OPTIMAL STABILIZATION IN A

GENERAL EQUILIBRIUM FINANCIAL MODEL[1]

by

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I. Introduction

The purpose of this research is the development of a general equilibrium financial model capable of examining optimal financial policy in a stochastic environment. The historical foundation of the model is drawn from two lines of monetary research: the important development in the mid 1960's of the general equilibrium financial model by James Tobin and William Brainard (1964, 1965) and the optimal policy rules derived for a simple ISLM framework by William Poole (1973). The integration of their models into the one presented here is made possible by the reduction of the multiple equation equilibrium system developed by Brainard-Tobin into one possessing only two equilibrium conditions. Despite this simplification the model presented here retains the ability to analyze the effect of regulatory and structural change on various classes of intermediaries and on the financial system as a whole.

A major goal of the model is the derivation of policy rules which minimize the variability of the price level for a given structure of shocks affecting asset demands. These policy rules include structural changes in the regulatory environment of the banking system as well as stabilization rules which peg asset supplies to endogenous variables. The conditions under which such policies reduce the variability of the unanticipated component of prices are carefully specified, since much of the current research in macroeconomics (Lucas (1973), Barro (1977)) stresses the role of unanticipated movements in prices in generating business cycles. Finally, the influence of reserve levels of financial institutions on price variability is examined and criteria for determing such levels are derived.

Both the deterministic and the stochastic representations of the model are developed in Section II. The third section derives optimal Federal

Reserve policy both with respect to regulatory policy and monetary "stabilization rules" assuming a given level of reserves for each class of intermediary. This section also discusses the ability of the government to control the unexplained variation in prices. The fourth section derives the optimal level of reserves on various types of deposits in order to minimize price level uncertainty. A simulation of optimal policy and reserve levels is presented for a given error structure among classes of intermediary assets.

II. The Model

A. General Framework

The basic framework of the model consists of four sectors: the household sector (h), the banking sector (b), the firm sector (f), and the government sector (g). There are n+1 assets in the economy: high-powered money (H) issued by the government which also serves as currency, n-2 financial deposits ($A_2, \ldots A_{n-1}$) issued by banking institutions, bonds (B) issued by government, and equity (E) issued by firms.

Each sector's demands are limited by budget constraints. For the household sector, real asset demand is constrained by total wealth so that

$$(1)\quad H_h^d + \Sigma A_{ih}^d + B_h^d + E_h^d = W = K + H/P + B_g^s$$

where W = wealth is total firm capital, real high-powered money, and exogenous government bonds.[2] The price level is defined in terms of the exchange rate between money and currently produced output, which in a one-good model applies to either consumption or capital goods.

Each financial institution of class i (b_i) is considered to be an entirely deposit financed institution whose only earning assets are high-powered money (reserves) and bonds (loans) so that

(2) $$H^d_{b_i} + B^d_{b_i} = A^s_{b_i} \qquad i = 2,........n-1,$$

where superscript s indicates supply. Finally firms obtain funds from issuing equity to finance their fixed capital so that

(3) $$E^s_f = K.$$

The general model contains n+1 equilibrium conditions:

(4) $$H^d_h(r^*_i, r_b, r_e, W) + \Sigma H^d_{b_i}(r^*_i, r_b, A^s_{b_i}) = H^s_g/P$$

(5) $$A^d{}_{b_i}(r^*_i, r_b, r_e, W) = A^s_{b_i}(r^*_i, r_b, r_e, W) \quad i = 2....n-1$$

(6) $$B^d_h(r^*_i, r_b, r_e, W) + \Sigma B^d_{b_i}(r^*_i, r_b, r_e, W) = B^s_g$$

(7) $$E^d_h(r^*_i, r_b, r_e, W) = K$$

There are n+1 endogenous variables: the n-2 intermediary deposit rates, indicated by the vector r^*_i, the bond rate r_b, the equity rate r_e, and the price level P. The exogenous variables are the government supply of high-powered money and bonds, H^s_g and B^s_g, and the capital stock, k. The supply curve of each class of itermediary is horizontal at a given deposit rate which is, at most, a function of the bond rate such that

(8) $$r_i = r_i(r_b),\ dr_i/dr_b \geqq 0, \qquad i = 2,.....n-1.$$

This will be true if either r_i is an exogenous policy parameter of the central bank which is set below its equilibrium value or there are constant returns to scale in the banking sector. In these cases, the quantity of deposits supplied by the bank is determined solely by the quantity demanded by households.[3]

It is also assumed that the demand for high-powered money on the part of each intermediary is determined as a ratio, k_i, of deposits issued and a function of the bond rate such that

(9) $$k_i = k_i(r_b),\ dk_i/dr_b \leqq 0, \qquad i=2,\ldots.n-1.$$

The effect of these assumptions is to contract our n+l equilibrium conditions to the following three:

(10) $$H_h^d + \Sigma k_i A_{ih}^d = H_g^s/P$$

(11) $$B_h^d + \Sigma(1-k_i)A_{ih}^d = B_g^s$$

(12) $$E_h^d = K$$

where each demand function is dependent on all the rates of return and wealth, and the endogenous variables are r_b, r_e, and P.

This model is still valid if the government (or firms) supplies no bonds. In that case the net quantity of liquid assets is identical to high-powered money and the households' demand for bonds is negative and equal to their demand for loans from financial institutions. Since there are many economic models that claim that neither the firm sector, via its financing decisions (Modigliani-Miller (1958)) nor government, via its bonds issues (Barro (1974)) creates any net liquid assets, it is important that this model can handle such views. Section III.B. below deals more explicity with these issues.

Since the markets are dependent through the budget constraint, Eq. (1), the model may be reformulated in terms of any two equilibrium conditions. It shall be convenient to consider (10), the high-powered money equation and equations (10) plus (11) as the two independent equilibrium conditions.

Equations (10) plus (11) denote the aggregate currency, deposits, and bonds which we shall term liquid assets.

