Regulation of Bank Capital
and
Portfolio Risk

by

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ABSTRACT

This paper examines the impact of bank portfolio behavior when banks are subject to capital regulation. It employs comparative static analysis of bank portfolio risk when regulators increase the minimum level of required capital relative to assets. The results show, under plausible assumptions, that an increase in the required capital-asset ratio can lead to a higher portfolio risk and to a higher probability of failure. For the system as a whole, the results of a higher required capital-asset ratio in terms of the average probability of failure is ambiguous, while the intra industry variance of probability of failure unambiguously increases. This result leads us to question the viability of regulating commercial banks in terms of a capital requirement. Thus, serious consideration should be given to the discontinuance of regulation of bank capital via ratio constraints and allowing the financial markets to control the risk behavior of bank management. Alternatively, such regulation should be imposed on a much more selective basis.

INTRODUCTION

Recent large bank failures (e.g., Franklin National), coupled with an unstable economic environment, have rekindled the controversy over the question of the adequacy of bank capital. Regulators such as the Federal Reserve have indicated their concern over the trend towards the reduction of capital relative to assets by requesting banks to improve their capitalization, and by enforcing a "go slow" policy towards bank expansion. However, bank management complains that such regulation restricts its investment opportunities, thereby reducing bank profitability. It argues that the low return on capital due to such restrictions is the ultimate cause of failure and that the financial markets should be the sole mechanism for the control of bank capital.

There is, of course, an abundance of literature on both sides of the bank capital issue. On the side of the regulators, for example, such authors as Wallich (19), Shay (17), and Watson (20) have pointed out that the cost of bank failure is not simply the loss to equity holders. They assert that bank failure disrupts the payments mechanism, and thus generates a far greater cost than the capital markets would consider. Conversely, authors such as Pringle (11), Vojta (18) and Robinson and Pettway (13) argue that the capital markets should be free to maximize market value within an unconstrained environment.

Implicit in most of the literature on bank capital is the attempt to define the amount of capital that would be "adequate" in light of the economic environment and the cost of bank failure. Santomero and Watson (16) use a macro general-equilibrium model to demonstrate that an optimal capital structure can be found in terms of the capital-asset ratio. Their model considers the social cost of bank failure as well as the cost of over-capitalization before determining the optimal capital-asset ratio. Using such a model as a basis, one can argue that bank capital may require some regulation and that capital structures should be held to the optimal amount.

Noticeably absent from the literature arguing for and against regulating bank capital, however, is a detailed consideration of the impact of such regulation on individual bank behavior and whether the regulation actually achieves its desired result.[1] Regulaton is assumed to function in an essentially ceteris paribus environment, reducing risk by the mere addition of capital to the bank's balance sheet. The purpose of this paper is to explicitly examine the issue of portfolio reaction to capital requirements by investigating the effect of capital ratio regulation on the portfolio behavior of a commercial bank.[2] It develops a precise specification of the bank's portfolio decision that flows directly from a risk averse portfolio allocation decision by the firm. Next, using comparative static analysis, it examines the effects on bank portfolio risk of a regulatory increase in the minimum capital asset ratio that is acceptable to the supervisory agency. It concludes that for the system as a whole, the results of a higher required capital-asset ratio in terms of the average probability of failure is ambiguous, while the intra-industry dispersion of the probability of failure unambiguously increases. This result leads us to question the viability of regulating commercial banks in terms of a capital requirement. Thus, serious consideration should be given to the discontinuance of regulation of bank capital via ratio constraints and allowing the financial markets to control the risk behavior of bank management. Alternatively, such regulation should be imposed on a much more selective basis.

The analysis will proceed in the following steps. Section I will enumerate the assumptions concerning bank behavior and the bank environment. Section II will derive the analytic model. Section III will carry out comparative statics when the capital-asset ratio is increased. Section IV will examine these results and relate them to a measure of the probability of failure.[3] The final section will summarize the results.

I. ASSUMPTIONS OF THE MODEL

The approach taken here is to examine the portfolio response of the commercial bank, when faced with a regulatory change. The major assumptions used in order to derive the model and to carry out the analysis are listed below. In general, they require that the bank be viewed as an expected utility maximizer in the capital market. Its objective function is maximized subject to the capital market opportunity set and the regulated ratio of the bank's capital to total assets.

The assumptions may be enumerated as:

a. Total bank size is assumed to be under the control of bank management. That is, the amount of deposits raised by management to support the purchase of risky assets is a choice variable,[4] as is the total amount of capital. However, it will be assumed that the ratio of capital to assets is effectively constrained by regulation.

b. For simplicity, there does not exist a risk-free asset for purchase by the bank, but the return paid to depositors is riskless.[5]

c. The bank acts as if it is a risk averse expected utility maximizer.[6] This assumption is well known in the literature (see D. Pyle (12), for example). Furthermore, Ross (14) has shown that an optimal fee schedule can be derived such that the agent, i.e., bank management, acting so as to maximize its expected utility, will maximize owners' expected utility. It is further assumed that the objective function can be approximated by a Taylor series expansion of a general class of risk averse utility functions, trunicated after the second moment. Thus, the allocation across assets becomes the choice variable deriving the optimal mean rate of return per unit of capital and the variance of that return.

d. The bank makes its portfolio decisions for a single period only. Thus, the portfolio decision is made at the beginning of a period and the portfolio is held until the end of the period.

e. All risky assets are traded in competitive markets so the individual bank treats the rates of return on each asset as given. That is, the bank is a price taker. This assumption could be criticized for certain asset markets (e.g., small commercial loans), but generally would seem to hold as a first approximation in every asset market. It should be pointed out that introducing market imperfections in the analysis (e.g., a monopoly position of the bank in an asset market) does not change the results.[7] Market imperfections do, however, complicate the derivation so they will subsequently be ignored in this analysis.

