# OPTIMAL MULTI-PERIOD INSURANCE CONTRACTS

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## Abstract

In multi-period insurance contracts, the premiums that the insured must pay increase whenever he files a claim. Hence in this case the buyer of insurance faces a problem which does not exist in one period contracts. Namely: he must decide for which damages he should file a claim and for which he should not, bearing in mind that whenever he makes a claim, his future rates will rise. We show that the results of Arrow [1963], [1974] and Mossin [1968] are valid for this case too. That is: optimal multi-period insurance contracts must provide the insured full insurance above a strictly positive deductible.

# OPTIMAL MULTI-PERIOD INSURANCE CONTRACTS

## I. Introduction

In multi-period insurance contracts, the premiums which the insured must pay increase whenever he files a claim. This practice is very popular in automobile insurance, but can also be found in medical and theft insurance. In multi-period insurance contracts, the buyer faces a problem which does not exist in one-period contracts. Namely: the buyer must decide for which damages he should file a claim, and for which he should not, bearing in mind that if he makes a claim, his future rates will increase. This type of insurance, and the optimal strategy of the buyer have been analyzed by some authors. Molnar and Rockwell [1966], Lemaire [1976], [1977], Seal [1969], and Von Lanzenauer [1971], investigated several aspects of dynamic automobile insurance contracts. Keeler, Newhouse, and Phelps [1977] analyzed dynamic models in medical insurance. Most of these authors assumed that the insurance contracts are of the form of full coverage above a deductible (henceforth FCAD), and in practice, indeed, most insurance contracts are of this form. However, whereas it has been proved (by Arrow [1963], [1974]) that optimal one-period contracts provide full coverage above a deductible, it has only been conjectured (see, e.g., Keeler, Newhouse, and Phelps [1977], p. 641) that the same holds true also for dynamic models. It is thus the purpose of this paper to extend Arrow's results to the case of dynamic insurance contracts. In our analysis we shall mainly concentrate on the case of automobile insurance.

The paper is constructed as follows: In section II we describe multi-period contracts, and argue that the practice of increasing the premiums of the buyer whenever he files a claim follows from the learning behavior of the sellers. The optimal claims strategy of the buyer is presented in section III. In section IV we show that the optimal policy gives full coverage above a deductible. It is also shown that the deductible must be strictly positive.

## II. Structure of Multi-Period Insurance

Buyers of automobile insurance are usually classified by the sellers according to risk categories. The risk category of each buyer is determined by some social and economic characteristics such as age, occupation, driving experience, make of car, and also by his past history of claims. We consider buyers of insurance with some given social and economic characteristics, and analyze the effect of their past behavior of claims on the insurance premiums that they pay. It is usually assumed by the seller that if the buyer makes a claim, the probability rises that he will make other claims in the future, and hence he should be moved to a higher risk category. This behavior of the seller can be explained by the following Bayesian behavior: Suppose that the probability $\tilde{p}$ that a buyer will make a claim is unknown. Suppose further that this probability is considered by the seller as a random variable with a distribution function $f(\tilde{p}|h)$, where h denotes the relevant claims history of the buyer. Each period of time say, t, the seller updates this distribution by observing if a claim has been made in the previous period or not. If a claim has been made, $f(\tilde{p}|h)$ will change so as to reflect an increase in risk, (or a decrease in risk if a claim has not been made). For example, suppose that $\tilde{p}$ is Beta distributed with parameters k and n $(k < n)$ at the beginning of period t. The claims behavior of the buyer during period t can be considered as a Bernoulli experiment with a probability $\tilde{p}$ of making a claim, and a probability of $(1-\tilde{p})$ of not making a claim.[1] It then follows (see, e.g., Winkler [1972], p. 153) that the distribution of $\tilde{p}$ at time (t+1) will be Beta with parameters k+1 and n+1 (with mean $\frac{k+1}{n+1} > \frac{k}{n}$) if the buyer has filed a claim, and it will be Beta with parameters k and n+1 (with mean $\frac{k}{n+1} < \frac{k}{n}$) if the buyer has not filed a claim during period t. The above model can partially explain the behavior of insurance companies in some countries where the insurance rate of the buyer rises or falls every year according to whether he has made a claim or not. (In most countries however, there are upper and lower limits to the amount that the buyer can pay. This implies that either premiums are not fair, or that there are transaction costs, or that the range of $\tilde{p}$ is bounded by values other than zero and 1). Clearly, other Bayesian models can also explain the rate determination by insurance companies, and the above serves as just one possible explanation. The exact Bayesian model which explains the learning process of the seller is beyond the scope of this paper. Here it is only assumed that the seller indeed revises the probability of future claims on the basis of the past claims behavior of the buyer. From this assumption it follows that if the seller charges premiums according to the expected damages, he must increase the premiums whenever a claim is made.

