GENERAL EQUILIBRIUM WITH FINANCIAL MARKETS: EXISTENCE, UNIQUENESS, AND IMPLICATIONS FOR CORPORATE FINANCE

By

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In this paper we set out a general equilibrium model with financial markets. To our knowledge, it is the first model of this type which takes as a starting point the prices of securities and bonds;[1] we shall show that a consistent and logical theory of the firm may be delineated in the context of the model. We shall thus provide the first general equilibrium examination of some critical issues in corporate finance. In the course of this paper we accomplish three objectives:

1. We are able to show how a general equilibrium with financial markets of the kind we describe (a description which roughly fits that of "real world" financial markets) relates to Arrow-type markets of the kind discussed in [1]. In particular we establish a basic equivalence theorem for a general equilibrium with financial markets and Arrow markets with state-dependent prices.

2. We establish the existence of an equilibrium in a general equilibrium model with financial markets.

3. We discuss the two famous results of Modigliani and Miller [7,8] in the context of our markets. While the MM dividend irrelevance theorem is true--though only in a specific sense-the MM capital structure irrelevance theorem is shown to be true only rarely. Specifically, the debt/equity ratio is fixed in equilibrium in the presence of the possibility of firm bankruptcy, no matter what the probability of such a bankruptcy, and without any attendant non-perfect market phenomen such as bankruptcy costs or secured debt.

While some of these results have been discussed previously, in less general contexts,[2] most of them are new.

Starting with a two-period, one-commodity model (this latter assumption is merely simplifying and is not necessary to the results of the paper), we outline the nature of production: Firms purchase inputs today in order to produce from them tomorrow, but production tomorrow from today's inputs is uncertain, as it depends on the state of nature at the time of production. This state of nature affects the production function. To finance today's purchases of inputs, each firm may sell either new equity or bonds. The former merely dilutes the earnings of equity holders in the second period. Bonds, on the other hand, have a prior claim on firm income in the second period: If the income from production suffices to cover repayment of the bonds (which are sold in the first period at a discount from their nominal value), then the remainder of production income accrues to those who purchased shares of the firm in the first period. Otherwise the firm is bankrupt, and all production income belongs to the bondholders.

After setting out the general structure of our model in sections 1-3, we discuss both consumer and firm maximization in sections 4 and 5. Consumer choice leads to a set of implicit prices which may be used by firms in maximizing the returns of initial shareholders (section 5). In section 6 we define equilibrium and in section 7 we prove a basic equivalence theorem: In every market it is possible to derive state prices so that firms maximize net profits subject to those prices; thus in some respects a general equilibrium with financial markets is similar to an Arrow equilibrium. The equivalence theorem allows us to give simple proofs that the MM irrelevance theorem on debt/equity ratios is not true in the presence of bankruptcy (section 8). Finally, in section 9, we prove that a general equilibrium with financial markets indeed exists.

1. Prices, States, and Preliminary Notation

We suppose there to be only one physical good, whose price will always be one. There are I individuals (consumers) and J firms. There are two periods; where the need exists, "today" will be subscripted by o, and states tomorrow will be indicated by m, m=1, ..., M. Thus, for example, the consumption of consumer i will be denoted by $x_i = (x_{io}, x_{i1}, \ldots, x_{iM})$, where the first component of the vector denotes today's consumption, and the other components denote consumption tomorrow if state m obtains. The securities of each of the J firms are traded today (tomorrow the firms liquidate, and there is consequently no trade in their securities). The vector of security prices will be denoted by p^s, $p^s = (p_1^s, \ldots, p_J^s)$. In addition, each of the firms may issue bonds, and consumers may sell bonds to each other, as well as buying firm bonds. A vector of bond prices will be indicated by $p^b = (p_o^b, p_1^b, \ldots, p_J^b)$, where the first component of the vector indicates inter-consumer bonds. The distinguishing factor between inter-consumer bonds and firm bonds is that the former will always be riskless. Firm bonds may be risky; firms may go bankrupt, and this risk will cause the bond prices to differ from the interconsumer bond price. We note that the bond prices are the inverse of the one period interest rate plus one: An individual buying b_{ij} of firm j's bonds today will pay $p_j^b b_{ij}$, $0 \leq p_j^b \leq 1$; in the absence of bankruptcy he will then receive b_{ij} from the firm in the next period.

2. Firms and production

Each of the J firms is distinguished by a stochastic production technology. Having purchased inputs $z_j \geq 0$ in the first period, firm j will produce $y_{jm}(z_j)$ units of the commodity in the second period if state m obtains. The functions y_{jm} will be assumed to be real-valued, non-negative, continuous, and concave.

Suppose that prices $p = (p^s, p^b)$ are given. Denote by b_j the nominal value of the one-period bonds issued by firm j and by a_j new equity issued by the firm. We shall constrain new equity and bonds to be non-negative, and impose a budget constraint on the firm: at current prices, new equity and bonds issued must pay for inputs purchased. Denote the budget set of firm j by

$$B_j(p^s, p^b) = \{k_j = (z_j, a_j, b_j) \quad a_j \geq 0,\ b_j \geq 0,\ z_j \geq 0;$$
$$p_j^b b_j + p_j^s a_j - z_j \geq 0\} \tag{1}$$

The firm may pay dividends to its shareholders in both the first and second periods. There is, however, a difference between dividends paid in the first period and those paid in the second: The former are paid to initial shareholders, while the latter are paid to those who bought shares in the first period. As we shall see in section 5, the firm's primary responsibility is to its initial shareholders.[3] We denote dividends paid by firm j in the first period by r_{jo}^s, where

$$r_{jo}^s = p_j^b b_j + p_j^s a_j - z_j \tag{2}$$

Dividends in the second period depend on the cash flow in that period: Suppose that state m obtains. The cash flow in that state, assuming firm j has chosen program k_j, will be

$$r_m(k_j, p) = y_{jm}(z_j) - b_j \tag{3}$$

The firm's first responsibility in the second period is to pay back its bondholders. It can do this in state m if $r_m(k_j,p) \geq 0$. If this is indeed so, then $r_m(k_j,p)$ is the dividend paid to shareholders. Otherwise, the firm is <u>bankrupt</u>, and shareholders get nothing. Writing r^s_{jm} for the total dividend paid by firm j at m, we see that

$$r^s_{jm} = \begin{cases} r_m(k_j,p) & \text{if } r_m(k_j,p) \geq 0 \\ 0 & \text{otherwise} \end{cases} \tag{4}$$