By considering currency as the first deposit with a reserve requirement of unity, and bonds as the n^{th} deposit with a reserve requirement of zero, the equilibrium conditions for high-powered money and liquid assets can be written

$$\Sigma k_i(r_b)A_i^d(r_i^*, r_b, r_e, W) = H_g^s/P \qquad (13)$$

$$\Sigma A_i^d(r_i^*, r_b, r_e, W) = H_g^s/P + B_g^s. \qquad (14)$$

Denoting H as the excess demand for high-powered money and A the excess demand for liquid assets, (13) and (14) can be written

$$H(r_b, r_e, P; r_i^*, K, H_g^s, B_g^s) = 0 \qquad (15)$$

$$A(r_b, r_e, P; r_i^*, K, H_g^s, B_g^s) = 0. \qquad (16)$$

Given (8) which determines the intermediary rates given the bond rates; (15) and (16) constitute two independent equilibrium conditions in three unknowns: r_b, r_e, and P, with intermediary rates the capital stock, high-powered money and government bonds parameters. In order to close the model it would be necessary to add another equation equilibrating the demand and supply for the stock of capital which would involve the Fisherian concepts of time preference and productivity. Instead, two alternative specifications will be analyzed. The first, which I term neoclassical, will involve fixing the equity return at r_e, the fixed marginal product of capital, and describing the equilibrium in terms of P and r_b. The second specification, termed "Keynesian," will fix the price level, but allow the equity rate to vary in order to equilibrate the market.[4]

In the neoclassical representation the model can be linearized around the bond rate, r_b and the log of the price level, p, to obtain

$$(17) \qquad H_b\tilde{r}_b + H_p\tilde{p} = 0,$$

$$(18) \qquad A_b\tilde{r}_b + A_p\tilde{p} = 0,$$

where (˜) represent deviations from equilibrium values and the subscripts b and p represent the partial derivatives taken with respect to the bond rate and the log of the price level respectively. For simplicity in the initial presentation of the model, it is assumed that the deposit rates and reserve levels are totally exogenous. Section IIIB. will endogenize these variables. The standard assumptions are made, i.e., each asset is a gross substitute and non-negatively dependent on wealth. Because of these assumptions and the fact that the reserve requirement on bonds, $k_N = 0$,

$$(19) \qquad H_b = \Sigma k_i \partial A_i / \partial r_b < 0$$

$$(20) \qquad A_b = \Sigma \partial A_i / \partial r_b > 0$$

$$(21) \qquad H_p = (H/P)(1 - \Sigma k_i \partial A_i / \partial W) > 0$$

$$(22) \qquad A_p = (H/P)(1 - \Sigma \partial A_i / \partial W) > 0, \quad H_p \geq A_p,$$

so that the locus of equilibrium points in the market for high-powered money and for liquid assets is represented by the HH and AA curves respectively, depicted in Figure 1. A unique equilibrium is achieved at r_b^* and p^*.

Comparative static shifts can be easily analyzed by this model (see Siegel (1977)). Any change which induces an excess supply of high-powered money will shift the HH curve rightward. Such changes include the increase in high-powered money or the lowering of the level of reserve requirements.

Any change which increases the supply of liquid assets will also shift the AA curve rightward. An increase in high-powered money will shift both the AA and HH curves rightward by equal amounts, since (15) and (16) are homogenous of degree zero in H and P. This is illustrated in Figure 1. An increase in deposit rates must increase the demand for liquid assets but the effect on the demand for high-powered money is ambiguous (see Tobin and Brainard (1963) and Section II.B. below). If high-powered money and financial deposits are net substitutes, the HH curve will shift rightward, but leftward in the case of complements.

The Keynesian respresentation of the model is similarly derived. The linearized version is represented by

(23) $$H_b\tilde{r}_b + H_e\tilde{r}_e = 0$$

(24) $$A_b\tilde{r}_b + A_e\tilde{r}_e = 0$$

where

(25) $$H_e = \Sigma k_i \partial A_i/\partial r_e < 0$$

(26) $$A_e = \Sigma \partial A_i/\partial r_e < 0, \quad H_e < A_e .$$

The locus of equilibrium points in the market for high powered money and liquid assets in the Keynesian Model is depicted in Figure 2, where a unique equilibrium r_b^*, r_e^* is achieved.

In the Keynesian model any comparative static change which induces an excess supply of high-powered money will shift the HH curve leftward, while an increase in the supply of liquid assets will shift the AA curve leftward. The homogeneity property that is present in the neoclassical model does not exist in the Keynesian specification. A unit increase in high-powered money will shift the HH curve to the left by $(1-\Sigma k_i\, A_{iw})/H_e$

which is greater than the AA shift of $(1-\Sigma A_{iw})/A_e$. This is depicted in Figure 2. Hence both the bond rate and equity rate will fall. An increase in deposit rates must increase the demand for liquid assets, but the effect on high-powered money is ambigiuous, as in the neoclassical model.

II. B. Stochastic Behavior of the Model

Although the model can be used to study comparative static shifts in the variables, this paper concentrates on optimal policy in a stochastic environment. It is assumed that through time each asset is subject to a zero mean random demand shock ε_i, i = 1,....n, with a stationary variance covariance matrix $V = [\sigma_{ij}]$[5]. To avoid the problem of inflationary expectations, it is assumed that such shocks follow a random walk through time. Hence,

$$\varepsilon_A = \Sigma\varepsilon_i \; ; \; \sigma_A^2 = \Sigma\Sigma\sigma_{ij}$$

$$\varepsilon_H = \Sigma k_i \varepsilon_i \; ; \; \sigma_H^2 = \Sigma\Sigma k_i k_j \sigma_{ij} \qquad (27)$$

$$\sigma_{AH} = \Sigma k_i \sigma_{ij}$$

Although disturbances in general affect both the high-powered money and liquid asset market, it is possible for shocks to affect each market independently. Disturbances which involve switching among liquid assets of differing reserve ratios, so that $\Sigma\varepsilon_i = 0$, involve shocks only to the HH equilibrium. Switching between bonds and equity affect only the liquid asset equilibrium.

If the monetary authorities had perfect knowledge of the excess demand functions and had information on the bond rate and price level without a lag, it could offset any shifts in the curves and completely control the bond rate and price level through its policy instruments of deposit and reserve rates and open market policy. When one of the variables, say the price level, is only known with a lag, and the sources of the disturbances are not known, complete control is foregone.[6] In this case the central bank can only operate to minimize the fluctuations in the price level by structuring the financial environment so as to minimize the effect of such disturbances on the economy.

