IMPLICIT CONSUMER VALUATIONS, BANKRUPTCY, THE VALUE OF THE FIRM, AND DIVIDEND POLICY¹

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1. Introduction

In this paper we argue that the debt/equity ratio is fixed in the presence of bankruptcy. Under conditions somewhat more limited than ours, Stiglitz [8] has proved a similar proposition; in its present generality, however, our result is new. As far as we are aware, the method of our proof--deriving firm valuations from implicit consumer valuations, and then evaluating the changes in firm values from the changes in the implicit valuations--is completely new in this context. A great simplification in exposition is achieved by this method. Finally, we show that our results imply that the MM dividend irrelevance proposition [6] is true in a limited sense only: In the presence of bankruptcy, dividend changes achieved by altering the amount of new equity issued are indeed irrelevant, but dividend changes achieved by altering the amount of debt issued are not irrelevant. This is also a new proposition.

In order to put or results in somewhat clearer perspective, we state first what we mean by a general equilibrium with financial markets: Our model is a simple, two-period model. In the first period consumers trade initial share and commodity endowments and firms buy inputs for stochastic production in the second period; additionally, firms issue bonds and new equity. In the second period firms produce and consumers consume their second-period endowments plus additional consumption bought with the retruns from share and bond portfolios purchased in the first period. Firms may go bankrupt; a firm goes bankrupt if its second period production income is less than what is owed to bondholders. In this case, bondholders

acquire the production income of the firm, and shareholders from the first period get nothing. Market equilibrium consists of a vector of security prices (for both bonds and shares), a vector of consumption and portfolio plans (for both bonds and shares) for each consumer, and a vector of input-bond-share plans for each firm, having the following properties: Given prices and firm plans, each consumer has maximized a utility function defined over feasible consumption plans; and given prices and consumer plans, each firm has maximized the total returns of initial shareholders.

There is a circularity inherent in the above definition: Given prices for shares and bonds (we may assume these prices to be called out by an "auctioneer"), firm plans depend on consumer plans and vice versa. Indeed, we show that--given prices and firm plans--consumer decisions lead to implicit (shadow) prices for all securities in the market, and that changes in firm plans may affect these implicit prices. We argue that, were firms to know the implicit consumer valuations of their stocks and bonds, they should take account of these valuations in determining how to change their plans. We examine the implicit consumer valuations and show how changes in firm issuance of bonds and shares affect these valuations. A change in implicit valuations of individuals will cause changes in their demand for shares and bonds, and even if no individual can affect the market price of a firm's equity or bonds, all individuals together can (through changes in the excess demand) affect market prices. Given consumer plans, but assuming that market prices follow implicit valuations, we then show how the value of the firm changes with the change in firm plans.

A number of results follow from this approach: If at equilibrium a given firm is seen to incur no risk of bankruptcy in any second-period state, then changes in new equity and bond issues do not affect the value

of the firm; this is the classic result of Modigliani and Miller [5] extended to the generalized uncertainty case proved by Stiglitz [7]. If, however, at equilibrium a given firm runs the risk of bankruptcy, then any change in the amount of bonds issued by the firm will decrease the returns to initial shareholders (and indeed, decrease the firm's value). This proposition generalizes a proposition of Stiglitz [8]. Finally, we show that our results imply that the firm's dividend policy is irrelevant in only a limited sense in equilibrium: A risky firm may change its dividends by altering the amount of new equity issued, but not by altering the amount of debt issued. The MM indeterminancy result [6] applies in its fullest generality only when there is no risk of bankruptcy.

Our results differ somewhat from those of other authors. In particular, we shall see that the relevance of debt/equity ratio holds even in complete markets; this contradicts statements by both Stiglitz [8] and Fama and Miller [4].

Finally, we note that nowhere in this paper do we prove the existence of an equilibrium. In a forthcoming paper [1], we shall indeed prove that an equilibrium such as we have described it always exists, but this paper concentrates only on the properties of such an equilibrium.