If firm j is not bankrupt at m, it can pay off its bondholders in full. If firm j is bankrupt, it defaults on its legal obligation to the bondholders, and these become the owners of the firm, receiving all proceeds from production. Denoting by r^b_{jm} the total payments to bondholders by firm j at state m, we get,

$$r^b_{jm} = \begin{cases} b_j & \text{if } r_m(k_j,p) \geq 0 \\ y_{jm}(z_j) & \text{otherwise} \end{cases} \tag{5}$$

3. <u>Consumers</u>

Each of the I consumers receives an endowment of the commodity in both the first period and every state of the second period. For consumer i, denote this endowment by $\bar{\omega}_{ih}$, h=o,1, ..., M. We shall assume that all endowments are strictly positive. In addition to his endowment, consumer i will be assumed to possess an initial portfolio of shares at the beginning of the first period. Denoting this portfolio by $\bar{f}_i = (\bar{f}_{i1},\ldots,\bar{f}_{iJ})$ we shall assume:

$$\bar{f}_{ij} \geq 0, \text{ for every i and every j,} \tag{6}$$

$$\sum_i \bar{f}_{ij} = 1, \text{ for every } j. \tag{7}$$

Given firm programs, k_j, for each j, consumer i chooses a consumption vector $x_i = (x_{io}, x_{i1}, \ldots, x_{iM})$, a share portfolio, $f_i = (f_{i1}, \ldots, f_{iJ})$, and a bond portfolio $b_i = (b_{io}, b_{i1}, \ldots, b_{iJ})$. The first coordinate of the bond portfolio, b_{io}, represents the amount of inter-consumer loans contracted by i; if $b_{io} > 0$, consumer i is a net lender to other consumers, while $b_{io} < 0$ means that i is a net borrower. As we shall see when we describe the budget set (8), consumers are constrained to repay their inter-consumer loans. Thus while our model has firm bankruptcy, it does not have consumer bankruptcy. The last J coordinates of the bond portfolio represent amounts bought by consumer i of firm bonds. Within the constraints set by the budget set, $B_i(p,(k_j))$, we shall allow short sales of both bonds and stocks:

$$B_i(p,(k_j)) = \{\ell_i = (x_i, b_i, f_i) \quad x_i \geq 0, \tag{8}$$

$$\sum_j p_j^s(\bar{f}_{ij} - f_{ij}) + \sum_j \bar{f}_{ij} r_{jo}^s - \sum_j p_j^b b_{ij} + \bar{\omega}_{io} - x_{io} \geq 0$$

$$\sum_j \frac{1}{1+a_j} f_{ij} r_{jm}^s + \sum_j \frac{b_{ij}}{b_j} r_{jm}^b - b_{io} - x_{im} + \bar{\omega}_{im} \geq 0\}$$

The first inequality in the budget set represents the non-negativity of consumption; the second inequality is the budget constraint for the first period; and the third inequality is the budget constraint for the second period. Note that if firm j has issued a_j new equity, the purchase by consumer i of f_{ij} of shares entitles him to only $f_{ij}/(1+a_j)$ of the dividends paid by the firm. A similar argument explains the term b_{ij}/b_j.

4. Consumer maximization and implicit prices

Each consumer i maximizes a utility function defined on his consumption vector x_i. We shall assume that this function, $U_i(x_i)$, is continuous, concave, and increasing in each argument. Thus, we assume that

$$\partial U_i/\partial x_{ih} > 0, \text{ for all } h=0,1,\ldots,M. \tag{9}$$

Now assume that given prices p^s and p^b and firm plans k_j, $j=1,\ldots,J$, that consumer i maximizes U_i at (x_i^*, f_i^*, b_i^*). Then strict equality must exist in the budget constraint equations, and we may thus write U_i as a function of the portfolios,

$$U_i(x_i^*) = \bar{U}_i(f_i^*, b_i^*) \tag{10}$$

Differentiating $\bar{U}_i$ with respect to the portfolios, we get

$$\frac{\partial \bar{U}_i}{\partial f_{ij}} = 0 \Rightarrow p_j^s = (\sum_m \rho_m^i r_{jm}^s) \frac{1}{1+a_j} \tag{11}$$

$$\frac{\partial \bar{U}_i}{\partial b_{ij}} = 0 \Rightarrow p_j^b = \frac{1}{b_j} \sum_m \rho_m^i r_{jm}^b \tag{12}$$

and

$$p_o^b = \sum_m \rho_m^i. \tag{13}$$

where

$$\rho_m^i = \frac{\partial U_i/\partial x_{im}}{\partial U_i/\partial x_{io}}, \tag{14}$$

and this is evaluated at x_i^*.

Thus consumer maximization leads to implicit valuations of firm share and bond prices which coincide with market prices. Note that the right-hand side of (11) and (12) depend on i. Accordingly, we shall label it p_j^{si} and p_j^{bi} respectively. Given k_j for each j, we have thus shown that the consumer maximizes utility until $p_j^{si} = p_j^s$, and likewise for bond prices.*

5. <u>The Firm objective function</u>

If market prices $p = (p^s, p^b)$ prevail, and the firm decides on program k_j, then in equilibrium the total return to initial shareholders of j will be the sum of the first period dividend plus the returns to these shareholders from the sales of their shares:

$$a_j p_j^s + p_j^b b_j - z_j + p_j^s = (1+a_j)p_j^s + p_j^b b_j - z_j \qquad (15)$$

If at current market prices and firm plans, consumers plan ℓ_i, then this expression becomes (taking account of consumer demands and implicit valuations):

$$\Psi((k_j), (\ell_i), p) = \sum_i f_{ij} p_j^{si} + \sum_i b_{ij} p_j^{bi} - z_j \qquad (16)$$