The distribution $f(\tilde{p}|h)$ together with the conditional distribution of damages (conditional that a damage has occurred), determine the unconditional distribution $F_t^j(\ )$ of damages. The index j in $F_t^j(\ )$ denotes the number of claims that have been filed until time t. Thus j is a measure of the riskiness of the buyer.[2] Following the practice of most insurance companies, it is assumed that there is a finite number, say J, of risk categories, to which a buyer can be classified.[3] Each category j, determines the premium rates of the buyers in that category. The total premium $r_j(I)$ depends on the category j and on the insurance contract I which the buyer purchases. The insurance contract is a function I(x) specifying the amount of money the buyer will receive if he makes a claim of x (e.g., if the policy gives full coverage above a deductible, say m, then $I(x) = 0$ if $x \leq m$, and $I(x) = x-m$ if $x > m$). Since a higher j implies a higher risk, the functions $r_j(I)$ are increasing in j. If a buyer of category j, $1 \leq j < J-1$ makes a claim, he is moved in the next period to category j+1.[4]

In the next section we shall investigate for which damages the buyer should make a claim and for which he should not.

## III. The Optimal Claims Strategy of the Buyer

It is assumed that the multi-period utility function of the buyer is additive of the form

$$U(c_1,\ldots,c_t,\ldots) = \sum_{t=1}^{\infty} \beta^t u(c_t) \tag{3.1}$$

where $c_t$ denotes consumption at time t, the function $u(\cdot)$ is a concave and twice differentiable one-period utility function, and $\beta$ is a time preference discount factor. The income of the buyer at time t is $A_t$, and it is assumed that there are no savings and that all damages are repaired. It thus follows that

$$c_t = A_t - \tilde{r}(I_t) - \tilde{x}_t + I_t(\tilde{x}_t)\Delta_t$$

where $\tilde{x}_t$ is a random variable denoting damages at time t and $\Delta_t$ is one or zero depending on whether or not a claim is made at time t and $\tilde{r}_t(I_t)$ is a random variable denoting the premium rate at time t as seen at the origin (it is a random variable since at the origin one does not know how many claims will be filed until t); the insurance contract I also has a subscript t since the amount of insurance that the buyer purchases is not the same at all periods. Consider now some buyer of risk category j that has some insurance contract I at period t. Denote by $V_{jt}(x)$ the maximum expected discounted multi-period utility of the insured at time t, subject that he sustained a damage of amount x, that his risk category is j, and that he follows an optimal claims and insurance purchase strategy. Denote also by $W_{jt}$ the expectation of $V_{jt}(\tilde{x})$, i.e.

$$W_{jt} = E[V_{jt}(x)],$$

where the expectation E is taken with respect to the distribution $F_t^j(\cdot)$;[5] $W_{jt}$ thus represents the welfare (infinite-period expected utility) of the insured at time t, assuming that he pursues an optimal claims policy, and that his risk category is j.