Figures 3 and 4 present a an example of the effect of a policy on the values of the endogenous variables. Any policy which changes the sensitivity of the response of high-powered money or liquid assets to the bond rate will alter the slope of HH or AA locus. Since the slope of the HH curve equals $-H_p/H_b$, a policy change which makes the market for high-powered money more sensitive to the interest rate will flatten the HH curve to H'H'. Since H_p and A_p are constant, any given disturbance in either market results in the same horizontal displacement of the curves. A disturbance in the market for high-powered money (indicated in Figure 3) results in a smaller change in the price level and interest rate (Q'_1) after the policy change than before (Q'_2). On the contrary, as indicated in Figure 4, disturbances in the market for liquid assets result in a greater change in the price level although a smaller change in the interest rate (Q'_2) after the structural shift than before.

III. Optimal Federal Reserve Control

A. General Solution

Given the characteristics of the disturbances in both markets, one can determine the optimal slopes of the HH and AA curves in order to minimize the variance of the price level and the interest rate. The results are derived for the neoclassical version of the model. Analogous solutions for the Keynesian model involve the substitution of the equity rate for the price level. Since

$$(28)\ p = (H_b\varepsilon_A - A_b\varepsilon_H)/(H_pA_p - H_pA_b),$$

$$(29)\ r_b = (A_p\varepsilon_H - H_p\varepsilon_A)/(H_bA_p - H_pA_b),$$

the variances of the endogenous variables are

$$(30)\ \sigma_p^2 = \frac{H_b^2\sigma_A^2 + A_b^2\sigma_H^2 - 2H_bA_b\sigma_{AH}}{(H_bA_p - H_pA_b)^2}\text{ , and}$$

$$(31)\ \sigma_{r_b}^2 = \frac{A_p^2\sigma_H^2 + H_p^2\sigma_A^2 - 2A_pH_p\sigma_{AH}}{(H_bA_p - H_pA_b)^2}$$

It is easily seen that (30) is homogenous of degree zero in H_b/A_b and hence the optimum condition for minimizing the variance of the price level can be written

$$(32)\ (H_b/A_b)^* = \frac{A_p\sigma_H^2 - H_p\sigma_{AH}}{A_p\sigma_{AH} - H_p\sigma_A^2}\ .$$

A policy instrument that operates on either H_b or A_b is capable of achieving an optimum.

When (32) is substituted into (30), one finds that the minimized variance of the price level is

$$(33)\ \sigma_p^{2*} = \frac{\sigma_H^2\sigma_A^2 - \sigma_{AH}^2}{A_p^2\sigma_H^2 + H_p^2\sigma_A^2 - 2H_pA_p\sigma_{AH}} = \frac{\sigma_H^2\sigma_A^2\ (1-\rho^2)}{\text{var}(H_p\varepsilon_A - A_p\varepsilon_H)}$$

where ρ is the correlation coefficient between ε_A and ε_H. The higher the degree of correlation between the two disturbances, the smaller the minimized flucuations of the price level. In fact, if disturbances only arise in one market, or the correlation between the two disturbances equals ±1, the monetary authority can completely eliminate the fluctuations in the price level.

From (31) it is easily seen that the conditions to minimize the variance of the interest rate are

$$(34)\ H_b \rightarrow -\infty \text{ or } A_b \rightarrow -\infty \text{ so that } \sigma^2_{r_b} \rightarrow 0.$$

To achieve this goal the monetary authorities must make either the HH or the AA curve horizontal. It is possible, theoretically, for both objectives, the minimization of the variance of the price level and the minimization of the variance of the interest rate to be achieved simultaneously. This is accomplished by making the high-powered money market and the liquid asset market increasingly sensitive to the bond rate while maintaining the ratio (32). The next section discusses the policies which can influence the interest sensitivities of these markets.

III. B. Policies which Alter H_b and A_b

There are basically two types of policies which influence the sensitivity of the excess demand for high-powered money and liquid assets to the bond rate and hence the slopes of the HH and AA loci. The first involves changing banking regulation and the second links to asset supplies to the bond rate.[7]

The first and simplest structural change to analyze is the effect of allowing the aggregate reserve ratios, k_i, to be flexible or responsive to changes in the bond rate so that dk_i/dr_b is negative. If the reserve level were made sensitive to the bond rate then the responsiveness of high-powered money to the bond rate,

$$(35)\ H_b = \Sigma k_i \partial A_i/\partial r_b + \Sigma A_i dk_i/dr_b < \Sigma\, k_i \partial A_i/\partial r_b,$$

would become more sensitive than in the case of fixed reserves and the HH locus flatter. At the level of the individual bank this can be accomplished in two ways. The first is to ease the penalties for deficiencies in reserves and to lessen the opportunity cost to banks for maintaining excess reserve positions. This can be accomplished by such measures as allowing a greater reserve averaging period or more liberal carry-forward provisions, or by paying interest on reserves, a plan now under study by the Federal Reserve System. A second method would be the elimination of mandatory reserve ratios, allowing the banks to choose their own profit-maximizing reserve level. Although this would lower the level of reserves, such reserves would be sensitive to the bond rate since this rate represents the cost of an input in the bank's production of deposit services.

On the aggregate level, reserves would be made more sensitive to the bond rate if banks could more easily shift from Federal Reserve membership to non-member status in response to changing market rates. This would occur since non-member reserve levels in terms of high-powered money, are often far lower than those of member banks.

A second regulatory change is greater flexibility in deposit rates. This could be accomplished either by the government setting the r_i functions of Eq. (8) such that statutory limits on deposit rates are made more responsive to open market rates or by allowing the intermediaries to set their own deposit rates competitively (see footnote 3 above). If this occurs then the sensitivity of liquid assets to the bond rate must become greater, since

$$(36)\quad A_b = \sum_{ij}\sum \partial A_i/\partial r_j \cdot dr_j/dr_b > \sum \partial A_i/\partial r_b .$$

This is a direct consequence of the gross substitutability property between assets. Flexible deposit rates must increase A_b and make the AA curve flatter in both the neoclassical and Keynesian models.

