# Consumption and Saving in Economic Development

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Working Paper No. 11-77

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From the viewpoint of the economy as a whole, interest in the relationship of consumption or saving to income stems from both cyclical and secular considerations. The consumption function, relating consumption to income and other appropriate variables, plays a central role in most models of national income determination, used both for forecasting short-term fluctuations in economic activity and for assessing the impact of alternative monetary and fiscal policies. More important probably for developing nations is the role played by saving, or abstinence from current consumption, in fostering economic growth and hence future consumption.

Though there is substantial disagreement on the relative importance for growth of non-human capital accumulated through saving (and on the welfare implications of government intermediation in the economic process to appropriately affect the saving-income, or its complement the consumption-income, ratio), most economists including the authors would agree that economic development can be promoted by increased saving. The size of the effect is likely to vary with the stage of economic development and with the magnitude of the change in the saving ratio. The potential impact on growth of increases in this ratio is probably relatively large in the earlier stages of economic development, at least so long as the increases in the saving ratio do not bear down excessively on the previous standard of living or otherwise raise political difficulties. Even in the earlier

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states of economic development, the effect of increased savings and non-human wealth on economic growth may be minimal unless there are institutional arrangements to channel saving into appropriate forms of investment and unless there exists the ability to make use of available technological advances.

In general private real investment is undertaken only if the expected rate of return exceeds the rate which must be paid to the suppliers of funds. Governments may influence the latter in a favorable direction by policies which encourage saving, which facilitate the flow of funds from savers to investors or which provide some degree of insurance against investment risks to individual enterprises and their owners and creditors. The marginal efficiency of investment may also be influenced by the government. Rapid growth of consumption tends to generate attractive investment opportunities, as does direct government investment of the kind associated with development of the infrastructure. Unfortunately policies that tend to raise the savings rate will have adverse effects on the growth of consumption; and unless strong growth in other sectors (e.g., export and governmental demand) compensates to some degree for declining growth in consumption, the reduction in cost of financing associated with a higher savings rate may be more than offset by a reduction in the volume of investment opportunities promising high returns. Governmental policies which tend to direct investment into those capital goods that contribute most substantially to increasing labor productivity, rather than into housing and consumer durables, may promote a high growth rate in labor income and hence in consumer demand.

This paper will discuss the theoretical bases of the optimal allocation of income between consumption and saving (Section 1); consider the limitations of data available for empirical analysis of the determinants of consumption and saving, provide a simplified theoretical framework for analyzing such data and review prior empirical studies (Section 2); present an analysis of new Columbian household data for the years 1967-68 which attempts to minimize data limitations (Section 3); and will finally summarize the methodological and policy implications of the preceding analysis (Section 4). We had originally hoped to analyze new household data for other South American countries but they were not available in time.

## 1. Optimal Consumption Behavior

The theoretical problem of the optimum allocation of income between consumption and asset accumulation cannot be separated from the question of what motivates the holding of assets and the related question of what determines the desired level of the asset stock. We consider first the case in which assets are held only for the purpose of reallocating consumption over time or to build an estate.

### Saving to Reallocate Consumption Over Time

If an individual has an additive separable utility function, with consumption in the current and a finite number of future periods as arguments, and if wealth at the end of any of these periods serves only the purpose of redistributing consumption over time, then it is relatively easy to determine the optimal time pattern of end-of-period wealth, given income expectations and the transformation possibilities for converting current into future consumption. It is also possible to represent the multiperiod optimization problem by a series of

single-period decisions allocating resources between current consumption and wealth at the end of the current period. The marginal utility of terminal wealth at the end of any period is defined as the sum of increments in utility obtainable by distributing an infinitesimal increment in that wealth in the optimal way over consumption in future periods.