We may now define the set of programs $\hat{k}_j$ preferred by firm j to its current program, k_j: Given firm plans (k_h), h=1,...,J, individual plans (ℓ_i), i=1,...,I, and market prices $p = (p^s, p^b)$ firm j will prefer plan $\hat{k}_j$ if

*The analysis of this section owes much to Baron [2].

$$\Psi((k_h),\ (\ell_i),p) + \frac{\partial\Psi}{\partial z_j}\,(\hat{z}_j - z_j) + \frac{\partial\Psi}{\partial b_j}\,(\hat{b}_j - b_j) \qquad (17)$$

$$+ \frac{\partial\Psi}{\partial a_j}\,(\hat{a}_j - a_j) > \Psi((k_h),\ (\ell_i),p)$$

The preference set defined above makes it clear that, given market prices, individual plans, and other firm plans, firm j will always prefer those plans which give its initial shareholders a greater return. Unfortunately, the dependence of firm j's preferences on the choices of other agents' choices, and *vice versa*, introduces a circularity which makes our model somewhat difficult to work with. In section 7 we shall prove a theorem which makes this less of a problem. But first we must define what we mean by an equilibrium.

6. The Definition of Equilibrium

A general equilibrium with financial markets is defined by a vector of financial asset prices $p^* = (p^{s*}, p^{b*})$, vectors of individual plans $\ell_i^* = (x_i^*,\ b_i^*,\ f_i^*)$ for each i=1,...,I, and firm plans $k_j^* = (z_j^*,\ a_j^*,\ b_j^*)$ such that:

(E.1) Commodity supply equals demand:

$$\sum_i \bar{\omega}_{io} - \sum_j z_j^* - x_{io}^* = 0$$

$$\sum_j y_{jm}(z_j^*) - \sum_i x_{im}^* + \sum_i \bar{\omega}_{im} = 0,\ m=1,\ldots,M$$

1.1. It fulfills (E.1)-(E.4).

1.2. There exist $(\rho_1,\ldots,\rho_M)$ such that for every j, $k_j^* \varepsilon B_j(p^*)$ and k_j^* maximizes

$$\sum_m \rho_m y_{jm}(z_j) - z_j \tag{18}$$

subject to

$$p_j^{s*} = \frac{1}{1+a_j^*}(\sum_m \rho_m r_{jm}^s) \tag{19}$$

$$p_j^{b*} = \frac{1}{b_j^*}\sum_m \rho_m r_{jm}^b \tag{20}$$

Proof:

Necessity is established as follows: Since $p_j^{si} = p_j^s$ and $p_j^{bi} = p_j^b$ for all i and all j, we get that

$$\psi_j((k_j^*), (\ell_i^*), p^*) = \sum_i f_{ij}^* p_j^{s*} + \sum_i b_{ij}^* p_j^{b*} - z_j^* \tag{21}$$

$$= (1+a_j^*)p_j^{s*} + b_j^* p_j^{b*} - z_j^*$$

Summing this last expression over all i, and substituting $p_j^{si} = p_j^{s*}$ and similarly for bond prices, we get,

$$I \cdot \psi_j((k^*), (\ell_i^*), p) = (1+a_j^*)\sum_i p_j^{si} + b_j^* \sum_i p_j^{bi} - z_j^*$$

$$= \sum_{im}\sum \rho_m^i y_{jm}(z_j^*) - z_j^*. \tag{22}$$

(E.2) Supply of financial assets equals demand:

$$\sum_i f^*_{ij} = 1 + a^*_j, \ j=1,\ldots,J$$

$$\sum_i b^*_{ij} = b^*_j \qquad , \ j=1,\ldots,J$$

$$\sum_i b^*_{io} = 0$$

(E.3) All firm and individual plans are feasible:

$$\ell^*_i \ \varepsilon \ B_i(p^*, \ (k^*_j)), \ i=1,\ldots,I.$$

$$k^*_j \ \varepsilon \ B_j(p^*), \ j=1,\ldots,J$$

(E.4) ℓ^*_i maximizes U_i for all feasible consumer plans.

(E.5) There exists no $k_j \varepsilon B_j(p^*)$ such that k_j is preferred to k^*_j at prices p^* and consumer plans (ℓ^*_i).

7. A basic equivalence theorem

In Section 5 we made mention of the circularity inherent in the definition of a financial equilibrium. The following theorem characterizes equilibria in financial markets, and will also serve to illuminate some of the conundra of financial theory, most notably those posed by the two famous Modigliani and Miller theorems [7,8].

Theorem 1: p^*, (ℓ^*_i), (k^*_j) is a general equilibrium with financial markets if and only if:

Setting

$$\rho_m = \frac{1}{I} \Sigma \rho_m^i, \quad m=1,\ldots,I, \tag{23}$$

necessity follows.

Sufficiency is established in a similar manner: If $k_j^* \varepsilon B_j(p^*)$, and (1.1) and (1.2) hold, then since $p_j^{si} = p_j^{s*}$ and $p_j^{bi} = p_j^{b*}$, for every i and every j, it follows that

$$\psi_j(k_h^*), (\ell_i^*), p^*) = \sum_m \rho_m y_{jm}(z_j^*) - z_j^*. \tag{24}$$

QED

Since Arrow's paper [1], there has been a tendency to view financial markets primarily in the light of the distinction between complete and incomplete markets. This distinction is valid in view of the <u>function</u> of financial markets, which is to regulate intertemporal consumption. We claim that it is not the right starting point for an analysis of what counts in financial markets, primarily because the <u>nature</u> of these markets dictates that we must start with an array of security and commodity prices, specify the nature of the market uncertainty, the actions of market agents (firms, consumers) in light of the actions of other agents and in light of prices--and, having stated all these, see where we are led. Only then can we return to the Arrow structure of complete markets and compare results. Theorem 1 does this; in an Arrow equilibrium with complete markets, firms maximize net production profits (18) subject to state prices $(\rho_1,\ldots,\rho_M)$. If markets are complete, however,

$$\rho_m^i = \rho_m \tag{25}$$

for every i. The easiest way to see this is to assume that for every m there exists a security j which pays off only in state m, and with a payoff of 1. Then since $p_j^{si} = p_j^s$ for every i, we get

$$p_j^{si} = \rho_m^i = p_j^s. \tag{26}$$

Equation (23) now shows that $\rho_m^i = \rho_m$ for every i. Theorem 1 thus shows that every general equilibrium with financial markets has a property which close resembles an Arrow equilibrium. But a general equilibrium with financial markets is not fully specified by (18); we need equations (19) and (20) in order to complete our description of equilibrium.[4]

8. Uniqueness Properties of financial equilibria

Two theorems of Modigliani and Miller [7,8] assert the irrelevance of debt/equity ratios and dividend policy, respectively. In this section we show that, while the latter result is generally true (though not without qualification), the former holds only in the absence of bankruptcy.