Unlike the case of flexible reserve level, flexible deposit rates may also affect the HH curves. Although there is an increase in the demand for total liquid assets under flexible deposit rates compared to fixed rates when the bond rate rises, the demand for some monetary assets may actually fall under flexible rates. This occurs when the substitution effects between assets outweighs the own effect of a rise in the deposit rate. When these intra-asset shifts are weighted by reserve ratios, then the sensitivity of the demand for high-powered money to the bond rate may either rise or fall. The change in the demand for base money can be decomposed into its components,

$$(37)\quad H_b = \sum\sum k_i \partial A_i/\partial r_j \cdot dr_j/dr_b = \sum k_i \partial A_i/\partial r_b + \sum\sum k_i \partial A_i/\partial r_j \cdot dr_j/dr_b .$$

The first term on the right is the change in high-powered money occasioned by a change in the bond rate holding all other deposit rates constant, while the second term measures the change in the demand for high-powered money, resulting from intra-asset shifts, caused by changes in the deposit rates, holding the bond rate fixed. We shall term the liquid assets and high-powered money <u>substitutes</u> if the second term on the right of (37) is negative and complements if it is positive. The HH curve under deposit rate flexibility is flatter than under fixed rates if liquid assets and high-powered money are substitutes and steeper in the case of complements.

III. C. Optimal Stabilization Rules

There is second and more direct means of influencing the sensitivity of high-powered money and liquid assets to the bond rate. The Central Bank can simply choose to issue these assets as a function of the bond rate. Assume a high-powered money policy H^s such that

$$(38)\ H^s/P = H_o/P + H^s(r_b)/P.$$

where $(\partial H^s/\partial r_b)/P = H_b^s \gtrless 0$. Substituting (38) into (15) and (16), H_b and A_b, the derivative of the excess demand for high-powered money and liquid assets with respect to the bond rate transform to

$$(39)\ H_b = H_b^o - H_b^s(1-\Sigma k_i A_{iw})$$

$$(40)\ A_b = A_b^o - H_b^s(1-\Sigma A_{iw}),$$

where H_b^o and A_b^o represent the interest sensitivities of the excess demand functions when the government is maintaining a constant level of high-powered money. Substituting (39) and (40) into (31), it can be shown that the variance of the interest rate is invariant to H_b^s, the high-powered money rule. This occurs because of the homogeneity properties of money and the fact that inflationary expectations are zero in this model.[8] The variance of the bond rate is not invariant to the high-powered money rule in the Keynesian representation of the model, since the homogeneity property does not prevail.

The optimal high-powered money rule, H_b^{s*} which achieves the minimized price level variance indicated by (33) is

$$(41)\quad H_b^{s*} = \frac{A_b^o (H_b/A_b)^* - H_b^o}{(1 - \Sigma A_{iw})(H_b/A_b)^* - (1 - \Sigma k_i A_{iw})}$$

which differs from zero insofar as H_b^o/A_b^o differs from $(H_b/A_b)^*$ represented in (32).

The usual method of introducing high-powered money into the economy is through open market operations where liquid government securities are purchased for newly created base money. This method of money creation, according to some authors (Tobin (1969)), leads to markedly different implications for monetary policy than does the outright infusion of high-powered money. Since the government bonds represent a stream of future tax liabilities, other authors (notably Barro (1974)) have claimed that government bonds do not represent any net wealth to the community. A related, but distinct question, is whether government bonds can be considered net liquidity to the economy even if they do not represent net wealth. If individuals view the future tax liabilities engendered by future interest payments as uncertain, then an open market operation trades a liquid asset for a less liquid liability. This view may find particular justification within a tax system where future tax liabilities are linked to future realized income, which for the individual is uncertain. Even though the economy as a whole must with certainty bear the tax liabilities incurred through debt issue, an individual will only bear that fraction of the tax associated with his future income <u>relative</u> to the entire economy. In effect, the issuance of government debt associated with an income-linked tax system is a form of insurance that the government provides individuals for specific employment risk.[9]

In order to capture these effects, let α_W be the percent of government debt that the public perceives as net wealth and α_L be the fraction they perceive as net liquidity.[10] $\alpha_W = \alpha_L = 0$ is the polar case where government debt represents a liquid tax liability equivalent to the bond issued, while $\alpha_W = \alpha_L = 1$ indicates that debt is totally undiscounted by the public.[11] With this notation it is straightforward to specify the effect of an open market operation, where high-powered money is exchanged for government bonds, on the equilibrium loci. An open market purchase which increases high-powered money by one unit will increase the wealth perceived by the economy by $1 - \alpha_W$ units, and hence the demand for high-powered money rises by $(1 - \alpha_W)\Sigma k_i A_{iW}$, and the HH locus shifts rightward by $1 - (1 - \alpha_W)\Sigma k_i A_{iW}$.

In the liquid asset market, an open market purchase will increase the supply of net liquidity by $1 - \alpha_L$. In addition, since wealth increases by $1 - \alpha_W$, the demand for liquid assets will rise by $(1 - \alpha_W)\Sigma A_{iW}$. Hence the effect on the excess supply of liquidity of an open market operation is $1-\alpha_L-(1-\alpha_W)\Sigma A_{iW}$. Therefore open market operations transform the excess demand functions into

$$(42)\quad H_b = H_b^o + H_b^s((1-\alpha_W)\Sigma k_i A_{iW} - 1)$$

$$(43)\quad A_b = A_b^o + H_b^s((1-\alpha_W)\Sigma A_{iW} - 1 + \alpha_L)$$

In contrast to the simple injection of high-powered money, the swap of bonds for money can affect the variance of the bond rate if either α_W or α_L is positive. The optimal high-powered money rule to minimize price level variations under a system of open-market operations is

$$(44)\quad H_b^{s*} = \frac{A_b^o(H_b/A_b)^* - H_b^o}{(1-\alpha_W)\Sigma k_i A_{iW}-1-((1-\alpha_W)\Sigma A_{iW}-1+\alpha_L)(H_b/A_b)^*}$$

When $\alpha_W = \alpha_L = 0$, (44) reduces to (41). If $\alpha_W = \alpha_L = 1$, so that government debt is not discounted at all, (44) reduces to

$$(45)\ H_b^{s*} = H_b^o - A_b^o(H_b/A_b)*$$

Open market operations will in this case only affect the equilibrium in the market for high-powered money. In general, for a given deviation of H_b^o/A_b^o from the optimal level, it is not possible to determine whether open market operations require a more or less interest-sensitive high-powered money rule than a policy which involves the simple injection of money.

