PARTIAL ADJUSTMENT IN THE DEMAND FOR MONEY: THEORY AND EMPIRICS

by

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I. INTRODUCTION

Central to the notion of money as the medium of exchange is the concept that all transactions are conducted using money balances. This notion, expounded most explicitly by Clower (1967, 1970), has led to the development over the past decade of interesting models of the transactions and precautionary demand for money.[1] These have examined optimal behavior of the economic agent in steady state and assumed that the economic time series can be explained by this desired balance scenario.

Although there has been much gained by examining money balance behavior in this way, money demand models so far have missed one aspect of the behavior of money, i.e. its role during periods of disequilibrium. As the basic payment mechanism, money is used out of equilibrium as well as in it. Therefore an adequate understanding of total money balances is possible only by incorporating both equilibrium and disequilibrium behavior into an overall view.

The empirical literature attempts to capture disequilibrium behavior by incorporating a lagged adjustment term in actual estimation. Partial adjustment models of money demand are usually of the form

$$M_t^D = M_{t-1} + \psi(M_t^* - M_{t-1}), \tag{1}$$

where M^D is current money demand, M is actual money holdings, M^* is equilibrium money demand, and ψ is an adjustment parameter. Two issues arise with respect to such models. First, there is the problem of the theoretical justification for this specification of the time series behavior. In general, partial adjustment is assumed to be a result of adjustment costs, usually not specified but supposedly captured in the parameter ψ. These adjustment costs are viewed as transaction costs such as those underlying the transactions demand for money models. Yet, no attempt so far has been made to analyze how such costs affect money balances and how economic variables, such as interest rates and income, affect the adjustment speed.[2]

The second problem that arises from the use of equation (1) is essentially econometric. As Laidler (1969) explains, one can not distinguish, using this formulation, between the disequilibrium adjustment model assumed above and one deriving from a money demand model based on permanent income, in which current income and lagged money appear because of the Koyck transformation used to proxy permanent income. In fact, when Feige (1967) performs a joint test of the partial adjustment and permanent income hypotheses of money demand, using annual data, he rejects the former and accepts the latter. The implication is that rather than capturing disequilibrium money balance behavior, the regression is merely supporting the use of permanent income in demand for money studies.[3] Consequently, it is questionable whether empirical studies have treated properly the partial adjustment of money balances.

In the present study, we address both the theoretical foundation and empirical importance of the partial adjustment effect. We construct a model of optimal behavior by the economic agent, showing the necessary conditions for the existence of partial adjustment and indicating how the speed of adjustment behaves over time and in response to changes in economic variables. This appears to be the first such attempt in the money demand literature, although there is related work in the investment literature on the flexible accelerator. We test the model on U.S. quarterly data to estimate the existance and magnitude of partial adjustment. We find a significant but small effect for both M_1 and M_2; furthermore, the effect dies out within one and two quarters for the respective definitions of money. The results thus support those of Feige, but disagree in magnitude with those of Darby (1972). The latter found a larger and longer-lasting partial adjustment effect. Finally, we find that regressions of money demand on current income and lagged money should be interpreted as tests of permanent income rather than partial adjustment models of money demand. Thus Goldfeld (1973) appears to interpret his results improperly, assuming them to follow from a partial adjustment model. Indeed, if his results are interpreted as following from a permanent income model, they agree quite closely with the results reported below.

II. A SYNTHESIS MODEL OF MONEY DEMAND

II.1. Equilibrium Behavior. Assume the individual determines equilibrium cash balances according to precautionary demand theory, as expounded by Whalen (1966).[4] As the result of an unspecified intertemporal maximization subject to lifetime wealth, the individual has determined a fixed expected real consumption expenditure. However, because of random elements in the billing process, in the availability of goods, and so on, the actual consumption expenditure may deviate from the expected, causing unexpected expenditures. These unexpected expenditures have a zero mean and some positive variance σ^2. As the volume of payments increases, so does σ^2, the nature of the increase depending on the probability distribution involved. The individual holds precautionary balances to cover consumption expenditures. In determining his optimal level of precationary balances, the individual must balance the costs and benefits of such balances. On one hand, the cost of precautionary balances is the interest earnings foregone by not holding a savings asset. If savings assets carry a nominal interest rate of r_A and money a nominal rate of r_M, then foregone interest earnings equal $M\ (r_A - r_M)$, where M is the amount of precautionary balances. On the other hand, the benefit of precautionary balances is avoidance of at least some illiquidity cost. If c is the dollar value of the illiquidity cost and p is the probability of illiquidity, then the expected illiquidity cost is p c. By Chebyshev's inequality, the probability that a variable deviates from its mean by more than k times its standard deviation is less than or equal to $1/k^2$. Defining $k \equiv M/\sigma$, then $p = \sigma^2/M^2$, assuming equality to be conservative.

The individual's problem now is to minimize the expected loss function.[5]

$$(2) \quad L = M\ (r_A - r_M) + \frac{\sigma^2}{M^2}\ c.$$

the optimal value of M, denoted M*, is

$$(3) \quad M^* = \left(\frac{2\ \sigma^2\ c}{r_A - r_M}\right)^{1/3} .$$

<u>II.2. Disequilibrium Behavior</u>. As noted above, the equilibrium cash balances of equation (3) obtain when income and expenditure plans are consistent with expectations. In all other cases, these balances may not result. Specifically, if the economic agent receives transitory income, which is assumed to come in the form of money, the equilibrium behavior above would be amended to incorporate the new cash flow. In accordance with the permanent income hypothesis, the individual would like to convert his transitory income into savings assets; however, he does not do so immediately. By definition, transitory income is unanticipated;

consequently, upon receiving it, the individual is unprepared to dispose of it immediately and must spend time searching for appropriate assets to buy. Until such assets are found, the individual continues to hold his transitory income as money. We refer to these extra money balances as disequilibrium balances.[6] These extra balances gradually decline as desirable assets are found.

Given that the individual has determined his optimal equilibrium balances by considering, among other things, the return on alternative assets to money, there naturally arises the question of why the individual has to search out alternative assets. Shouldn't he already know quite a lot about them? At least two answers can be given.

First, if precautionary balances are held as a complement to transactions balances (which we discuss later), as the models of Whalen (1966) and Tsiang (1969) assume, then presumably the only asset the individual knows much about and considers an alternative to cash is the short asset (such as a passbook account) that he regularly uses to store transactions balances for short periods of time.[7] This is not the asset in which he would want to store transitory income; for this purpose, long assets and durable goods would be more appropriate. Because these assets are infrequently used, the individual in general will know little about them when he receives his transitory income and thus will be unprepared to buy or sell particular ones immediately.

Second, if information is costly to obtain, we may assume that the individual solves his loss-minimization problem at discrete points in time rather than continuously. In the interviewing intervals, asset returns may vary both absolutely and relatively. If an infusion of transitory income in the form of money is received during one of these intervals, the individual will be unwilling to commit such a relatively large amount of cash to any particular assets without first finding out what has happened to their rates of return.[8]

In this section of the paper, we present a precise theory of how assets are searched out and how transitory money holdings are worked off.

