Risk and Capital Asset Pricing

by

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In a recent paper, the authors presented new evidence on the capital asset pricing model (CAPM) which filled in one of the most serious gaps in prior tests of that model, i.e., the omission of data on bond returns in estimating the relation between expected returns on individual assets and expected returns on the market portfolio of all risky assets.[1] We found that our new and more comprehensive analysis, like earlier empirical investigations based on stocks only, yielded a return - risk relationship which implied a riskless rate of return inconsistent with any reasonable measure of the actual risk-free rate of return, and therefore concluded that the evidence against the original Sharpe-Lintner version of the CAPM was quite strong. However, unlike other recent tests based on stocks, we found that the residual variance or standard deviation of returns in the CAPM (σ_r^2 or σ_r) was fully as important in explaining returns on risky assets as beta (β) or covariance measures of risk in the two five year periods covered by our analysis. This result taken _prima facie_ is inconsistent not only with the Sharpe-Lintner or Black mean variance versions of the CAPM but also with the one-factor arbitrage pricing model proposed by Ross.[2] Moreover our results did not appear particularly

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[1] I. Friend, R. Westerfield, and M. Granito, "New Evidence on the Capital Asset Pricing Model," _Journal of Finance_, June 1978.

[2] Nor do the two-factor versions of the model tested seem to fare better. Our analysis specifically tested the recent Litzenberger-Kraus two-factor version of the CAPM which introduces co-skewness in addition to beta into the return-risk relationship and obtained results inconsistent with that version as well. In a paper not yet published (Westerfield and Friend) we also obtained results inconsistent with the Merton two-factor version of the CAPM.

[3] The market index was constructed with varying estimates of the relative values of long term marketable government debt, corporate debt and common stocks.

sensitive to different construction of the market index.[3]

Though our finding that $\sigma^2_{r.i}$ or ($\sigma_{r.i}$) seemed fully as important as β_i in explaining average return ($\bar{R}_i$) on the i'th risky asset raises a fundamental question about the validity and descriptive usefulness not only of the Sharpe-Lintner version of the CAPM but also of the more current revised versions, there are two reasons why this nihilistic result requires further testing. First, it is based on a relatively small number of observations--quarterly data for two five year periods, 1964-1968 and 1968-73. Second, a quite different result had been obtained earlier by Fama and MacBeth[4] who found over a much longer time interval, covering monthly observations for the period January 1935-June 1968, that $\sigma_{r.i}$ unlike β_i was not significantly (and positively) related to $\bar{R}_i$ for the period as a whole.[5] The Fama-MacBeth analysis deviated from ours not only in the longer time span, but also in its narrower coverage of assets (only stocks), its use of a market proxy consisting of an equally weighted index of stocks versus our value-weighted index of stocks and bonds, its use of different grouping and estimation procedures and its use of predictive rather than contemporaneous values of portfolio betas and standard deviations (i.e., they use prior period risk measures to explain current period returns). The objective of the present paper is to determine more definitively whether the central role ascribed by CAPM theory to the beta coefficient as the determinant of expected return can be justified by a more comprehensive and more satisfactory examination of the available data than has been carried out heretofore.

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[4]Eugene Fama and James MacBeth, "Risk Returns and Equilibrium: Empirical Tests," Journal of Political Economy, May 1973.

[5]However, the statistical significance of the relationship between $\bar{R}_i$ and β_i is far from strong and when the 1935-68 period is broken down into six sub-periods, the relationship is statistically significant for only one of these sub-periods (1961 to mid-1968). As a result of their analysis, they concluded that their proxy for the market portfolio was approximately *ex ante* efficient. It is obvious that the proxy used by Fama-MacBeth was not *ex post* efficient in the sample period used for testing.

The Model

The Sharpe-Lintner version of the CAPM asserts that the expected one-period return on asset i (i=1,2,...,N) is given by

$$(1)\quad E(R_i) = R_f + \beta_i[E(R_m) - R_f]$$

where R_i is the random return on security i, R_m is the random return on a market value-weighted portfolio of all marketable assets, R_f is a risk-free rate, and $\beta_i = \mathrm{cov}(R_i, R_m)/\sigma^2(R_m)$. In the Black version , where there is no risk-free asset, R_f would be replaced by the expected return on a zero-beta asset. Both versions of this model assume that the only relevant measure of risk of the i'th asset is β_i since any unique or residual risk, measured in this model by $\sigma_{r.i}$ or $\sigma^2_{r.i}$, could be diversified away. Thus expected returns can be estimated from an equation of the form[6]

$$(2)\quad \bar{R}_i - R_f = \gamma_o + \gamma_1 \hat{\beta}_i + \gamma_2 \hat{\sigma}^2_{r.i} \qquad i=1,2,\ldots,N$$

where the null composite hypothesis is $\gamma_o = 0$, $\gamma_1 > 0$ and $\gamma_2 = 0$, which assumes both the validity of equation (1) and of a return generating model which permits the substitution of mean realized returns for expected returns. $\hat{\beta}_i$ is the coefficient from the regression of $\tilde{R}_i$ on $\tilde{R}_m$, and $\hat{\sigma}^2_{r.i}$ is an estimate of the variance of the residuals of the regression. In this hypothesis it is predicated that R_m is generated from an _ex ante_ efficient portfolio. An alternative hypothesis is that $\gamma_1 > 0$ and $\gamma_2 > 0$, and the market portfolio is not _ex ante_ efficient. In the alternative hypothesis, individual asset variances are also relevant measures of risk, thus providing a potentially important role for $\hat{\sigma}^2_{r.i}$ as well as $\hat{\beta}_i$.[7]

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[6] For a complete discussion of the development of equation (2) see Fama and MacBeth.

[7] For a development of the alternative hypothesis see papers by Levy and by Mayshar.

Tests of the CAPM Based on a Value-Weighted Index of Stock and Bond Returns

To construct an overall market return index which would be expected to be more satisfactory for testing the CAPM than the usual stock index, it was necessary to obtain appropriate market indexes for the major classes of marketable assets and then to apply the relevant market weights.[8] We used the Standard & Poors Composite Index to cover all common stocks; the Salomon Brothers Total Performance bond index to cover all bonds other than U.S. Governments from 1969 through 1976, and the Moody's composite bond index from 1947 through 1968; and a U.S. Government bond index developed by John Bildersee[9] to cover long-term marketable U.S. Government issues. The weights applied in estimating the overall market return (R_m) for 1973 from the three constituent returns were 60% for corporate equities (with a return R_s), 30% for bonds other than U.S. Governments (R_b), and 10% for long-term marketable U.S. Government issues (R_g). These weights which varied from year to year were obtained from the annual Federal Reserve Board Flow of Funds data on the market value of stocks and bonds held by U.S. individuals and financial institutions. A potential limitation of the R_m index as an estimate of return on all stocks and bonds is the absence of a satisfactory index for returns on municipal bonds, which account for about 10% of the value of all stocks and bonds held by individuals and institutions. Municipal returns have been assumed to move in the same fashion as corporate bonds and the weight on corporate bonds has been increased accordingly.

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[8] Correct weights on various classes of securities are difficult to determine because of problems associated with the treatment of government debt, financial intermediation and non-marketable assets. As a result several different sets of weights were tested. Of course, all of our tests hypothesize that our composite index of stocks and bonds is *ex ante* efficient and close to the true market portfolio.

[9] J. Bildersee, "Some New Bond Indexes," *Journal of Business*, October 1975.