Theorem 2: If p^*, (k_h^*), (ℓ_i^*) is a general equilibrium with financial markets, then

(2.1) z_j^* is unique

(2.2) a_j^* and b_j^* are unique if $p_j^{b*} < p_o^{b*}$.

Proof:

Since we are at an equilibrium, there exist $\rho_1,\ldots,\rho_M$, as in Theorem 1. Since z_j^* maximizes a concave function, its uniqueness is immediate.

The best way to show the uniqueness of p_j^{b*} is graphically (Fig. 1): By Theorem 1 we know that b_j must be chosen so that

$$p_j^{b*} = \frac{1}{b_j} \Sigma \rho_m r_{jm}^b \tag{27}$$

But the right-hand side of the above equation is a non-increasing function of b_j. In fact, given z_j^*, there exists a value of b_j, call it α, such that for $b_j \leq \alpha$, the right hand side above is constant. Since this range of values for b_j corresponds to the non-bankruptcy of firm j for all m, and since from $p_j^{b*} < p_o^{b*}$ we learn that firm j goes bankrupt for at least one m, we must conclude that $b_j^* > \alpha$. For these values of b_j, the right-hand side of (27) is strictly decreasing, and b_j^* is thus unique.

Figure 2 illustrates the uniqueness of a_j^*: Given b_j^* and z_j^*, we must choose a_j^* so that

$$p_j^{s*} = \frac{1}{1+a_j} \sum_m \rho_m r_{jm}^s \tag{28}$$

But the right-hand side of (28) is strictly declining as a function of a_j, and so, given b_j^* and z_j^*, a_j^* is unique.

QED

<u>Corollary 3 (Modigliani-Miller)</u>: If p^*, (k_h^*), (ℓ_i^*) is a general equilibrium with financial markets and $p_j^{b*} = p_o^{b*}$, then there exists α such that if $0 \leq \hat{b}_j \leq \alpha$, and

$$\hat{a}_j = \frac{(b_j^* - \hat{b}_j)}{p_j^{s*}} p_j^{b*} + a_j^* \tag{29}$$

then $\hat{k}_j = (z_j^*, \hat{a}_j, \hat{b}_j)$ is part of an equilibrium in which prices are unchanged, consumer utility levels are unchanged, and firm programs fulfill (1.2) of Theorem 1.

Proof:

The proof is well known (see, for example, Stiglitz [10,12]). Since $p_j^{b*} = p_o^{b*}$, j is a riskless borrower. By Theorem 1, z_j^* is unique, but referring to Figure 1, we see that we must be on the linear portion of $\frac{1}{b_j} \Sigma \rho_m r_{jm}^b$. Let α be that value of b_j for which this curve is no longer linear, i.e.,

$$\alpha = \max_{b_j} \quad \{y_{jm}(z_j^*) - b_j \geq 0,\ m=1,\ldots,M\} \tag{30}$$

Suppose the firm chooses $\hat{b}_j \leq \alpha$ instead of b_j^*, and $\hat{a}_j$ as defined above. For each consumer i, redefine

$$\hat{b}_{ij} = \frac{b_{ij}^*}{b_j^*}.\hat{b}_j \quad , \qquad \hat{b}_{io} = b_{io} + \frac{b_{ij}^*}{b_j^*}(b_j^* - \hat{b}_j). \tag{31}$$

and

$$\hat{f}_{ij} = \frac{f_{ij}^*}{1+a_j^*}.(\hat{a}_j + 1) \tag{32}$$

The new equilibrium may be seen to have all the required properties.

QED

Neither Theorem 2 nor Corollary 3 completely solve the Modigliani-Miller paradox with regard to capital structure. Each may be thought to hark back to our original definition of equilibrium with financial markets: Since financial markets have the property that the plans of one set of agents (firms) depend on the plans of another set of agents (consumers), and vice versa, one type of question we may ask about these markets is the following:

> Keeping market prices constant, what kind of adjustments in firm debt/equity ratios and consumer portfolios are compatible?

The answer is that changes in firm debt/equity ratios are only compatible with existing market prices if the firm is not currently a risky borrower. This is an important point, since many MM proofs rely on the substitution of "homemade" leverage for firm leverage (see, for example [14]).

Corollary 4 is a dividend counterpart of Theorem 2 and Corollary 3. It states that, given prices, firm production plans, and consumption plans, dividend policy may be changed only when the firm is a riskless borrower.

Corollary 4: If p^*, (k_j^*), (ℓ_i^*) is a general equilibrium with financial markets, then a change in dividend policy without a corresponding change in market prices is possible only for those firms which are riskless borrowers.

Proof:

The first period dividend is given by

$$r_{jo}^s = a_j p_j^s + b_j p_j^b - z_j \tag{33}$$

By Theorem 2 and Corollary 3, changes in a_j and b_j are possible, at current market prices, only if the firm is a riskless borrower.

QED

As we have seen from our definitions, the first period dividend may be changed by altering the amount of new equity issued. As long as prices are kept fixed, Corollary 4 shows that only firms which are riskless borrowers may change their first-period dividend policy. Provided we allow for adjustment in market prices, our next theorem shows that first-period dividend policy is irrelevant. If firms alter the amount of new equity they issue, and we allow prices to adjust in accordance with the equations of Theorem 1, then a new equilibrium is established in which consumers have purchased the same proportion of each firm's equity, at the same total expense. Dividends have changed, but nothing else has (not even the debt/equity ratio).