II. THE EFFICIENT OPPORTUNITY SET UNDER CAPITAL REGULATION

Given that the regulator fixes the capital to asset ratio, denoted $c \equiv K/A$, the choice problem facing the bank is to determine (a) its optimal scale, i.e., the amounts of both deposits and capital to issue, and (b) the optimal allocation of this asset pool over the available risky asset set. However, given the competitive market assumption, with the absence of monopoly positions in any financial market, no unique optimal bank size exists. The bank may issue more equity and deposits in the ratio of c to (1-c) respectively and proportionately increase total earnings.[8] Return and risk per unit of capital are unaffected by bank scale in the competitive market. Therefore, the analysis will be developed in terms of risk and return per unit of capital with no loss in generality.

The bank will be assumed to maximize expected utility given a general risk averse function in which the first two moments of the return per unit of capital are arguments. The particular portfolio chosen, described by the quantities held relative to capital, will be from the mean-variance efficient set.[9]

To find the entire efficient investment set the bank has the following quadratic programming problem, described in depth by Merton (6):

(1) $\underset{\{x_i\}}{\text{minimize}}$

$$\tfrac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n} x_i x_j \sigma_{ij} - \lambda_o E_p \qquad \text{for all } \lambda_o,\ 0 \leqq \lambda_o \leqq \infty$$

subject to

(2)
$$1 = \sum_{i=0}^{n} x_i$$

(3)
$$x_0 \geqq 1 - \frac{1}{c}$$

with

(4)
$$E_p = x_0 R + \sum_{i=1}^{n} x_i E_i$$

where

(a) E_i is the expected return on the i^{th} asset

(b) x_i is the percentage of equity value, as distinct from total portfolio value, invested in the i^{th} asset, i=1,2,...,n.

(c) σ_{ij} is the covariance of returns between the i^{th} and j^{th} assets, and the variance of return on the i^{th} asset is $\sigma_{ii} = \sigma_i^2$. We assume that the variance-covariance matrix is positive-definite.

(d) N is the total number of risky assets available for purchase by the bank.[10]

(e) E_p and σ_p^2 are the expected return and variance of return, per unit of capital, on the bank portfolio.

(f) c is the capital-asset ratio $(\frac{K}{A})$ which is regulated by bank supervisors.

(g) x_0 is the percentage of capital held in the negative asset (deposits) paying the risk-free rate R.

(h) λ_0 is the real tradeoff between variance and expected return at any point on the efficient investment frontier, i.e., $\lambda_0 = \frac{d\sigma_p^2}{dE_p}$. It ranges from 0 to ∞.

Given that the bank is interested in generating the entire opportunity set, it will set λ_0 equal to particular values within the relevant range and solve for the corresponding efficient portfolios. Obviously, for $\lambda_0 =$ 0 the resulting portfolio will be the minimum variance efficient portfolio. Conversely, for $\lambda_0 = +\infty$, the corresponding portfolio will be the efficient portfolio with highest expected return. For values of λ_0 lying between these extremes, the solution to equation (1) will yield efficient portfolios with intermediate values of E_p and σ_p. We should note here that the value of λ_0 which corresponds to the portfolio that maximizes the banks' expected utility is related to the risk aversion of the bank. Thus, a high value of λ_0 corresponds to a less averse attitude towards risk taking.

Equation (4) represents the leverage or capital constraint imposed by regulation. As such it needs more explanation. As the value of c varies from zero to one, the leveraging potential of the bank varies from the bank's unconstrained optimum to an amount equal only to its equity capital. Thus, when c=1, the bank acts like a mutual fund unable to issue deposits of any kind. Likewise, if c=.10, then the maximum proportion of equity which can take the form of deposit liabilities in the bank portfolio is:

$$x_0 = -9.$$

That is, the bank can issue liabilities up to 900% of its equity capital, and thus leverage its risky asset portfolio to an amount far greater than its equity. Restating the above problem using Lagrangean multipliers yields:

$$(5) \quad \min \left[\tfrac{1}{2} \sum_{i=1}^{n} \sum_{j=1}^{n} x_i x_j \sigma_{ij} - \lambda_0 \left[x_0 R + \sum_{i=1}^{n} x_i E_i \right] \right]$$

$$+ \lambda_1 \left[1 - x_0 - \sum_{i=1}^{n} x_i \right] + \lambda_2 \left[1 + Z_0 - x_0 - \frac{1}{c} \right]$$

where

(a) Z_0 = dummy variable

(b) λ_1, λ_2 = Lagrangean multipliers.

In general, x_0 is a choice variable, and, therefore, the restriction on the capital-asset ratio c may never constrain the bank from reaching its desired size if $x_0^* > 1 - \frac{1}{c}$ and $Z^* > 0$. However, since interest centers upon the impact of a marginal change in c on bank portfolio behavior, it will be assumed that x_0 is constrainted to $1 - \frac{1}{c}$ and $Z_0 = 0$, i.e., the leverage of the bank is constrained by c. This assumption appears realistic, given bank management complaints that capital requirements are too restrictive and that less capital should be allowed to support a given level of risky assets.