II.2.A. Behavioral Relations. The individual is assumed to have an objective function which orders the choice set facing him in his attempt to allocate his transitory income among alternative financial assets. The first argument of this objective function is the loss that is experienced whenever actual average money holdings $M \geq 0$ differ from desired holdings M^*. Denote these deviations, or excess money holdings, by $\tilde{M}$,

$$(4) \qquad \tilde{M} \equiv M - M^* .$$

Further, denote the difference between actual cash management loss L and the minimum loss L^* by $\tilde{L}$,

$$(5) \qquad \tilde{L} \equiv L - L^* .$$

After substitution from equations (2) and (3) and some manipulation (See Appendix), $\tilde{L}$ may be written:

$$\tilde{L} = \tilde{M} \; [(r_A - r_M) - \sigma^2 \; c \; \frac{2M^* + \tilde{M}}{[M^*(M^* + \tilde{M})]^2} \;].$$

The first term inside the brackets, the interest rate differential, is the opportunity cost of holding money. The second term is the reduction in expected illiquidity cost that increased money holding brings; the larger the average money holding, the fewer the occasions when unexpected expenditures exceed balances on hand. It can be shown (See Appendix) that, when $\tilde{M} > 0$, the first term inside the brackets exceeds the second and the bracketed term is positive; when $\tilde{M} < 0$, the opposite relation holds and the bracketed term is negative. Consequently, $\tilde{L}$ is always non-negative, i.e.

(5B) $$\tilde{L} = g(\tilde{M}, \ldots) \geq 0,$$

with the partial derivatives:

$$g_1 = (r_A - r_M) - \sigma^2 c \frac{2M^* + 4\tilde{M}}{[M^*(M^*+\tilde{M})]^2} \gtrless 0 \text{ as } \tilde{M} \lessgtr 0$$

$$g_{11} = -\sigma^2 c \left(\frac{-4M^*\tilde{M}}{[M^*(M^*+\tilde{M})]^3}\right) \gtrless 0 \text{ as } \tilde{M} \gtrless 0.$$

In the expression for g_1, the first term is the interest cost of a marginal dollar of money, and the second term is the reduction in excess loss caused by that dollar. Because these derivatives switch sign with $\tilde{M}$, it is convenient to define $\hat{M}$ as the absolute value of $\tilde{M}$,

(6) $$\hat{M} = |\tilde{M}|,$$

and express L as a function of $\hat{M}$:[9]

(5C) $$\tilde{L} = h(\hat{M}, \ldots),$$

$$h_1 > 0, \; h_{11} > 0.$$

Thus whenever actual holdings deviate from desired, there are excess cash management losses.

To eliminate the excess loss $\tilde{L}$, the individual must eliminate excess money holdings $\hat{M}$ by buying or selling assets. In a world of costless information, such adjustments would occur immediately; however, when information is costly and when there are many assets with different characteristics,[10] the individual must spend time looking for appropriate assets to buy or sell. This search time is the second argument of the utility function. The individual derives utility from leisure and therefore attributes disutility to search.

The individual's utility at time t thus is a function of excess loss and search intensity; it is represented by:

$$(7) \qquad V = V[\tilde{L}(t),\ S(t)],$$

where S(t) is the rate of search, measured in hours per unit of time (e.g., hours per week). Using (5c), this function can be rewritten as a function of $\hat{M}(t)$ and S(t); thus utility at time t can be written as

$$(7A) \qquad U = U[\hat{M}(t),\ S(t), \ldots].$$

We assume that V and, therefore, U are additively separable and concave in their arguments and obey the usual Inada conditions, viz.,

$$U_1 < 0,\ U_{11} < 0,$$

$$U_2 < 0,\ U_{22} < 0,$$

$$U_{12} = U_{21} = 0,$$

$$\lim_{\hat{M}\to 0} U_1 = 0,\ \lim_{\hat{M}\to\infty} U_2 = -\infty,$$

$$\lim_{S\to 0} U_2 = 0,\ \lim_{S\to x} U_2 = -\infty,$$

where x is the number of hours in the unit of time (e.g., 168 hours in a week).

The individual can reduce $\hat{M}$ only by buying savings assets. For analytical ease, it will be assumed that these assets all have the same risk-adjusted rate of return, r_A, but have other relevant characteristics that differ among assets.[11] The individual can discover these characteristics only by searching over assets. The harder he searches, the sooner he learns of these characteristics and the faster he can dispose of his excess money balances. Consequently, the behavior of $\hat{M}$ over time is governed by the following equation:[12]

$$(8) \qquad \frac{d\hat{M}}{dt} = f(\hat{M},S) \leq 0,$$

where

$$f(0,S) = 0 = f(\hat{M},0),$$

$$f_1 < 0,\ f_{11} = 0,$$

$$f_2 < 0,\ f_{22} < 0,$$

$$f_{12} = f_{21} < 0.$$

Equation (8) really should be treated as a stochastic differential equation, but we assume the individual takes the expected value of the appropriate stochastic equation and then treats this expected value as the rate of change of $\hat{M}$ that will prevail. Equation (8) represents this expected value and thus is the expected rate of change of $\hat{M}$.

The assumption that $f_{11} = 0$ implies constant returns to scale of $\hat{M}$ in the $d\hat{M}/dt$ process. The assumption that $f_{22} > 0$ implies diminishing returns to search. A simple example of a function satisfying these properties is

(8A) $d\hat{M}/dt = -\alpha_0 \hat{M} S^{\alpha_1}, \; 0 < \alpha_1 < 1,$

which is the standard geometric decay equation modified by search intensity. It also is a generalization of the common partial adjustment equation[13]

(8B) $d\hat{M}/dt = -\psi(\bar{M} - \bar{M}^*),$

with $\psi = \alpha_0 S^{\alpha_1}$. Thus examination of how S depends on various variables will reveal how the usual partial adjustment parameter also depends on those variables. In what follows, the more general specification of equation (8) is used.

II.2.B The Realism of The Behavioral Relationships. We are now ready to solve the individual's optimization problem and thereby derive the optimal speed of adjustment. But before doing that, we wish to comment on two aspects of the foregoing model of the individual's economic decision process.

First, the theory requires a rather strong assumption for there to be an empirically relevant partial adjustment of narrowly - defined money, i.e., M_1: the individual must not convert transitory M_1 balances to a short savings asset for an appreciable length of time. This necessary assumption seems rather unrealistic, but one could argue that for disturbances arising from transitory income an immediate conversion may not happen. Under reasonable assumptions about the time-cost of transfers, for observed magnitudes of interest rates, and for small disturbances, the interest earned during the interval that the transitory income would be on deposit might not justify the cost of a special trip to the savings bank.[14] Note in addition, for the individual, the period over which these transitory balances are held may be fairly short, perhaps a few weeks; but that, when these balances are spent, they become transitory income for someone else who then in turn holds them for a while, and so on. By such a multiplier process, the aggregate period of transitory money holding may become appreciable. Indeed, as discussed in the next section, Darby (1972) finds the period to be several years long. Thus the assumption of slow transfers between M_1 and savings accounts cannot be dismissed out of hand. In any event, our explicit theory makes it clear that, at least within our theoretical framework, this assumption is required for partial adjustment in M_1 demand. The assumption is less of a problem, of course, for M_2 demand. In the empirical work of section III, we examine and compare the partial adjustment equation for M_1 and M_2.

Second, the model implies that excess money balances are disposed of gradually. It might seem more reasonable to assume that they are disposed of all at once after a delay: the individual spends some time searching out assets, selects those he wants, and then expends his excess balances on them, eliminating all excess balances in one grand transaction. In fact, given costly information, the optimality of this once-and-for-all procedure is not obvious. The optimality of a single-purchase strategy depends on transaction costs, distribution functions, etc., and its nature is not obvious *a priori*. Even if the once-and-for-all procedure should prove optimal, aggregation could justify the present framework as describing a "representative" or "average" individual's behavior.