III.D. Effect of Policies on Anticipated and Unanticipated Price Level Changes

Since the shocks to the economy are random walks, the best estimate of this period's price level is lost period's, i.e., $E(P_t^e) = P_{t-1}$. Hence Eq. (30) can also be interpreted as the unconditional variance of the error in price forecast, $\sigma^2(P_t - P_t^e)$. As described above, this unconditional variance attains its minimized value, Eq. (33), when the slopes are set at their optimum levels indicated by Eq. (32). However, if we assume that the bond rate is revealed to economic agents at time t, before the price level, then the appropriate measure of prices variability would be the variance of the price level conditional on the bond rate. Knowledge of the bond rate reveals some information about the source of the shocks in the economy and hence permits a decrease in the forecast variance.

It can be shown that the variance of the price level conditional on the bond rate $\sigma^2(P_t - P_t^e\ r_{b,t})$ is exactly Eq. (33), the minimized variance of prices, <u>no matter what</u> the values of H_b and A_b. Therefore, if the bond rate is known prior to the price level, no policy change which alters interest rate sensitivities of the excess demand functions can change the variance of the unanticipated price level. The best that policy can do is to neutralize the anticipated portion of price variability. Although this may have beneficial effects (see Fischer (1977)), it would not influence the outcome of the rational expectations models of income determination where output is a function of the difference between actual and anticipated price levels.

If the bond rate is not known prior to the price level, the variance of both the anticipated and unanticipated price level is described by Eq. (30). Optimal stabilization rules which depend on the current values of the bond rate cannot exist. However, structural changes, such as making reserve levels and deposit rates more flexible, act as automatic stabilizers, altering the responsiveness of the system to shocks and reducing unanticipated price level variability.

The stochastic structure of the model can alternatively be described assuming economic agents possess current knowledge of the bond rate and the price level, but the money supply affects the economy with a one period lag. Money rules dependent on the bond rate can be derived which minimize the variance of anticipated prices but have no effect on price level uncertainty (if the public knows the money rule). However structural and regulatory changes, since they react instantaneously with the shocks, can be utilized to reduced the variance of both anticipated and unanticipated price level changes.

IV. Optimal Setting of Reserve Requirements

A. Theory and Example

The previous section discusses policies which change the sensitivity of the excess demand functions to the bond rate for given reserve levels. The government can also influence the variance-covariance structure of the error terms given by (27) by controlling the level of reserve requirements. This leads to the determination of the optimal level of reserves on various types of deposits, a problem that has long interested monetary economists.[12]

Formally, let K be the row vector of reserve ratios on the N liquid assets such that $k_1 = 1$ (currency) and $k_n = 0$ (bonds). If V is the variance-covariance matrix of the shocks, then (27) can be rewritten

$$(46)\qquad \sigma_A^2 = 1V1'$$

$$\sigma_H^2 = KVK'$$

$$\sigma_{AH} = KV1',$$

where $1 = (1,1,\ldots.1)$ is the unit row vector. For simplicity the price level is normalized and it is assumed $H_p = A_p = 1$. The formal problem for the monetary authorities is to minimize (33), the minimized variance of the price level with respect to H_b and A_b, with respect to K, i.e.,

$$(47)\quad \min_{\{K\}} \frac{KVK'1V1' - (KV1')^2}{1V1'+KVK'-2KV1'}$$

such that $k_1 = 1$ and $k_n = 0$ and $1 \geq k_i \geq 0$.[13]

Let us illustrate the solution by choosing a four-asset economy; currency, deposits, with reserve requirement k, bonds and equity. For simplicity, it is assumed there are no shocks to the bond market, so that $\varepsilon_b = 0$ and ε_C and ε_D represent shocks to currency and deposits respectively. From (27),

$$
\begin{aligned}
(48)\qquad \sigma_H^2 &= \sigma_C^2 + 2k\sigma_{CD} + k^2\sigma_D^2 \\
\sigma_A^2 &= \sigma_C^2 + 2\sigma_{CD} + \sigma_D^2 \\
\sigma_{AH} &= \sigma_C^2 + (1+k)\sigma_{CD} + k\sigma_D^2.
\end{aligned}
$$

Substituting (48) into the minimized price variance (33) yields

$$
(49)\quad \sigma_p^{2*} = \frac{\sigma_C^2\sigma_D^2 - \sigma_{CD}^2}{\sigma_D^2}
$$

which is independent of the reserve requirement! The variance of the bond rate under the optimal policy is, from (31),

$$
(50)\quad \sigma_{rb}^2 = \frac{(\sigma_{CD} + \sigma_D^2)^2}{A_b^2\ \sigma_D^2}
$$

which is also independent of the reserve level. These results do not mean that the optimal Central Bank policy is independent of the reserve requirement on deposits since, from (32),

$$
(51)\quad (H_b/A_b)^* = \frac{\sigma_{CD} + k\sigma_D^2}{\sigma_{CD} + \sigma_D^2}\ .
$$

The above indicates that when that optimum policy is set, the variance of the price level is dependent only on the variance-covariance structure of currency and deposits. Hence, in response to a changed reserved ratio, the Central Bank need only target the bond rate so as to maintain its former variance, and the variance of the price level will remain at it minimized level.

Unfortunately, this result does not readily generalize to the case of many deposits. Let us define

$$
\begin{aligned}
C_1 &= (\sigma_{12}, \ldots\ldots\ldots \sigma_{1\ N-1}) \\
C_N &= (\sigma_{N1}, \ldots\ldots\ldots \sigma_{N\ N-1}) \\
V_r &= [\sigma_{ij}] \quad 2 \geqq i,\ j \geqq N-1 \qquad (52) \\
K_r &= (k_2, \ldots\ldots k_{N-1}).
\end{aligned}
$$

Then the implicit solution of (47) is

$$
(53)\quad K_r^* = \frac{(\sigma_{AH}-\sigma_H^2)\cdot 1 - (\sigma_A^2+\sigma_H^2-2\sigma_{AH})V_r^{-1}C_1 - (\sigma_H^2-\sigma_{AH})V_r^{-1}C_N}{\sigma_A^2-\sigma_{AH}}
$$

The only time when the optimal reserve requirements are identical is when $C_1 = C_N = 0$, and even then the minimized variance is not independent of the reserve level chosen.

B. Computer Simulations

Computer simulations do suggest that when there are two types of deposits, say, demand and time, and the variance of the bond shocks is zero, the minimized price variance does not involve a unique set (k_D, k_{TD}) of reserve requirements. The set of such optimized reserve levels exhibits a nearly linear relationship, although this has not yet been proved.