The first order conditions for a minimization of equation (5) are:

$$(6) \quad \sum_{j=1}^{n} x_i \sigma_{ij} - \lambda_0 E_i - \lambda_1 = 0 \qquad i=1,2,\ldots,n$$

$$(7) \quad -\lambda_0 R - \lambda_1 - \lambda_2 = 0$$

$$(8) \quad 1 - x_0 - \sum_{i=1}^{n} x_i = 0$$

$$(9) \quad 1 - x_0 - \frac{1}{c} = 0$$

The solution to the above system of equations is unique and will minimize equation (1) by the assumption that the variance-covariance matrix is positive-definite. The particular portfolio chosen by the bank, given that it is constrained by c, is obtained by the simultaneous solution of equations (6) to (9) and the equality of λ_0 to the marginal rate of substitution between risk and return dictated by the bank's objective function.

To obtain a unique portfolio of the firm, it will be assumed that the bank has a general risk averse utility function in end-of-period capital. For a given value of initial capital, the objective function can be written as:

$$(10) \qquad U = U(K+R_p K)$$

Taking a Taylor expansion of equation (10) around initial capital of the bank, and neglecting higher order terms, see Pratt (10), one obtains

$$(11) \qquad U = U(K) + U'(K)KR_p + \tfrac{1}{2} U''(K)K^2 R_p^2$$

where U(K), U'(K) are positive constants dependent upon initial capital. The expectation of equation (11) can be written as

$$(12) \qquad E(U) = U(K) + U'(K)[E(R_p)-b\{(E(R_p))^2 + \sigma_p^2\}]$$

where $b = -\dfrac{U''(K)K}{2U'(K)}$ and is the coefficient of relative risk aversion of the underlying utility function. For this objective function, the marginal rate of substitution between risk defined as the variance of portfolio return, and expected return is,

$$(13) \qquad \left.\frac{d\sigma_p^2}{dE_p}\right|_{u=\bar{u}} = \frac{1}{b} - 2E_p = MRS_{\sigma_p^2,\, E_p}.$$

Since optimality requires equality of the MRS to the objective tradeoff of risk and return, then the following condition must be satisfied for the optimal portfolio, viz.

$$\frac{1}{b} - 2E_p = \lambda_0.$$

Hence, the optimal solution for x_i, $i=1,2,\ldots,n$ in the institution's portfolio can be written as the following simplified system.

(14) $$\sum_{j=1}^{n} x_i \sigma_{ij} - \lambda_0((E_i - R) + \lambda_2 = 0 \qquad i=1,2,\ldots,n$$

(15) $$\sum_{i=1}^{n} x_i = \frac{1}{c}$$

(16) $$\lambda_0 = \frac{1}{b} - 2[R + \sum_{i=1}^{n} (E_i - R)x_i]$$

Equations (13) to (15) are linear in the x_i's and thus from equation (13):

(17) $$x_i = \lambda_0 \sum_{j=1}^{n} v_{ij} \; (E_j - R) - \lambda_2 \sum_{j=1}^{n} v_{ij} \qquad i=1,2,\ldots,n$$

where v_{ij} is the ij^{th} element of the inverse of the variance-covariance matrix.

Following Merton (5) and eliminating λ_0 and λ_2, using equations (14) to (16), the optimal ratio of asset i to capital, $x_i(i=1,2,\ldots,n)$, held in each asset given the constraint $x_0 = (1 - \frac{1}{c})$ is:

$$(18)\ x_i^* = \frac{AC(\frac{1}{b} - 2R) - C(1 + B)\frac{1}{c}}{C + D}\left[\frac{\sum_{j=1}^{n}(E_j - R)v_{ij}}{A} - \frac{\sum_{j=1}^{n} v_{ij}}{C}\right]$$

$$+ \frac{\sum_{j=1}^{n} v_{ij}(E_j - R)\frac{1}{c}}{A} \qquad i=1,2,\ldots,n$$

where:[11]

$A = \sum_{ij}\sum v_{ij}(E_j-R)$,

$B = \sum_{ji}\sum v_{ij}(E_i-R)(E_j-R) > 0$,

$C = \sum_{ij}\sum v_{ij} > 0$,

$D = BC - A^2 > 0$

Equation (18) expresses the optimal ratio of each risky asset in the portfolio to equity capital, chosen in terms of the parameters of the joint distribution of returns, the value b of the utility function and the capital asset constraint, c.

III. IMPACT ON THE COMPOSITION OF THE OPTIMAL BANK PORTFOLIO OF A CHANGING CAPITAL CONSTRAINT

Before the imposition of the increased capital constraint, the balance sheet of the bank in equilibrium is made up of a fixed ratio of deposit liabilities and capital. The deposits are assumed to be homogenous and pay a risk free rate R. The left hand side of the balance sheet is made up of n risky assets held in proportion described in equation (18). This particular portfolio is efficient.

Now consider an increase in c. From the budget constraint, equation (8), and the capital constraint, equation (9), it is clear that the bank will be unable to leverage its capital to the degree it had prior to the increase in c. That is, the imposition of an increased c restricts the leveraging capability of the bank, and its efficient investment frontier falls downward and to the left for any given value of capital. Thus, over the entire frontier[12]:

$$\frac{\partial \sigma_p^2}{\partial c} < 0, \quad \frac{\partial E_p}{\partial c} < 0.$$

This implies unambiguously that over the entire permissable efficient frontier the total variance of the portfolios fall and the return on each set declines.