II.2.C. The Optimization Problem. The individual's problem is to maximize his lifetime utility, given by:[15]

(9) $\int_T^D U[\hat{M}(t), S(t), \ldots]dt,$

subject to equation (7) and the initial condition

(10) $\hat{M}(T) = M_T,$

where T is the initial time, D is the time of death, known to the individual, and M_T is the initial value of $\hat{M}$, also known to the individual.[16]

The solution to this problem can be characterized with Pontryagin's maximum principle.[17] The Hamiltonian is

(11) $H = U(\hat{M}, S, \ldots) + \lambda f(\hat{M}, S).$

Optimality requires that the following conditions hold:

(12) $\partial H/\partial S = U_2 + \lambda f_2 = 0$

(13) $d\hat{M}/dt = \partial H/\partial \lambda = f$

(14) $d\lambda/dt = -\partial H/\partial \hat{M} = - U_1 - \lambda f_1$

(15) $\lambda(D) = 0$

and that (10) hold. Because of the concavity of equation (11) in $\hat{M}$, these conditions are also sufficient. Equation (12) states that the marginal benefits and costs of search must be equal. Equation (13) is simply a restatement of (7). The adjoint variable λ, which is negative, is the marginal value of $\hat{M}$, and equation (14) describes its motion over time. Equation (15) is the transversality condition.

Total differentiation of (12) yields the implicit function

$$(16) \qquad S = \underset{(+)}{k(\hat{M}}, \underset{(-)}{\lambda)},$$

where the sign under a variable is the sign of the partial derivative with respect to that variable. Equation (16) describes how S changes as $\hat{M}$ and λ move through time along their optimal paths. From (13), $\hat{M}$ steadily decreases, which, from (16), tends to cause a steady decrease in S. From (14), it is not clear whether λ initially grows or falls; however, from (15), λ eventually must grow to zero. Thus eventually the behavior of λ must reinforce the effects of the behavior of M on S, so that eventually S must fall, reaching zero exactly at time D. Initially, though, λ might fall, perhaps overriding the effect of the falling $\hat{M}$ and causing S to rise for a while.

In any event, the important points are that excess money balances are worked off only gradually and that the speed of adjustment changes over time.

II.2.D. Comparative Statics. We now examine how S responds to certain exogenous shocks. Such shocks change the entire optimal plan, including the function k itself. The individual can be thought of as making a new plan at every instant. If at time (t + dt) all variables have changed as anticipated at time t, then the plan chosen at (t + dt) coincides on the interval [t + dt, D] with that chosen at time t; otherwise, the new plan differs from the old one. It is the nature of this difference that is of interest here.[18]

Consider first an unexpected increase in $\hat{M}$. Such an increase raises the absolute magnitude of f_2 and throws (12) into inequality. It also increases the growth rate of λ, from (14). In response to this latter effect, λ must be made more negative. However, increasing the absolute magnitude of λ in this way exacerbates the inequality of (12). To restore inequality to (12), then, S must be increased. Thus in response to an unexpected increase in $\hat{M}$, S must increase. Exactly the opposite results hold for a decrease in $\hat{M}$.

One particular way that $\hat{M}$ may increase, of course, is through a transitory income shock. Thus S is positively related to transitory income; the larger the transitory income shock, the harder the individual works to eliminate it.

Consider now a change in the interest rate differential $(r_A - r_M)$. Such a change affects the individual's optimal plan by altering $\tilde{L}$ just as changes in $\hat{M}$ do. However, the effect on $\tilde{L}$ and thus on the individual's response depends on whether M or $\tilde{M}$ is held constant. Both cases are interesting. If M, actual cash balances, is held constant, then (See Appendix)

$$(17) \qquad \left.\frac{\partial U}{\partial (r_A - r_M)}\right|_M = \frac{\partial V}{\partial \tilde{L}} \left.\frac{\partial \tilde{L}}{\partial (r_A - r_M)}\right|_M \lesseqgtr 0 \text{ as } \tilde{M} \gtreqless 0.$$

In contrast, if $\tilde{M}$, the difference between actual and desired cash holdings, is held constant, then (See Appendix)

$$(18) \qquad \left.\frac{\partial U}{\partial (r_A - r_M)}\right|_{\tilde{M}} = \frac{\partial V}{\partial \tilde{L}} \left.\frac{\partial \tilde{L}}{\partial (r_A - r_M)}\right|_{\tilde{M}} < 0.$$

The same kind of arguments that were used above to analyze the response of S to changes in $\hat{M}$ can be used here. If M is fixed, then, as shown in the Appendix, an increase in $(r_A - r_M)$ increases $\tilde{L}$ if $\tilde{M} > 0$ and decreases $\tilde{L}$ if $\tilde{M} < 0$. Thus if $(r_A - r_M)$ increases, S increases if $\tilde{M} > 0$ and decreases if $\tilde{M} < 0$. If $\tilde{M}$ is fixed, then an increase in $(r_A - r_M)$ increases $\tilde{L}$ irrespective of the sign of $\tilde{M}$, so that S also increases irrespective of the sign of $\tilde{M}$.

For analysis of individual behavior, (17) is a more useful result than (18) because it seems reasonable to suppose that M rather than $\tilde{M}$ remains fixed when $(r_A - r_M)$ changes. For the aggregate empirical work that follows in section III, however, (18) is the more useful result because it shows how monetary adjustment is affected by changes in $(r_A - r_M)$ for a given amount of monetary disequilibrium.

Turning to the remaining exogenous variables, it is easily shown that the effect of changes in σ^2 and c on utility levels and search intensity is dependent upon the sign of $\tilde{M}$ when M is held fixed but not when $\tilde{M}$ is held fixed. The Appendix demonstrates that:

(19A) $$\left.\frac{\partial U}{\partial \sigma^2}\right|_{M} \gtrless 0 \text{ as } \tilde{M} \gtrless 0,$$

(19B) $$\left.\frac{\partial U}{\partial \sigma^2}\right|_{\tilde{M}} > 0,$$

(20A) $$\left.\frac{\partial U}{\partial c}\right|_{M} \gtrless 0 \text{ as } \tilde{M} \gtrless 0,$$

(20B) $$\left.\frac{\partial U}{\partial c}\right|_{\tilde{M}} > 0.$$

Thus, in response to an increase in either c or b, S decreases if $\tilde{M} > 0$ and decreases if $\tilde{M} < 0$ when M is held fixed, and S always decreases when $\tilde{M}$ is held fixed.

These results are summarized in the following equation:

$$(21) \qquad S = S^E(\underset{(+)}{\hat{M}},\ \underset{(\mathrm{sgn}\,\tilde{M})}{(r_A - r_M)|\bar{M}},\ \underset{(+)}{(r_A - r_M)|\tilde{M}},\ \underset{(-\mathrm{sgn}\,\tilde{M})}{c|M},\ \underset{(-)}{c|\tilde{M}},\ \underset{(-\mathrm{sgn}\,\tilde{M})}{b|M},\ \underset{(-)}{b|\tilde{M}}),$$

where the superscript E is a reminder that these results pertain to exogenous changes in the variables concerned, the symbol x y means "x given y", and sgn is the signum function. In section III, a simplified version of (21) is incorporated into empirical estimates of the demand for money.