Table 1 presents return-risk cross-section relationships in which the average monthly differences $\overline{(R_i - R_f)}$ between returns on individual New York Stock Exchange (NYSE) common stock issues and on one-month Treasury bills (which are assumed to be the risk-free asset) are regressed on $\hat{\beta}_i$ and $\hat{\sigma}^2_{r.i}$ for five different 60 month periods from January 1952 through December 1976.[10] The returns for individual stocks were obtained from a data tape of the Rodney L. White Center for Financial Research covering all NYSE issues, but for each 120 month period only those issues for which data were available for every month were included in the regressions.[11] The beta coefficients computed for these stocks have been adjusted for "order bias" using procedures originally suggested by Vasicek.[12]

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[10] Returns are dividend-adjusted investment relatives. The regression analysis takes place in two stages. The first stage consists of computing estimates of beta and residual variance from a time series of monthly returns regressed on the composite index. In the second stage, in a cross-sectional analysis, the mean risk premium $\overline{(R_i - R_f)}$ is regressed on the first stage estimates of beta and residual variance.

[11] This requirement can introduce some <u>ex post</u> selection bias in exaggerating the expected positive relationship between return and risk. However, an analysis carried out in Friend, Westerfield and Granito suggests that the qualitative results are not likely to be affected. The 120 month period corresponds to that defined for the grouping procedure discussed in subsequent pages.

[12] O.A. Vasicek, "A Note on Using Cross-Sectional Information in Bayesian Estimation of Security Beta," <u>Journal of Finance</u>, December 1973. Also, see M.E. Blume, "Beta and Their Regression Tendencies," <u>Journal of Finance</u>, June 1975. The exact procedure is found on pages 790-91 of Blume, except that sample means replace the population means and the error in measuring beta for each asset is not assumed to be the same.

The regressions in Table 1 indicate that the null hypothesis, which assumes the CAPM is true and that realized returns are a satisfactory proxy for expected returns, is consistent with the estimates for only one period, 1952-1956, though even for this period it is inconsistent with the Sharpe-Lintner version of the model. In every other period the estimates are inconsistent with the CAPM since either $\hat{\gamma}_1 < 0$, or $\hat{\gamma}_1 > 0$ and $\hat{\gamma}_2 > 0$. The alternative hypothesis -- that variance is a meaningful measure of risk -- is consistent with the estimates in two periods, 1962-1966 and 1967-1971, where $\hat{\gamma}_1 > 0$ and $\hat{\gamma}_2 > 0$. The estimates in still another period, 1957-61 where again $\hat{\gamma}_2 > 0$, are also consistent with the interpretation of variance as a measure of risk.

Since the CAPM is always cast in terms of expectations of returns, $E(R_i)$ and $E(R_m)$, our interpretation of the parameter estimates of equation 1 rely upon the validity of substituting $\bar{R}_i$ and $\bar{R}_m$ for $E(R_i)$ and $E(R_m)$. For a more satisfactory hypothesis explaining observed risk differentials in the theoretical framework of the CAPM, a stochastic return generating model can be specified relating market returns to expected returns on the i'th asset: $R_i = E(R_i) + \beta_i \pi + \varepsilon_i$ where ε_i is orthogonal to π, and $E(\varepsilon_i) = E(\pi) = 0$, $i=1,2,\ldots,N$. With this formulation it can be shown that under the CAPM, $\gamma_1 = R_m - R_f$ (or $R_m - E(R_0)$) in every sample. Under the joint null hypothesis of the CAPM and the market model, the coefficient of $\gamma_1(\beta_i)$ is both statistically significant and has the right sign in one period, 1952-1956, but is statistically significant with the wrong sign in three other periods -- 1957-1961, 1962-1966 and 1972-1976.[13]

[13] The signs of the coefficients of β_i and $\sigma^2_{r.i}$ are expected to be positive when $R_m > R_f$ and at least for β_i, and probably also for $\sigma^2_{r.i}$, negative when $R_m < R_f$. (For 1972-76, however, it should be noted that R_m is not significantly larger than R_f). Discussion of this point appears later in the paper.

Thus, so far as these tests based on individual stocks are concerned, $\sigma^2_{r.i}$ seemsto be somewhat more useful than β_i in the explanation of observed risk differentials on common stocks over the post-World War II period, so that the sample data appear to indicate that variance is a meaningful measure of risk and the market portfolio proxy is not ex ante efficient. However, to reach more definitive conclusions on the relative efficacy of β_i and $\sigma^2_{r.i}$, it is desirable to refine our analysis in several respects. First, it is useful to re-estimate our return-risk relationships from grouped data rather than relying on our analysis based on individual assets alone. The reason is that grouping provides an independent check on the adequacy of the Vasicek adjustment which we made to take care of the substantial measurement errors in estimating β_i from observations on individual stocks, and no adjustment has been made for measurement error in estimating $\sigma_{r.i}$ from the data on individual stocks. Nonetheless, before proceeding to the tests based on grouped data, it should be noted that tests of capital asset pricing theory that rely only on grouped data, to the exclusion of tests based on individual assets, are not completely satisfactory, since it is the returns on individual assets which the theory is trying to explain, and some kinds of individual asset deviations from linearity may cancel out in the formation of portfolios.[14] Second, the statistical analysis in Table 1 assumes that the regression coefficients are constant or stationary over each 60 month period (but not from one period to the next) and it would be desirable to test, and if necessary allow, for possible non-stationarities in the values of these coefficients within a 60 month period.[15]

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[14]See papers by Levy and by Roll.

[15]For example, if the market model is assumed to be valid, it would be expected that $\gamma_1 = R_m - R_f$ (or $R_m - E(R_o)$).

Table 2 presents two sets of risk-return relationships, both based on grouped data.[16] The first set makes the same assumption of stationarity of regression coefficients as in Table 1, while the second allows for non-stationarities in these coefficients.[17] In both cases, the grouping procedure followed was to rank the individual stocks first by beta quintile on the basis of monthly data for the preceding 60 month period and then within each quintile by residual variance into 5 sub-groups, resulting in a total of 25 groups. Each equation was then estimated using subsequent beta and residual variance estimates for each group. As a result, only those issues for which returns data were available for 120 months were included in these regressions.

The grouped regressions in Table 2 assuming stationarity of the regression coefficients (S) indicate that $\sigma^2_{r.i}$ both adds significantly to the explanation of the risk differentials in common stocks and has the expected sign (under the alternative hypothesis that variance is a meaningful measure of asset risk) for three out of the five periods covered -- 1957-61, 1967-71

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[16]The values of β_i and $\sigma^2_{r.i}$ in the grouped regressions represent group means. They are weighted averages of the values for the individual assets in the group.

[17]For each month in a five year period the cross-section or individual common stock returns are regressed on the estimated betas and residual variances to obtain the regression coefficients in the relationship $\bar{R}_i = \hat{\gamma}_0 + \hat{\gamma}_1 \hat{\beta}_i + \hat{\gamma}_2 \hat{\sigma}^2_{r.i} + \hat{\varepsilon}_i$. This procedure results in a time series of observations in the estimates ($\hat{\gamma}_0$, $\hat{\gamma}_1$ and $\hat{\gamma}_2$) and averaging these 60 cross-sectional estimates for each of the regression coefficients provides an estimate of the risk-return trade-off. Standard errors of these averages are taken from the time series of $\hat{\gamma}_0$, $\hat{\gamma}_1$ and $\hat{\gamma}_2$, thus incorporating the variability of the risk-return trade-off. $\bar{R}^2$ in these regressions are an average of the monthly values. An F-test of the stationarity of $\hat{\gamma}_1$ for each of the five 60 month periods, estimated by dividing the variance of the 60 values of $\hat{\gamma}_1$ by the average of the square of the standard errors of the 60 values of $\hat{\gamma}_1$, points to non-stationarity at the .95 level of significance for all five periods.

and 1972-76. For the other periods the coefficient of $\sigma^2_{r.i}$ is not statistically significant. The coefficient of β_i is statistically significant with positive sign ($\hat{\gamma}_1>0$) in two periods but has a sign consistent with the CAPM and market model in only one period, 1952-56. It is statistically significant with the wrong sign in three other periods -- the same result obtained in the Table 1 regressions based on individual stocks. Thus, in this analysis based on grouped data, $\sigma^2_{r.i}$ seems to be statistically more useful than β_i as a measure of risk.