In addition, with the decrease in leverage, the bank reacts by reshuffling the composition of its efficient asset portfolio. The effect on the composition of the bank's portfolio, given a small change in the required capital-asset ratio, can be evaluated by differentiating equations (18) with respect to c. If it is assumed that the utility function exhibits constant relative risk aversion,[13] the result may be written as:

$$(19) \quad \frac{\partial x_i^*}{\partial c} = \frac{-\frac{1}{c^2}\, C(1+B)}{C+D} \left[\frac{\sum_{j=1}^{n} (E_j - R) v_{ij}}{A} - \frac{\sum_{j=1}^{n} v_{ij}}{C} \right]$$

$$- \frac{1}{c^2} \frac{\sum_{j=1}^{n} v_{ij} (E_j - R)}{A}, \qquad i=1,2,\ldots,n$$

or, in elasticity form,

$$(20)\quad \varepsilon_{x_i,c} = \frac{1}{x_i}\,\frac{(\frac{1}{b} - 2R)(AC)}{C + D}\left[\frac{\sum_j^n v_{ij}(E_j - R)}{A} - \frac{\sum_j^n v_{ij}}{C}\right] - 1$$

$$i=1,2,\ldots,n$$

Equation (20) is the elasticity of proportional demand for the i^{th} asset with respect to the capital-asset constraint, evaluated at the optimum.

It is easy to show that the relative size of the elasticity expression for every asset is determined by the asset's risk contribution to the optimal portfolio. Risk contribution is defined as σ_{ip} so that $\varepsilon_{x_i,c} > \varepsilon_{x_j,c}$ if $\sigma_{ip} > \sigma_{jp}$. This may be demonstrated as follows.

As is well known, any portfolio on the efficient frontier of risky assets can be generated by two frontier portfolios or mutual funds made up of the n risky assets. This is the Mutual Fund Theorem of Merton (6) or Black (1). The proportions held of each risky asset in each fund is independent of investor preferences and the proportions of each fund chosen by the investor is independent of the parameters of the joint distribution of returns. In the case of the bank, the mix chosen of the two funds will be a function of the parameter of its utility function. The mix will generate the optimal portfolio with proportions described in equation (18).

Since any two efficient portfolios can generate all other efficient portfolios, assume the two funds are the minimum variance portfolio and the fund on the efficient frontier that is tangent to the slope $\frac{E_p - R}{\sigma_p}$. The proportions of the i^{th} asset $(i=1,2,\ldots,n)$ held in the two portfolios can be shown to be respectively:

(21) $$\beta_i = \frac{\sum_{j}^{n} v_{ij}}{C}$$

(22) $$\alpha_i = \frac{\sum_{j=1}^{n} v_{ij} (E_i - R)}{A}$$

The characteristics of these two portfolios are such that

(23a) $$\sum_{i=1}^{n} \alpha_i E_i > \sum_{i=1}^{n} \beta_i E_i$$

or equivalently

(23b) $$\sum_{i=1}^{n} (\alpha_i - \beta_i) E_i > 0$$

and

(24a) $$\sum_{i}^{n}\sum_{j}^{n} \alpha_i \alpha_j \sigma_{ij} > \sum_{i}^{n}\sum_{j}^{n} \beta_i \beta_j \sigma_{ij}$$

or equivalently

(24b) $$\sum_{i=1}^{n} (\alpha_i - \beta_i) \sigma_{ip} > 0$$

Using equations (21) and (22) the elasticity expression, equation (20), can be rewritten as

$$(25) \qquad \varepsilon_{x_i,c} = \frac{1}{x_i} \frac{(\frac{1}{b} - 2R)AC}{C + D} (\alpha_i - \beta_i) - 1. \qquad i=1,2,\ldots,n$$

Equations (23) and (24) imply that $(\alpha_i - \beta_i)$ will be positive in equation (25) for those assets that have a relatively large risk contribution, σ_{ip}, to the optimal portfolio. For example, for any two assets i and j, $(\alpha_i - \beta_i) > 0$ and $(\alpha_j - \beta_j) < 0$ imply that $\sigma_{ip} > \sigma_{jp}$. Therefore, for any asset i that contributes relatively more risk than asset j, its elasticity will be greater than the elasticity of asset j. Thus in general,

$$(26) \qquad \varepsilon_{x_i,c} > \varepsilon_{x_j,c} \text{ if } \sigma_{ip} > \sigma_{jp}, \text{ i, j}=1,2,\ldots,n.$$

It should be noted that if all assets are uncorrelated, then

$$\varepsilon_{x_i,c} > \varepsilon_{x_j,c} \text{ if } \sigma^2_i > \sigma^2_j, \quad \text{i, j}=1,2,\ldots,n.$$

This result gives the following interpretation to equation (20) or (25). Subsequent to the imposition of a decreasing leveraging capability, i.e., an increasing c, the reaction of the bank is to reschuffle its portfolio. The composition of the resulting equilibrium portfolio after the increase in c is described by relatively more risky assets than before the increase.[14] The effect of an increased c on the bank portfolio is, thus, somewhat contrary to the desired result.