<u>II.2.E. Transactions Demand Models</u>. The foregoing results have been obtained under the assumption that equilibrium balances are determined by precautionary motives. It appears that essentially the same results can be obtained using a transactions demand model instead; however, formal proofs are more difficult.

The difficulties arise from an asymmetry in the loss function. Consider first a positive transitory income shock. The individual can either hold the transitory income as cash or make a special transaction to deposit the income in the short savings asset. For there to be partial adjustment, the latter possibility must be ruled out (or at least delayed), in which case the loss associated with the transitory income shock is simply the foregone interest earnings on that income. But now consider a negative transitory income shock. The individual cannot simply "hold" this shock as reduced balances; he must either reduce his consumption, which has a one-to-one relation with money balances in the deterministic transactions model, or make a special transaction out of his savings asset to replenish his cash balances. In either case, the loss is of a different nature than when transitory income is positive. This is the asymmetry that causes trouble.

The asymmetry problem, of course, does not occur if the individual holds both transaction and precautionary balances as in Tsiang (1969). In this case the formulation, while considerably more cumbersome than the model above, is equivalent to the precautionary approach of Section II.1. Preliminary analysis of this and the pure transactions demand case suggest that they produce virtually all the qualitative results obtained for the precautionary demand case. Thus it appears that our results are fairly general.

II.2.F. Implications of the Model. The foregoing model provides a general theory of partial adjustment by money demanders. When an individual receives transitory income, he does not convert it immediately into long term assets because he has incomplete information on the assets available. Instead, he gradually reduces excess money holdings as suitable assets are found for transitory income. Thus the individual's demand for money consists of two parts: a desired level, based on precautionary motives, and a partial adjustment component, reflecting gradual adjustment to transitory income shocks. In the limit, i.e. in steady state, money balances follow the precautionary demand models previously developed in the literature. However, outside of this position, money balances include the pure precautionary balances plus the balances associated with previous transitory shocks to the income stream. The latter component decreases with time and can be modelled by a decaying lag on previous economic disturbances.

The speed with which the individual reduces excess money holdings and approaches equilibrium changes with the time elapsed since the initial disturbance, as shown in (16), and varies in response to exogenous shocks to important economic variables, as shown in (21). Thus if disturbances to money holdings arising from transitory income are modelled as a distributed lag on transitory income, then the properties of that lag should change as the independent variables in (21) change. In particular, as $\hat{M}$ and $(r_A - r_M)\tilde{M}$ increase, the weights on the later terms of the lag should decrease, indicating faster adjustment. For increases in $(r_A - r_M)$, the length of the lag may shorten, too. For increases in $\hat{M}$, the change in the length of the lag is less clear. The increase in S would tend to shorten the lag, but the increase in $\hat{M}$ itself tends to lengthen the lag.

Although the model has been formulated in terms of money and money demand, it apparently could be generalized quite easily for use in discussing partial adjustment in the demand for any asset. Indeed, as suggested by the discussion in section II.2.B above, it may be that the model should only be used for a broader monetary aggregate. Because this is primarily an empirical issue, we now attempt the estimation of a simplified empirical version of the model using both the M_1 and M_2 definitions of money balances.

III. ESTIMATION

III.1. Estimating a Simplified Model. The foregoing theory suggests that the demand for money function consists of two parts, one part being the equilibrium money demand and the other part being excess holdings resulting from partial adjustment to past transitory income shocks. Consequently, an additive functional form for current real money demand seems appropriate:

$$(22) \qquad M_t^D = M^*(Y_{Pt}, (r_A - r_M)_t, c_t) + M^T (Y_{Tt}, (r_A - r_M)_t, c_t),$$

where M_t^D is real short run money demand, Y_P is real permanent income, M^T is real transitory balances resulting from partial adjustment, and Y_T is real transitory income.

Because each term on the right side of equation (22) is nonlinear in the independent variables, this additive form is not amenable to the usual linear estimation techniques. Consequently, the following log-linear approximation is posited:

$$(22A) \qquad M_t^D = M_t^*(1 + D),$$

where the disequilibrium term D is a distributed lag on past monetary disturbances.[19]

Disturbances arise from variations in income from its expected, or permanent, level. This could occur for two relatively independent reasons. First, deviations of demand for output will result in income levels different from permanant income. Accordingly one element of unexpected income will deviations of observed income from its expected value. Second, unexpected money supply changes can alter income. This point has been emphasized recently by Carr and Darby (1978) and Laidler (1979). The argument is that unexpected money balances which are placed in the hands of the public will be treated as an element of unexpected income. The notion that changes in money supply should be treated as a component of income dates back to Tobin (1965). Here we use the unexpected component as the determinant of short run money demand. Combining these two elements D will be distributed lag on deviations of income from its expected value:

$$D = \sum_{j=0}^{N} Z^j \left[\frac{(Y_{t-j} - Y_{Pt-j}) + (M^S_{t-j} - M^S_{t-1-j})}{Y_{Pt-j}} \right]$$

where Y_t is current real income, M^S_t is the real money supply, and the parameters N and Z must be found by estimation. The first term inside the parentheses is a standard measure of transitory income converted to proportional terms. The second element captures the change in real money supply. If nominal money supply changes are anticipated, price adjustment will occur so as to maintain real money at its long run level, assuming the demand for money is stable (an assumption with considerable empirical support). If, however, nominal money supply changes in an unexpected manner, prices will not adjust immediately and the real supply will differ from its long run level. The second term captures this unexpected supply effect. This functional form for D assumes that all transitory income shocks, irrespective of source, have the same impact on money demand.[20] A more refined analysis would differentiate sources and effects but also would require a considerably more complex model. As it is, this approach is more general than those reported by Carr and Darby (1978) and Laidler (1979) in that it includes both real and monetary disturbances effects on the demand for money.[21]

Thus the proposed equation to be estimated is

$$(23)\quad M^D_t = \alpha Y^{\beta}_{Pt}(r_{1t} - r_{Mt})^{\gamma_1} \dots (r_{mt} - r_{Mt})^{\gamma_m}\left[1 + \sum_{j=0}^{N} Z^j \left(\frac{(Y_{t-j} - Y_{Pt-j}) + (M^S_{t-j} - M^S_{t-1-j})}{Y_{Pt-j}}\right)\right]^{\delta}$$

where r_i is the nominal rate of return on the ith asset and the following parameter signs are expected:

$$\alpha, \beta, \delta, > 0,$$

$$\gamma_1, \ldots \gamma_m < 0.$$

In the absence of the last term, (23) would be a standard form of the money demand function. The distributed lag term on transitory income attempts to capture the partial adjustment to past income disturbances that Darby (1972) calls the shock absorber effect. According to the theory of the previous section, the shape, length, and perhaps even elasticity (represented by Z, N, and δ, respectively) of the distributed lag term should depend on all the interest rate differentials $(r_i - r_M)$ and the magnitude of transitory income. A term with these properties would be difficult to formulate and estimate, so the simple form in (23) is used. Given this formulation, the null hypothesis on the lag elasticity is that $\delta < 1$, making the shock absorber term concave in transitory income. In this case, changes in transitory income have a less than proportional effect on disequilibrium money balances implying that more effort is devoted to quickly (i.e., within the current period) disposing of large shocks than to disposing of small ones, as implied by the theory.