The grouped regressions allowing for non-stationarities in the regression coefficients (NS) are obviously characterized by much larger standard errors of these coefficients so that $\sigma^2_{r.i}$ is statistically significant and has the right sign in only one period, 1957-61, but β_i now is not significant with the right sign (assuming the market model) in any period. For the five periods as a whole, neither $\sigma^2_{r.i}$ nor β_i is impressive in explaining the observed risk differentials. The standard errors of the $\sigma^2_{r.i}$ and β_i coefficients in these regressions, it should be noted, test the significance of the departures from zero of the mean values of the coefficients of $\sigma^2_{r.i}$ and β_i over the indicated period of time and hence reflect the variance of the regression coefficients over time as well as the variance of the estimation error terms. The results of the NS regressions make no special assumption about the stochastic process generating returns and tend to understate the significance of the β_i (and possibly the $\sigma^2_{r.i}$) coefficients within the time period covered (e.g., for individual months within a five year or longer period of time) if the market model is assumed. The S regressions, in contrast, may tend to overstate significance. A somewhat more satisfactory analysis consisting of a detailed analysis of the risk-return relationship for individual months will be presented later in this paper.

Stocks and Bonds

It is possible for a more limited period to provide additional evidence on the relative explanatory power of beta and residual standard deviation[18] in determining returns on risky assets by using the available quarterly data on two samples of individual corporate bonds as well as common stocks. The first sample of 891 individual bonds was obtained from a data tape compiled by the Rodney L. White Center containing quarterly rates of return from the fourth quarter of 1968 through the third quarter of 1973 for every corporate bond listed on the NYSE, after removing a small number of bonds for which satisfactory price and interest data were not available for as many as 10 quarters. The same data tape was used to obtain for this period an equally-weighted quarterly index of market return on bonds (the RLW index) based on all issues covered by this tape. To compare the risk-return relationships for bonds with those for stocks, the quarterly rates of return for the same period were obtained for 867 NYSE common stocks from a second Center tape.[19] The second sample of 86 individual bonds covering the period from the first quarter of 1964 through the third quarter of 1968 consisted of a 10% sample of 891 bonds covered in the subsequent period, except that not more than one bond was included from a single corporation. The S&P Composite AAA Bond Price Index was used to obtain for this period a quarterly index of market return on bonds. Again, to compare the risk-return relationships for

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[18]Residual standard deviation was used in this earlier analysis (in lieu of the probably theoretically preferable residual variance) for compatability with published studies. However, the tests which were carried out indicated that the substitution of $\sigma^2_{r.i}$ for $\sigma_{r.i}$ did not materially affect the results.

[19]The common stocks were required to have 19 quarters of return data in each period from the 1st quarter 1964 to the 2nd quarter 1968 and from the 3rd quarter of 1968 to the 2nd quarter of 1973. Again this requirement can introduce some ex post selection bias which is not likely to affect the qualitative results.

bonds with those for stocks, the quarterly rates of return over this period were obtained from the relevant RLW data tape for 802 NYSE stocks having continuous return data from the 2nd quarter 1959 to the 3rd quarter of 1968. The appropriate overall quarterly market return indexes, both for 1964-68 and 1968-73, were then constructed following the same general procedures as for the 1952-76 monthly returns described earlier.

Table 3 presents risk-return relationships separately for stocks, for bonds, and for stocks and bonds combined, assuming stationarity of the regression coefficients, for each of the two approximately five-year periods, 1964-68 and 1968-73.[20] For the first of these periods, $\sigma_{r.i}$ is not only statistically significant (with the theoretically expected sign) for stocks, for bonds, and for stocks and bonds combined but in all three instances $\sigma_{r.i}$ is more significant than β_i. These estimates are consistent with the hypothesis that variance is a meaningful measure of risk for individual assets. For the second of these periods, the results are mixed, and are consistent with expectations only if the return generating function implies that asset returns are negatively related to risk when the market rate of return (R_m) is less than the risk-free rate (R_f). It should be noted that the explanatory power ($\bar{R}^2$) of these return-risk relationships is very much higher in the 1964-68 period than for 1968-73 when the average rate of return on the market was less than the average risk-free rate. The ability of the 1964-68 results to discriminate between the competing hypotheses may reflect a closer coincidence between <u>ex ante</u> and <u>ex post</u> returns (or the inadequacies of an expectational type of hypothesis or of the return generating hypothesis implied by the regressions fitted).

Similar results were obtained for the 1964-68 and 1968-73 periods when an identical analysis was carried out for returns on individual assets rather

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[20] $R_m > R_f$ in 1964-68 and $R_m < R_f$ in 1968-73.

than groups of assets. Nor were these results qualitatively changed when different corporate bond indexes were used (such as Moody's Composite bond index or Salomon Bros. Total Performance Bond Index), when bonds were reduced in weight to 20% (vs. 80% for equities) or increased to 50% (vs. 50% for equities), or when the logarithms of return relatives were substituted for the returns themselves.

Predictive Measures of Risk

So far in our analysis, we have followed customary practice and used contemporaneous rather than predictive values of betas and residual variances in testing CAPM theory; i.e., we have used current rather than prior period risk measures to explain current period returns. Fama and MacBeth (and on an earlier occasion Pettit and Westerfield) used predictive values on the grounds they were interested in testing the CAPM as a normative rather than merely as a positive theory. While our own primary interest in the CAPM (and apparently that of most of the earlier academic studies) is in its validity as a positive theory, there is a possible statistical advantage in utilizing predictive risk measures since their use in testing the CAPM minimizes the danger of introducing any spurious correlation between the mean and standard deviation of returns as a result of skewness in the distribution of returns.[21] Such skewness would be expected to be substantially lower in monthly than in quarterly or annual data but might still cause difficulties. Thus, in view of their desirable statistical qualities and to permit comparison with the Fama-MacBeth results, we have introduced predictive values of β_i and $\sigma^2_{r.i}$ in the return-risk relationships

[21] Roll [Part II] maintains that the Fama-MacBeth reliance on grouped data effectively eliminates the skewness problem, but this assertion seems to be without any basis, since they like ourselves estimate β_i and $\sigma^2_{r.i}$ for the i'th group as averages of the values for individual assets in that group.

presented in Table 4 and the following tables,[22] though the significance of the regression coefficients as a test of positive CAPM theory probably tends to be understated in these analyses.[23]

Table 4 and 5 present, for individual common stocks and groups of stocks respectively, return-risk relationships which both allow for non-stationarities in the regression coefficients and incorporate predictive rather than contemporaneous risk measures. With these changes in the

[22]The procedures followed for estimating the predictive values of the risk measures are similar to those described in Fama and MacBeth. Groups: (a) Form portfolios on the basis of estimates of betas and re-residual variances for individual securities in an initial five year period and then estimates of portfolio betas and residual variance are computed by reestimating betas and residual variances for individual securities in a subsequent five year period. The portfolio betas and residual variances are equal weighted averages taken from the individual securities and are formed from ranking securities on estimated beta (from high to low) and dividing them into five groups and within each group ranking the securities on estimated residual variance (from high to low) and dividing the group into five subgroups. Thus 25 groups of securities are obtained. (b) Monthly returns are calculated for these groups for the twelve months subsequent to those used in estimating the betas and residual variances. (c) In each month these returns are regressed on the group betas and residual variances. The process is repeated in each year. The average of the monthly OLS estimates γ_1 (β_i) and $\gamma_2(\sigma^2_{r.i})$ from the cross-sectional regression are the estimates of the risk-return trade-off. Note 132 months of return data are required for a set of regression estimations. See table 5.
Individual securities: The procedures for the cross-sectional estimates involving predictive measures of risk for individual securities closely resemble those just described, except (a) Estimates of betas and residual variances are computed by OLS for all (NYSE) securities with continuous data in an initial five year period, (b) A'Vasicek' weighting procedure is applied to each estimated beta, (c) Monthly returns are calculated for the next twelve month period for each security and the cross-sectional regression model is estimated. The process is repeated in each year. Now 72 months of return data are required for a set of regression estimates. See table 4.