It can be seen from equation (25) that the degree to which this reshuffling occurs is dependent upon the risk aversion coefficient, b. For highly risk averse institutions, the elasticity value of high risk assets is less than the elasticity for other institutions possessing less risk aversion. Likewise, the conservative institution will shift its portfolio from low risk to a lesser degree than its risk taking counterparts. It follows, therefore, that institutions which initially held relatively more risky assets per unit of capital, shift to offset the capital restriction to a greater extent than do the more conservative group of firms. Consequently the dispersion of risk taking across the banking industry expands. Relatively conservative institutions somewhat offset the capital restrictions. Their more risky counterparts, on the other hand, reshuffle their balance sheet to an even greater extent and therefore increase the variance of total risk for the entire industry. The effect of this adjustment on the industry's probability of failure will be the next area of concern.

IV. CAPITAL REGULATION AND BANKRUPTCY

It will be assumed here that the central purpose of bank regulation is to reduce the riskiness of the bank portfolio per se, but only insofar as it affects the probability of failure. While some arguments to the contrary may be offered, the alleged purpose of regulatory control over these institutions has been to increase stability and viability. Accordingly, the works of Santomero and Watson (16) and Blair and Heggestad (2), among others, have made this explicit assumption as well.

An explicit relationship between the risk of the bank portfolio, the amount of bank capital held and the chance of bankruptcy must, therefore, be obtained to evaluate the result of bank capital regulation. Such a relationship may be accomplished once the characteristics of the distribution of the returns from bank operations has been defined. There are essentially two choices. Either the distribution can be assumed to be known, e.g., using a normality assumption, or left indeterminant, e.g., using some general probability characteristics. In either case, the analytic results indicated below will immediately follow. For simplicity, therefore, the major part of the analysis will make reference only to Chebyshev's Inequality. The normal distribution assumption will be left to the interested reader.

The capital-asset ratio, the expected return of the portfolio, and the variance of the return can be utilized, via the Chebyshev Inequality, to estimate the upper bound of the probability of failure, as suggested by Roy (15) and Blair and Heggestad (2).[15] Given the characteristics of the bank portfolio described by E_p and σ_p, this upper bound may be written as:[16]

$$\text{(27)} \qquad PR\ [\tilde{R}_p < -c] \leqq \frac{\sigma_p^2}{(E_p + c)^2} = P,$$

with

$$\frac{\partial P}{\partial \sigma_p^2} > 0, \ \frac{\partial P}{\partial E_p} < 0, \ \frac{\partial P}{\partial c} < 0.$$

As expected, an increase in variance increases the probability of failure, while an increase in returns or the capital ratio will, _ceteris paribus_, decrease failure risk. This is indicated by the partial derivatives of equation (26).[17]

Graphically, this upper bound on the probability of failure can be seen as the square of the reciprocal of the slope of a ray in mean variance space.[18] The ray has an intercept of [-c] in Figure 1 and is denoted as D_0D_0. It intercepts the efficient frontier at point Z_0, where the optimal allocation is indicated by the tangency of the objective function. The