The Logarithmic Utility Function. Consider for simplicity the particular case of a logarithmic utility function,[1] with terminal wealth, $W_L$, assumed fixed:

$$U_{1,L} = \sum_{i=1}^{L} \alpha_i \log C_i + B.$$

Here the planning horizon is L periods, B is the utility of $W_L$, and the $\alpha_i$ reflect inter alia the personal rate of time preference. The wealth constraint is

$$W_o(1+r)^L + \sum_{j=1}^{L} Y_j(1+r)^{L-j} = \sum_{j=1}^{L} C_j(1+r)^{L-j} + W_L,$$

where $W_o$ is net worth at the beginning of period 1, $Y_j$ is earned income in the $j^{th}$ period, $W_L$ is wealth at the end of L periods and r is the lending (borrowing) rate.[2] Maximizing $U_{1,L}$ subject to this constraint we obtain the following first order conditions:

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[1]This function is exhaustively considered in W.H. Somermeyer and R. Bannink, A Consumption-Savings Model and its Applications, North-Holland Publishing Co., 1973.

[2]Both consumption and income are assumed to occur at the end of the period.

$$C_i^* = \alpha_i \frac{W_o(1+r)^L + \sum_{j=1}^{L} Y_j(1+r)^{L-j} - W_L}{(1+r)^{L-i} \sum_{j=1}^{L} \alpha_j}, \quad i=1, 2...L \qquad (1)$$

Thus under present assumptions, optimum consumption in each time period is a fraction (varying among periods) of the total resources available for consumption over the planning horizon. Equivalently, if we factor out $(1+r)^L$ from the numerator of (1), we may express each $C_i^*$ as a fraction of initial wealth plus the present value of future earnings less the present value of terminal wealth. This is, of course, the familiar Modigliani-Brumberg-Ando conception. Note that we have assumed here that earned income is known with certainty over the L periods and that r is also known with certainty. We defer to a later point the treatment of uncertainty as to future income, as well the introduction of real assets with uncertain return as an investment alternative to debt assets (which are taken to be risk-free).

The equations (1) may generate consumption levels early in the working life that require borrowing against future earnings; and this may not be generally feasible under existing capital markets or may occur only at rates above the lending rate available to consumers. A high value of the personal rate of time preference relative to the interest rate, a low level of initial earnings relative to that expected in the middle years and a weak bequest motive relative to inheritances received are all conducive to such an outcome. While there is little empirical evidence on the incidence of the first or the third condition, the second is widely applicable, although the individual consumer -

once we relax the assumption that future earnings are certain - may be relatively reluctant to borrow against hypothetical future increases. In those cases where early borrowing against future earnings is implied by (1) and capital market constraints restrict such borrowing, the model is inadequate and (1) does not represent an attainable optimum.

The equations (1) describe a multi-period consumption plan. Only the first step of the plan is undertaken in the current period; however, and in practice the remaining steps are not likely to be carried through without modification since earnings and interest rate expectations, as well as tastes, change over time. It is convenient for some purposes (in particular graphical ones) to restate the determination of $C_1$ in terms of a single-period tradeoff between $C_1$ and wealth at the end of the first period, $W_1$. Since each additional dollar of $C_1$ requires the sacrifice of one dollar of $W_1$, the single period optimum may be obtained by equating the marginal utility of consumption in period 1 to the marginal utility of wealth at the end of period 1, with the latter of course depending on the satisfactions expected from the subsequent use of $W_1$.

The value of $C_1$ that maximizes $U_{1,L}$ subject to the L-period resource constraint must also maximize the related function

$$U'_{1,L} = \alpha_1 \log C_1 + U_{2,L}^{max} \qquad (2)$$

subject to the constraint

$$W_1 = W_o(1+r) + Y_1 - C_1,$$

where $U_{2,L}^{max}$ is the maximum contribution to utility that can be obtained

from consumption in periods 2,..,L under the (L-1) period resource constraint:

$$W_1(1+r)^{L-1} + \sum_{j=2}^{L} Y_j(1+r)^{L-j} - W_L = \sum_{j=2}^{L} C_j(1+r)^{L-j}.$$

By the same argument used earlier to derive (1), the consumption levels, $C_2^*$, $C_3^*$, ... $C_L^*$, that produce such a maximum are linear in the resources available, represented by the left hand side of the constraint equation above.