2. THE MODEL

There are two periods; by 1 we denote the first period, and by m denote events at the second period. In both periods, a single physical good is available for both consumption and production; for simplicity we shall take its price as unity.

In the first period each of J firms purchases inputs from which outputs will be produced by the firm in the second period. For firm j, denote the inputs by \mathbf{z}_j . Production is stochastic, and the outputs produced by j at event m if inputs of \mathbf{z}_j from the first period are available will be denoted by

$$y_{mj} = y_{mj}(z_j) \tag{1}$$

In addition to purchasing outputs, each firm j may engage in two kinds of financial activities in the first periods: It may sell bonds, which are to be repaid in the second period, and it may sell new equity. Let $p^S = (p_1^S, \dots p_J^S)$ be the vector of share prices in the market, where p_j^S is the price of firm j's equity in the first period. Furthermore, let $p^b = (p_0^b, p_1^b, \dots, p_J^b)$ be the vector of bond prices which prevail. The subscript zero denotes the price of inter-consumer loans (to be discussed shortly), and the other subscripts denote the price of firm bonds. If the firm j issues p_j^b nominal bonds, its net receipts from these bonds will be p_j^b . Barring bankruptcy, it will owe the bondholders the face value of the bonds at every event in the second period. We shall assume that all bond prices lie between zero and one; there are no negative interest rates.

We shall assume that initially there is one share of firm stock outstanding and that its price is p_j^s . Each of the I consumers has an initial endowment of the shares of each firm; the endowment of consumer i of firm j's shares will be denoted by f_{ij} , $0 \le f_{ij} \le 1$. The initial endowments of shares in any firm are assumed to sum to unity, i.e.,

$$\Sigma_{i} f_{ii} = 1 \tag{2}$$

A firm j which issues new equity a_j in period 1 may use the income from this equity. If the value of the firm's equity is p_j^s , the income to the firm from issuing a_j is $\frac{a_j}{1+a_j}$ p_j^s . This is so, since by issuing a_j , a proportion of the firm has effectively been taken from initial stockholders and sold on the market. Thus net firm income in the first period is

$$r_{1j}^{s} = \frac{a_{j}}{1 + a_{j}} p_{j}^{s} + p_{j}^{b} b_{j} - z_{j}$$
 (3)

If expression (3) is positive, it represents the surplus of first-period receipts over disbursements; i.e., a positive (3) will denote first-period dividends to initial shareholders. We shall constrain (3) to be non-negative; this is equivalent to requiring that inputs purchased during the first period be financed from sales of bonds and new equity. (We note that if the firm were to produce in the first period, we could require the sum of production income plus income from sales of new equity and bonds to be greater than or equal to $\mathbf{z_j}$. Thus no generality is lost in our procedure, and none of our results depend on the absence of production income in the first period.)

The dividends the firm pays in the second period depend on its second period revenues. Given event m and firm j, the revenue of j at m will be denoted by r_{mj} , where

$$\mathbf{r}_{\mathbf{m}i} = \mathbf{y}_{\mathbf{m}i} - \mathbf{b}_{i}. \tag{4}$$

Firm j will make a positive payoff to stockholders only if r_{mj} is non-negative. Otherwise, the firm is bankrupt, and what income there is from production goes to bondholders. Denote by r_{mj}^s the dividend to stockholders, and by r_{mj}^b the returns of bondholders. Then

$$r_{mj}^{s} = \begin{cases} r_{mj} & \text{if } r_{mj} \geq 0 \\ 0 & \text{otherwise} \end{cases}$$
 (5)

and

$$r_{mj}^{b} = \begin{cases} b_{j} & \text{if } r_{mj} \geq 0 \\ \\ r_{mj} + b_{j} & \text{otherwise} \end{cases}$$
 (6)

3. Consumers

Each consumer, in addition to having an endowment of shares, has an endowment of the physical good in both the first period and every state of the second period; we denote this endowment by (w_{1i}, w_{mi}) . Given share prices p^s and bond prices p^b , each consumer i must make three choices:

- (i) He must choose a consumption vector (x_{1i}, x_{mi}), where x_{1i} denotes consumption of the physical good in state m of the second period.
- (ii) He must choose a share portfolio, $f_i = (f_{i1}, \dots, f_{iJ})$. We shall assume that $f_{ij} > 0$ is the amount of shares

consumer i purchases in firm j. Thus, given j's issue of new shares (a,), the proportion of j held by consumer

i is
$$\frac{f_{ij}}{1+a_{i}}$$
.