Figure 5 displays isovariance price curves for an arbitrarily chosen variance-covariance matrix shown in (54). The isovariance curves indicate identical price level variances for various combinations of reserve requirements on time and demand deposits. Let there be currency (C), demand deposits (D), time deposits (T), and bonds (B). Let

$$(54) \qquad W = \begin{matrix} \sigma_{CC} & \rho_{CD} & \rho_{CT} & \rho_{CB} \\ & \sigma_{DD} & \rho_{DT} & \rho_{DB} \\ & & \sigma_{TT} & \rho_{TB} \\ & & & \sigma_{BB} \end{matrix} \qquad \begin{matrix} 1 & -.28 & -.04 & 0 \\ & 10 & -.36 & -.08 \\ & & 5 & -.16 \\ & & & 20 \end{matrix}$$

where $\rho_{ij} = \sigma_{ij}/\sigma_i\sigma_j$. For this matrix, the minimized variance of the price level is .895884 which is found at the optimum $k_D^* = .100$ and $k_T^* = .066$. It is certainly not true that minimized variance is achieved at zero ($\sigma_p^2 =$ .9611) or unity ($\sigma_p^2 > 8$) reserve ratios. It also appears from this simulation that the further the reserve requirements are from their optimum, the greater is the change of the variance of prices for any given change in reserve requirements. Other simulations indicate that the more negative the covariance matrix, the higher are the optimal reserve requirements and lower is the minimized price variance. The optimal set of reserve ratios may not be positive, particularly if the covariance structure is strongly positive. This is unlikely since the adding up constraint on the errors require that the sum of all covariances between <u>all</u> the assets (including equity) must equal the negative of the sum of all the variances.

V. Conclusions

A major conclusion of this study is that policy rules and regulatory changes can be devised which minimize the variance of the price level. Whether these policies affect the unexpected variation of prices depends both on the amount of information available to economic agents and the type of policies pursued. In general, structural changes in the regulatory environemnt, such as greater flexibility of reserve levels and deposit rates, may be more effective at reducing the unexplained variation of prices than policy rules pegging money to interest rates. The interest rate and its variability can only be influenced by money rules in the flexible price version of the model if individuals regard government debt as providing some net liquidity to the private sector.

The government can also influence the structure of shocks affecting the economy and hence the variability of prices by setting the level of reserves on financial deposits. The optimal levels depend, in general, on the variance-covariance structure of asset shocks, but in some special cases the price level variance is independent of such level if the central bank is following an optimal high-powered money rule. These optimality conditions can be readily applied to an empirically estimated financial model.

Figure 1

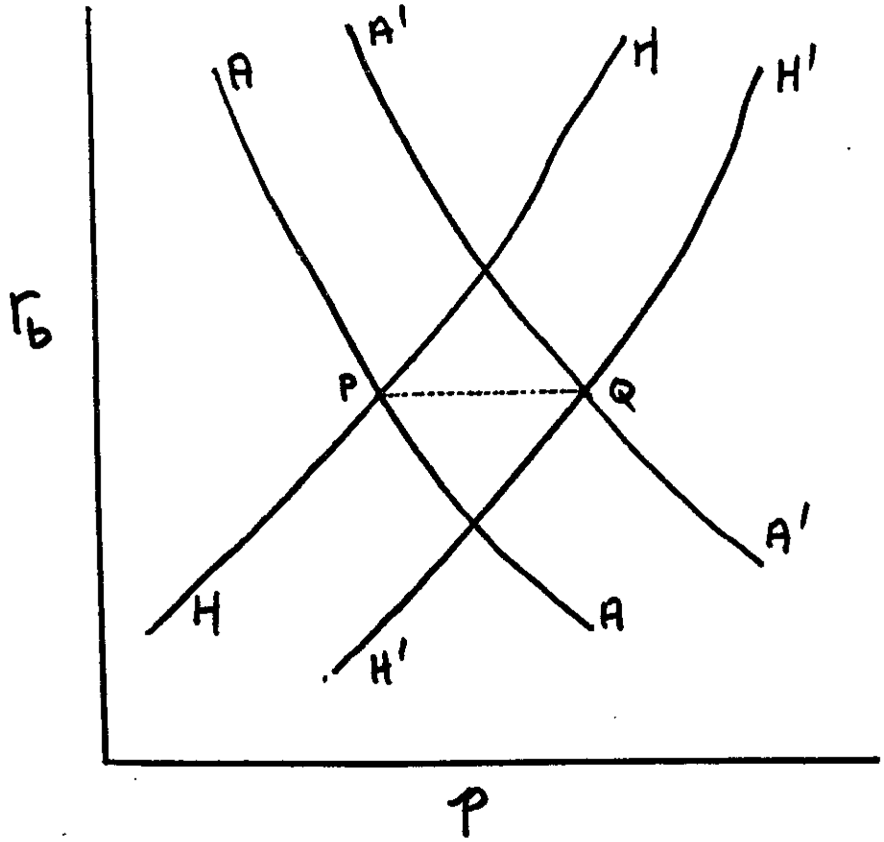


Figure 1. The shift in the neoclassical equilibrium from P to Q as a result of an increase in high-powered money.