Theorem 5: At an equilibrium, all consumers are indifferent to the level of first period dividends.

Proof:

Let p, (k_h), (ℓ_i) be an equilibrium. Define $\hat{p}$, $(\hat{k}_h)$, $(\hat{\ell}_i)$ by

$$\hat{z}_j = z_j, \hat{b}_j = b_j \tag{34}$$

$$\hat{x}_i = x_i, \hat{b}_i = b_i, \frac{\hat{f}_{ij}}{1+\hat{a}_j} = \frac{f_{ij}}{1+a_j}$$

$$\hat{p}^s_j = \frac{1}{1+\hat{a}_j} \sum_m \rho_m r^s_{jm}, \quad \hat{p}^b_j = p^b_j$$

It is easily confirmed that this defines an equilibrium (the proof is a direct consequence of Theorem 1). Furthermore, the dividend level has now become

$$\frac{\hat{a}_j}{1+\hat{a}_j} \sum_m \rho_m r^s_{jm} + p^b_j b_j - z_j, \tag{35}$$

and this may be changed by suitable manipulation of the first term.

QED

What motivates our proof? Dividends may be changed by increasing or decreasing the amount of new equity issued, a_j. But changes in the amount of new equity issued do not affect the total returns of initial shareholders; rather, they affect the proportion of the total returns received through the first period dividend and the proportion received by the direct sale of initial shareholder equity. Thus all initial shareholders are indifferent to first period dividends. The indifference of new shareholders (those consumers i for whom $f_{ij} \neq 0$) stems from a different source: New equity issues do not affect the distribution of returns at all; r^s_{jm} is determined solely by the amount of debt and the level of initial inputs. Thus, as long as the price is changed inversely to the amount of new equity issued, purchasers of the firm's equity may continue to purchase the same proportion of the equity that they originally purchased in equilibrium. Thus all are indifferent to dividends.

<u>Theorem 6</u>: If p, (k_h), (ℓ_i) is an equilibrium, then for every j,

$$\sum_m \rho_m y'_{jm} = 1. \tag{36}$$

Proof:

By Theorem 1, at an equilibrium we have that k_j maximizes

$$\sum_m \rho_m y_{jm}(z_j) - z_j \tag{37}$$

subject to

$$p_j^s - \frac{1}{1+a_j} \sum_m \rho_m r_{jm}^s = 0 \tag{38}$$

$$p_j^b - \frac{1}{b_j} \sum_m \rho_m r_{jm}^b = 0 \tag{39}$$

$$z_j - a_j p_j^s - b_j p_j^b \leq 0. \tag{40}$$

Case 1: At a maximum, the first period dividend is positive. Then the third constraint is not operative and since the Kuhn-Tucker conditions apply (see [4], p. 199), we may maximize the Lagrangean

$$\sum_m \rho_m y_{jm}(z_j) - z_j + \lambda_1 (p_j^s - \frac{1}{1+a_j} \sum_m \rho_m r_{jm}^s) \tag{41}$$

$$+ \lambda_2 (p_j^b - \frac{1}{b_j} \sum_m \rho_m r_{jm}^b)$$

Taking derivatives with respect to a_j and b_j, we get

$$\lambda_1 (\frac{1}{1+a_j} p_j^s) = 0 \tag{42}$$

$$\lambda_1\left(\frac{-1}{1+a_j} \sum_A \rho_m \right) - \lambda_2\left(\frac{1}{(b_j)^2} \sum_{\bar{A}} \rho_m y_{jm} \right) = 0, \tag{43}$$

where A is the set of those m for which the firm does not go bankrupt, and $\bar{A}$ is the complement of A. Thus $\lambda_1 = \lambda_2 = 0$. Our problem is thus one of unconstrained maximization of (37), and this proves our claim for this case.

Case 2: The first period dividend is zero. By the previous theorem, such an equilibrium is fully equivalent to one in which the firm chooses the same input vector, individuals choose the same consumption vectors, but the firm pays a positive dividend. We thus reduce this case to that of case 1.

QED

Theorem 6 asserts the efficiency of any financial equilibrium. This contradicts an assertion of Stiglitz [16]. The difference may be explained in that Stiglitz asserts that firms take their debt/equity ratios into account in making investment decisions. This is true if first period dividends are equal to zero. The reader may confirm this by solving the constrained maximization problem posed in (37)-(40) when all three constraints are tight; however, since dividends are irrelevant by Theorem 5, we may always assume that the third constraint (40) is not fulfilled, and thus our problem is seen to boil down to one of unconstrained maximization. Theorem 6 thus extends to the general case the results of Diamond [3].

We may now turn our attention to a broader statement of the relevance of the debt/equity ratio. Note first that there are several ways in which the MM irrelevance theorem may be expressed. The narrowest statement of the theorem is as follows:

1. Keeping prices, consumption plans, and firm investment plans constant, any firm may change its equilibrium debt/equity ratio by issuing more debt and less new equity; consumer adjustment to such a change consists only in changes of holdings of the firm's debt and equity and holdings of the riskless asset.

We have (in Theorem 2) disproven this statement in all cases except where the firm is a riskless borrower in equilibrium. The reasons for this are simple: We showed that given debt and equity prices are consistent (except in the case of riskless firm borrowing) with only one debt/equity policy, and so this version of the theorem cannot, in general, hold. There is a second, weaker, form of the theorem, in which we relax the condition that prices remain unchanged:

2. A firm j may change its equilibrium debt/equity ratio by issuing more debt and less new equity (or vice versa); if prices adjust accordingly, then consumers will make changes only in their holdings of j's debt and equity and in their holdings of the riskless asset.