(iii) He must choose a bond portfolio,

$$b_i$$
 = $(b_{io}, b_{i1}, \dots, b_{iJ})$. Here $b_{ij}, j=1, \dots, J$, is the proportion of firm j's nominal bonds purchased by consumer i. b_{io} will represent consumer i's net lending or borrowing from other consumers; if $b_{io} \geq 0$, consumer i is a net lender to other consumers.

Constraints on consumption are given by the following equations:

$$x_{1i} \leq \sum_{j} p_{j}^{s} \left(\frac{\bar{f}_{ij} - f_{ij}}{1 + a_{j}} \right) - \sum_{j} b_{ij} b_{j} p_{j}^{b} - p_{o}^{b} b_{io} + w_{io} + \sum_{j} f_{ij} r_{1j}^{s}$$

$$(7)$$

$$x_{mi} \leq \sum_{j} f_{ij} \left(\frac{r_{mj}^{s}}{1 + a_{j}} \right) + \sum_{j} b_{ij} r_{mj}^{b} + b_{io} + w_{mi}$$
 (8)

We shall assume that consumer i maximizes a state-dependent utility function of his consumption at both periods of the form:

$$U^{i}(x_{1i}, x_{mi}) = U^{i}_{1}(x_{1i}) + \sum_{m} \Pi^{i}_{m} U^{i}_{m}(x_{mi})$$
 (9)

Here Π_m^i represents i's assessment of the probability of the occurrence of state m; we shall assume that all such probabilities are strictly positive and sum to one for each i:

$$\Pi_{\mathbf{m}}^{\mathbf{i}} > 0, \ \Sigma_{\mathbf{m}}^{\mathbf{n}} \Pi_{\mathbf{m}}^{\mathbf{i}} = 1, \quad \mathbf{i} = 1, \dots, \mathbf{I}.$$
(10)

Furthermore, we assume everywhere positive marginal utilities:

$$\frac{dU_1^i}{dx_{1i}} > 0, \quad \frac{dU_m^i}{dx_{mi}} > 0 \tag{11}$$

Now assume that given prices p^s and p^b and firm plans (y_{mj}, z_j, a_j, b_j) for $j=1, \ldots, J$, consumer i maximizes U^i at $(x_i^{\sharp}, f_i^{\sharp}, b_i^{\sharp})$. Then strict equality must exist in the equations (7) and (8) above, and we may thus write U^i as a function of the portfolios,

$$U^{i}(x_{1i}^{*}, x_{mi}^{*}) = \bar{U}^{i}(f_{i}^{*}, b_{i}^{*})$$
 (12)

If $f_{ij}^{\star}>0$ and $b_{ij}^{\star}>0$, we may differentiate \bar{U}^{i} with respect to the portfolios, getting

$$\frac{d\bar{U}^{i}}{df_{ij}} = 0 \implies \frac{-dU_{1}^{i}}{dx_{1i}} \frac{p_{j}^{s}}{1+a_{j}} + \sum_{m} \prod_{m}^{i} \frac{dU_{m}^{i}}{dx_{mi}} \frac{r_{mj}^{s}}{1+a_{j}} = 0$$
 (13)

$$\frac{d\tilde{U}^{i}}{db_{ij}} = 0 \implies \frac{-dU_{1}^{i}}{dx_{1i}} p_{j}^{b} b_{j} + \sum_{m} \Pi_{m}^{i} \frac{dU_{m}^{i}}{dx_{mi}} r_{mj}^{b} = 0$$
 (14)