[23]This is true if the measurement error in $\hat{\beta}_{i(t-1)}$ contains random estimation error and "true" non-stationarity error (see Blume, 1975).

analysis, which may understate the significance of the relevant regression coefficients, neither β_i nor $\sigma^2_{r.i}$ has a statistically significant effect on returns for the 1952-76 period as a whole. Within five year sub-periods, only β_i is ever statistically significant but then more often with the wrong than with the right sign (assuming the market model).

In view of the fact that virtually all tests of the CAPM as well as those in this paper can be considered joint tests of the CAPM theory and of a return generating process, we have in Table 6 broken down the predictive risk results for 1952-76 as a whole both for individual stocks and groups of stocks into two periods -- one including all months in which the market rate of return was higher than the risk-free rate ($R_m > R_f$) and the other including all months in which the reverse was true ($R_m < R_f$).[24] Presumably in the first of these periods (all months where $R_m > R_f$) there would be a closer coincidence between <u>ex ante</u> and <u>ex post</u> returns.

The results of this breakdown of the entire period 1952-76 into two sets of months, depending on the relationship between R_m and R_f, are quite striking. Unlike the insignificant results for the period as a whole, in the months when $R_m > R_f$, β_i and to a lesser extent $\sigma^2_{r.i}$ make a significant contribution to the explanation of returns in the individual regressions, and the signs of their regression coefficients are consistent with the hypothesis that both are meaningful measures of risk. In the months when $R_m < R_f$, and it is necessary to incorporate some plausible return generating function to make the transition from an <u>ex ante</u> or expected relationship between returns and risk to an <u>ex post</u> or observed relationship, β_i is significantly and -- as would be expected from the market model customarily used -- negatively related to

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[24]The standard errors are computed from the resulting time series of coefficients.

returns in the individual regressions, while $\sigma^2_{r.i}$ is also significantly and negatively related to returns. It is not clear what return generating function would best reflect the incorporation of residual variance or some similar measure in the explanation of the difference between observed and anticipated returns, but it seems plausible to expect a positive correlation between observed returns and residual variance (or standard deviation) when $R_m > R_f$ and a negative correlation when $R_m < R_f$.[25] However, while both β_i and $\sigma^2_{r.i}$ add significantly to the explanation of returns in the individual asset regressions when periods of relatively up and down markets are separated, only β_i is significant in the group regressions (though the t-values of the $\sigma^2_{r.i}$ coefficient are one or above and in the expected direction).

Another approach to examining the CAPM using predictive risk measures, which avoids some of the difficulties with the tests of significance employed in earlier procedures, is provided by a comparison of the number of months for which the regression coefficients of β_i and $\sigma^2_{r.i}$ (in the monthly regressions $R_{it} = \gamma_o + \gamma_1\beta_{it} + \gamma_2\sigma^2_{r.i}$) have the theoretically expected signs over the longest period covered by our analysis, as well as separately for months in which the market return is higher than the risk-free rate and months in which it is lower. The sample includes all individual issues of NYSE common stock with continuous dividend-adjusted monthly return data for five years prior to the first month used in the cross-section OLS estimation. The estimates of β_i

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[25]It is easy to show that these results would be obtained if, as has been suggested in Elton and Gruber, investors generally consider the correlation coefficient between individual stocks and the market as a whole to be a constant. Under such circumstances, only standard deviation would determine relative stock returns. While formally the CAPM might still be regarded as valid, the relevant measure of risk in differentiating among returns would be standard deviation or variance rather than beta. With these assumptions, it is not possible from the data on stock returns to discriminate between standard deviation as a proxy for beta or as an independent measure of risk. However, the results of the return-risk regressions for stocks and bonds combined, which included two classes of assets with clearly different correlations with the market as a whole, suggest that variance or standard deviation reflects an independent element of risk (Table 3).

and $\sigma^2_{r.i}$ are computed in each month and the sample is revised each year (and thus is static for six years). The results of such a test presented in Table 7 show that in the regressions based on individual stock observations there were 227 months in which the signs of the β_i coefficients were consistent with expectations and only 73 months in which they were inconsistent, while the signs of the $\sigma^2_{r.i}$ coefficients were consistent with expectations in 181 months and inconsistent in 118 months. In the regressions based on grouped observations (using procedures described previously), the corresponding numbers were 207 and 93 for the β_i coefficients and 158 and 142 for the $\sigma^2_{r.i}$ coefficients. Thus, in these non-parametric as well as the earlier tests using predictive risk measures, β_i seems more useful than $\sigma^2_{r.i}$ in explaining returns, but $\sigma^2_{r.i}$ again makes a contribution which is statistically significant for the regressions based on individual stocks. Moreover, it might be noted that even in the non-parametric tests using predictive risk measures based on grouped observations, $\sigma^2_{r.i}$ contributes significantly to the explanation of returns in periods when $R_m > R_f$ though β_i is clearly more important.

Our Findings on Beta and Residual Sigma Compared with Earlier Tests

The previous results for individual assets and grouped data, covering stocks and bonds separately and jointly, provide fairly strong evidence against the central role ascribed to the beta coefficient by CAPM theory to the exclusion of residual variance or standard deviation. The analyses using contemporaneous measures of risk suggested that $\sigma^2_{r.i}$ was fully as significant as β_i in explaining returns on risky assets. The analyses using predictive measures of risk which combined periods with $R_m > R_f$ and $R_m < R_f$ suggested that neither $\sigma^2_{r.i}$ nor β_i made a useful contribution to the explanations of returns. However, in the analyses using predictive measure of risk, when periods with $R_m > R_f$ were separated from those with $R_m < R_f$, a type of analysis not carried out in previous studies, β_i appeared more significant than $\sigma^2_{r.i}$. Nevertheless, $\sigma^2_{r.i}$ still seemed to contribute to the explanation of returns.

It is still necessary, therefore, to reconcile these results with those obtained in two recent studies by Fama and MacBeth and by Foster[26] both of which essentially concluded, on the basis of an analysis of grouped data, that while beta systematically affected average returns on risky assets there was no evidence that residual risk had any effect. Neither of these studies distinguished between periods with $R_m > R_f$ and those with $R_m < R_f$. Although there are other methodological differences between our analysis and each of these earlier studies, the main common difference between our analysis and both of the others is their reliance on a market proxy consisting of stocks only versus our use of a value-weighted index of stocks and bonds.

Obviously, the first question to be answered in explaining this difference in findings is whether it would disappear if the market proxy in our analysis is confined to stocks alone rather than to the more theoretically satisfactory stocks and bonds. Table 8, therefore, duplicates the return-risk

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[26] G. Foster, "Asset Pricing Models: Further Tests," Journal of Financial and Quantitative Analysis, March 1978.

relationships for groups of common stocks corresponding to those presented in Table 1 and 5, except that now the market proxy is a value-weighted stock index (the Standard and Poor's Composite Index) rather than the value-weighted index of stocks and bonds combined described in the preceding section. The new stationary regression estimates incorporating contemporaneous measures of risk still point to the statistical superiority of $\sigma^2_{r.i}$ over β_i in explaining rates of return on risky assets with the expected sign of the regression coefficient, but the new nonstationary regression incorporating predictive measures of risk now points to a slight superiority of β_i though neither coefficient is statistically significant. As a consequence, it appears that the more comprehensive market proxy explains only part of the difference between our results and those obtained by Fama-MacBeth and Foster.[27]

Another potentially important explanation for the difference between our and the Fama-MacBeth results are that they formed groups based on betas in previous periods while we formed groups (as did Foster) based on both betas and residual standard deviations in previous periods. Thus the Fama-MacBeth grouping procedures might tend to understate the influence of residual standard deviation as compared to the influence of beta since they permit a much greater variation in group betas than in group residual standard deviations. Even our approach discriminates slightly against $\sigma_{r.i}$ since we first rank by β_i and then by $\sigma_{r.i}$ within beta classes, but this effect should be relatively minor.