Given the interest rate, future earnings, and terminal wealth--which are all determined independently of the optimization process--the term $U_{2,L}^{max}$ becomes a function of $W_1$ alone:

$$U_{2,L}^{max} = \sum_{i=2}^{L} \alpha_i \log C_i^* = \sum_{i=2}^{L} \alpha_i \log(a_i W_1 + b_i),$$

where the $a_i$ and $b_i$ are constants wth respect to the optimization process. Thus $U'_{1,L}$ depends only on $C_1$ and $W_1$; and the consumption decision in period 1 may be determined by the tangency of a two-dimensional indifference curve to the appropriate two-dimensional linear constraint.

The Constant Elasticity Utility Function. Another simple utility function, which--like the logarithmic--is characterized by constant

relative risk aversion but appears to be somewhat more plausible on empirical grounds,[1] is:

$$U_{1,L} = -\sum_{i=1}^{L} \alpha_i (C_i)^{1-\gamma}, \quad \gamma>1. \tag{3}$$

In this case also, it may be shown that consumption in each period is linear in $W_o$, $W_L$ and the $Y_j$. Assuming that interest is not earned on the current period's saving (or lost on the current period's dissaving), we have the following constraints

$$W_i = (1+r)W_{i-1} + Y_i - C_i, \quad i = 1, 2,...L$$

or

$$C_i = (1+r)W_{i-1} + Y_i - W_i.$$

Then (3) may be written

$$U_{1,L} = -\sum_{i=1}^{L} \alpha_i \left[(1+r)W_{i-1} + Y_i - W_i\right]^{1-\gamma}.$$

Equating to zero the partial derivatives of $U_{1,L}$ with respect to the L-1 unknown $W_i$, we may determine the optimal time pattern of wealth at the end of each period (and implicitly of consumption within each period):

$$\frac{\partial U_{1,L}}{\partial W_i} = \alpha_i(1-\gamma)\left[(1+r)W_{i-1}+Y_i-W_i\right]^{-\gamma} - \alpha_{i+1}(1-\gamma)(1+r)\left[(1+r)W_i+Y_{i+1}-W_{i+1}\right]^{-\gamma} = 0$$

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[1] Irwin Friend and Marshall Blume, "The Demand for Risky Assets," The American Economic Review, December, 1975.

or

$$(1+r)W_i + Y_{i+1} - W_{i+1} = \left[\frac{\alpha_i}{(1+r)\alpha_{i+1}}\right]^{-1/\gamma} \left[(1+r)W_{i-1} + Y_i - W_i\right].$$

Letting $m_i = [\alpha_i \div (1+r)\alpha_{i+1}]^{-1/\gamma}$ and rearranging in standard form, we obtain the following L-1 equations in the L-1 unknowns $W_1, W_2, \ldots W_{L-1}$

$$\begin{aligned}
(1+r+m_1)W_1 - W_2 &= m_1 Y_1 - Y_2 + m_1(1+r)W_o \\
-m_i(1+r)W_{i-1} + (1+r+m_i)W_i - W_{i+1} &= m_i Y_i - Y_{i+1}, \quad i=2,3,\ldots,L-2 \\
-m_{L-1}(1+r)W_{L-2} + (1+r+m_{L-1})W_{L-1} &= m_{L-1} Y_{L-1} - Y_L + W_L
\end{aligned}$$

The solutions for the $W_i$, and hence for the $C_i$, are linear in $W_o$, $W_L$ and the $Y_j$, with coefficients depending on the utility function parameters, $\gamma$ and $\alpha_i$, the interest rate r and the planning horizon L. It can in fact be shown that the $C_i^*$ are linear in the expression

$$W_o(1+r)^L + \sum_{j=1}^{L} Y_j(1+r)^{L-j} - W_L,$$

which represents maximum potential lifetime resources for consumption.[1]

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[1]This can be done most easily by assuming that the entire present value of all resources available for consumption is on hand initially. We then redefine initial wealth as

$$W_o^* = W_o + \sum_{j=1}^{L} Y_j(1+r)^{-j} - W_L(1+r)^{-L},$$

and the constraints become

$$W_i^* = (W_{i-1}^* - C_i)(1+r), \quad i = 1, 2, \ldots, L,$$

$$W_L^* = 0,$$

if we now assume that funds allocated to consumption in a given period earn no interest during that period.