If the insured had an accident involving damage of value x, he can either file a claim or not. If he files a claim, he will receive I(x), and his one-period utility at t will be $u[A_t - r_j - x + I(x)]$.[6] Since starting next period his risk category will be (j+1), his infinite period utility at t+1 will be $W_{j+1,t+1}$, and consequently his infinite-period expected utility at time t is $u[A_t - r_j - x + I(x)] + \beta W_{j+1,t+1}$. If the insured does not file a claim, his one-period utility at time t will be $u(A_t - r_j - x)$, his future multi-period expected utility will be $W_{j,t+1}$, and his current infinite-period expected utility is $u[A_t - r_j(I) - x] + \beta W_{j,t+1}$. (Note, from the structure of the $r_j$'s and $F^j$'s that $W_{j,t+1} < W_{j+1,t+1}$).

It follows, since $V_{jt}(x)$ represents an optimal policy, that

$$V_{jt}(x) = \max\{u(A_t - r_j - x) + \beta W_{j,t+1},\ \ u[\pi_j(x)] + \beta W_{j+1,t+1}\} \qquad (3.2)$$

where the function $\pi_j(x)$ defined by

$$\pi_j(x) = A_t - r_j - x + I(x)$$

denotes consumption at time t, if a claim is made at that time.[7] Thus, the optimal policy of the buyer can be obtained from (3.2), since a claim should be filed only if the damage, x, satisfies

$$u(A_t - r_j - x) + \beta W_{j,t+1} \leq u[\pi_j(x)] + \beta W_{j+1,t+1}. \tag{3.3}$$

If we further assume that $0 < I'(x) \leq 1$ (if $I' > 1$ the marginal payments exceed the marginal damages and a moral hazard problem may emerge), then the continuity of u implies that there exists a critical value, $y^*_{jt}$, such that a claim will be filed only if $x \geq y^*_{jt}$. The critical value satisfies

$$u(A_t - r_j - y^*_{jt}) + \beta W_{j,t+1} = u[\pi_j(x)] + \beta W_{j+1,t+1}, \quad j = 1,\ldots,J-1. \tag{3.4}$$

Since $W_{jt} = E[V_{jt}(\tilde{x})]$, it follows that $W_{jt}$ satisfies

$$W_{jt} = \int_0^{y^*_{jt}}[u(A_t - r_j - x) + \beta W_{j,t+1}]dF^j_t(x) + \int_{y^*_{jt}}^{\infty}\{u[\pi_j(x)] + \beta W_{j+1,t+1}\}dF^j_t(x), \quad j = 1,\ldots,J-1. \tag{3.5}$$

The $W_{jt}$'s and hence the $y^*_{jt}$'s can be obtained as follows. Suppose that the planning horizon of the buyer is some finite period T. Then at time

$t > T$ the claims strategy of the buyer is always to claim. Also, the amount of insurance, I, that he buys can be determined as in the one period models (see, e.g., Mossin [1968], Gould [1969], Pashigian, Schkade, and Menfee [1969], Smith [1968]). It is thus easy to compute $W_{jT}$, for $j = 1,\ldots,J$ since

$$W_{jT} = \int_0^\infty u[A_T - r_j(I) - x + I(x)]dF_T^j(x), \qquad j = 0,1,\ldots,J-1$$

$$W_{JT} = \int_0^\infty u[A_T - x]dF^J(x)$$

Based on the $W_{jT}$'s, the $W_{jt}$'s and $y^*_{jt}$'s can be computed recursively for $t < T$ using (3.5) and (3.4). The optimal insurance contract at each step can be determined as shown in section IV.

In the sequel we shall consider the case where $F_t^j(\cdot)$ and $A_t$ are the same for all t (the stationary case).[8] In this case the subscript t can be omitted and (3.4) and (3.5) are written as

$$u(A - r_j - y^*_j) + \beta W_j = u[\pi_j(x)] + \beta W_{j+1}, \qquad j = 1,\ldots,J-1 \tag{3.6}$$

$$W_j = \int_0^{y^*_j}[u(A - r_j - x) + \beta W_j]dF^j(x) + \int_{y^*}^\infty \{u[\pi_j(x)] + \beta W_{j+1}\}dF^j(x) \tag{3.7}$$

$$j = 1,\ldots,J-1.$$

In the next section we show that, for a risk averse buyer, an optimal insurance policy gives full coverage above a deductible.