Solving the above two equations for prices p_j^b and p_j^s , we get

$$p_{j}^{s} = \sum_{m} n_{m}^{i} \frac{dU_{m}^{i}/dx_{mi}}{dU_{1}^{i}/dx_{1i}} r_{mj}^{s} = \sum_{m} n_{m}^{i} \rho_{m}^{i} r_{mj}^{s}$$
 (15)

$$p_{j}^{b}b_{j} = \sum_{m} \prod_{m}^{i} \rho_{m}^{i} r_{mj}^{b}, \quad j=1,...,J$$
 (16)

$$p_o^b = \sum_m \prod_m^i \rho_m^i \tag{17}$$

Since inter-consumer lending is riskless, equation (17) represents the riskless interest rate in terms of consumer i's marginal rates of substitution between periods.

Equilibrium: We shall say that equilibrium exists if given securities prices p^s and p^b , firm plans (z_j, a_j, b_j) , and consumer plans (x_i, f_i, b_i) , the following hold: Given prices and consumer plans, firms maximize total returns (the sum of r_{1j}^s plus the value of equity endowments) to initial shareholders; given prices and firm plans, consumers maximize their utilities; and consumer demand for bonds and shares equals the supply:

$$\Sigma_{i}^{b}_{ij} = 1, j = 1, \dots, J$$
 (18)

$$\Sigma_{i}f_{ij} = 1 + a_{i}, j = 1, \dots, J.$$
 (19)

Note that equilibrium in the securities market (equations (18) and (19)), coupled with (11) implies equilibrium in the commodity market (Walras's Law).

We note that in our model the terms $\Pi_m^i \rho_m^i$ play the role of individual state prices. Each individual's consumption decision determines ρ_m^i , and since state probabilities Π_m^i are fixed, consumption decisions for each individual determine the manner which firms are priced. Moreover, since the market provides separate prices for each firm's debt and equity, these separate prices are compared with the implicit consumer evaluations to determine portfolio decisions. What differentiates our market from a complete market? If all consumers agreed on $\Pi_m^i \rho_m^i$, we should be correct in calling our market complete. That is, if for every two consumers i and h, and for all states m, we have $\Pi_m^i \rho_m^i = \Pi_m^h \rho_m^h$, then our markets would be complete. None of the results we are about to prove below are changed by such an assumption, and previous authors are thus wrong in assuming that MM always holds in complete markets. 4

4. The effect of changes in new-shares issued and bonds issued on firm value

In what follows we shall make the standard assumption that all consumers evaluate changes in firm plans by their marginal utilities. The fundamental equations to which we shall refer will be equations (15) -(17). It is worthwhile repeating the meaning of these equations: The right-hand side of the equations is the shadow price to consumer i of the firm's shares, its debt, and consumer debt, respectively. The left-hand side is the market price. Equality need not obtain; only if a share or bond of firm j is held in the portfolio of individual i does equality obtain in equations (15) and (16) respectively. If we allow for short sales of firm bonds and shares, however, equality will always obtain. Since we have, in effect, allowed for short sales of consumer bonds, we may state:

<u>Lemma 1</u>: Given prices p^s and p^b , all individuals have the same implicit evaluation of the riskless interest rate. I.e.,

$$p_o^b = \sum_m \prod_m^i \rho_m^i$$
, for every i. (20)

Proof:

Since individuals may both borrow and lend risklessly, they will borrow if the left-hand side above exceeds the right-hand side, and they will lend otherwise, until equality is attained.

QED

In order to establish the central result of this paper, we wish to evaluate the changes in implicit prices which follow from the changes in firm plans. Denoting by A those second period events m for which firm j will not be bankrupt at current plans, and denoting the complement of A by \bar{A} , we get:

$$\frac{\partial p_{j}^{s}}{\partial a_{j}} = 0 \tag{21}$$

$$\frac{\partial p_{j}^{s}}{\partial b_{j}} = - \sum_{A} \Pi_{m}^{i} \rho_{m}^{i}$$
 (22)