This version of the theorem, in order that it be distinct from the previous theorem, automatically requires the firm j in question to be a risky borrower in equilibrium; otherwise we are back to version 1 of the theorem. We shall now show that this second version of the MM theorem is, in general, not true:

<u>Theorem 7</u>: Let p, (k_h), (ℓ_i) be an equilibrium. Suppose that

(7.1) $p_j^b < p_o^b$

(7.2) there exists i such that $\frac{b_{ij}}{b_j} \neq \frac{f_{ij}}{1+a_j}$

(7.3) $y_{jn}(z_j) \neq y_{jm}(z_j)$ for every $m \neq n$

(7.4) there exist two states for which firm j is not bankrupt and there exist two states for which firm j is bankrupt at plans k_j and prices p.

Then there exists no other equilibrium which fulfills (7.4) and having the following properties:

(7.5) Consumption vectors of consumers remain unchanged.

(7.6) Firm j has changed the amount of debt and new equity issued, but no other firm has changed its debt and equity plans.

(7.7) Consumer portfolios of all firms except j and the riskless asset are unchanged. I.e., the only changes in consumer portfolios are those in holdings of the riskless asset and firm j's debt and equity.

Note: Before we prove Theorem 7, the most general form in which we shall prove the relevance of the debt/equity ratio, we make the following remarks:

1. There is a yet more general form of the MM irrelevance theorem to be disproved: What if all firms are allowed to make the changes described in the theorem, and consumers are allowed to change all their portfolios? To date I have not been able to prove this form of the theorem.

2. Using risk-neutrality and non-homogenous expectations, Stiglitz [16] proved a somewhat similar statement. Our theorem is more general than his, and corresponds to a fuller, general equilibrium, proof of the relevance of debt/equity ratios. Stiglitz was using a modified mean-variance model, in which homogenous expectations cause condition (7.2) to be violated (see Mossin [9]). Condition (7.2) thus corresponds to Stiglitz's assumption of heterogenous expectations. This condition is necessary to establish the relevance of the debt/equity ratio; if all consumers hold the same equilibrium proportions of every firm's debt and equity, then no change in the debt/equity ratio can affect their consumption.

3. I suspect that (7.4) may be relaxed, though I have not been able to prove the theorem in this form. In particular, I suspect that it is enough to require that both before and after the change in debt/ equity ratios, there be at least one non-bankrupt (and one bankrupt) state.

4. Condition (7.3) may be relaxed somewhat: For purposes of the proof it is enough that in the two states n and m which saw bankruptcy both before and after the change in the debt/equity ratio, $y_{jn} \neq y_{jm}$. Whether the theorem is true for all production functions remains an open question. In any case, even the strong form of (7.3) is not particularly cumbersome.

Proof:

We note first that although the theorem makes no statement about firm inputs under the new debt/equity ratio, theorems 1 and 6 show that these inputs will be unchanged for every firm. The total returns to be divided among debt and equity holders in the next period are unchanged, therefore.

Since by the conditions of the theorem only holdings of firm j's debt and equity and the riskless asset are changed, we shall, for consumer i write these as $\hat{f}_{ij}$, $\hat{b}_{ij}$, $\hat{b}_{io}$ respectively. By the budget constraints for each m, we thus have

$$x_{im} - \left(\frac{f_{ij}}{1+a_j} r^s_{jm} + \frac{b_{ij}}{b_j} r^b_{jm} + \bar{\omega}_{im} \right) = \tag{44}$$

$$\frac{\hat{f}_{ij}}{1+\hat{a}_j} r^s_{jm} + \frac{\hat{b}_{ij}}{\hat{b}_j} \hat{r}^b_{jm} + \hat{b}_{io}$$

Call the left-hand side of the above expression A_m. We may then write,

$$\hat{b}_{io} = A_m - \frac{\hat{f}_{ij}}{1+\hat{a}_j} r^s{}_{jm} - \frac{\hat{b}_{ij}}{\hat{b}j} \hat{r}^b_{jm} \tag{45}$$

Substitute this expression in the budget constraint for another state n:

$$\frac{\hat{f}_{ij}}{1+\hat{a}_j}(\hat{r}^s_{jn} - \hat{r}^s_{jm}) + \frac{\hat{b}_{ij}}{\hat{b}_j}(\hat{r}^b_{jn} - \hat{r}^b_{jm}) = A_n - A_m \tag{46}$$

Now suppose that n and m are two states for which firm j does not go bankrupt at either its old or its new debt/equity ratio. Then (46) becomes

$$\frac{\hat{f}_{ij}}{1+\hat{a}_j}(y_{jn} - y_{jm}) = A_n - A_m \tag{47}$$

and this may be solved to give

$$\frac{\hat{f}_{ij}}{1+\hat{a}_j} = \frac{A_n - A_m}{y_{jn} - y_{jm}} \tag{48}$$

Similarly, by assuming that n and m are two states for which firm j goes bankrupt at both its old and its new debt/equity ratio, we find that

$$\frac{\hat{b}_{ij}}{\hat{b}_j} = \frac{A_n - A_m}{y_{jn} - y_{jm}} \tag{49}$$

Solving equations (44)-(49), only this time for the initial portfolio of consumer i, will lead us to the same solution for the proportion of equity and debt purchased. We may thus conclude that

$$\frac{\hat{b}_{ij}}{\hat{b}_j} = \frac{b_{ij}}{b_j} \quad \text{and} \quad \frac{\hat{f}_{ij}}{1+\hat{a}j} = \frac{f_{ij}}{1+a_j} \tag{50}$$

We now return to solve for $\hat{b}_{io}$. Suppose first that m is a state for which, neither at its old nor its new debt/equity ratio, firm j goes bankrupt. Let $\beta = \hat{b}_j - b_j$. Substituting (50) in (45), we find that

$$\hat{b}_{io} = A_m - \frac{f_{ij}}{1+a_j} (r^s_{jm} - \beta) - \frac{b_{ij}}{b_j} (r^b_{jm} + \beta) \tag{51}$$

$$= A_m - \frac{f_{ij}}{1+a_j} r^s_{jm} - \frac{b_{ij}}{b_j} r^b_{jm} + \beta(\frac{f_{ij}}{1+a_j} - \frac{b_{ij}}{b_j})$$

$$= b_{io} + \beta(\frac{f_{ij}}{1+a_j} - \frac{b_{ij}}{b_j}).$$

and we may thus conclude (by assumption (7.2) of this Theorem), that $\hat{b}_{io} \neq b_{io}$. On the other hand, suppose m to be a state for which, both at its old and its new debt/equity ratio, firm j is bankrupt. Then (45) becomes,

$$\hat{b}_{io} = A_m - \frac{b_{ij}}{b_j}(y_{jm}(z_j)) = b_{io}. \tag{52}$$

This is a contradiction, and our theorem is proved.