## IV. The Structure of the Optimal Multi-Period Contract

In this section we concentrate on the case of an insured of category omit this index and hence $W_j$ will be replaced by $W$ and for convenience we shall replace $W_{j+1}$ by $M$). We briefly review the methodology that Arrow [1963] used to prove the optimality of FCAD contracts. Arrow showed that if a contract is not of this form, another contract with the same actuarial value (i.e., with the same expected payments) can be constructed which the risk averse buyer prefers. Here a similar methodology is used, except that there are two added complications: 1) the dynamic nature of the problem must be considered; and 2) when constructing an improved policy using Arrow's approach, the actuarial value of the improved contract differs from that of the original one since the critical value corresponding to the former contract differs from the one corresponding to the latter. Thus, when constructing an improved policy, its effects on the critical value must be considered.

Theorem 4.1 An optimal insurance policy provides the buyer full insurance above a deductible.

Proof: For the proof we show that if an insurance contract is not of the FCAD form, then another contract can be drawn which is preferred by both the buyer and the seller.

We first note that if a policy gives full coverage above a decuctible, say m, then the optimal critical value must satisfy $y^* > m$. We also observe that for any $x \geq y^*$, $I(x) = x-m$ and hence $\pi(x) = A-r-m$. That is, the FCAD insurance has the property that $\pi(x)$ is the same for all $x \geq y^*$ (in the one period model $\pi(x)$ is the same for all $x \geq m$). Moreover, since FCAD insurance is the only insurance with this property, it suffices to show that the optimal policy has it.

Suppose the contrary, i.e. suppose that for the optimal contract, I, $\pi(x)$ is not the same for all x. In this case there exist $x_1$, $x_2$, such that $\pi(x_1) > \pi(x_2)$. We now construct a contract $\bar{I}$ which is preferred to I by both seller and buyer, thus contradicting the optimality of I. Let $\bar{I}(x)$ be the same as I(x) except that the payments to the buyer are smaller by an amount $p_2\varepsilon$ in the interval $[x_1,x_1+\delta]$, and larger by $p_1\varepsilon$ in the interval $[x_2,x_2+\delta]$. The small numbers $\varepsilon$, $\delta$, were chosen so that $I(x) > 0$ for $x \in [x_1,x_1+\delta]$, and $\pi(x') < \pi(x)$ for $x' \in [x_2,x_2+\delta]$ and $x \in [x_1,x_1+\delta]$.[9] The $p_i$'s denote the probability that the damage will lie in the interval $[x_i,x_i+\delta]$, i = 1,2. It is easy to verify that if I and $\bar{I}$ had the same critical values, they would have the same actuarial value.

We now show that the buyer prefers $\bar{I}$ to I. For this we define by $W(\bar{W})$ the expected discounted utility of the buyer given that he is offered the contract $I(\bar{I})$. We also denote by $W^*$ the expected discounted utility of the buyer if he were offered $\bar{I}$ but maintained the critical value $y^*$ corresponding to I. Then clearly $\bar{W} \geq W^*$. Since we show in the Appendix that $W^* > W$, it follows that $\bar{W} > W$, and the buyer therefore prefers $\bar{I}$ to I.

We next show that the critical value $\bar{y}^*$ corresponding to $\bar{I}$ is higher than the one corresponding to I. Hence, since a higher $y^*$ implies a lower probability that a claim will be made, also the seller prefers $\bar{I}$ to I.