III.2. Results for M_1. In estimating (23) it will be assumed that price movement is sufficiently rapid to eliminate deviations between demand and supply of money within the quarter. Therefore, real money balances may be used as a proxy for the short run demand for money. The rate of return on money r_M is set equal to zero; estimates including non-zero remission rates as a positive return to money balances proved unsatisfactory.[22] As the rates of return on alternative financial assets two interest rates are used, the 4-6 month commercial paper rate and the commercial bank passbook rate. This is done to facilitate comparison with other demand for money functions, most notably Goldfeld (1973). All data is quarterly and is from the MPS model data bank obtained from the Federal Reserve Bank of Philadelphia.[23]

Estimation is by the Cochrane-Orcutt technique; OLS results suggested severe autocorrelation of the residuals. The parameters N and Z are found by iterative search to maximize the adjusted R^2. The results for the period 1952II - 1972IV are, in log form,[24]

$$\text{(24)} \qquad \log M_t^D = \underset{(0.49)}{2.08} + \underset{(0.075)}{0.47} \log Y_{Pt} - \underset{(0.005)}{0.014} \log RCP_t - \underset{(0.02)}{0.051} \log RCBP_t$$

$$+ \underset{(0.04)}{0.32} \log \left[1 + \sum_{j=0}^{1} (0.6)^j \frac{Y_{t-j} - Y_{Pt-j}}{Y_{Pt-j}}\right]$$

$$\bar{R}^2 = 0.9924, \ SSQR = 0.001888, \ \rho = 0.967, \ D.W. = 1.2,$$

where RCP and RCBP are the rates of return on 4-6 month commercial paper and on time deposits.[25] Standard errors are in parentheses below the estimated coefficients. All variables are significant.

The Durbin-Watson statistic is still low, even after first order correction, suggesting higher order serial correlation of errors. This would be expected with an independent variable constructed from a distributed lag series. Examination of the serial pattern of the errors strongly suggested a second order process, so Durbin's method for second order correction was applied. The resultant coefficients were virtually the same as those reported in (24), including an almost-unchanged Durbin-Watson statistic. Thus, it appears that higher order correlation than a second order process is present in the data. Given this result, we decided to accept the results even though the standard errors may be underestimated. It should be recalled that, even though the reported standard errors may be too small, the point estimates themselves are unbiased. Also, as discussed below, these estimates are close to those of Goldfeld (1973) and Lieberman (1977b), both of whom obtained acceptably high Durbin-Watson statistics.

The significance of the coefficient on the distributed lag supports the existence of partial adjustment in general and of Darby's shock absorber effect in particular. However, the shock absorber term is short-lived, lasting only two quarters; adding additional terms to the distributed lag worsened the fit. Because $\delta = 0.32 < 1$, the shock abosrber term is concave in transitory income in accordance with the null hypothesis that large transitory shocks will be disposed of more quickly than small ones. Interestingly, the shock absorber effect is of fairly small magnitude, even at its largest. Transitory income is never more than 10 percent of permanent income (and only rarely more than 6 percent). Even if transitory income were as large as 10 percent of permanent income two quarters in a row, the total percentage change in money balances in the first period is 3.1%. Next period money balances rise by an additional 1.8%, so that the total shock absorber effect would increase the demand for money by only 4.9 percent.

The coefficient on permanent income is much smaller than one, suggesting large economies of scale in money holding. This finding is at variance with the results of many other money demand studies, although it agrees with several more recent contributions, such as Goldfeld (1973) and Lieberman (1977b). One interpretation of current models using transactions and precautionary demand theory is that the income elasticity should be less than one. However, if the expected illiquidity cost is the time involved in restoring liquidity or is proportional to total expenditures, then the income elasticity should be closer to unity for households and firms The results reported here suggest that other sorts of costs may be pertinent or perhaps that technological innovation which is essentially a trend variable and therefore correlated with income over time, is reducing the coefficient on income.[26]

The coefficient on RCBP is interesting. Its value of 0.05 is one-third the 0.15 value found in most earlier money demand studies [Laidler (1969)]. This reduction in the short interest rate elasticity stems entirely from serial correlation correction. In the uncorrected OLS fit, the RCBP elasticity was the usual 0.15. Lieberman (1977b) obtains similar large reductions in interest rate elasticities by correcting serial correlation in earlier studies.

III.3. Comparison with Darby's Results. Several of the findings differ considerably from Darby's (1972) original shock absorber results. Most important are the differences in the magnitude and duration of the shock absorber. Darby reports that

> [t]ransitory income increases the demand for money by about 40 percent of the amount of transitory income, and transitory money balances are worked off at a rate of about 20 percent per quarter. (p.934)

The implication of Darby's results is that if transitory income were 10 percent of parmanent income, money demand would increase by 20 percent, even ignoring the effects of previous quarter's transitory income. As mentioned above, the results of the present study indicate that, when transitory income is 10 percent of parmanent income, the total increase in money demand is 4.5 percent. Thus Darby's results indicate a much stronger shock absorber effect than we find.[27] Also, Darby's 20 percent decay rate for transitory money balances indicates a much longer duration for the shock absorber than found in this study. According to Darby's results, it takes 11 quarters for transitory money balances to fall below 10 percent of their original level, whereas we find that transitory balances have completely disappeared within 2 quarters. Our finding of so short a duration for the shock absorber effect is especially important in light of Barro's (1978) paper which relies on Darby's long-lasting effect to explain price behavior. Other differences between Darby's results and those reported here concern the income elasticity, which Darby finds to be one (double ours), and the short interest rate elasticity, which Darby finds to be 0.026 (half ours).

The major source of difference apparently is the form of the estimating equation. Darby's equation is linear, whereas ours is log linear.[28] Because the latter follows from the equilibrium behavior of cash balances and has been used extensively in the literature, we favor the log form.

III.4. Comparsion with Goldfeld's Results. It is informative to compare the present results with those of Goldfeld (1973). Goldfeld obtains the following estimate for the period 1952II - 1972IV:

$$\text{(25)} \qquad \log M_t^D = 0.271 + 0.193 \log Y_t + 0.717 \log M_{t-1} - 0.019 \log RCP_t - 0.045 \log RCBP_t .$$

$$\bar{R}^2 = 0.995,\ SSR = 0.001197,\ \rho = 0.472,\ DW = 1.73$$

Note that Goldfeld uses current rather than permanent income and includes lagged money as an explanatory variable. There are two possible reasons for including these variables. One reason is partial adjustment. It may be that people have a desired level of money balances, dependent on current income. When displaced from this desired level, they return to it only gradually because of adjustment costs. One might postulate the following model, as did Goldfeld:

(26) $$\log M_t^* = \alpha + \beta \log Y_t + \gamma_1 \log RCP_t + \gamma_2 \log RCBP_t,$$

(27) $$\log M_t^D = \log M_{t-1} + \lambda(\log M_t^* - \log M_{t-1}),$$

where M^* is desired money balances. Substituting the first equation into the second yields

(28) $$\log M_t^D = \lambda\alpha + \lambda\beta \log Y_t + \lambda\gamma_1 \log RCP_t + \lambda\gamma_2 \log RCBP_t + (1 - \lambda) \log M_{t-1},$$

which is the form estimated in equation (25).