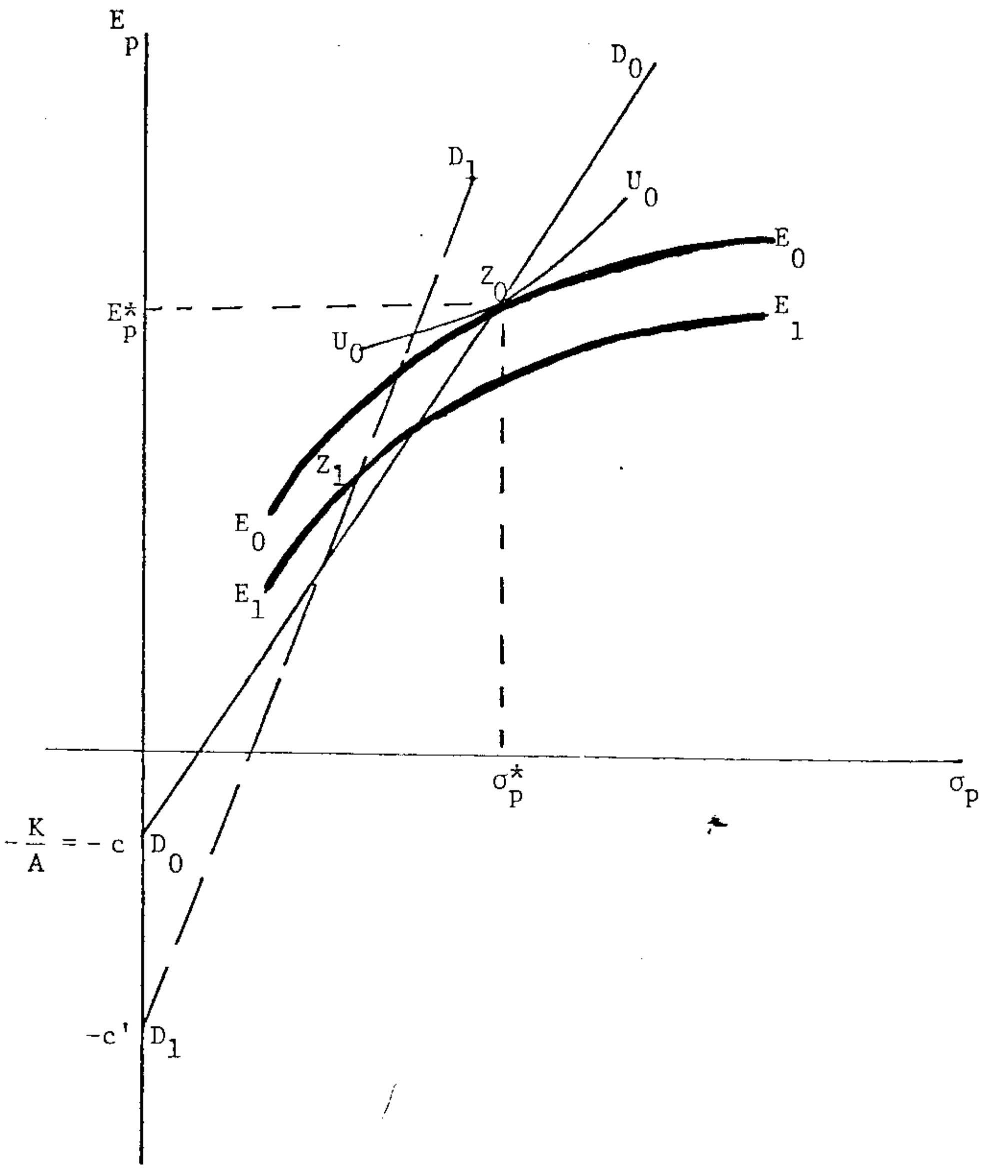

FIGURE I

The effect of a reduction in allowable leverage, if no portfolio adjustment occurs.

E_p = the return per unit of capital on the portfolio.

σ_p = standard deviation per unit of capital on the portfolio.

E_oE_o = initial allowable efficient frontier

E_1E_1 = newly constrainted efficient frontier

Z_o = initial optimum

Z_1 = point on E_1E_1 with identical MRS as Z_o.

portfolio at point Z_0 is characterized by the proportions $x_i (i = 1,\ldots,n)$ or equation (18), and E_p^* and σ_p^*. The firm, therefore, can be described as possessing a portfolio that has an upper limit on its probability of failure of P_D, which is constant along the D_0D_0 ray.

If bank regulators feel, given the current bank portfolio Z_0, that the chance of failure is too high, then a higher capital-asset ratio is imposed. As c increases, the frontier moves down and to the left and the intercept moves down to (-c') on the vertical axis in Figure I. If the bank settles for the same risk-return trade-off in its portfolio as initially, the portfolio set will shift to Z_1 on the new constrained frontier. Since the slope of D_1 through Z_1 is greater than the slope of D_0, the upper bound of the probability of failure decreases and the regulators achieve their desired result.

However, as was demonstrated earlier, a bank satisfying the conditions set above, will not move to point Z_1. That is, the portfolio Z_1(identical in risk-return trade-off to portfolio Z_0) will not become the equilibrium portfolio of the bank. Recall that subsequent to the increase in c, the bank reshuffles its portfolio towards relatively more risky assets. That is, the new equilibrium portfolio is composed of relatively larger quantities of riskier assets than before. This may be seen as a shift of the tangency point from Z_1 in Figure I to a point of higher risk and return. The exact point will, as noted above, be dependent upon the risk aversion factor of the institution's preference function.

Two cases can be distinguished and are pictured in Figure II. In the first of these the new portfolio choice results in a reduction in the probability of failure, even though some asset reallocation has occurred. This is denoted as Z_2 and lies to the left of DD. Locus DD maps a proability of failure that is equal to that which obtained on D_0D_0, the original locus. Its intercept is below the initial locus, given that higher capital ratios are required in the new regime, but is everywhere parallel to D_0D_0. The fact that Z_2 lies to the left of DD implies that the overall probability of failure for the institution has declined. On the other hand, Z_3 lies to the right, indicating that, for this institution, the probability of failure has increased as a result of increasing required capital.

To explain the differing results, reference must be made to the portfolio decision model of the previous section. Examination of equation (25) reveals that, for an institution with a smaller value of b, the bank's reaction to an increase in c is a larger shift towards riskier assets which more than offsets the effect of increasing c. Therefore, the chance of failure increases. Conversely, if the bank is sufficiently risk averse, i.e., b is sufficiently large, then the movement towards riskier securities will be small, relative to the increase in c. In this case, the chance of failure decreases.[19]

This point may be seen analytically by examining the effect of variance reduction for institutions with different risk aversion coefficients, viz.,

$$\frac{d\left(\frac{d\sigma_p^2}{dc}\right)}{db} > 0.$$

Thus, σ_p^2 declines for every bank's portfolio, but the absolute magnitude of the decline depends upon b. If b is sufficiently small, the fall in E_p along with the reshuffling of the portfolio implies that the chance of failure increases.

Notice that the rank ordering of the increase in the relative holdings of the risky assets associated with a higher capital requirement is identical to the risk ordering of the institutions' portfolios before regulation. Firms with low values of b, the risk aversion factor, will initially possess larger quantities of risk assets in their portfolio than institutions with a higher degree of conservatism. A frequency distribution of the probability of failure for the entire industry will accordingly be equivalent to a frequency distribution of risk aversion coefficients. If regulation is now imposed so as to uniformly increase the capital requirements of the industry, offsetting reshuffling of the risk-asset components of the portfolio will occur based upon this same value, b. Risky institutions will react more to thwart the regulatory intent than relatively safer institutions.