$$\frac{\partial p_{j}^{b}}{\partial a_{j}} = 0 (23)$$

$$\frac{\partial \mathbf{p}_{j}^{b}}{\partial \mathbf{b}_{j}} = \frac{-1}{(\mathbf{b}_{j})^{2}} \Sigma_{\bar{\mathbf{A}}} \Pi_{m}^{i} \rho_{m}^{i} \mathbf{y}_{m}, \quad \text{Since } \mathbf{p}_{j}^{b} = \frac{1}{\mathbf{b}_{j}} (\Sigma_{\mathbf{A}} \Pi_{m}^{i} \rho_{m}^{i} \mathbf{b}_{j} + \Sigma_{\bar{\mathbf{A}}} \Pi_{m}^{i} \rho_{m}^{i} \mathbf{y}_{mj})$$

Equation (21) states that the equity value of the firm is not affected by the amount of new shares issued; neither do new equity issues affect the price of the firm's bonds (equation 23). However, the equity value of the firm is affected by changes in the bond issues of the firm (equation 22), and the price of the bonds is, under certain circumstances, also affected (equation 24, about which more later). Now suppose that firm j currently has plans (a_j, b_j, z_j) and that it changes the amount of debt issued by β . Furthermore, suppose that consumers and other firms do not change their plans; in particular, each consumer i will still purchase proportion $\frac{f_{ij}}{1+a_j}$ of the firm's shares and proportion b_{ij} of its bonds. Before the changes,

of the firm's shares and proportion b_{ij} of its bonds. Before the changes, the value of the firm was

$$\Sigma_{i} \frac{f_{ij}}{1+a_{j}} p_{j}^{s} + \Sigma_{i} b_{ij} b_{j} p_{j}^{b}$$
(25)

The change in firm plans will cause a change in the market valuation of the firm which, ignoring second order changes, will now be

$$\Sigma_{i} \frac{f_{ij}}{1+a_{j}} \{p_{j}^{s} - \beta \Sigma_{A} \Pi_{m}^{i} \rho_{m}^{i}\}$$
(26)

+
$$\Sigma_{i}b_{ij}(b_{j}+\beta) \{p_{j}^{b} - \frac{\beta}{(b_{j})^{2}} \Sigma_{\bar{A}} \Pi_{m}^{i} \rho_{m}^{i} y_{mj}\}$$

The following propositions now follow:

Theorem 2 (Modigliani-Miller): Suppose one of the two conditions (i) or (ii) hold:

(i) $\bar{A} = \emptyset$: i.e., firm j will not go bankrupt at current plans for any state m.

or

(ii) For every m ε \bar{A} , $y_{mj}=0$: i.e., for every state m for which firm j goes bankrupt, bondholders get nothing.

Then a change in the debt issued by the firm has no effect on the value of the firm if the market for the firm's shares and bonds was in equilibrium at plans (a_i, b_i, z_i) .

Proof:

Since there was equilibrium at previous plans,

 Σ_{i} f_{ij} = 1+a_j and Σ_{i} b_{ij} = 1. If β is the change in debt issued, and under conditions (i) or (ii) of the Theorem, (26) becomes

$$\{p_{j}^{s} - \beta \sum_{A} \prod_{m}^{i} \rho_{m}^{i}\} + (b_{j} + \beta)p_{j}^{b}$$
 (27)

We now examine two cases, described by (i) and (ii):

Case (i):
$$\bar{A} = \emptyset$$
. Then (27) becomes

$$p_{i}^{s} - \beta p_{o}^{b} + (b_{i} + \beta)p_{o}^{b} = p_{i}^{s} + p_{o}^{b}b_{i}$$

Case (ii): If condition (ii) holds, it follows from (16) that

$$p_j^b = \sum_A \prod_m^i \rho_m^i.$$

Therefore (27) becomes

$$p_{j}^{s} - \beta \Sigma_{A} \Pi_{m}^{i} \rho_{m}^{i}$$
+ $(b_{j} + \beta) \Sigma_{A} \Pi_{m}^{i} \rho_{m}^{i} =$

$$= p_{j}^{s} + p_{j}^{b} b_{j}.$$

In both cases, the Theorem is proved.

Q.E.D.