The second possible reason for inclusion of Y_t and M_{t-1} in (25) is that money demand depends on permanent income, which is measured as a distributed lag on past incomes levels. Thus the underlying model in this case consists of two equations:

(29) $$\log M_t^D = \alpha + \beta \log Y_{Pt} + \gamma_1 \log RCP_t + \gamma_2 \log RCBP_t$$

(30) $$\log Y_{Pt} = \log Y_{Pt-1} + \lambda(\log Y_t - \log Y_{Pt-1}).$$

Applying the Koyck transformation then yields the estimating equation

(31) $$\log M_t^D = \lambda\alpha + \lambda\beta \log Y_t + \gamma_1 \log RCP_t - (1 - \lambda)\gamma_1 \log RCP_{t-1} + \gamma_2 \log RCBP_t - (1-\lambda)\gamma_2 \log RCBP_{t-1} + (1 - \lambda) \log M_{t-1}.$$

Equation (25) can be considered a version of (31) in which the lagged interest rate terms erroneously have been omitted.[29]

The interpretation of the estimated coefficients in (25) depends on whether permanent income or partial adjustment is considered the reason for including lagged money. In the permanent income case, the estimated coefficients on the interest rates are the true interest rate elasticities themselves, plus whatever bias results from the omission of the lagged correlated interest rate terms. If this bias is small and permanent income is the appropriate reported independent variable, Goldfeld's results agree quite closely with those in (24). In the partial adjustment case, the estimated coefficients in (25) must be divided by λ to give the interest rate elasticities, as can be seen from (28). The RCP and RCBP elasticities implied by (25) are then 0.067 and 0.159, the latter being very close to the value often found for short rates [Laidler (1969)].

Goldfeld uses the partial adjustment model and ignores permanent income; consequently, Goldfeld would accept the larger interest elasticities gotten by dividing the estimates in (25) by λ. The present study suggests that permanent income and partial adjustment are both relevant but that, if a model like (25) is used, it would be more nearly correct to interpret it as a permanent income model. First, the significance of the shock absorber term in (24) clearly means that there is partial adjustment. However, this partial adjustment term is very small and dies out very quickly, so that, if the shock absorber were the only source of partial adjustment then λ in (25) would be close to one, which it is not. Of course, there are other possible sources of monetary disequilibrium, notably interest rate changes, to which individuals respond only partially in any given period; but there seems no reason to believe the transactions costs to making these adjustments are substantially different than those in responding to transitory income shocks, so there seems little reason to believe that other partial adjustments will be any slower or longer lasting. Second, when equation (23) is estimated without the distributed lag term but including permanent income, one gets elasticities very similar to those in (29), which are very close to those obtained by Goldfeld. The implication is that Goldfeld's equation would be interpreted more appropriately as deriving from the permanent income rather than the partial adjustment model.[30]

III.4. Results for M2. As remarked in section II.2.B, for our model to permit empirically significant partial adjustment of M_1 demand, one must make the strong and perhaps unrealistic assumption that the individual does not transfer transitory balances to a short savings asset quickly. Our empirical results suggest that this assumption may not hold. The assumption obviously is less important for partial adjustment of M_2 demand, so one might expect to see slower adjustment of these balances to transitory income shocks. Interestingly, the partial adjustment results are quite close to those for M_1. In particular, over the period 1952II-1976III, our estimate of the short run demand for M_2 balances is

$$(33)\ \log M_t^D = \underset{(0.27)}{-0.318} + \underset{(0.05)}{0.99} \log Y_{Pt} - \underset{(0.005)}{0.03} \log RCP_t - \underset{(0.028)}{0.044} \log RCBP_t$$

$$+ \underset{(0.04)}{0.37} \log \left[1 + \sum_{j=0}^{2} (0.5)^j \frac{Y_{t-j} - Y_{Pt-j}}{Y_{Pt-j}}\right]$$

$$\bar{R}^2 = 0.9991,\ SSQR = 0.003506,\ \rho = 0.932,\ D.W. = 1.08$$

All variables are significant. The parameters of disequilibrium term are fairly close to the values obtained for M_1, implying a small shock absorber effect of short duration (three quarters in this case instead of two). If transitory income is as high as 10 percent of permanent income three quarters in a row, the anti-log of the disequilibrium term is 1.061, implying a 6.1 percent increase in money demand.

IV. CONCLUSIONS

In this paper we have constructed a general model of partial adjustment by money demanders. The major implications of the theory are that partial adjustment is theoretically reasonable and that the speed of adjustment is not constant but changes over time and in response to exogenous shocks, as summarized in equations (16) and (19).

Empirical examination of a simplified version of the theoretical model finds support for the theory. Transitory disturbances have a less than proportional effect on money demand, suggesting that more effort is spent on correcting large shocks than is spent on small shocks, as implied by the theory.

Partial adjustment of both M_1 and M_2 money demand is found to be statistically significant but to have a small effect that dies away within two or three quarters. These results conflict with those of Darby, who found a stronger and longer-lasting effect, but agree with those of Feige (1967), who found that partial adjustment was an insignificant influence in annual models of money demand. Comparison of these results with Goldfeld and with results obtained by omitting partial adjustment from the reported equation suggest that partial adjustment is not a very important force even in quarterly models of money demand. These results invalidate Barro's (1978) explanation of long lags in the response of prices to unanticipated money growth, which relied on Darby's long-lasting shock absorber effect.

FOOTNOTES

[1]See Miller and Orr (1966), Barro and Santomero (1972), Santomero (1974) on transactions models and Whalen (1966) and Tsiang (1969) on precautionary models.

[2]See Seater (1979) for additional theoretical criticisms of (1).

[3]No test similar to Feige's has been done on quarterly data, for the U.S. However, Laidler and Parkin (1970) do interpret their results using quarterly data for Britian as supporting the Feige conclusion. Also, Frenkel (1977, 1979), using different methods and monthly data, finds almost full adjustment within one quarter for hyperinflation Germany.

[4]In an earlier version of their paper, we assumed that equilibrium balances were determined according to transactions demand theory. That approach led to results essentially the same as those reported in the present paper, but because of certain mathematical difficulties, the transactions approach is more cumbersome to treat than the precautionary approach. Later in the paper we will comment further on the transactions approach.

[5]In fact, equation (2) should be solved simultaneously with the unspecified equations determining income and consumption. Such a solution is fairly difficult to carry out, so the simplifying assumption of sequential problems, frequently encountered in money demand models, is adopted here.

[6]The term disequilibrium has at least two meanings, only one of which is relevant here. One meaning, the one <u>not</u> relevant to the present paper, is that supply does not equal demand because prices are slow to clear the market. The other meaning, which we are using, is that, even though the market may be clearing, it has not reached its steady state. In particular, we assume that, because of adjustment costs, the individual chooses not to move his money balances immediately to their steady state value so that a meaningful distinction between short and long run money demand can be made. We model short run money demand and its convergence to its long run or steady state value. We do not explain how real balances themselves move from one level to another (which would require analysis of price dynamics); rather, we explain how the <u>demand</u> for real balances moves.

[7]Santomero (1974) has shown that there is in fact only one such asset in the standard transactions demand framework.

[8]The possibility that transitory income is shifted to a short asset such as a passbook account for storage is treated both theoretically and empirically below. Essential to the scenario in the text is the assumption that individuals do not allocate transitory balances immediately to their optimal long run positions.