From the above argument, therefore, it is clear that there exists some value of b, denoted b^*, below which any increase in c will <u>increase</u> the probability of bankruptcy, rather than decrease it as assumed. Thus as the capital constraint increases, the probability of failure will decrease, increase or remain unchanged as b is greater than, less than or equal to the critical value b^*. The final distribution of risk of failure for the banking industry will therefore possess a higher dispersion than before the imposition of regulation. Essentially, the relatively safe banks become safer, while risky institutions increase their risk position. The mean of the industry is dependent upon the underlying distribution of risk aversion, b.

If capital constraints are only imposed on a certain subset of the industry, the prognosis is no better. The banks likely to be subjected to a higher required c are ones that are less risk averse, i.e., a low value of

b. In other words, one would expect that a less risk averse bank would hold risky securities and therefore would be closely watched by regulators. However, the imposition of a higher required capital-asset ratio on these banks may well have a perverse effect. For such banks, it would seem that the regulators should find some other instrument to control the probability of failure, such as asset restrictions, or abandon attempts that essentially prove counterproductive. In general, regulators do not know *a priori* the degree of the bank's risk aversion, the size of b. Hence they do not know the affect of an increase in the required c. Regulating bank capital through ratio constraints, therefore, appears to be an inadequate tool to control the riskiness of banks and the proability of failure.

V. SUMMARY

Regulatory constraints that impinge upon bank behavior are designed -- excluding from consideration questions of monetary policy -- (a) to protect depositors, and (b) to protect the banking system as a whole and therefore, the health of the economy.

The rationale given for these restrictions including capital constraints usually points to the lower probability of failure that will result when these constraints are binding. However, as was demonstrated above, this is not what necessarily obtains. In fact, a case could be argued that the opposite result can be expected to that which is desired when higher capital requirements are imposed.

It is interesting to note that a number of researchers have not found an explicit relationship between bank failure and the amount of capital held.[20] The model presented above explains why this might be the case, for it includes consideration of the reaction of the institution to new regulatory stringency. This implies that the result of such regulation may be the opposite of regulatory intent.

FOOTNOTES

[1]The only exception to this is a paper by Mingo and Wolkowitz (8). However, this work neglects several essential features by assuming a linear utility function and a non-stochastic profit function. Their work, which uses a loan quality index and a certainty model, may, therefore, be viewed merely as a first step in our understanding of this problem.

[2]We will assume that regulators can enforce capital requirements and that the measure of adequacy used is the capital-risk asset ratio. See Santomero and Watson (16) and Wolkowitz (21) for a discussio of capital enforcement and the use of the capital-asset ratio as a useful measure of capital adequacy. It should be pointed out here that we do not defend the capital-risk asset ratio as the appropriate measure of bank risk but only attempt to understand bank behavior if regulators enforce such a measure. Throughout the paper, then, the capital-risk asset ratio will refer to the ratio of equity and debt capital to assets subject to either interest rate risk or default risk.

[3]The measure of the probability of failure comes from Roy (15). Using Roy's model, Blair and Heggestad (2) have examined bank portfolio restrictions and their impact on solvency. The present paper exploits this relationship for the purposes of capital regulation.

[4]It will be assumed the bank holds enough cash to satisfy reserve requirements as well as working cash needs. Cash holdings will subsequently be ignored.

[5]Effectively the bank issues the risk free asset in this model. If, alternatively, there existed another risk free asset, the portfolio decision as structured by traditional theory would reach a bounding conclusion. If the deposit rate were less than the risk free rate, the bank would grow to infinite size. If the converse existed, the risk free asset would not be purchased. To arrive at an interior solution would require that the deposit rate fluctuate with size due to, e.g., default risk. However, this leads the analysis far afield and will be omitted here.

[6]See Michaelsen and Goshay (7). There may be a question as to whether a bank should act in accordance to the market value rule or act as if it maximized a concave utility function. If we assume that banks are closely held, then assuming risk aversion is not unreasonable. Furthermore, without a well-defined general equilibrium model of the capital markets, it is not clear what determines market value. See D. Pyle (12) and O. Hart and D. Jaffee (4) for examples of the use of utility functions to describe intermediary behavior. See also J. Cozzolino and T. Taga (3).

[7]See James (5) for a discussion of the impact of market imperfections on the mean-variance efficient frontier.

[8]In a more general model the existence of monopoly profits in some markets could derive an optimal bank size. For the present purposes,

however, only the asset choice per unit of capital is required for the analysis. Therefore the added complexity necessary to jointly derive scale and composition is avoided in the text.

[9]The efficient frontier is made up of portfolios with maximum expected return for all possible levels of variance. See Merton (6).

[10]We will neglect the effects on solvency and bank behavior if certain assets are restricted from purchase by regulation.

[11]See Merton (6) for proof of the following inequalities. The inequalities result from the fact that the inverse of the variance-covariance matrix is positive-definite.

[12]To show the sign of these derivatives, differentiate σ_p^2, and E_p with respect to c, using the definitions of σ_p^2 and E_p and equation (18).

[13]An increase in the required c ratio will reduce the value of current period wealth, K, defined as the capitalized returns from bank equity ownership. If the utility function exhibits constant relative risk aversion, the MRS from equation (13) will be unaffected by the regulation change. The implications of a utility function with decreasing relative risk aversion are treated below. See footnote 14.