Relationship of Terminal Wealth to Earnings. If we consider the entire span of an individual's life from the time he starts to work until death, $W_L$ must be interpreted as the amount desired for estate purposes, while $W_o$, if not zero, is an amount received by inheritance or gift. Under the present assumption of certainty as to future income, it is interesting to note that so long as $W_L$ (planned bequest) is not very different from $W_o(1+r)^L$ (inheritance received invested at compound interest over expected life), the optimal levels of wealth in intermediate periods will be more closely related to the variability among periods of future earned income and desired future consumption than to the average level of expected earned income. $W_i$ will exceed $W_o(1+r)^i$, i=1, 2, ... L-1, only to the extent that it is optimal to transfer consumption from the periods preceding i to the periods subsequent to i. This occurs only if income in periods preceding i is high relative to income in periods subsequent to i or if consumption needs are relatively urgent in the later as compared with the earlier period or if the interest rate is high relative to the rate of time preference. Thus a low-income person anticipating a declining trend in income or relatively heavy consumption needs at some future date might well hold more assets than a high-income person anticipating a rising income trend or having relatively urgent needs for current consumption. More generally, if the optimal $(Y-C)_i$ are uncorrelated with time, the income effect on wealth held for reallocation purposes will be minimal. Only if $W_L$ is fixed at a level substantially above $W_o(1+r)^L$, suggesting that the estate motive is significant, and if this level is correlated with income, as we might reasonably expect, will there be a clear case for a substantial income effect on the wealth held during intermediate periods.

However, if the normal life cycle pattern entails rising income over the earning years, with a sharp decline after retirement, and if individuals select their retirement date as the planning horizon, L, relevant to their savings decisions, then both terminal and intermediate wealth holdings will be closely related to earnings, even in the absence of a strong estate motive. In this case $W_L$ is interpreted as an amount sufficient to generate the desired bequest[1] plus the capital value of an annuity of the desired size to provide for the years of retirement. The post-retirement period is then one of capital consumption, in which a sum equal to the cost of the annuity is dissipated over the remaining life of the individual.

It is reasonable to assume that the desired level of post-retirement consumption, and hence the size of the annuity to be purchased at retirement, is primarily dependent on lifetime earnings and the standard of living which these earnings have permitted. The simplest assumption is that the desired annuity, as well as the desired bequest, is roughly porportional to average income during the working years, but it may be somewhat more plausible to assign to $W_L$ an income elasticity greater than unity. If the planned reallocation of consumption among periods 1 through L is small and/or uncorrelated with time but the retirement and estate motives are significant, then the optimal time pattern of end-of-period wealth will reflect the gradual buildup of $W_o$ to $W_L$; and the $W_i$ will have an income elasticity similar to that of $W_L$.

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[1] Allowing for the accumulation of compound interest from the time of retirement to the time of death.

We consider the implications of this model for individual and aggregate saving, defining normal income, $Y_N$, to be weighted average of future earned incomes, with weights $(1+r)^{L-j}$

$$Y_N = \frac{1}{\sum_{j=1}^{L} (1+r)^{L-j}} \sum_{j=1}^{L} Y_j(1+r)^{L-j}.$$

Again utilizing the log utility function as an example and making the further assumption that

$$\alpha_i = \frac{\alpha}{(1+r)^i} \quad ,$$

it follows from (1) that optimal consumption during the earning years is then given by

$$C_i^* = \frac{1}{\sum_{j=1}^{L} (1+r)^{L-j}} [\sum_{j=1}^{L} Y_j(1+r)^{L-j} - W_L + W_o(1+r)^L].$$

Expressed in terms of normal income, $Y_N$, this becomes

$$C_i^* = Y_N - \frac{r}{(1+r)^L - 1} [W_L - W_o(1+r)^L].$$

In this special case we note that, for each individual, $C_i^* = C_j^* = C$ for all periods within the planning horizon, while for all earners of the same age (i.e. with the same number of years, L, to retirement) con-

sumption is equal to mean future earnings less a fixed linear function, given r, of desired wealth at retirement and current wealth.