[9]Equation (5C) can be used for qualitative purposes only. Quantitatively, it would be misleading because $\tilde{L}$ is not symmetric in $\tilde{M}$. Positive values of $\tilde{M}$ cause larger losses than negative values do. (However, for small values of $\hat{M}$, this asymmetry is small.)

[10]The agent may buy either a real or a financial asset. A dispersion of characteristics are easy to imagine for durable goods: horsepower, color, landscaping, etc. For financial assets, such things as risk, maturity, and income vs. growth potential seem important.

[11]If assets had different rates of return, then the excess loss function would be

$$\tilde{L} = \tilde{M}\left[\sum_{i=1}^{m} (r_{Ai} - r_M)\delta_i - \sigma^2 c \frac{2M^* + \tilde{M}}{[2M^*(M^* + \tilde{M})]^2} \right],$$

where m is the number of alternative savings assets, r_{Ai} is the return on the ith such asset, and δ_i is the fraction of excess money $\tilde{M}$ going into (or coming out of) the ith asset. Presumably, each δ_i would depend on economic variables, most notably all the interest differentials. Assuming identical interest rates appears to impose no important qualitative restrictions but does result in considerable mathematical simplification.

[12]The explicit relation between the distribution of asset characteristics and both S and $d\hat{M}/dt$ can be derived as in Seater (1977). Such a derivation is not carried out here.

Note that an aspect of adjustment is ignored in eq. (8) and again in the optimization solution of section II.2.C below. As shown later, an increase $\tilde{L}$ increases S, which increases the depletion of $\hat{M}$, as shown in (8). In addition, an increase in $\tilde{L}$ also leads the individual to settle for a lower value of the characteristics being searched out. This latter element also would increase the depletion of $\hat{M}$ but is held fixed in (8) instead of being determined as part of the optimization in section II.2C. For most of the effects considered in this paper, the qualitative results appear to be unaffected by this simplification.

[13]Because $\bar{M} = \tilde{M} + \bar{M}^*$,

$$d\bar{M}/dt = d\tilde{M}/dt + d\bar{M}^*/dt$$
$$= d\tilde{M}/dt,$$

so that, by the partial adjustment equation,

$$d\tilde{M}/dt = -\psi(\bar{M}-\bar{M}^*)$$
$$= -\psi\tilde{M}.$$

But $\hat{M} = \pm\tilde{M}$ as $\tilde{M} \gtrless 0$, so that

$$d\hat{M}/dt = \pm d\tilde{M}/dt \text{ as } \tilde{M} \gtrless 0$$
$$= \pm(-\psi\tilde{M})$$
$$= -\psi\hat{M}.$$

[14]For a detailed discussion and example of this case, which is covered in an earlier version of this paper, contact the authors.

[15]Insertion into (9) of the usual discount factor $e^{-\delta t}$ does not change the results substantially. However, when the time horizon is known and finite, the discount rate neither is needed to guarantee existence of the integral nor has the natural interpretation that arises when the time horizon is infinite. Consequently, we omit it.

[16]It is somewhat inaccurate to solve the individual's search problem in isolation from his consumption and equilibrium cash balances problems. The correct procedure would be to solve everything simultaneously, but this general approach is simply too difficult. We thus proceed sequentially. This issue already has been discussed in footnote 5.

[17]Solving the problem this way treats (4) as a deterministic equation. In fact, as Seater (1977) shows, it is stochastic so that the optimization problem should be solved with stochastic control methods. Our use of deterministic control amounts to imposing certainty equivalence and ignores interesting stochastic aspects of the problem, such as behavior toward risk.

[18]Examining the effects of unanticipated shocks gives some insight into the stochastic aspects of the problem that were alluded to in Note 12.

[19]Equation (22A) assumes that all disequilibrium in money balances is a result of transitory income. Further it assumes that M_t^T is proportional to M_t^* and that their elasticity with respect to exogenous variables is likewise proportional. As such, it is only a crude approximation of the general theoretical model of Section II.

[20]One particular source of transitory income that could be especially troublesome is random shifts in money demand. However, a number of studies have shown the money demand function to be stable so that this source may be ignored.

[21]We have examined the model with disturbances occuring only as a result of deviations of income from its permanent level; our results were essentially the same or those reported below.

[22]Three series on this implicit rate of return on money are offered in the literature, viz, Barro and Santomero (1972), Klein (1974), and Becker (1975). See Santomero (1979) for a detailed analysis of the characteristics and quality of each series.

[23]Permanent income is generated by the usual ad hoc equation:

$$Y_{Pt} = a\ Y_t + (1-a)(1+b)Y_{Pt-1},$$

where a = 0.026 is taken from Darby (1974) and b = 0.0089 is the quarterly growth rate of real GNP, found by regressing Y_t on time:

$$\log Y_t = c + b\ \text{TIME}.$$

[24]Attempts to extend the model beyond 1972 failed because of the well-known shift in money demand that occurred around 1973. Investigation of this shift produced results no different from those of Goldfeld (1976), Carr and Darby (1978), or virtually any other study of the post-1972 period.

[25]Note that our dependent variable is real balances. Were we trying to analyze the movement of real balances themselves, we would have to analyze price dynamics. However, we are analyzing only the demand for real balances, which does not require analysis of price dynamics.

[26]Lieberman (1977a) finds that time, when entered as an independent variable to capture changes in technology, has a significant negative coefficient but apparently of a magnitude too small to explain our failure to obtain a unitary elasticity on income. Barro and Santomero (1972) find evidence of a larger technological effect but still apparently not large enough to account for our small income elasticity.

[27]Failure by Darby to correct for serial correlation apparently is not the source of these differences, for Darby checks his results using a generalzed least squares estimate.

[28]There is some difference in estimation periods as well. Darby ends his sample at 1966IV whereas our sample period ends with 1972IV.

[29]A direct test of whether (28) or (31) is the correct form is precluded because of the high serial correlation of quarterly interest rates.

[30]One caution here is that our estimate of ρ in (24) is almost unity. A possible reason for this result is that lagged M should be present, perhaps for partial adjustment reasons, so that our results might understate the extent partial adjustment. Further research, perhaps along the lines of those followed by Feige (1967), would be worthwhile.

BIBLIOGRAPHY

1. Barro, Robert J. (1974), "Review," Journal of Money, Credit, and Banking 6 (February), pp. 125-140.

2. Barro, Robert J. (1978), "Unanticipated Money, Output, and the Price Level in the United States," Journal of Political Economy 86 (August), pp. 549-580.

3. Barro, Robert J. and Santomero, Anthony M. (1972) "Household Money Holdings and the Demand Deposit Rate," Journal of Money, Credit and Banking 4 (May), pp. 397-413.

4. Baumol, William J. (1952), "The Transactions Demand for Cash: An Inventory Theoretic Approach," Quarterly Journal of Economics 66 (November), pp. 545-556.

5. Becker, William J. (1975), "Determinants of the United States Currency - Demand Deposit Ratio," Journal of Finance (March), pp. 57-74.

6. Carr, Jack and Darby, Michael R. (1978),"The Role of Money Supply Shocks in the Short-Run Demand for Money," National Bureau of Economic Research discussion paper number 98.

7. Clower, Roert A. (1967), "A Reconsideration of the Micro Foundations of Monetary Theory," Western Economic Journal (December), pp. 1-8.

8. Clower, Robert "Introduction" (1970), Monetary Theory, R. Clower editor.

9. Darby, Michael R. (1972), "The Allocation of Transitory Income Among Consumers' Assets," American Economic Review 62 (December), pp. 928-941.