Figure 2

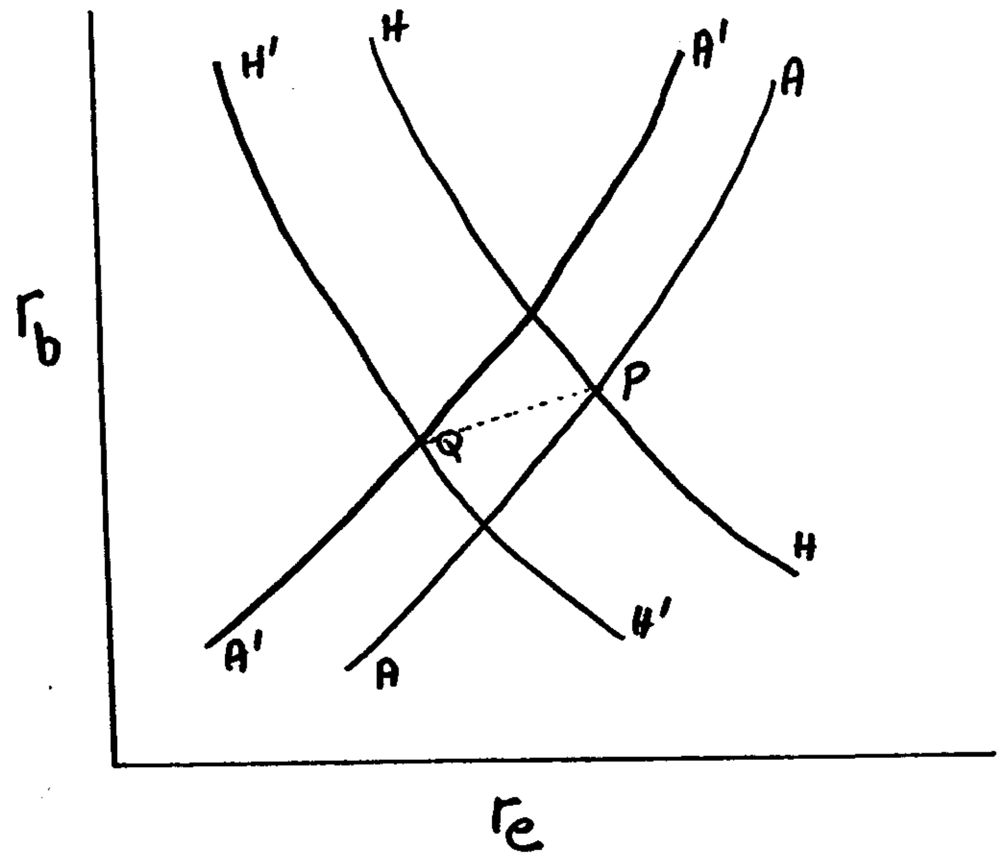
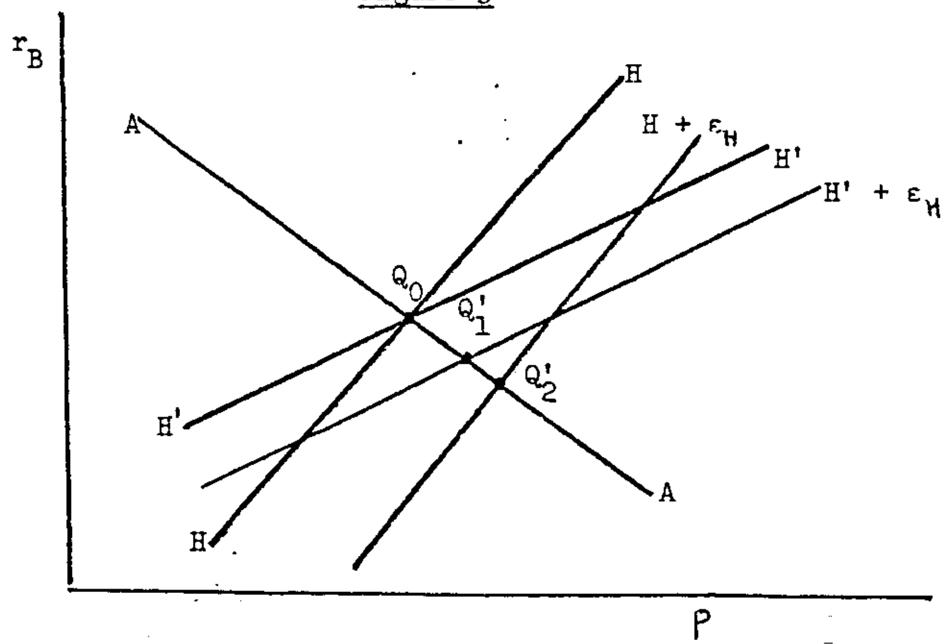


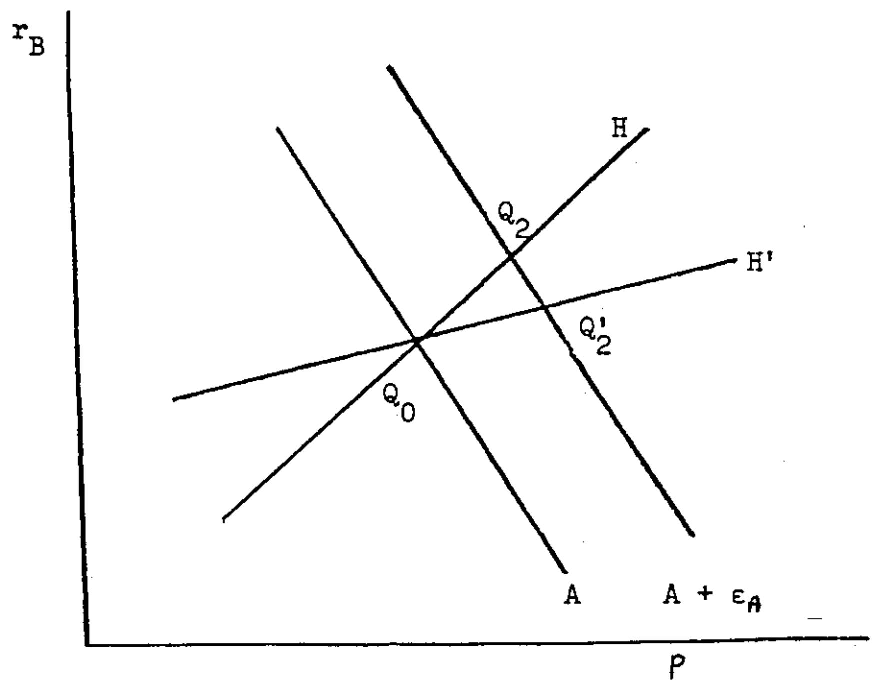
Figure 2. The shift in the Keynesian equilibrium from P to Q as a result of an increase in high-powered money.