Optimal saving by earners in period 1 may then be written as the difference between current and mean future earned income plus a fraction (dependent on age and the interest rate) of the difference between desired wealth at retirement and the value that can be generated by investing current wealth at compound interest to the date of retirement:

$$S_1 = (Y_1 - Y_N) + f(r,L)\ [W_L - W_o(1+r)^L]. \qquad (4)$$

We observe that under present assumptions, with future earnings certain, transitory income, defined as $Y_1 - Y_N$, is fully saved.

For a household with given tastes at the beginning of the working span it is plausible to treat $W_L$ as depending only upon average life time earnings:

$$W_L = \phi(Y_N). \qquad (5)$$

For households approaching retirement, this model holds only in the unlikely event that the accumulation of wealth to date is consistent with the plan made at the beginning of the working life. A more generally appropriate assumption for such households is that $W_L$ depends on total resources available for pre-retirement consumption and annuity purchase:

$$W_L = \phi\left[\sum_{j=1}^{L} Y_j(1+r)^{L-j} + W_o(1+r)^L - Q\right], \qquad (6)$$

where Q is the amount to be set aside at retirement for bequest purposes. Non-human wealth is a negligible component of total resources

for most households at the beginning of their working life but becomes increasingly important as the consumer unit ages and perhaps predominant in the last few years preceding retirement.

Let us consider a young household that begins earning at age 25, with $Q = W_o = 0$ and a planned retirement fund at age $(25 + L)$ of $W_L = bY_N$. Then under a logarithmic utility function with $\alpha_i = \frac{\alpha}{(1+r)^i}$,

$$C = Y_N - \frac{r}{(1+r)^L - 1} (b\ Y_N) = F(r,b)\ Y_N.$$

If income and consumption both proceed according to plan over the first J years and if a fraction $d_J$ of the present value of lifetime earnings is obtained in the first J years, then $W_J$, where

$$W_J = \sum_{i=1}^{J} (Y_i - C)(1+r)^{J-i},$$

is proportional to $Y_N$, with the factor of proportionality depending on $d_J$ as well as on r and b. If the household at age (25+J) were now to recompute its optimal plan, it would presumably take account of $W_J$, as well as its expected earnings stream over the (L-J) remaining years to retirement, in computing the desired retirement fund $W_L'$:

$$W_L' = b' \left[ \sum_{i=J+1}^{L} Y_i (1+r)^{L-i} + W_J (1+r)^{L-J} \right].$$

But the expression in brackets remains proportional to $Y_N$, so that --when expectations are realized precisely--the household of age (25+J), like the household of age 25, chooses a retirement fund based only on lifetime earnings.[1] In fact, since nothing has happened to cause the household to modify its plan, we expect

$$W_L' = W_L = bY_N.$$

The situation is different, however, if either income or consumption or both have deviated from the initial plan over the first J years of the household's earning life. Then we find

$$W_J = \sum_{i=1}^{J} (Y_i^o - C^o)(1+r)^{J-i} + \sum_{i=1}^{J} D_i(1+r)^{J-i}$$

where $Y_i^o$ represents the earnings initially expected in the $i^{th}$ year of the working span, $C^o$ is consumption as initially planned and $D_i$ is the deviation of actual saving in the $i^{th}$ year, $S_i$, from the planned saving, $Y_i^o - C^o$. $D_i$ may also reflect the impact on non-human wealth of rates of return different from expectations and in particular of unanticipated capital gains.