10. Darby, Michael R. (1974), "The Permanent Income Theory of Consumption - A Restatement," Quarterly Journal of Economics 88 (May), pp. 228-250.

11. Feige, Edgar L. (1967), "Expectations and Adjustments in the Monetary Sector," American Economic Review 57 (May), pp. 462-473.

12. Frenkel, Jacob (1977), "The Forward Exchange Rate, Expectations, and the Demand for money: the German Hyperinflation," American Economic Review, 67(September), pp. 653-670.

13. Frenkel, Jacob (1979), "Further Evidence on Expectations and the Demand for Money During the German Hyperinflation," Journal of Monetary Economics 5 (January), pp. 81-96.

14. Goldfeld, Stephen M. (1973), "The Demand for Money Revisited," Brookings Papers on Economic Activity 3, pp. 577-646.

15. _______________, (1976), "The Case of the Missing Money," Brookings Papers Economic Activity 3, pp. 683-789

16. Klein, Benjamin (1974), "Competitive Interest Payments on Bank Deposits and the Long-Run Demand for Money," American Economic Review 64 (December), pp. 931-949.

17. Laidler, David E. W. (1969), "The Demand for Money: Theories and Evidence (Scranton: International Textbook Co.).

18. Laidler, David (1979), "The Demand and Supply of Money Yet Again" Carnegie-Rochester Conference paper, April

19. Laidler, David E. W. and Parkin, Michael (1970), "The Demand for Money In the United Kingdom 1956-1967: Preliminary Estimates" The Manchester School, pp. 187-205.

20. Leiberman, Charles (1977a), "The Transactions Demand for Money and Technological Change," Review of Economics and Statistics (August), pp. 307-317.

21. Lieberman, Charles (1977b), "Chow's Long Run and Short Run Demand for Money: An Extension," presented before the Southern Economic Association, New Orleans.

22. Meltzer, Allan H. (1963), "The Demand for Money: A Cross-Section Study of Business Firms," Quarterly Journal of Economics 77 (August), pp. 405-421.

23. Miller, Merton H. and Orr, Daniel (1966), "A Model of the Demand for Money by Firms," Quarterly Journal of Economics 80 (August), pp. 413-435.

24. Santomero, Anthony M. (1974) "A Model of the Demand for Money by Households," Journal of Finance 29 (March), pp. 89-102.

25. Santomero, Anthony M., (1979) "The Role of Transaction Costs and Rates of Return on the Demand Deposit Decision" Journal of Monetary Economics, forthcoming

26. Seater, John J. (1977), "A Unified Model of Consumption, Labor Supply, and Job Search," Journal of Economic Theory 14 (April), pp. 349-372.

27. Seater, John J. (1979), "The Effects of Monetary Operations in the Presence of Accurate Perceptions, Rationality, and Costly Adjustment," Research paper #37, Federal Reserve Bank of Philadelphia.

28. Tobin, James (1956), "The Interest Elasticity of the Transactions Demand for Cash," Review of Economics and Statistics 38 (August), pp. 241-247.

29. Tobin, James (1965), "Money and Economic Growth" Econometrica (October)

30. Tsiang, S.C. (1969), "The Precautionary Demand for Money: An Inventory Theoretical Analysis," Journal of Political Economy 77 (January), pp. 99-117.

31. Whalen, Edgar L. (1966), "A Rationalization of the Precautionary Demand for Cash, "Quarterly Journal of Economics 80 (May), pp. 314-324.

APPENDIX

The loss function to be minimized in determining equilibrium precautionary balances is

$$L = M\ (r_A - r_M) + \frac{\sigma^2 c}{M}.$$

From the definitions of $\tilde{L}$ and L^*:

$$\text{(A.1)} \qquad \tilde{L} = \tilde{M}[(r_A - r_M) - \sigma^2 c \frac{2M^*+\tilde{M}}{[M^*(M+\tilde{M})]^2}].$$

Note that maximization of L requires

$$\frac{\partial L}{\partial M} = (r_A - r_M) - \frac{2\sigma^2 c}{M^3} = 0.$$

Thus, the sign of the bracketed term in (A.1) equals sign $\tilde{M}$.

For $\tilde{M}$ small relative to M^*, $\tilde{L}$ has the following derivatives:

$$\text{(A.2)} \qquad \frac{\partial \tilde{L}}{\partial \tilde{M}} = [(r_A - r_M) - \sigma^2 c \frac{2M^* + 4M}{[M^*(M^*+\tilde{M})]^2}] \gtrless 0 \text{ as } \tilde{M} \gtrless 0.$$

$$\text{(A.3)} \qquad \left.\frac{\partial \tilde{L}}{\partial (r_A - r_M)}\right|_M = - \frac{\partial M^*}{\partial (r_A - r_M)} [r_A - r_M) - \sigma^2 c \frac{2M^*+\tilde{M}}{[M^*(M^*+\tilde{M})]^2}] + \tilde{M}\ [1 + \frac{2M^{*2}+2M^*\tilde{M}-M^*}{6M^{*2}+12M^*\tilde{M}+6\tilde{M}^2}]$$

$$\text{(A.4)} \qquad \left.\frac{\partial \tilde{L}}{\partial (r_A - r_M)}\right|_{\tilde{M}} = \tilde{M}\ [1 - \frac{M^{*3}+M^{*2}\tilde{M}+(\frac{1}{3})\tilde{M}^2 M^*}{(M^*+\tilde{M})^3}] > 0.$$

$$\text{(A.5)} \qquad \left.\frac{\partial \tilde{L}}{\partial \sigma^2}\right|_M = - \frac{\partial M^*}{\partial \sigma^2} [(r_A - r_M) - \sigma^2 c \frac{2M^*+\tilde{M}}{[M^*(M^*+\tilde{M})]^2}] - \frac{c\tilde{M}}{[M^*(M^*+\tilde{M})]^2} [2M^*+M-2\frac{M^*+M}{M^*M^2}]$$

$$\lessgtr 0 \text{ as } \tilde{M} \gtrless 0.$$

(A.6) $$\left.\frac{\partial\tilde{L}}{\partial\sigma^2}\right|_{\tilde{M}} = - \frac{\tilde{M}^2 c}{[M^*(M^*+\tilde{M})]^2} \left(\frac{M^*+(\frac{1}{3})\tilde{M}}{M^*+\tilde{M}}\right) < 0.$$

(A.7) $$\left.\frac{\partial\tilde{L}}{\partial c}\right|_{M} =- \frac{\partial M^*}{\partial c} \left[(r_A - r_M) -\sigma^2 c \frac{2M^*+\tilde{M}}{[M^*(M^*+\tilde{M})]^2}\right]- \frac{\sigma^2\tilde{M}}{[M^*(M^*+\tilde{M})]^2}\left[2M^*+M-2 \frac{M^*+M}{M^*M^2}\right]$$

$$\lessgtr 0 \text{ as } \tilde{M} \gtrless 0.$$

(A.8) $$\left.\frac{\partial\tilde{L}}{\partial c}\right|_{\tilde{M}} =- \frac{\tilde{M}^2 \sigma^2}{[M^*(M^*+\tilde{M})]^2} \left(\frac{M^*+(\frac{1}{3})\tilde{M}}{M^*+\tilde{M}}\right) < 0.$$