[14]If the utility function exhibits decreasing relative risk aversion, this shift into riskier assets is reinforced as the reduced value of expected terminal capital reduces b in equation (12) above.

[15]Blair and Heggestad's model is an application of Roy's (15) Safety First Model.

[16]Following Blair and Heggestad the Chebyshev Inequality states

$$PR\ [\ \tilde{R}_p - E_p \ > k\sigma_p] \leq \frac{1}{k^2}$$

$$PR\ [\tilde{R}_p < (E_p - k\sigma)] \leq \frac{1}{k^2}$$

$$\text{Let } -c = E_p - k\sigma_p, \text{ so } k = \frac{E_p + c}{\sigma_p}$$

Substitution of k yields equation (27).

[17]The necessary results to carry out the analysis conducted below are the partial derivative signs of equation (27). This will allow for a monotonic failure locus and all other results obtained. A normal distribution, as well as most others used in portfolio theory, would satisfy these conditions.

[18]See Roy (15) for a proof of this, or Blair and Heggestad (2) for a thorough discussion.

[19]As noted in footnote 14, b falls as c is increased in the case where the utility function exhibits decreasing relative risk aversion. This fosters even further movement up the new efficient frontier and favors the more extreme readjustment indicated by a shift to point Z_3 in Table II.

[20]See S. Peltzman (9) for example.

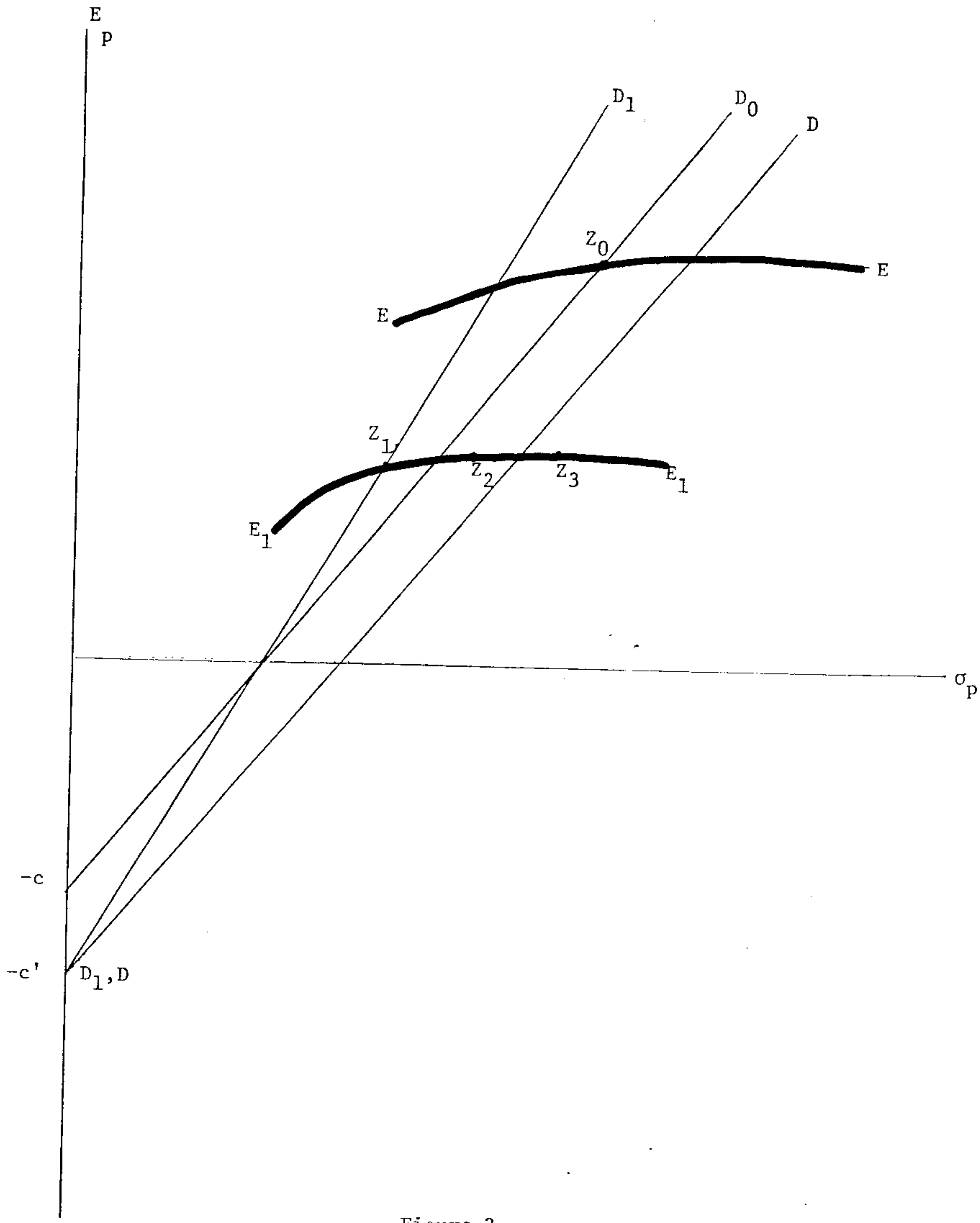

Figure 2

Alternative Portfolios After a Reduction In Allowable Leverage
See Figure 1 for legend.

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