QED

A careful look at our proof will reveal that what motivates it is a very simple mechanism: If equilibrium is established with bankruptcy, and one firm changes its debt equity ratio, it is easily established that former bond and share holders should continue to purchase the same proportion of the firm's debt and equity they purchased before. The question now is how much of the riskless asset consumers should purchase. An increase in the debt/equity ratio, ceteris paribus, means that in any state for which the firm does not go bankrupt, the payoff to equity holders has been decreased. This means (equation (51)) that if a consumer holds more (proportionally) of the firm's debt than he does of its equity, that the net receipts in non-bankrupt states have gone down. To make up for this, the consumer must purchase more of the riskless asset. On the other hand, the increase in the debt/equity ratio does not at all affect the payoff in states where the firm has gone bankrupt. This would imply that, ceteris paribus, the amount of riskless assets purchased does not change. The contradiction reached is sufficient to establish our theorem.

9. The Existence of Equilibrium

While the above theorems and corollaries show some of the properties of equilibrium, none establishes its existence. In this section we show that a general equilibrium with financial markets indeed exists.

Lemma 8: Let p^*, (k_j^*), (ℓ_i^*) be an equilibrium. Then there exist bounds W_o, W_m such that

$$z_j^*, \; x_{io}^* \leq W_o \tag{53}$$

and

$$x_{im}^* \leq W_m \tag{54}$$

for all j and all i.

Proof:
Let $W_o = \sum_i \bar{\omega}_{io}$, and let $W_m = \sum_j y_{jm}(W_o) + \sum_i \bar{\omega}_{im}$

QED

Lemma 9: Let p^*, (k_j^*), (ℓ_i^*) be an equilibrium. Then there exists $\bar{B}_j \geq 0$ such that if $b_j^* > \bar{B}_j$, then firm j will go bankrupt for every second period state m.

Proof:
Let $\bar{B}_j = \max_m y_{jm}(W_o)$.

QED

By the above lemmas, we may safely bound firm budget sets by constraining $b_j \leq \bar{B}_j$ and $z_j \leq W_o$. We do not wish to constrain firms to the amount of new equity they may issue, but for the purposes of our proof we shall require bounded budget sets. Instead of writing a_j, therefore, we shall write

$$q_j = \frac{a_j}{1+a_j} \ . \qquad (55)$$

For all $a_j \geq 0$, $0 \leq q_j \leq 1$.

In order to bound the purchases and sales of bonds by consumers, we note that the maximum income of any agent in the market in the second period is

$$X = \max_m \{y_{jm}(W_o) + \sum_i \bar{\omega}_{im}\} \qquad (56)$$

No promise to deliver more than X in the second period can therefore be credible, and we are safe in bounding $b_{ij} \leq X$.

Finally, in order to bound sales and purchases of shares of firms, we shall resort to the standard convention that short sales are sales of borrowed stock. No consumer may therefore sell short more than all the stock in a firm, since it is impossible to borrow more than this amount of stock. Similarly, it is impossible to purchase more than all the shares in a firm. Denoting

$$q_{ij} = \frac{f_{ij}}{1+a_j} \ , \qquad (57)$$

we thus bound

$$-1 \leq q_{ij} \leq 1 . \tag{58}$$

Now define the bounded budget set for each consumer i,

$$\hat{B}_i(p(k_j)) = \{(x_i,q_i,b_i) \quad (x_i,f_i,b_i)\varepsilon B_i(p,(k_j)) \tag{59}$$

$$0\leq x_{io}\leq W_o,\ 0\leq x_{im}\leq W_m,\quad q_{ij} \leq 1,\quad b_{ij} \leq X\}$$

For each firm j, define the bounded budget set by

$$\hat{B}_j(p) = \{(z_j,q_j,b_j) \quad (z_j,a_j,b_j)\varepsilon B_j(p),\ 0\leq z_j\leq W_o, \tag{60}$$

$$0\leq q_j\leq 1,\ 0\leq b_j\leq \bar{B}_j\}$$

Now define <u>price sets</u> as follows: For security prices, define

$$\Delta^s = \{(p_1^s,\ldots,p_J^s) \quad \sum_j p_j^s = 1\} \tag{61}$$

For bond prices, define,

$$\Delta^b = \{(p_o^b,p_1^b,\ldots,p_J^b) \quad 0\leq p_j^b\leq 1,\ j=0,1,\ldots,J\} \tag{62}$$

Finally, define the <u>excess demand</u>: Given consumer plans ℓ_i, and firm plans k_j, for every i and every j, the excess demand for bonds is defined as

$$E_j^b = \sum_i b_{ij} - b_j,\ j=1,\ldots,J \tag{63}$$

$$E_o^b = \sum b_{io} \quad ,\ j=0 \tag{64}$$

The excess demand for stocks is defined by

$$E_j^s = \sum_i f_{ij} - (1+a_j),\ j=1,\ldots,J \tag{65}$$

The excess demand for commodities is defined by

$$E_o^c = \sum_i x_{io} - \sum_i \bar{\omega}_{io} + \sum_j z_j \tag{66}$$

$$E_m^c = \sum_i x_{im} - \sum_i \bar{\omega}_{im} - \sum_j y_{jm}(z_j) \tag{67}$$

Finally, we are able to prove that an equilibrium exists:

Theorem 10: A general equilibrium with financial markets exists.

