# BANK CREDIT RATIONING AND THE CUSTOMER RELATION

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## I. Introduction

A substantial literature has developed in recent years around the central premise that banks will preferentially lend to their larger "prime" customers during periods of credit restraint in order to protect the value to the bank of this group's high deposits and strong intertemporal demand.

The importance of the "customer relation" to bank lending policy was first highlighted by Hodgeman (1963) and has been an implicit or explicit assumption of much of the literature of the past decade.[1] Nevertheless, despite its widespread acceptance, the underlying theory has never been satisfactorily proven nor explicitly developed. This paper provides an analytical framework within which the meaning and importance of customer relation effects may be examined and shows that the arguments relating these effects to credit rationing are incomplete. In particular, it is shown that they could not result in the pattern of credit rationing suggested by proponents of this position.

The investigation of credit allocation under conditions of rationing, of course, presumes that periods of rationing do occur. One argument for the occurrence of such non-price rationing suggests that the behavior may result from the inability of banks to fully adjust their interest rate to uncertain demand within a competitive environment due to discrete transaction costs or lags in the implementation of such a change.[2] Either of these imperfections will lead to disequilibrium periods of excess demand when rationing, rather than price variation, is used as a temporary method for controlling supply. In the absence of such imperfections, however, such behavior would be irrational, as was noted quite pointedly by Samuelson (1952). The importance of such rationing is, therefore, dependent upon the perception of financial markets.

different risk if their risk adjusted returns are equal.

(5) The marginal unit of lending is "a customer loan" rather than part of a loan. Rationing therefore implies turning down a customer rather than just scaling down the size of his loan demand.

## III. The Effect of Elasticity on the Rationing Decision

Before examining the impact of customer relation effects, the rationing decision is first analyzed in the situation where customer groups differ only in their elasticity. This will allow the latter presentation to proceed in a more direct fashion.

In its simplest form the bank's total profit from its loan portfolio is given by revenues less costs across all loan groups, i.e.

$$(1)\quad \pi = \sum_i r_i \ell_i - c\sum_i \ell_i$$

where $r_i$ is ther interest rate on the $i^{th}$ loan, $\ell_i$, and c is the unit (average) cost of funds which is dependent upon quantity but invariant to allocation.

Within a given group, the homogeneity assumption allows the bank to formulate an optimum interest rate for all customers in that group, dependent on the elasticity of the group's aggregate demand curve. Thus we may rewrite Equation (1) as

$$(2)\quad \pi = \sum_j r_j \ell_j - c\sum_j \ell_j$$

where $r_j$ and $\ell_j$ represent the interest rate and total loan volume of the $j^{th}$ customer group.

Profit maximization using the rate charge each customer as a control variable implies, for all j, that the first order condition be satisfied, viz., may be written as

$$(8) \quad \phi_j = \frac{r_c}{\eta_j - 1} \quad .$$

Taking the derivative of equation (8), the sign of $\frac{\partial \phi_j}{\partial \eta_j}$ is unambiguously negative. Thus, ceteris paribus, the more elastic market will always have the lower cost of rationing and will therefore be rationed first. This follows logically from the observation that the interest rate is highest, and the monopolistic profits greatest, in the least elastic market.

Since rationing at the optimum loan volume is unprofitable, it is of course improbable that such rationing would occur in the absence of any external constraint. However, in Section I it was suggested that rationing may occur in disequilibrium, as when, following an increase in the total demand for funds, the current interest rate, $r_j$, does not immediately rise in response to an increase in the cost of funds. Consider the case where the marginal cost of funds increases from $r_c^o$ to $r_c^o + \varepsilon$, where superscripts refer to the initial value. From equation (7) the cost of rationing in this case will be

$$(9) \quad \phi_j = r_j^* - [r_c^o + \varepsilon] = \phi_j^o - \varepsilon$$

where $\phi_j^o$ refers to the cost of rationing in the previous equilibrium, with rationing profitable only for $r_c^o + \varepsilon > r_j^*$. The comparative cost of rationing any two markets, given by equation (10), is then the same as the difference in costs in the equilibrium situation,

$$(10) \quad \phi_I - \phi_{II} = [\phi_I^o - \varepsilon] - [\phi_{II}^o - \varepsilon] = \phi_I^o - \phi_{II}^o \; .$$