Theorem 2 is the familiar Modigliani-Miller result in its fullest generality. The debt-equity ratio is irrelevant if and only if two conditions hold: Either the firm at current plans will not go bankrupt, or, for every state at which the firm will go bankrupt at current plans, the bondholders will receive nothing. In addition, we must require that the market for the firm's bonds and shares clears at current plans. We note that our proof is motivated differently from other proofs (for instance, that of Stiglitz [7]). The standard motivation for the MM theorem is that individual and firm leverage are perfect substitutes when there is no bankruptcy risk. This is, of course, true in our case also, but we need have no recourse to this kind of logic. We suppose throughout this paper that changes in prices follow changes in implicit valuations. Under the conditions of Theorem 2, there is no change in the implicit valuation of the firm, and hence the MM theorem holds.

Our last theorem complements Theorem 2. That theorem stated that if the firm is at equilibrium, a change in its bond sales will not affect firm value, provided that at current plans there is no risk of firm bankruptcy, or bondholders expect to get nothing in the case of bankruptcy with current plans, and if the firm is currently in equilibrium, then a change in bond plans will decrease firm value. Before we state, and prove, Theorem 3, we note that it provides an important statement about the stability of any equilibrium when there is risk of firm bankruptcy. Any such equilibrium is stable with respect to changes in additional debt issued. This may provide some theoretical support for a maxim of practical finance, namely that an

engoing firm should not change its debt-equity ratio (and, by implication, its dividend policy).

Theorem 3: Suppose the following conditions hold:

- (i) \bar{A} is non-empty, and there exists at least one me \bar{A} such that at current firm plans $y_{mj} \neq 0$.
- (ii) The market is in equilibrium.

Then any change in bonds issued will decrease the value of the firm.

Proof:

If there is a change in b_{i} , (26) becomes

$$\{p_{j}^{s} - \beta \Sigma_{A} \Pi_{m}^{i} \rho_{m}^{i}\}$$

$$+ (b_{j} + \beta) \{p_{j}^{b} - \frac{\beta}{(b_{j})^{2}} \Sigma_{\overline{A}} \Pi_{m}^{i} \rho_{m}^{i} y_{mj}\} =$$

$$= p_{j}^{s} + p_{j}^{b} b_{j} - \frac{\beta^{2}}{(b_{j})^{2}} \Sigma_{\overline{A}} \Pi_{m}^{i} \rho_{m}^{i} y_{mj}.$$

and this expression is maximal when $\beta = 0$.

QED

Theorem 3 holds under conditions which exactly complement the conditions of theorem 2. Given these conditions, the debt/equity ratio of the firm is important. Moreover, given that equilibrium existed at previous firm plans, any change in the firm's debt/equity ratio will decrease the value of the firm. There is another way of stating this result: Suppose that firms try to maximize their values given consumer plans, and suppose that for a given firm plan, value is maximized and security market

equilibrium is achieved. Then if at equilibrium firm j is a risky borrower, the firm value is unique with respect to the debt/equity ratio. Since equilibrium implied that firm value was maximized, uniqueness must imply that any change in bonds issued decreases firm value.

We note that Theorems 2 and 3 together imply that the conditions of Theorem 2 are both necessary and sufficient for the MM irrelevance results. Finally, we note that our previous comment about Arrow-Debreu prices applies: Completeness has no effect on our results.

5. Dividend policy and debt/equity policy

Our definition of a full general equilibrium (section 3) states that firms try to maximize the total returns to initial shareholders. These returns are the sum of the dividend paid to initial shareholders (equation (3)) plus the market value of the shares currently held by them, $\frac{1}{1+a} p_j^s.$ Summing these two terms, we get

$$p_{j}^{s} + p_{j}^{b}b_{j} - z_{j}$$
 (28)

The theorems we have proven show that maximization of this expression does not depend on the debt/equity ratio only in the case of non-bankruptcy. Thus, any attempt to increase dividends to initial shareholders by selling more debt will actually fail to increase the total returns of these shareholders, since the value of (28) will fall. On the ther hand, equation (3) shows that dividends to initial shareholders may be changed by issuing more (or less) new equity; since the value of equation (28) is independent of a, this does not affect returns to initial shareholders.