Note that the critical value $y^*$ satisfies

$$u(A - r - y^*) + \beta W = u[A - r - y^* + I(y^*)] + \beta M \tag{4.1}$$

Consider now the small change in policy from I to $\bar{I}$. This change can be viewed as a change of $\delta$ from zero to some positive constant and its effect on $y^*$ can be studied by examining the total differential of (4.1). Since $x_1$,

$x_2 > y^*$, the function $\bar{I}(x)$ is the same as $I(x)$ in a small enough neighborhood of $y^*$. Thus, from the total differential of (4.1), we obtain

$$-u'(A - r - y^*)dy + \beta dW = -[1 - I'(y^*)]u'[A - r - y^* + I(y^*)]dy,$$

where dy and dW are the changes in W and $y^*$ due to the change in policy. Using some algebra, we obtain

$$dy = -\beta dW \cdot \{[1 - I'(y^*)]u'[A - r - y^* + I(y^*)] - u'(A - r - y^*)\}^{-1}.$$

Since the buyer is risk averse and since by assumption $0 \leq I'(y^*) \leq 1$, and we have already shown that $dW > 0$, it follows that $dy > 0$ and hence $\bar{y}^* > y^*$. Q.E.D.

The implication of Theorem 4.1 is that, for any j, the optimal policy must provide full coverage for any loss above a deductible. Here, however, unlike the one-period models the insured will file a claim only if it is "sufficiently" higher than the deductible.

Having established that the optimal policy gives full insurance above a deductible, we proceed to show:

<u>Theorem 4.2</u> If the premium is some multiple (larger than or equal to one) of the actuarial value of the contract, the optimal deductible is strictly positive.

<u>Proof</u>: Denote the deductible by m. For any premium r, a deductible m, and a critical value y (not necessarily optimal) the expected utility of the buyer is given by

$$\hat{W}(y) = \int_0^y [u(A - r - x) + \beta\hat{W}(y)]dF(x) + \int_y^\infty [u(A - r - m) + \beta M]dF(x). \tag{4.2}$$

The premium, r, is assumed to satisfy

$$r = (1 + \lambda) \int_y^\infty (x - m)dF(x), \tag{4.3}$$

where $\lambda$ is some nonnegative loading factor. The optimal critical value, $y^*$, is obtained by maximizing (4.2) subject to (4.3). Equation (4.2) can, however, be written in the form

$$\hat{W}(y) = [1 - \beta F(y)]^{-1}\{\int_0^y u(A - r - x)dF(x) + [u(A - r - m) + \beta M]G(y)\}. \tag{4.4}$$

where $G(y) = 1 - F(y)$.

The optimal critical value is obtained by differentiating (4.4) with respect to y, under the constraint (4.3), and equating the derivative to zero.

We wish to establish that $dW/dm = d\hat{W}(y^*)/\partial m > 0$ at the point $m = 0$. This derivative, however, can be written as

$$\frac{dW}{dm} = \frac{d\hat{W}(y^*)}{dy^*} \cdot \frac{\partial y^*}{\partial m} + \frac{d\hat{W}(y^*)}{dm} .$$

Since from the first order conditions of maximum $d\hat{W}(y^*)/dy^* = 0$, it follows that $dW/dm = d\hat{W}(y)/dm$ evaluated at $y^*$. We therefore proceed to evaluate this derivative. By the chain rule of differentiation:

$$d\hat{W}(y)/dm = \partial\hat{W}(y)/\partial r \cdot \partial r/\partial m + \partial\hat{W}(y)/\partial m.$$

However

$$\partial\hat{W}/\partial r = [1 - \beta F(y)]^{-1}[-\int_0^y u'(A - r - x)dF(x) - G(y)u'(A - r - m)],$$

$$\partial r/\partial m = -G(y) - \lambda G(y),$$

$$\partial\hat{W}/\partial m = [1 - \beta F(y)]^{-1}[-G(y) \cdot u'(A - r - m)].$$

Hence

$$\frac{dW}{dm} = G(y^*)[1 - \beta F(y^*)]^{-1}[\int_0^{y^*} u'(A - r - x)dF(x) + G(y^*)u'(A - r - m)$$

$$- u'(A - r - m)] + \lambda G(y^*)[1 - \beta F(y^*)]^{-1}[\int_0^{y^*} u'(A - r - x)dF(x)$$

$$+ G(y^*)u'(A - r - m)]$$

$$= G(y^*)[1 - \beta F(y^*)]^{-1} \{\int_0^{y^*} [u'(A - r - x) - u'(A - r - m)]dF(x)$$

$$+ \lambda[\int_0^{y^*} u'(A - r - x)dF(x) + G(y^*)u'(A - r - m)]\}.$$

We note that when $m = 0$, $dW/dm > 0$ if $u$ is concave (whether $\lambda > 0$ or $\lambda = 0$). Thus, the optimal deductible cannot be zero. Q.E.D.