Hence, in a situation where it is profitable to ration one or both markets, there will always be an additional cost (smaller benefit) to rationing the and the first order condition for profit maximization yields

$$(12)\quad \frac{\partial\pi}{\partial\ell_j} = [r_j + \ell_j\frac{\partial r_j}{\partial\ell_j}] + Q_j r_c - r_c = 0.$$

Comparing equation (12) with equation (3) at equilibrium, the marginal cost of funds is now set equal to the marginal interest revenue from loans plus the additional marginal revenue obtained from reinvestment of the associated deposits. The equilibrium interest rate can be obtained by sbustituting for the elasticity in equation (12) to yield

$$(13)\quad r^*_j = \frac{\eta_j r_c(1-Q_j)}{\eta - 1} \quad .$$

The impact of the deposit relationship on the optimal rate charged each group may be seen by taking the partial derivative of $r^*_j$ with respect to $Q_j$, i.e., $\frac{\partial r_j}{\partial Q_j} < 0$. Thus in the presence of associated deposits, the bank increases its profits by offering a lower interest rate to customers with a deposit-loan relationship and extending a larger volume of loans.

Turning to the cost of rationing, equation (11) indicates that the loss in profit from rationing at the equilibrium interest rate is

$$(14)\quad \left.\frac{\partial\pi}{\partial\ell}\right|_{r^*_j} = \phi_j = r^*_j - r_c(1 - Q_j)$$

which, substituting for $r^*_j$ from above, may be written as

$$(15)\quad \phi_j = \frac{r_c(1 - Q_j)}{\eta_j - 1} \quad .$$

The partial derivatives of equation (15) indicate the effect of elasticity and a deposit relationship on the cost of rationing any customer group and is a set of combinations of required balances and interest rates that have the same effective cost to the borrower. Assuming that the demand was a function of the total cost, the bank would be unable to profitably increase its deposit requirements without being forced by competition to lower its interest rate in compensation. The most profitable required balance would then be the actual customer's natural deposit level.

Hodgeman (1961) on the other hand contends that competition for the deposits of prime borrowers is inhibited by the "prime rate convention", regarded by him as a price fixing arrangement that provides an "... agreed minimum interest rate for bank loans to deposit customers." In this situation large depositors would be unable to get interest rate concessions from any bank, and the additional revenue from investment of their deposits would accrue as excess profits to the banks. Thus, in the presence of such collusion the profitability of these customers could be high enough to offset the lower elasticity of the non prime customers and lead to the latter group being rationed first.

The assumption of such collusion does not, however, seem well justified. Davis and Guttentag (1963) suggest that the compensating balance requirement, rather than being a method for increasing the loan cost above the competitive level, is instead merely a qualification for membership of the "high deposit" group, whose members receive a lower interest charge in return. Under these conditions the results of the earlier analysis are valid, and high deposit customers would not receive preferential treatment during periods of credit rationing.

## V. An Intertemporal Model of the Bank and Its Implication for Credit Rationing

So far the analysis has taken into account only income derived from the customer in the period in which the loan is outstanding. However, there

Similarly, if in the aggregate for a given group, the expected deposit level in any period is a function of loan volume in previous periods, the deposit relationship may be written as

$$(18)\quad D^j_t = D^j(\ell^j_t, \ell^j_{t-1}, \ell^j_{t-2}, \ell^j_{t-3}, \ldots)$$

or once again assuming geometrically decling weights

$$(19)\quad D^j_t = q\ell_t + dq\ell^j_{t-2} + d^2q\ell^j_{t-2} + d^3q\ell^j_{t-3} \ldots$$

where q, as before, is the fraction of loan volume kept on deposit and d is the probability that the deposit will remain one period beyond the loan, $d^2$ the probability of it remaining two periods, etc..