We may thus gain a clearer understanding of the MM dividend irrelevance theorem [6]: Since changes in amount of new equity issued are

changes in "accounting units," they may be made at will by the firm. Shareholders counteract these changes by buying the same proportion of the firm's equity they purchased before, and though dividends change, the total return to initial shareholders remains constant. Since these changes do not affect the value of the firm's equity (equation (21)), they do not affect the debt/ equity ratio. On the other hand, Theorem 3 shows that changes in debt issued do, in general, affect the value of the firm, and hence (in the presence of bankruptcy) changes in the debt issue by the firm will affect (28).

The MM dividend irrelevance theorem is true, therefore, only in a very limited sense: Dividends are irrelevant, in the presence of bankruptcy, if they are changed by altering the amount of new equity the firm issues. But they are not irrelevant if their alteration is accomplished by changing the amount of debt issued, unless the firm is a riskless borrower.

6. Conclusion

We have presented a model which examines the shadow prices for a firm's shares and bonds derived from the consumer's maximization problem. Changes in market prices may be approximated by changes in implicit prices; it may then be shown that the debt/equity ration is determinate in equilibrium when the risk of bankruptcy exists. The same reasoning shows that the firms which are risky borrowers in equilibrium cannot change their dividends by changing the amount of debt issued without diminishing returns to initial shareholders. Neither of the MM irrelevance results [5,6] is thus generally true in the presence of bankruptcy.

Footnotes

I am indebted to Irwin Friend and Jeff Jaffee for a number of helpful comments. All mistakes are, of course, mine.

²Stiglitz showed that risk-neutrality, bankruptcy risk, and heterogeneous expectations lead to the non-irrelevance of the debt/equity ratio. We shall show that risk-aversion and bankruptcy lead to the same result (no matter what expectations are).

³For a clear exposition of the difference between complete and incomplete markets, see Baron [2]. Implicit valuations were also used by Diamond [3].

⁴A <u>locus classicus</u> of such an error is Fama and Miller [4]. On pages 158-59, they deduce the irrelevance of financial decisions in complete markets from the fact that firm value (equity plus present value of bonds) in such markets does not include any financial term. The reader may verify that, if $f_{ij}^{\star} > 0$, and $b_{ij}^{\star} > 0$, equations (15) and (16) imply that

$$p_j^s + p_j^b b_j = \sum_m \prod_m^i \rho_m^i y_{mj}$$
, for every i.

This does not, however, imply that individual valuations of each firm's equity and debt, taken separately, are independent of financial decisions. Since the nature of financial markets is to give separate prices for firm debt and equity, the non-dependence of total firm value on financial decisions does not imply their irrelevance.

BIBLIOGRAPHY

- 1. Benninga, S. "Competitive Equilibrium with Bankruptcy in a Sequence of Markets." unpublished working paper.
- 2. Baron, D.P. "Default Risk and the Modigliani-Miller Theorem: A Synthesis." American Economic Review 66 (1976), 204-212.
- 3. Diamond, P.A. "The Role of a Stock Market in a General Equilibrium Model with Technological Uncertainty." American Economic Review 57 (1967), pp. 759-76.
- 4. Fama, E. and M. H. Miller. The Theory of Finance. New York: Holt, Rinehart, and Winston, 1972.
- 5. Modigliani, F. and M. H. Miller. "The Cost of Capital, Corporation Finance, and the Theory of Investment." American Economic Review 48 (1958), pp. 261-97.
- 6. "Dividend Policy, Growth, and the Valuation of Shares." Journal of Business 34 (1961), pp. 411-33.
- 7. Stiglitz, J. "On the Irrelevance of Corporate Financial Policy."

 <u>American Economic Review</u> 64 (1974), 851-67.
- 8. _____. "Some Aspects of the Pure Theory of Corporate Finance: Bankruptcies and Take-Overs." <u>Bell Journal</u> 3 (1972), 458-82.