Note that $dW/dm|_{m=0} > 0$ even if $\lambda = 0$. This result differs from that obtained for one-period contracts (see Mossin [1968], p. 562), where it was shown that a positive deductible is optimal only if $\lambda = 0$. The reason for this difference is the following. In the one-period model, filing a claim involves equals the actuarial value), the insured prefers as much insurance as possible and hence a zero deductible. If, in the one-period model, the contract is unfair (i.e., the premium is higher than the actuarial value), the buyer will buy less than the maximal insurance and so will prefer some positive deductible. In the multi-period case, filing a claim is always costly since it raises future rates. Hence, even if current premiums are fair, the buyer prefers not to insure against small risks, and therefore prefers a positive deductible.

## Footnotes

[1]We assume for simplicity that the buyer cannot make more than one claim at any period. This has no effect on the results of the paper.

[2]Few insurance companies classify buyers to risk categories also according to the size of claims they have made in the past.

[3]J can be interpreted as the highest rate for which there is demand for insurance, since for higher rates the buyer is better off without insurance.

[4]It is assumed, as is common in the United States, that the buyer is not automatically moved to a lower risk bracket if he does not make a claim. However, the results in the sequel hold, even if this assumption is dropped.

[5]In what follows, we assume that the distribution function is the same for both the buyer and the seller. This assumption could easily be removed without altering the results.

[6]We write $r_j$ for $r_j(I)$ whenever this does not lead to confusion.

[7]The subscript t is implicit in the definition of $\pi$.

[8]All our results are valid also in the nonstationary case.

[9]Invoking the continuity of I, such $\varepsilon$ and $\delta$ do exist.

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## Appendix

Lemma A.1  $W^* > W$.

Proof. By definition:

$$W^* = \int_0^{y^*}[u(A - r - x) + \beta\hat{W}]dF(x) + \int_{y^*}^{\infty}\{u[\bar{\pi}(x)] + \beta M\}dF(x). \qquad (A.1)$$

Since $\bar{\pi}(x) = \pi(x)$ outside the intervals $[x_i, x_i + \delta]$, $i = 1,2$, we obtain, by adding and subtracting

$$\int_{x_i}^{x_i+\delta} u[\pi(x)]dF(x), \quad i = 1,2, \text{ from both sides of (A.1) that:}$$

$$W^* = \int_0^{y^*}[u(A - r - x) + \beta W^*]dF(x) + \int_{y^*}^{\infty}\{u[\pi(x)] + \beta M\}dF(x) + K, \qquad (A.2)$$

where

$$K = \int_{x_1}^{x_1+\delta}\{u[\bar{\pi}(x)] - u[\pi(x)]\}dF(x) + \int_{x_2}^{x_2+\delta}\{u[\bar{\pi}(x)] - u[\pi(x)]\}dF(x). \qquad (A.3)$$

Expression (A.2) can be written as

$$[1 - \beta F(y^*)]W^* = \int_0^{y^*} u(A - r - x)dF(x) + \int_{y^*}^{\infty}\{u[\pi(x)] + \beta M\}dF(x) + K. \qquad (A.4)$$

However, rearranging terms in (3.6) we obtain that the first two terms on the right-hand side of (A.4) are equal to $[1 - \beta F(y^*)]W$,. Hence,

$$[1 - \beta F(y^*)]W^* = [1 - \beta F(y^*)]W + K.$$

It then follows from the concavity of u (see Arrow [1963], p. 971), that $K > 0$, and $W^* > W$. Q.E.D.