The present value of the bank's total profit stream is then given by the summation of the discounted revenue over cost each period, i.e.,

$$(20)\quad \pi = \sum_{t=1}^{\infty} [\sum_j \beta^{t-1}[r^j_t\ell^j_t - c_t(\ell^j_t - (1-\rho)D^j_t)]].$$

The profit maximizing behavior towards a given group is found by setting the partial derivatives with respect to the interest rate in each period equal to zero,

$$(21)\quad \frac{\partial\pi}{\partial r^j_t} = 0 = [\ell^j_t + (r^j_t - r_{c_t})\frac{\partial\ell^j_t}{\partial r^j_t} + \beta(r^j_{t+1} - r_{c_{t+1}})\frac{\partial\ell^j_{t+1}}{\partial r^j_t}$$

$$+ \beta^2(r^j_{t+2} - r_{c_{t+2}})\frac{\partial\ell^j_{t+2}}{\partial r^j_t} + \ldots] + [r_{c_t}\frac{\partial D^j_t}{\partial\ell^j_t} \cdot \frac{\partial\ell^j_t}{\partial r^j_t}(1-\rho)$$

$$+ r_{c_{t+1}}\beta(\frac{\partial D^j_{t+1}}{\partial\ell^j_t} + \frac{\partial D^j_{t+1}}{\partial\ell^j_{t+1}} \cdot \frac{\partial\ell^j_{t+1}}{\partial r^j_t})\frac{\partial\ell^j_t}{\partial r^j_t}(1-\rho)$$

$$+ r_{c_{t+2}}\beta^2(\frac{\partial D^j_{t+2}}{\partial\ell^j_t} + \frac{\partial D^j_{t+2}}{\partial\ell^j_{t+1}} \cdot \frac{\partial\ell^j_{t+1}}{\partial\ell^j_t} + \frac{\partial D^j_{t+2}}{\partial\ell^j_{t+2}} \cdot \frac{\partial\ell^j_{t+2}}{\partial r^j_t})\frac{\partial\ell^j_t}{\partial r^j_t}(1-\rho)\ldots].$$

loss of current and discounted future profits is given by equation (23).

$$(23)\quad \left.\frac{\partial \pi}{\partial \ell_t^j}\right|_{r_t} = \phi = (r_t^j - r_{c_t}) + \beta(r_{t+1}^j - r_{c_{t+1}})\frac{\partial \ell_{t+1}^j}{\partial \ell_t^j}$$

$$+ \beta^2(r_{t+2}^j - r_{c_{t+2}})\frac{\partial \ell_{t+2}^j}{\partial \ell_t^j} + \ldots + \frac{\partial D_t^j}{\partial \ell_t^j} r_{c_t} + r_{c_{t+1}}\beta[\frac{\partial D_{t+1}^j}{\partial \ell_{t+1}^j}$$

$$+ \frac{\partial D_{t+1}^j}{\partial \ell_{t+1}^j}\frac{\partial \ell_{t+1}^j}{\partial \ell_t^j}] + \ldots$$

Making an approximation that the interest rate moves directly to the long run optimum in period two,[6] equation (23) can be simplified, by the same method used above, to yield

$$(24)\quad \phi_j = \frac{r_c{}^j}{\eta_j - [\frac{1-2\beta\omega_j}{1-\beta\omega_j}]}\ [1 - \frac{(1-\rho)q_j}{(1-\beta d_j)}] \quad .$$

Analyzing the impact that the intertemporal effects has on the cost of rationing a customer group, results in

$$\left.\frac{\partial \phi_j}{\partial \eta_j}\right|_{q_j,\, d_j} < 0, \quad \left.\frac{\partial \phi_j}{\partial q_j}\right|_{d_j,\, \eta_j} < 0, \quad \left.\frac{\partial \phi_j}{\partial d_j}\right|_{q_j,\eta_j} < 0 \quad .$$