Figure 3



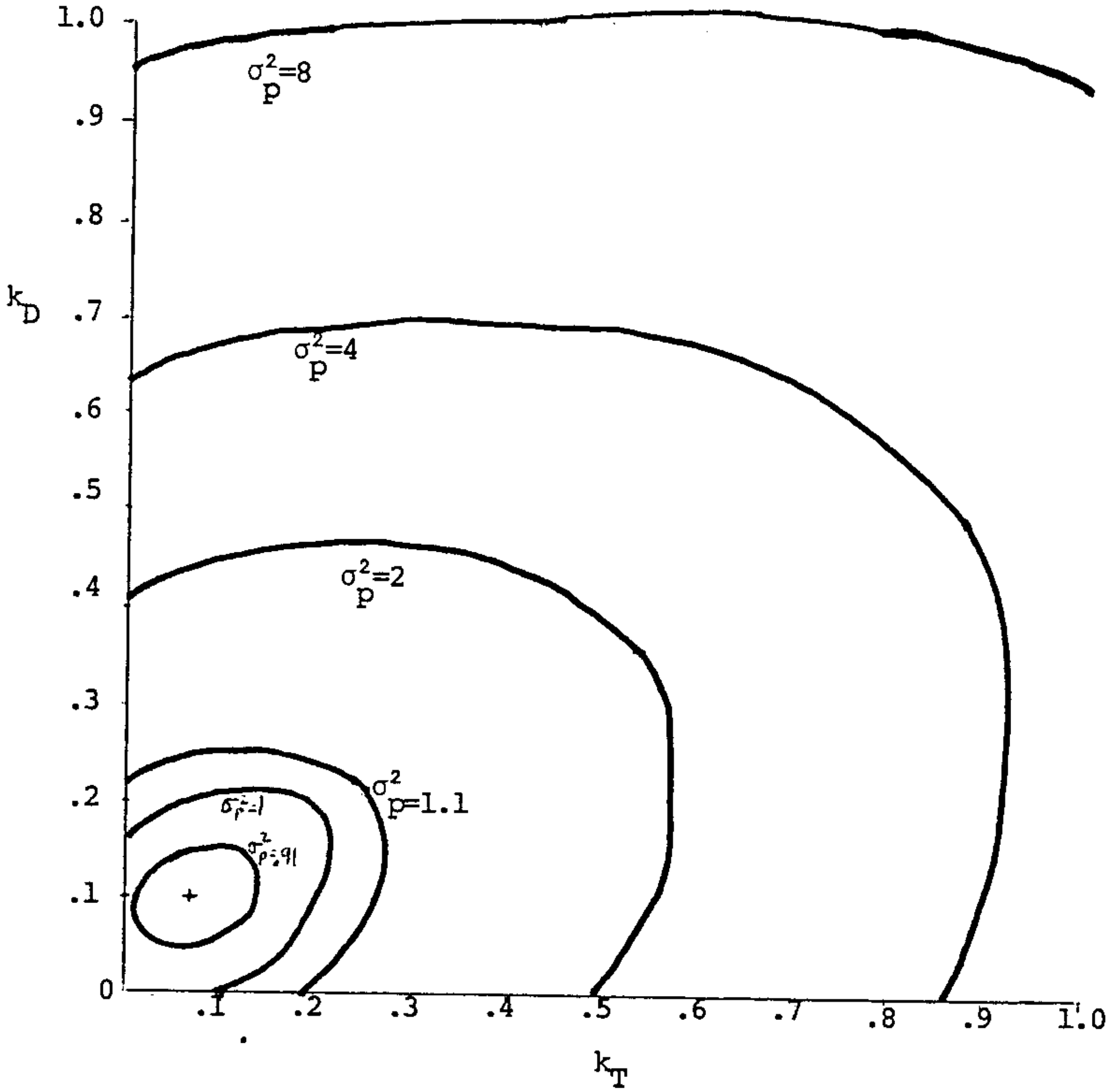
Graphical exposition of disturbance in HH curve under HH and H'H' policies.

Figure 4



Graphical exposition of disturbance of AA curve under HH and H'H policies.

Figure 5



Isovariance curves for W matrix. Minimized σ_p^2 =.895884 at k_D = .100, k_T = .066.

FOOTNOTES

[1]This paper represents the ongoing development of a general equilibrium financial model developed by Siegel (1977). Other aspects of the model are explored in Santomero and Siegel (1978) and Siegel (1978). My thanks to the Rodney White Center for financial support and Marcel Genet for computational assistance.

[2]Bonds are here considered as net wealth and liquidity. See Section III.B. below for alternative specifications. The supply of government bonds is assumed real. This is true if the bonds are indexed or if foreseen changes in the price level generate interest rate changes which are debt-financed. The substitution of non-indexed for indexed debt does not change any of the qualitative analysis.

[3]A common specification in the competitive case is $r_i = (1-k_i)r_b-c_i$, where k_i is the reserve ratio in terms of high-powered money and c_i is the per dollar cost of providing the deposit.

[4]The capital stock will still be valued at its real reproduction costs, and not its equity rate capitalized value. A generalization to allow for capital value changes can be easily incorporated without qualitatively changing the properties of the model.

[5]Supply shocks to high-powered money and government bonds are ignored but could be easily incorporated into the disturbance structure.

[6]Complete control is also impossible if policy affects the price level with a lag, even if the current values of all variables and functional forms are known. See Section III.D. below.

[7]Comparative static shifts are analyzed extensively in Santomero and Siegel (1978). It is conlcuded there that a rise in deposit rates, reserve requirements, or the institution of interest on reserves will generally raise the sensitivity of high-powered money to the bond rate, leaving the liquid asset market unchanged.

[8]These zero expectations are rational since the error structure given by (23) is intertemporally uncorrelated. Non-zero inflationary expectations could be introduced without altering the qualitative properties of the model.

[9]Of course, there is a question as to whether the government is more efficient than the private sector in providing such insurance. The stricter nature of the penalties imposed by the government for non-payment of taxes compared to non-payment of private debts may be one means the government has of providing net liquidity.

[10]If $\alpha_w \neq \alpha_L$, there exists a non-liquid liability that is not explicitly specified in the model. It may not be unreasonable to aggregate this liability with the residual equity market.

[11]It is the opinion of this author that $1>\alpha_L>\alpha_W>0$.

[12]In particular see Friedman (1959) and Carson (1973) for arguments for setting reserves at 100 and 0 percent. Kaminow (1977) presents a rigorous analysis of the control problem. See Guttentag (1966) for a study of the strategy of open market operations.

[13]It is possible for k_i to exceed unity. If we require an equity ratio for the intermediary, then high-powered money could exceed deposits. Negative reserve requirements would be indicative of allowing intermediaries to issue currency along with their deposits which are viewed as perfect substitutes for government currency by the households. These specifications require some generalization of the original model, however.

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