelter flow in

$$R_t - (1-T)L_t + TD_t + TR_t$$

or equivalently

$$R_t - (R_t + D_t)T = L_t - L_tT \ .$$

We are indebted to R.S. Bower for this comment.

4. Since it is not immediately obvious that Equations (13) and (14) yield in the the same solutions for L_t^*, a formal proof of their equivalence is given in the Appendix.

APPENDIX: Deviation of Equations

In this appendix we derive Equations (13) and (14) of the text, and show that they are equivalent, and therefore,/that they can be used interchangeably to evaluate a lease or buy decision.

Let us denote by B_t the balance of a loan outstanding at the end of period t. Hence, for any n period project, $B_n = 0$, and B_0 is the total amount borrowed. In the text it is argued that in order to equate the riskiness of the lease and buy options, the firm should borrow a sum that requires a total payments stream (repayment of principal and after-tax interest) which is just equal to

$$(1-T)L_t + TD_t \ . \qquad \text{(A-1)}$$

Thus the neutralization of the differential risk in year t implies that the following relationship should hold:

$$B_{t-1} - B_t + (1-T)rB_{t-1} = (1-T)L_t + TD_t \qquad \text{(A-2)}$$

That is, the debt repayment $(B_{t-1} - B_t)$ plus after-tax interest payment $(1-T)rB_{t-1}$ should equal the after-tax lease payment plus depreciation shelter. From Equation (A-2) we derive:

$$B_{t-1} = \frac{(1-T)L_t + TD_t + rTB_{t-1} + B_t}{1 + r} \ . \qquad \text{(A-3)}$$

Since, by definition $B_n = 0$, we obtain for t = n - 1:

$$B_{n-1} = \frac{(1-T)L_n + TD_n + rTB_{n-1}}{1 + r} \ . \qquad \text{(A-4)}$$

Using (A-3) and (A-4) we have

$$B_{n-2} = \frac{(1-T)L_{n-1} + TD_{n-1} + rTB_{n-2} + B_{n-1}}{1 + r} =$$

$$\frac{(1-T)L_{n-1} + TD_{n-1} + rTB_{n-2}}{1 + r} + \frac{B_{n-1}}{1+r}$$

Substituting the right hand side of Equation (A-4) for B_{n-1} yields,

$$B_{n-2} = \frac{L_{n-1}(1-T) + TD_{n-1} + TB_{n-2}\, r}{1 + r} + \frac{L_n(1-T) + TD_n + TB_{n-1}\, r}{(1 + r)^2}$$

which simplifies to

$$B_{n-2} = \sum_{t=n-1}^{n} \frac{L_t(1-T) + TD_t + TB_{t-1}\, r}{(1 + r)^{t-(n-2)}} . \qquad \text{(A-5)}$$

Continuing this substitution procedure we finally obtain

$$B_0 = \sum_{t=1}^{n} \frac{L_t(1-T) + TD_t + TB_{t-1}\, r}{(1 + r)^t} . \qquad \text{(A-6)}$$

To find the critical lease payment, L_t^*, which leaves the firm indifferent between the buy and lease options, we simply substitute $B_0 = I$ and solve for L_t^*,

$$I = \sum_{t=1}^{n} \frac{L_t^*\,(1-T) + TD_t + TB_{t-1}\, r}{(1 + r)^t} . \qquad \text{(A-7)}$$

Recalling that $TB_{t-1}r$ is identical to TR_t, where R_t is defined as the interest paid on the loan at the end of period t, Equation (A-7) is clearly the same as Equation (13) of the text.

In order to show that Equation (13) is equivalent to Equation (14) let us write (A-2) as follows:

$$B_{t-1}(1 + r) - TB_{t-1}\, r = L_t(1 - T) + TD_t + B_t . \qquad \text{(A-8)}$$

Hence:

$$B_{t-1} = \frac{L_t(1-T) + TD_t + B_t}{1 + (1-T)\, r} . \qquad \text{(A-9)}$$

Using the same substitution procedure as above, and recalling that $B_n = 0$, the critical lease payment L_t^* can be found from the following equation:

$$B_0 = I = \sum_{t=1}^{n} \frac{(1-T)L_t^* + TD_t}{\left[1 + (1-T)r\right]^t} \tag{A-10}$$

which is equivalent to Equation (14) of the text. Thus, neutralizing the risk differential of the lease and buy alternatives [see Equation (A-2] leads either to Equation (13) or Equation (14) which for the purpose of evaluation are fully equivalent, since both formulations yield the same estimate of the critical lease payment, L^*, and therefore lead to the same decision regarding the relative desirability of the lease and buy alternatives.

Exhibit 1

Lease or Buy Case Flows: A Numerical Example

Year	Loan balance outstanding at the beginning of year t,	Annual after-tax lease payment	SYD depreciation schedule	Annual depreciation tax shield	Total available for loan repayment (principal and interest)	After-tax interest payment on outstanding loan balance	Repayment of principal*	Loan balance at the end of year t,	Present value of total after-tax commitment (7)+(8) discounted at
t	B_{t-1}	$(1-T)L$	D_t	TD_t	$(1-T)L + TD_t$	$(1-T)rB_t$		B_t	$(1-T)r$
(1)	(2)	(3)	(4)	(5)	(6) = (3) + (5)	(7)	(8) = (6) - (7)	(9)	(10)**
1	$ 10,000	$ 758.75	$ 1818.3	$ 909.15	$ 1667.90	$ 500.00	$ 1168	$ 8832	$ 1588.60
2	8,832	758,75	1636.4	818.20	1576.95	441.60	1135	7697	1470.80
3	7,697	758.75	1454.5	727.30	1486.05	384.90	1101	6596	1283.80
4	6,596	758.75	1272.7	636.40	1395.15	329.85	1064	5532	1147.80
5	5,532	758.75	1090.9	545.50	1304.25	276.60	1028	4504	1021.90
6	4,504	758.75	909.1	454.50	1213.25	225.20	988	3516	905.30
7	3,516	758.75	727.3	363.60	1122.35	175.80	947	2569	797.60
8	2,569	758.75	545.4	272.70	1031.45	128.45	903	1666	698.10
9	1,666	758.75	363.6	181.80	940.55	83.30	857	809	606.30
10	809	758.75	181.8	90.90	849.65	40.45	809	0	521.60
Total			$ 10,000				$ 10,000		$ 10,000

*Numbers are rounded to the nearest dollar

**The total present value of the ten annual payments $TD_t + (1-T)L_t$ using an after-tax discount rate of $(1-T)r$ is equal to $10,000, as asserted by Equation (12).

Exhibt 2

Lease vs. Buy for Alternative Levels of Lease Payments

Annual lease payment	Size of loan required to equate the risks*	Profit (or loss) from buying rather than leasing
$ 1,000.00	$ 8,002	- $ 1,998
1,500.00	9,932	- 68
1,517.50	10,000	0
2,000.00	11,802	+ 1,862
2,500.00	13,792	+ 3,792

$$^{*}B_0 = \sum_{t=1}^{10} \frac{(1-T)L + TD_t}{\left[1+(1-t)r\right]} = 0.5L \sum_{t=1}^{10} \frac{1}{(1.05)^t} + \sum_{t=1}^{n} \frac{TD_t}{(1.05)^t} = L \times 3.86 + 4{,}142.35.$$