Thus, starting for a given elasticity, intertemporal loan demand and deposit level, movement towards a higher elasticity and stronger customer relation effects will actually reduce the cost of rationing borrowers. Consequently, even in a multiperiod model there is no reason to expect the bank to provide preferential credit for the prime customer group, possessing high elasticity and intertemporal aspects to its customer relations. While the presence of high deposits and intertemporal inertia increases the bank's aggregate profit, the bank optimally responds with a lowered interest rate which differentiating the profit expression at constant interest rate,

$$(27)\quad \phi_j = \left.\frac{\partial\pi}{\partial\ell_j}\right|_{r_j^*} = (r_j^* - r_c) + \psi_j' \quad .$$

The may be simplified by substituting for $r_j^*$ from equation (26) to yield,

$$(28)\quad \phi_j = -\ell_j \frac{\partial r_j^*}{\partial\ell_j} \quad .$$

Finally, substituting the elasticity expression gives the general expression for the cost of rationing any single group, here group j,

$$(29)\quad \phi_j = \frac{r_j^*}{\eta_j} \quad .$$

In fact it can be verified by inspection that the same relationship held in all previous cases.

The incorrect, yet traditional arguments relating to intertemporal loan demand and deposit levels have suggested that since the cost of rationing expression in equation (27) included the additional profit of the customer relation, a sufficiently large value of $\psi_j'$ could compensate for the lower interest rate of the more elastic market and lead to preferential treatment for prime market customers. However, if the bank sets the interest rate for each group at the profit maximizing level, the presence of customer relation effects will lower the interest rate $r_j^*$ below what the bank would have charged the customer group without the added relationship of future loan demand and deposit levels. This reduces the profitability of each customer - though increasing aggregate profits from the larger number of customers. Therefore, it is clear from equation (29) above that whatever the nature of $\psi_j$, the market with lower $r_j^*$ and higher $\eta_j$ will always be the least costly to ration.

In achieving this objective it is subject to the budget constraint,

$$(31)\quad X_t = (1 + rx_{t-1})X_{t-1} + \ell_t + B_t - (1 + r_{t-1})\ell_{t-1} - (1 + rb_{t-1})B_{t-1} .$$

The firm's decision variables are the capital investment, loan size, and amount of other borrowing.[8] The first order conditions of the maximization of equation (30) subject to equation (31) can be written as

$$0 = \frac{\partial E_{t+1}}{\partial X_t} = (r'_{x_t} + 1) - \mu_t ,$$

$$0 = \frac{\partial E_{t+1}}{\partial \ell_t} = -(r'_t + 1) + \mu_t ,$$

$$0 = \frac{\partial E_{t+1}}{\partial B_t} = -(r'_{b_t} + 1) + \mu_t ,$$

where $\mu_t$ is the Lagrangean multiplier, and $r'_{x_t}$, $r'_t$ and $r'_{b_t}$ represent respectively the marginal return on capital, the marginal cost of a bank loan and the marginal cost of other borrowing.[9]

From these first order conditions, the firm's optimum position is thus characterized by equality of the marginal return on capital and the marginal cost of funds, viz.,

$$(32)\quad r'_{x_t} = r'_t = r'_{b_t} = (\mu_t - 1).$$

If the bank now rations the amount of credit available to the firm that period, this optumum is not obtainable. Instead the firm's loan is limited to $\overline{\ell}_t$, less than the unconstrained level, and the firm must reoptimize subject to this constraint to find the second best solution. This may be represented by the addition of the extra constraint to equation (30), with Lagrangean Multiplier $\xi_t$.

$$(33)\quad \overline{E}_t = E_t + \xi_t(\overline{\ell}_t - \ell_t).$$

than the cost of expanding borrowings with the investment size held constant, or the cost of reducing investment with other borrowings unchanged.