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[27]The significance of the estimates of $\gamma_1(\beta_i)$ and $\gamma_2(\sigma^2_{r.i})$, are not, in general, sensitive to the composition of the market portfolio which implies to us (1) that the probability of the error of rejecting the CAPM when it is true is about the same using any plausible market index containing marketable common stocks and bonds, and (2) that if the differences in the mean-variance attributes of these indexes is approximately the same as the difference between these indexes and the "true" market portfolio, the probability of making this type of error would appear to be low. However, since we do not know the exact composition of the true market portfolio, an unequivocal statement about the probability of falsely rejecting the CAPM is not possible on the basis of these regression data.

To assess the effect of this difference in grouping procedures on the Fama-MacBeth and our results, we recomputed all the regressions in Table 2 and the 1968-73 stock regression in Table 3 using their procedure. For five of the six periods tested, our more satisfactory grouping procedure improved somewhat the relative importance of residual standard deviation vs. beta. In two instances the coefficient of residual standard deviation was transformed from an insignificant to a statistically significant value with the correct sign. This was true for the 1967-71 regression assuming stationarity (S) of the regression coefficients and the 1968-73 regression assuming non-stationarity (NS).[28] In the one period--1962-66--when residual standard deviation was statistically significant with an incorrect sign using Fama-MacBeth grouping (both in the S and NS regressions), statistical significance completely disappeared with our grouping, while for beta the situation was significantly worsened. Thus, another part of the difference between the Fama-MacBeth and our assessments of the relative importance of residual standard deviation and beta as measures of risk appears to be attributable to the difference in grouping procedures, with ours statistically preferable.

A further difference between the Fama-MacBeth and our procedures is our use of a value-weighted market index (to estimate systematic and residual risk) while they used an equally-weighted index (Fisher's Arithmetic Index).

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[28]Thus the 1967-71 regression with the Fama-MacBeth grouping procedure becomes

$$\bar{R}_i = \underset{(2.7)}{-.0057} - \underset{(0.0)}{.0001}\,\beta_i + \underset{(0.0)}{.0069}\,\sigma_{r.i}$$

which may be compared with the corresponding equation in Table 2. A similar comparison of the results obtained for 1968-73 from the two different grouping procedures is presented in Appendix I of Friend, Westerfield and Granito.

Though our approach is theoretically preferable, we have computed the non-stationary regressions of returns of groups of stocks on predictive values of β_i and $\sigma_{r.i}$ using first the S&P 500 value-weighted stock index and second the Fisher's equally-weighted stock index over the period January 1952 through December 1974, the last month for which the Fisher index was readily available. We have used $\sigma_{r.i}$ for this purpose instead of $\sigma^2_{r.i}$ to duplicate as closely as possible the Fama-MacBeth analysis though the period covered is still somewhat different. These regressions were estimated using both their and our grouping procedures, yielding four regressions differing either in the market index used or in the grouping procedure followed. None of these regressions gives statistically significant results. The difference between our finding using the Fama-MacBeth grouping procedure and market index and their result presumably reflects the difference in the periods covered.

Foster follows the same general procedures as Fama-MacBeth with three notable exceptions. He uses more correct grouping and weighting procedures which are similar to ours (in that part of the analysis where we confine the market portfolio to stocks only) but unlike the Fama-MacBeth and our analysis he uses an incorrect measure of the residual or unsystematic risk in regressing portfolio returns on portfolio betas and portfolio residual standard deviations. While a portfolio beta is an average of the betas for individual assets contained in that portfolio, a portfolio residual standard deviation is not an average of, and may have little relationship to, the residual standard deviations of the individual assets (and indeed in a large portfolio is more closely related to the betas of the individual assets). In testing the variables affecting capital asset pricing of individual assets through the use of portfolio or group data to minimize measurement errors, it is the average of betas and the average of residual risks of the individual assets rather than the corresponding portfolio measures which are relevant (though for betas there is no difference). Unfortunately, Foster uses the portfolio residual standard deviation rather than the average of residual standard deviations for the stocks in that portfolio. To the extent that any weight can be given the Foster findings, it may be noted that his regression of average returns on betas and residual standard deviations for the 1931-74 period, unlike the Fama-MacBeth result for 1935-68, finds residual risk more significant than beta and both positively related to average return realizations, with neither significant at the .05 level. However, he dismisses this result on grounds of multicollinearity. To empirically confirm his theoretical preconception that beta is the only meaningful measure of risk, he then uses a statistical procedure whose justification depends on the validity of his theoretical preconception. In any case, his use of an incorrect measure of residual risk in his empirical tests would appear to vitiate his findings.

From this discussion of the earlier literature covering the two extensive studies which obtain results that appear to differ from ours, it seems clear that our analysis has a firmer empirical foundation. Indeed, it seems fair to conclude from an examination of the relevant literature that there is no satisfactory regression analysis of the relationship between average returns and both betas and residual standard deviations which does not suggest that the latter may not be fully as important a measure of risk as the former, though much of this analysis raises questions about the importance of both measures.

Some Caveats

One point we have stressed is that our tests use ex post data and, thus, depend upon our null joint hypothesis of the validity of the CAPM and the market model of the return generating process. If the market model is not valid (and there is some evidence to support this view in Pettit and Westerfield), the presence of market-wide factors affecting all stocks and bonds may not be completely accounted for by the ex post realization of our proxy for the market portfolio, i.e., there may be additional sources of covariance among individual risky securities. It is conceivable that the additional sources of covariance will produce biased estimates of $\gamma_1(\beta_i)$ and that the residual variance values may be acting as proxies for the additional sources of covariance. Thus a multi-factor return generating function which could satisfactorily explain our results within the framework of the CAPM may exist even though no such function has yet been specified. However in the long run, if our proxy for the market portfolio is ex ante efficient, the effects of additional covariance terms would be expected to be small. Moreover, a predictable sign pattern to $\hat{\gamma}_2(\sigma^2_{r.i})$ might be detectable and possibly significant for samples of individual assets--but not for groups of assets, as it is in some of our results.[30]

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[30]If the measurement error of $\hat{\beta}_i$ is positively correlated to $\hat{\sigma}^2_{r.i}$, the coefficient on $\hat{\sigma}^2_{r.i}$ may be upward biased. The sign of $\hat{\gamma}_2(\hat{\sigma}^2_{r.i})$ would also be related to the realization of the return on the market portfolio as is the case for $\hat{\gamma}_1(\hat{\beta}_i)$ if the market model is valid. This type of effect should be greatly diminished for groups of individual securities.

A possible reason why the sample variance of residuals may be related to return is assymmetry in the population residual return generating process.[31] The sign pattern of $\hat{\gamma}_2(\sigma_{r.i})$ and $R_m - R_f$ seems to make this line of reasoning unlikely since it is not consistent with the assymmetry argument. In addition, the use of monthly data and predictive betas should eliminate the problem of skewness as discussed by Miller and Scholes.[32]

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[31] If residual returns are drawn from a lognormal distribution, the sample means and variances will tend to be positively related (regardless of the observed market risk premium, $R_m - R_f$). Miller and Scholes have demonstrated that the significance of $\gamma_2(\sigma^2_{r.i})$ in explaining returns on individual stocks can arise from the skewness of returns associated with lognormality.