Proof:

Define sets C_j and C_i for each j=1,...,J, and i=1,...,I respectively:

$$C_j = \{(z_j,q_j,b_j)\ \ 0\le z_j\le W_o,\ 0\le q_j\le 1,\ 0\le b_j\le \bar{B}_j\} \tag{68}$$

$$C_i = \{(x_i,q_i,b_i)\ \ 0\le X_{io}\le W_o,\ 0\le x_{im}\le W_m \quad q_{ij} \le 1,\quad b_{ij} \le X\} \tag{69}$$

Now define a function

$$\psi:\ \Delta^s \times \Delta^b \times \prod_j C_j \times \prod_i C_i \to \Delta^s \times \Delta^b \times \prod_j C_j \times \prod_i C_i \tag{70}$$

by

$$\psi(p^s,p^b,k_1,\ldots,k_J,\ell_1,\ldots,\ell_I), \tag{71}$$

where

$$\{\hat{\ell}_i\} \subseteq \hat{B}_i(p,(k_j)) \text{ is set of consumer i's plans which maximize } U_i \text{ given } k_1, \ldots, k_J, p^s, p^b \qquad (72)$$

$$\{\hat{k}_j\} \subseteq \hat{B}_j(p) \text{ is the set of firm j's plans which are preferred by j to } k_j \text{ given } p^s, p^b, \ell_1, \ldots, \ell_I. \qquad (73)$$

$$\{\hat{p}^s,\hat{p}^b\} = \{(\hat{p}^s,\hat{p}^b)\varepsilon(\Delta^s \times \Delta^b) \quad (\hat{p}^s,\hat{p}^b)(E^s,E^b)>(p^s,p^b)(E^s,E^b)\} \qquad (74)$$

Now ψ meets the conditions of a theorem of Shafer and Sonnenschein [13], and therefore a Nash equilibrium exists. From the consumer and producer budget sets we get that the value of excess demand must be zero at this equilibrium point.

Denote the Nash equilibrium by p^{s*}, p^{b*}, k_1^*, ..., k_J^*, $\ell_1^*, \ldots, \ell_I^*$. Fix i_o, $1 \leq i_o \leq I$, and define $\tilde{\ell}_i$ by

$$\tilde{b}_i = \begin{cases} b_i^* & i \neq i_o \\ b_{io} - E^b(p^*) & i = i_o \end{cases} \qquad (75)$$

$$\tilde{f}_i = \begin{cases} f_i^* & i \neq i_o \\ f_{io} - E^s(p^*) & i = i_o \end{cases} \qquad (76)$$

$$\tilde{x}_i = x_i^* \qquad (77)$$

It is easily verified that $(p^{s*}, p^{b*}, k_1^*, \ldots, k_J^*, \tilde{\ell}_1^*, \ldots, \tilde{\ell}_J^*)$ is an equilibrium.

QED

10. Summary

In this paper we have attempted to model a plausible picture of "real-world" financial markets, where both consumption and production takes place. A primary purpose of our model was to investigate the relation between "real-world" markets and markets where state-dependent prices exist. We succeeded in doing this in Theorem 1, which shows that in many respects these two models are equivalent. Theorem 1 proves an valuable tool for investigating the extension of the MM debt/equity irrelevance result to the case of bankruptcy. We have shown that this theorem is not in general extendable to this case; furthermore, the MM result on the irrelevance of dividends, while true even for firms which are risky borrowers, is true only in a specific sense: Firms which are risky borrowers in equilibrium cannot change their dividends by changing the amount of debt they issue. Finally, we have established the existence of equilibrium in our model.

It remains to extend our current model to the multi-period case. Here the problems of defining bankruptcy become much more complex (especially where multiperiod bonds exist). In a future paper we hope to explore this issue.

FOOTNOTES

[1]The Arrow theorems take as their starting point the existence of state-dependent prices for future consumption. These prices may be used to define both production objectives and stock prices. Unfortunately, ordinary financial markets work in reverse order: The market provides financial and spot commodity prices, and it is up to firms and consumers to use these prices to establish consumption and production plans. After developing the theory of consumer and firm behavior, we show, in Theorem 1, how these two market concepts are connected.

[2]Current finance texts (for example [18,19]) have sidestepped the MM irrelevance theorems by pointing out "practical" aspects which may mitigate the theoretical oblivion to which corporate finance has been consigned by MM. Most authorities on finance follow: The irrelevance of capital structure, it is pointed out, may not hold if there exists bankruptcy risk (but assuming consumers are risk-neutral) [16], or if there are bankruptcy costs [6], or if capital structure is held to signal the market concerning the firm's true state [11], or if there are option-type factors in operation [10], or if debt is secured [12].

[3]Our theory of production is based on the following scenario: As the firm enters the first period, it is wholly owned by its initial shareholders; these are the individuals i for whom $\bar{f}_{ij} > 0$. The decisions of the firm are symbolized by k_j, where k_j is a vector denoting the amount of inputs purchased for production in the next period, the amount of new equity purchased, and the nominal amount of one-period bonds issued. The decision k_j is the sole decision made by the firm in our model; since the owners of the firm are those individuals whom we have termed the initial shareholders, we shall maximize the returns of these shareholders. Returns to initial shareholders are the sum of the value of their equity plus the dividends paid to them by the firm in the first period; this is discussed in Section 5. Note that this does not imply that purchasers of the firm's equity in the first period are inferior to initial shareholders; indeed, all individuals who hold shares at the end of the first period (i.e., all individuals i for whom $f_{ij} > 0$) are on an equal footing with respect to firm dividends in the second period. Finally, note that though we have ignored production in the first period, its addition would have made no difference: We could easily assume that firms enter the first period with an endowment of inputs, and that this endowment is used for the production of goods in the first period. The only change in our model would be the addition of a constant to equation (2).

[4]Note that it follows from equations (11) and (12) that the equilibrium value of the firm

$$p_j^s + p_j^b b_j = \sum_m \rho_m^i y_{jm}(z_j).$$

Some authors have concluded that this is a general property of Arrow-type financial markets, and that it shows the irrelevance of financial structure in complete markets. In truth, neither of these statements is true: In any kind of market, complete or incomplete, the value of the firm (in or out of equilibrium) will not include a debt term. This, however, does not imply the irrelevance of the debt/equity ratio, as we shall show in Section 8. The <u>locus classicus</u> of this type of error is Fama and Miller [5].

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