The integrals of equation (37) can, however, be conveniently evaluated from the firm's bank loan demand schedule, which gives the interest rate, $r'(\ell_t)$, at which the firm would demand a loan of size $\ell_t$. From equation (32), in an unconstrained optimization the loan cost, $r'(\ell_t)$, will equal the marginal cost of other borrowings, $r'_{b_t}$, and the marginal return on capital, $r'_{x_t}$, when X and B are at their optimum levels, given $\ell_t$. Hence, if the firm is rationed to a loan of size $\bar{\ell}_t$, the interest rate, $r'(\ell_t)$, will again equal $r'_{b_t}$ and $r'_{x_t}$ at the constrained optimum values. Assuming loan demand falls to zero at some finite interest rate, denoted $r_0$, one can equate

$$(38)\quad \int_{B_t}^{\bar{B}_t} r'_{b_t}\,dB = \int_{X_t}^{\bar{X}_t} r'_{x_t}\,dX = \int_{\ell_t}^{0} r'(\ell_t)\,d\ell$$

The total cost of being rationed, equation (37), can, therefore, be written

$$(39)\quad \Lambda = -\int_{\ell_t}^{0} r'(\ell_t)\,d\ell + \int_{\ell_t}^{0} \bar{r}'_t\,d\ell$$

This is the traditional consumer surplus area of the loan demand curve.

The cost of being rationed, $\Lambda$, will clearly vary between firms according to the characteristics of their demand functions. Substituting for the elasticity in equation (39) indicates that this expression may be written as:

$$(40)\quad \Lambda = \int_{r_t}^{r_0} \ell\eta\,dr - \int_{\ell_t}^{0} r_t\,d\ell$$

and, at any current loan volume $\ell_t$ and interest rate $r_t$, the derivative is:

the bank would not, in this case, lower the interest rate for the high deposit firm since it would be accepting the deposit revenue in return for incurring an additional cost.

The analysis of the previous sections have shown that in the absence of such agreements the prime customers would be the first borrowers to be rationed and would therefore be the group desiring this protection. Since the group is assumed to be homogeneous, if one customer were able to negotiate a credit line all others in the prime group would also be willing to do so. However, in this case they would not need to compensate the bank for the cost of not rationing anyone, but only for the additional costs involved in rationing the non-prime customers instead. This net cost to the bank will be

$$\phi_{np} - \phi_p$$

per dollar of rationing in that period, and the minimum compensation for guaranteeing a loan to a prime customer one period in advance would be

$$P_B = \beta_B \ell (\phi_{np} - \phi_p)$$

where all terms have their previous definitions. A mutually beneficial bargain between the bank and its prime customers is thus possible if and only if

$$(42) \qquad \beta_f \ell_p \lambda_p > \beta_B \ell_p (\phi_{np} - \phi_p)$$

which reduces to

$$(43) \quad \lambda_p > (\phi_{np} - \phi_p)$$

if the two parties have the same expectations of the probability of rationing and the same discount factors.

However, if such a bargain were feasible, the previously unexposed not to the prime customer, but to the least elastic customer group who can pay the highest price for such commitments. Their existence reinforces the allocation of credit to the non-prime group, rather than reversing this result.

## IX. Conclusion

This paper has investigated the theory that preference in situations of bank rationing is given to those customers with the strongest customer relation. It has shown that, within a certainty model, neither deposit levels, intertemporal demand nor any other non-interest element of the loan vector would in fact produce the preferential treatment claimed. It has also been shown that the prime borrowers would be unable to bargain effectively for credit line protection from such occasions of credit restriction.

These results are not meant to suggest that prime customers would actually be rationed first; the available evidence lends support to the claim that discrimination against non-prime customers does in fact occur.[10] Rather it suggests that models based on the customer relation are inadequate to explain the special status of the prime customer, and other factors such as risk must properly be considered.[11] Indeed, the suggestion offered by the present study is that such rationing is totally a risk allocation matter.

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