[32] It should also be noted that the beta and residual variance effects were not markedly changed when a skewness variable was added to β_i and $\sigma^2_{r.i}$ for explaining returns in earlier regressions employing contemporaneous measures of risk, where some spurious correlation between the mean and standard deviation of returns may possibly be introduced as a result of remaining skewness in the distribution of returns even with monthly data. Thus, for the 1957-61 and 1967-71 periods the return-risk regressions for individual stocks adjusted for order bias as in Table 1, now reestimated to include a skewness effect, become

$$R_i = .0163 - \underset{(-5.2)}{.0046}\,\beta_i + \underset{(5.6)}{.6538}\,\sigma^2_{r.i} - \underset{(-0.9)}{.1225}\,Sk_i \qquad \bar{R}^2 = .07 \quad \text{and}$$

$$R_i = .0071 + \underset{(1.0)}{.0009}\,\beta_i + \underset{(1.9)}{.1965}\,\sigma^2_{r.i} + \underset{(3.1)}{.6642}\,Sk_i \qquad \bar{R}^2 = .06$$

where:

$$Sk_i = \sum_{t=1}^{T} (R_{it} - \bar{R}_{it})^3 \qquad \text{for } i = 1, 2, \ldots, N;\ t = 1, 2, \ldots, T$$

Another point, recently raised again by Roll (March 1977) in a more general context, concerns the possibility that we may have rejected the CAPM as a result of an inadequate ex ante proxy for the market portfolio perhaps confounded in our ex post tests by some peculiarity in the return generating process relating ex post to ex ante returns. We are inclined to doubt that the inadequacy of our market proxy is responsible for our findings since we have used different market-value weighted indexes of both stocks and bonds as well as of stocks alone and the results (notably the significant role of residual risk) do not appear sensitive to the index used so long as a market-value rather than an arbitrary weighting scheme is used. Nor have we (nor anyone else to our knowledge) been able to construct a plausible market portfolio which would validate the CAPM. As for the problems associated with the return generating process, the limited tests we have been able to carry out so far on the basis of ex ante data referred to in the following section seem to confirm the results of the more extensive analysis based on ex post data. In any case, it seems fair to conclude that to the extent the CAPM is testable, much of the empirical results appear to be inconsistent with the theory.

An additional reason why residual risk might be related to return is the inappropriateness of the assumption of homogenous expectations required for the derivation of the CAPM in its customary form. It can be shown that the more plausible assumption of heterogenous expectations (specifically as to expected returns on risky assets) can lead to a modified CAPM which relates returns to residual risks as well as to beta coefficients.[33] However, in the long-run the residual risk effects might be expected to be small and the sign pattern of $\gamma_2(\sigma_{r.i})$ and $R_m - R_f$ does not appear to be consistent with the heterogeneity argument.

A final caveat to our regression analysis is that it is conceivable that multicollinearity between the two risk measures may result in attributing to $\sigma^2_{r.i}$ part of the β_i effect. With sufficient multicollinearity it might be impossible to disentangle the effects of the two risk measures so that a decision as to the relative importance of the two measures would have to depend either on theoretical preconception or on other types of evidence. However, we have carried out a two stage analysis to determine whether the residuals (u_i) in the (OLS) regression of individual security returns on betas alone (i.e., $R_i = \gamma_o + \gamma \hat{\beta}_i + u_i$) are significantly related to total variance ($u_i = \theta_u + \theta_1 \hat{\sigma}^2_i + \delta_i$) and found that u_i is significantly related to σ^2_i, indicating that multicollinearity is not the answer to the apparent relevance of $\sigma^2_{r.i.}$[34]

[33] Joseph T. Williams, "Capital Asset Prices with Heterogeneous Beliefs," Journal of Financial Economics, November 1977.

[34] Thus, when non-stationary Vasicek-adjusted regressions for individual common stocks incorporating predictive risk measures are estimated for those months during January 1952 - December 1976 when $R_m > R_f$, the second-stage regression is $u_i = -.0024 + \underset{(2.7)}{.3262}\ \sigma^2_i$, and for months when $R_m < R_f$ the corresponding regression is $u_i = .0029 - \underset{(3.9)}{.4572}\ \sigma^2_i$. The sampling procedures are described earlier.

[35] Blume and Friend [1975].

Other Evidence on Beta vs. Residual Sigma

In addition to the regression tests discussed earlier in this paper, there are three other independent types of evidence, generally neglected in the CAPM literature, bearing on the relative importance of beta and residual risk in explaining returns (and prices) of risky assets.

The first of these, based on an analysis of the stock portfolios as well as of the major classes of assets and liabilities held by different individuals,[35] found that a surprisingly large proportion of portfolios and assets were highly undiversified. Thus, in 1971 approximately 60% of U.S. households owning stock had a level of diversification in their stock portfolios equal to or less than the level achieved in an equally weighted portfolio of two issues, and such households accounted for 46% of all stock owned directly by individuals; only 5% of the stockholders owning 14% of all stock had a level of diversification above that realized in an equally weighted portfolio of seven issues.[36] It was concluded from an examination of the other alternatives that the two most plausible explanations of these findings are either: first, that investors hold heterogeneous expectations as to expected return and risk and the short sales mechanism is imperfect; or, second, that they do not properly aggregate risks of individual assets to measure the risk of an entire portfolio. Both of these explanations conflict with important assumptions typically made in capital asset theory, but the second is obviously more fundamental and more relevant to the issue under examination in this paper since it raises questions about the justification for sole reliance on beta or covariance with the market return rather than on variance (or standard deviation) of the asset's own returns, or on a combination of beta and variance or residual variance, as measures of the market's appraisal of asset risk.

A second independent source of information on the relative importance of the different measures of risk is a survey of over 1000 individual stockholders in the fall of 1975 which found that when the 82% of stockholding families which customarily evaluated the degree of risk involved in purchasing stock were asked what measures of risk they used, 45% stated they used earnings volatility, 30% price volatility and 17% betas.[37] The answers to other questions also suggested that the preponderance of investors including those who were rich had very little conception of the relationship between

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[36]Similar results are obtained if mutual fund shares are excluded from shareholdings.

[37]Blume and Friend, The Changing Role of the Individual Investor, John Wiley and Sons, 1978. Weighting the replies by the value of a family's stock portfolio does not change these results substantially.

an asset's covariance with the market and its contribution to the riskiness of a portfolio. Thus this second study even more than the first raises questions about the centrality attributed to beta as a measure of asset risk in current capital asset pricing theory.

The third additional source of evidence comes from an analysis in Friend, Westerfield and Granito which substitutes ex ante (expected) for ex post (realized) measures of return in testing capital asset pricing theory. The ex ante data which were used were obtained for a sample of 49 to 66 common stocks from a sample of 21 to 33 financial institutions for each of three periods in 1974, 1976 and 1977. Such data permit a more direct test of theory explaining the relationship between expected or required rates of returns and risk than is possible when only actual rates of return are available since it is no longer necessary to predicate the nature of the relationship between expected and actual rates of returns. In this test, for each of the years covered the mean expected return for each stock (i.e., the mean of the institutional expectations) was regressed on its beta coefficient and residual standard deviation (with β_i and $\sigma_{r.i}$ obtained from the ex post distribution of returns). The findings of this analysis while not at all strong support the view that the residual standard deviation of return and related variance measures play fully as important a role in the pricing of risky assets as the beta coefficient.[38] It should be noted that as a result of the use of ex ante measures of return this analysis largely avoids the danger of

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[38]The only other available estimates of ex ante returns based on expectations of institutional investors, made by the Wells Fargo Bank (e.g., see the Security Market Line, February 1, 1978 distributed by their Institutional Counsel Service), cover a much larger sample of companies but reflect the expectations of only one investor. The relative impact of beta and residual standard deviation on expected stock returns as estimated by Wells Fargo has not been examined separately, but it is interesting to note that--as in the independent analysis of ex ante data referred to in the text--the zero - beta return implied by the intercept of the market line has consistently been very substantially above the risk-free rate. This is much more marked than is true for the ex post data used in this paper, and is even more inconsistent with the Sharpe-Lintner version of the CAPM.

introducing any spurious correlation between the mean (now based on ex ante data) and standard deviation of returns (based on ex post data) as a result of skewness in the distribution of ex post returns.