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/1 $$\sum_{i=J+1}^{L} Y_i(1+r)^{L-i} + (1+r)^{L-J}\left[\sum_{i=1}^{J} Y_i(1+r)^{J-i} - \sum_{i=1}^{J} C(1+r)^{J-i}\right]$$

$$= \sum_{i=J+1}^{L} Y_i(1+r)^{L-i} + \sum_{i=1}^{J} Y_i(1+r)^{L-i} - F(r,b)\, Y_N \sum_{i=1}^{J} (1+r)^{L-i}$$

$$= Y_N\left[\sum_{i=1}^{L} (1+r)^{L-i} - F(r,b) \sum_{i=1}^{J} (1+r)^{L-i}\right].$$

In this case $W_L'$ consists of two components, the first proportional to $Y_N$ as initially estimated (say $Y_N^o$) and the second proportional to the weighted sum of deviations from the original saving plan, reflecting the cumulative effect of unexpected transitory income and transitory consumption. For an individual household in the real world this cumulative effect may be very substantial and may indeed lead to a revision of the retirement target. As a first approximation, however, we shall assume that only large cumulative deviations affect retirement plans and that in the aggregate, the effect of upward adjustments roughly cancels out that of downward adjustments within each age cohort. Then $W_L$ can be taken to depend on $Y_N^o$ alone for older households on the average, as well as for households just beginning their working life.

Aggregate Saving. If the utility function is of the simple logarithmic type discussed above, if the target retirement fund is a function $\Phi(Y_N^o)$ of average lifetime earnings, and if the saving plan is recomputed annually in the light of new information, then it follows from (5) that current saving for a household with L years to retirement is given by

$$S_1 = (Y_1 - Y_N) + f(r, L)\ [\Phi(Y_N^o) - W_o(1+r)^L]$$

Aggregate saving over all age cohorts of earners (e.g. letting L = 1 for the 64 years olds up to, say, 45 for the 20 year olds) is then equal to the aggregate difference between current and mean future earned income plus a linear function of initial and desired terminal wealth:

$$\Sigma S_1 = \Sigma(Y_1 - Y_N) + \Sigma\Phi(Y_N^o)\Sigma V_L f(L,r) - \Sigma W_o \Sigma V_L' f(L,r)(1+r)^L$$

where the summations of saving, income and initial and desired terminal

wealth are over all earners, $V_L$ is the share of the $L^{th}$ age cohort in the aggregate fund desired for retirement purposes and $V_L'$ is the share of the $L^{th}$ age cohort in aggregate wealth at the beginning of the period. If the distributions, by age, of initial wealth and the desired retirement fund are stable, $V_L$ and $V_L'$ will be constant and $\Sigma_L V_L f(L,r)$ and $\Sigma_L V_L' f(L,r)(1+r)^L$ will be functions of r only.

Under the assumption that incomes are certain over the working life, the difference between current and average future earnings, $Y_1 - Y_N$, reflects only the life cycle pattern associated with maturation and aging. If the age distribution of earner households is stable, then in the aggregate $\Sigma(Y_1 - Y_N)$ may be taken as roughly proportional to aggregate normal income.

It should be noted, however, that once we permit incomes to deviate from expectations, $\Sigma(Y_1 - Y_N)$ takes on an entirely different character, primarily reflecting cyclical deviations of aggregate earnings from their long run trend. Nothing in the preceding analysis, which was based on a certain income stream, suggests that transitory income of this unexpected type is fully saved. Nor would there be any reason to posit a stable relationship to aggregate normal earnings.

Letting desired wealth at retirement be $\Phi(Y_N) = bY_N$ for all earners. where we now understand $Y_N$ to be average lifetime earnings as envisaged at the beginning of the working life (previously designated $Y_N^o$), we have for saving per earner in period t

$$\frac{1}{P_t}\Sigma S_t = a\frac{1}{P_t}\Sigma Y_{Nt} + bF(r)\frac{1}{P_t}\Sigma Y_{Nt} - F'(r)\frac{1}{P_t}\Sigma W_{t-1}$$

where $P_t$ is number of earners and the summations run over all earners.