Finally, reference should be made to the preliminary results of another analysis of the same ex ante data provided by a sample of institutional investors. These data were used to estimate the composition of ex ante efficient portfolios, with a reasonable range of values assumed for the risk-free rate and the variance of the market portfolio, using the average values of the expectational inputs. This analysis covering a number of different periods shows that with short sales allowed the customary capital asset pricing model implies that close to half of the large numbers of N.Y.S.E. stock covered would be held short by institutional investors as a whole, while with no short sales only a handful of stocks would be held in the over-all efficient portfolios. These are of course highly questionnable results for an equilibrium theory but are quite characteristic of the results usually obtained when doing portfolio optimization on ex ante data from individual investors. In an earlier unpublished paper, H. Levy obtained similar results from ex post data for all investors.

Conclusion

The regression analysis in this paper, earlier regression analysis after appropriate conceptual adjustments, and the other relevant available evidence all indicate that the beta coefficient may be no more significant and may be less significant than residual and total variance (or standard deviation) in explaining returns on risky assets. We conclude that our more comprehensive market proxy like the narrower proxies tested heretofore is ex ante inefficient and that to the extent it is possible with the available data to test capital asset pricing theory, this finding is inconsistent not only with the well-known Sharpe-Lintner version of the CAPM but with any one-factor version of that model and raises sufficient questions about multiple factor versions so that the burden of proof would seem to rest on the proponents of such models. Since the CAPM model, especially in its one factor form, has probably motivated more theoretical and empirical literature in finance than any other single subject over the past decade, and is now used extensively for both business and public policy purposes, it is difficult to understand the widespread application of this theory[39] when one considers its relatively weak empirical basis.

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[39] How widespread the acceptance of the CAPM has been is suggested by the Fama and MacBeth statement on p. 616: "For those accustomed to the portfolio viewpoint, this alternative model [incorporating residual risk as well as beta] may seem so naive that it should be classified as a straw man."

The obvious question that remains to be answered is the theoretical justification for investors demanding an additional premium to assume residual risk that CAPM tells us can be diversified away, at least in relatively large portfolios. There are several possible answers, but perhaps the most theoretically plausible is that the implicit CAPM assumption of zero transaction and information costs may not be acceptable even as a first approximation. If transaction and information costs are sufficiently significant, investors may in general concentrate on a relatively small subset of marketable assets so that residual risk measures may be as important as, or more important than, systematic risk (beta) in explaining individual asset returns. Thus Levy has constructed an equilibrium theory which posits that because of transaction and information costs most investors are constrained to hold highly undiversified portfolios, resulting in a capital asset pricing relation which ascribes major importance to the variance measure of risk and minimizes the importance of beta.[40]

A related question which can be (and has been) raised, if residual as well as systematic risk is positively correlated with return, is why some large institutions do not invest disproportionately in securities with large residual risk, since such a portfolio would be expected to outperform the market on the average, if performance is measured by portfolio return and risk. While we do not know that some institutional investors have not followed such a policy, there is no evidence to indicate that they have.

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[40]Also see Mayshar. The basic idea is that transaction costs tend to produce concentration in the portfolio composition of investors and imply some role for an asset's variance in predicting expected returns. The presence of transaction costs may in turn increase investor's holding period which would presumably be infinitely small in the absence of transaction costs.

However, though it is possible that large investors might profit from such an investment policy, this is not certain since even for these investors, transaction and information costs are not necessarily inconsequential, and both the expected gain (even before transaction and information costs) and the probability of achieving a gain in any particular period may be small.

To conclude with a caveat, while it is may be necessary to introduce residual risk into an acceptable theoretical explanation of capital asset pricing, as an empirical matter even the joint use of systematic and residual risk leaves much to be desired in the explanation of returns on risky assets.

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Table 1

Return-Risk Regressions for Individual Common Stocks[1/]
Adjusted for Order Bias, January 1952-December 1976

Period	Estimates of Regression Coefficients			$\bar{R}^2$	$\overline{R_m - R_f}$
	γ_o	γ_1	γ_2		
1952-56	.006	.0058 (8.2)	.0694 (0.8)	.08	.0085
1957-61	.0164	-.0047 (-5.3)	.6645 (6.6)	.06	.0056
1962-66	.0001	.0024 (2.6)	.8964 (11.6)	.19	-.0021
1967-71	.0063	.0014 (1.5)	2638 (3.0)	.03	.0021
1972-76	.0129	-.0029 (-3.4)	.0173 (0.3)	.01	.0005 [2/]

[1/] These are cross-section regressions of the general form

$\bar{R}_i = \gamma_o + \gamma_i \hat{\beta}_i + \gamma_2 \hat{\sigma}^2_{ri}$. The values of $\hat{\beta}_i$ and $\hat{\sigma}^2_{r.i}$ and obtained from a regression of R_i on R_m, where R_i and R_m are the monthly dividend-adjusted returns on an individual stock and on a composite value-weighted market index of stocks and bonds over a 60 month period. The t-test statistics are indicated by ( ). The number of observations is 808 for 1952-56, 832 for 1956-61, 791 for 1962-66, 732 for 1967-71, and 843 for 1972-76.

[2/] Not significantly different from zero at .05 probability level.

Table 2

Return - Risk Regressions for Common Stocks
Grouped to Minimize Measurement Errors,[1] January 1952 - December 1976

Period	Type of Reg.	Estimates of Regression Coefficients			$\bar{R}^2$	$\overline{R_m - R_f}$
		γ_0	γ_1	γ_2		
1952-56	S	.0059 (6.3)	.0050 (6.5)	-.0413 (-0.3)	.64	.0085
	NS	.0059 (2.8)	.0050 (1.4)	-.0413 (-0.0)		.0085
1957-61	S	.0196 (10.3)	-.0084 (-5.7)	.7814 (3.2)	.56	.0056
	NS	.0196 (5.3)	-.0084 (-2.0)	.7814 (1.8)		.0056
1962-66	S	-.0059 (-3.3)	.0064 (3.7)	.2591 (1.1)	.31	-.0021
	NS	-.0059 (-1.4)	.0064 (1.3)	.2591 (0.4)		-.0021
1967-71	S	.0054 (2.6)	-.0022 (-0.9)	.5265 (1.8)	.11	.0021
	NS	.0054 (1.0)	-.0022 (-0.3)	.5265 (0.9)		.0021
1972-76	S	.0078 (2.9)	-.0046 (-2.02)	.3638 (2.3)	.12	.0005
	NS	.0078 (0.9)	-.0046 (-0.5)	.3638 (0.8)		.0005

[1]The regression forms and symbols are identical to those described in Table 1. The regressions designated by S assume stationarity of the regression coefficients over the period indicated while those designated by NS allow for non-stationarity so that while the regression coefficients are the same, the t- statistics are not. Observations in this table are based upon the means for 25 groups of individual common stocks, grouped jointly by $\hat{\beta}_i$ and $\hat{\sigma}^2_{r.i}$, as estimated from data on returns for the preceding 60 months.

Table 3

Return - Risk Regressions for Common Stocks and Corporate Bonds Grouped to Minimize Measurement Errors, 1st Quarter 1964 - 2nd Quarter 1973

Period	Type of Asset	Estimates of Regression Coefficients			$\bar{R}^2$	$\overline{R_m - R_f}$
		γ_1	γ_2	γ_3		
$Q_1 64 - Q_3 68$	Stocks	-.011 (-3.4)	.004 (1.7)	.456 (15.7)	.77	0.0102
	Bonds	-.005 (-1.9)	.014 (1.8)	.204 (2.0)	.66	0.0102
	Stocks & Bonds	-.011 (-5.0)	.004 (2.0)	.453 (16.5)	.83	0.0102
$Q_4 68 - Q_2 73$	Stocks	.037 (8.4)	-.004 (-0.7)	-.284 (-3.6)	.64	-0.0038
	Bonds	.016 (8.9)	-.001 (-0.2)	.011 (0.3)	0	-0.0038
	Stocks & Bonds	.024 (24.0)	-.010 (-3.4)	-.099 (-2.3)	.71	-0.0038

[1]The regression forms and symbols are identical to those described in Table 1. Observations are based upon the means of groups of individual assets: 144 groups for stocks in 1964-68, 16 for bonds, and 160 for stocks and bonds combined; 50 groups for stocks in 1968-73, 50 for bonds and 100 for stocks and bonds combined. In obtaining these groups, individual assets were grouped jointly by $\hat{\beta}_i$ and $\hat{\sigma}_{r.i}$ as estimated for stocks from data on quarterly returns for the preceding five year period, and for bonds from regressions of β_i and $\sigma_{r.i}$ on the bond's Standard and Poor's quality rating, its maturity and its coupon. See Friend, Westerfield and Granito for further details.

Table 4

Non-Stationary Return-Risk Regressions for Individual Common Stocks[1/]

Adjusted for Order Bias and Incorporating Predictive Risk Measures

January 1952-December 1976

Period	Estimates of Regression Coefficients			$\bar{R}^2$	$\overline{R_m - R_f}$
	γ_o	γ_1	γ_2		
1952-56	.0074	.0040 (1.9)	-.0722 (-.3)	.003	.0085
1957-61	.0201	-.0057 (-2.1)	-.1652 (-.5)	.004	.0056
1962-66	.0030	.0030 (.9)	.1341 (.5)	.004	-.0021
1967-71	.0075	.0034 (.9)	-.2171 (-.8)	.005	.0021
1972-76	.0120	-.0040 (-1.1)	.2774 (.9)	.004	.0055
1952-76	.0100	.00014 (.1)	-.0086 (-.1)	.004	.0029

[1/] The regression forms and symbols are identical to those described on Table 1. Unlike the contemporaneous values of $\hat{\beta}_i$ and $\hat{\sigma}^2_{ri}$ used in Tables 1-3, the regressions in this table are based on predictive values, i.e., they use prior period $\hat{\beta}_i$ and $\hat{\sigma}^2_{ri}$ to explain current period returns. The number of observations in these regressions is 732 for 1952-56, 804 for 1957-61, 792 for 1962-66, 697 for 1967-71, 753 for 1972-76.

Table 5

Non-Stationary Return-Risk Regressions for Common Stocks[1/]
Grouped to Minimize Measurement Errors and Incorporating Predictive Risk Measures

January 1952 - December 1976

Period	Estimates of Regression Coefficients			$\bar{R}^2$	$\overline{R_m - R_f}$
	γ_o	γ_1	γ_2		
1952-56	.0099	.0032 (1.5)	-.3079 (-0.8)	.27	.0085
1957-61	.0196	-.0074 (-2.4)	.1267 (0.3)	.36	.0056
1962-66	-.008	.0123 (2.6)	.0529 (0.1)	.43	-.0021
1967-71	.0083	-.0018 (-0.4)	.6739 (1.1)	.45	.0021
1972-76	.0031	.0032 (0.5)	-.1203 (0.2)	.39	.0005
1952-76	.0066	.0020 (1.0)	.0850 (0.4)	.38	0.0029

1/ The regression forms are identical to those in Table 2 except that data for 25 groups of common stocks are used instead of data on individual stocks.

Table 6

Non-Stationary Return-Risk Regressions for Individual and Groups of Common Stocks 1/
Incorporating Predictive Risk Measures and Segregating Periods
with Positive and Negative Risk Differentials

January 1952 - December 1976

Period	Type of Asset	Estimates of Regression Coefficients			$\bar{R}^2$
		γ_o	γ_1	γ_2	
	Individual Stocks 2/				
$R_m > R_f$ 4/		.0177	.0120 (7.8)	.4836 (2.8)	.04
$R_m < R_f$ 5/		.0003	-.0148 (-8.1)	-.6266 (-3.8)	.05
Period as whole		.0010	.0001 (0.1)	-.0086 (-0.1)	.04
	Groups of Stocks				
$R_m > R_f$		.0110	.0170 (6.4)	.3449 (1.0)	.33
$R_m < R_f$		.0110	-.0154 (-6.5)	-.4012 (-1.3)	.44
Period as whole		.0063	.0026 (1.3)	.0141 (0.1)	.38

1/ The regression forms are identical to those described in Table 4. and 25 in the grouped regressions.

2/ Adjusted for order bias.

3/ Grouped to minimize measurement errors.

4/ Regression computed for months in which rate of return on market is higher than risk-free rate.

5/ Regression computed for months in which rate of return on market is less than risk-free rate.

Table 7

Signs of Monthly Return-Risk Regression Coefficients for Stock Regressions[1/] Incorporating Predictive Risk Measures and Segregating Periods with Positive and Negative Risk Differentials

January 1952 - December 1976

Period	Type of Asset	Number of Months with Indicated Signs of Regression Coefficients $\gamma_1(\beta_i)$		$\gamma_2(\sigma^2_{r.i})$	
		+	–	+	–
	Individual Stocks				
$R_m > R_f$		121	27	91	43
$R_m < R_f$		46	106	76	90
	Groups of Stocks				
$R_m > R_f$		117	43	85	60
$R_m < R_f$		50	90	82	73

[1/] See Table 6 for explanatory notes. The regression equations are the same as reported in earlier tables except that the sample used for individual securities was extended to include all common stocks that had dividend adjusted returns for five years preceeding the first month used in the monthly cross sections and 11 months after--a total of 72 months.

Table 8

Return-Risk Regressions for Individual and Groups of Common Stocks With Risk Measures Based on Value-Weighted Stock Index [1]

January 1952 - December 1976

Period	Type of Asset	Type of Regression	Type of Risk Measure	Estimates of Regression Coefficients γ_o	γ_1	γ_2	$\bar{R}^2$	$\overline{R_m - R_f}$
	Indiv. stocks [2]	S	Contemporaneous					
1952-56				.0060	.0099 (8.2)	.0593 (0.7)	.08	.0145
1957-61				.0168	-.0084 (-6.7)	.7281 (7.4)	.08	.0083
1962-66				-.0002	.0036 (3.0)	.8847 (11.4)	.19	.0024
1967-71				.0059	.0023 (2.0)	.2314 (2.6)	.03	.0035
1972-76				.0115	-.0027 (-2.3)	-.0115 (-0.2)	.01	.0001 [4]
	Groups of stocks [3]	N	Predictive					
1952-76				.0067	.0037 (1.4)	-.1141 (-0.7)		

[1] In prior tables, risk measures (β_i and σ^2_{ri}) were derived by regressing stock returns (R_{it}) on returns in a value-weighted index of stocks and bonds combined (R_{mt}). In this table R_{mt} is measured by the Standard and Poor Composite Stock Index.

[2] Adjusted for order bias.

[3] Grouped to minimize measurement errors.

[4] Not significantly different from zero at .05 probability level.