This implies that the aggregate savings ratio achieved as individual earners each pursue their own optimum is of the form

$$\frac{\Sigma S_t}{\Sigma Y_{Nt}} = a + bF(r) - F'(r)\frac{\Sigma W_{t-1}}{\Sigma Y_{Nt}}$$

Alternatively if we let $\Phi(Y_N) = AY_N^B$ for all earners, we may approximate saving per earner by

$$\frac{1}{P_t}\Sigma S_t \simeq a\frac{1}{P_t}\Sigma Y_{Nt} + AF(r)\left(\frac{\Sigma Y_{Nt}}{P_t}\right)^B - F'(r)\frac{1}{P_t}\Sigma W_{t-1}$$

We note that $\frac{1}{P_t}\Sigma Y_{Nt}^B \neq \left(\frac{\Sigma Y_{Nt}}{P_t}\right)^B$, but the two variables will be highly correlated since both are changed in the same proportion by those increases in normal income per earner in which the entire income distribution shifts by a constant factor.[1] The assumption $\Phi(Y_N) = AY_N^B$ implies that the aggregate savings

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[1] Let $P_{t+j} = P_t(1+p)^j$ and $\left(\frac{\Sigma Y_{Nt+j}}{P_{t+j}}\right) = \left(\frac{\Sigma Y_{Nt}}{P_t}\right)(1+q)^j$ where p is the growth rate of the labor force and q the growth rate of normal earnings per worker, assumed constant over the earnings distribution. We note that

$$\Sigma(Y_{Nt+j})^B = (1+p)^j\Sigma[Y_{Nt}(1+q)^j]^B$$

and

$$\frac{1}{P_{t+j}}\Sigma(Y_{Nt+j})^B = \frac{(1+p)^j(1+q)^{jB}}{P_t(1+p)^j}\Sigma Y_N^B = (1+q)^{jB}\frac{\Sigma Y_N^B}{P_t}.$$

On the other hand

$$\left(\frac{\Sigma Y_{Nt+j}}{P_{t+j}}\right)^B = (1+q)^{jB}\left(\frac{\Sigma Y_{Nt}}{P_t}\right)^B.$$

Thus

$$\frac{1}{P_{t+j}}\Sigma(Y_{Nt+j})^B \div \left(\frac{\Sigma Y_{Nt+j}}{P_{t+j}}\right)^B = \frac{1}{P_t}\Sigma Y_N^B \div \left(\frac{\Sigma Y_{Nt}}{P_t}\right)^B, \quad \text{for all } j.$$

ratio will rise with $Y_N$, if B>1 as empirical data suggest, and will be linear, given r, in the ratio of initial wealth to normal income.

Thus, we find aggregate saving per earner to be a linear function, given r, of desired wealth per earner at retirement and initial wealth per earner invested at compound interest to the date of retirement. If desired wealth at retirement is linearly related to lifetime earnings, then so is saving. On the other hand if desired terminal wealth depends on $Y_N^B$, then so does saving. Furthermore aggregate wealth per earner in any given time period, being largely a cumulation of previous saving, will depend linearly on aggregate $Y_N$ per earner under the first assumption and on $Y_N^B$ under the second.[1].

The above demonstration applies to a particularly simple logarithmic utility function. For a more plausible separable utility function, such as (3), we have seen that the $C_i^*$ remain linear functions of both normal income and $W_L - W_o(1+r)^L$, with the first exerting a positive and the second a negative effect. There is a reasonable presumption that the $S_i$, while negatively related to initial wealth, will be positive functions of normal income if $W_L = bY_N$ or of $Y_N^B$ if $W_L = AY_N^B$.

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[1] We have not considered the dissaving of the retired about which very little is known. Presumably under the present model the wealth at retirement would be divided into two parts, the capital value of the annuity purchased and the funds retained and invested at compound interest for estate purposes. Aggregate dissaving by the retired would then be approximated by aggregate annuity payments and may be taken as roughly proportional to $\Sigma W^R$, the aggregate wealth of the retired: $\Sigma S^R = -\Sigma V''(t)F''(t)\Sigma W^R$, where t is age, V''(t) is the fraction of $\Sigma W^R$ held by individuals of age t and F''(t) is the fraction, increasing with age, of wealth consumed by those of age t.

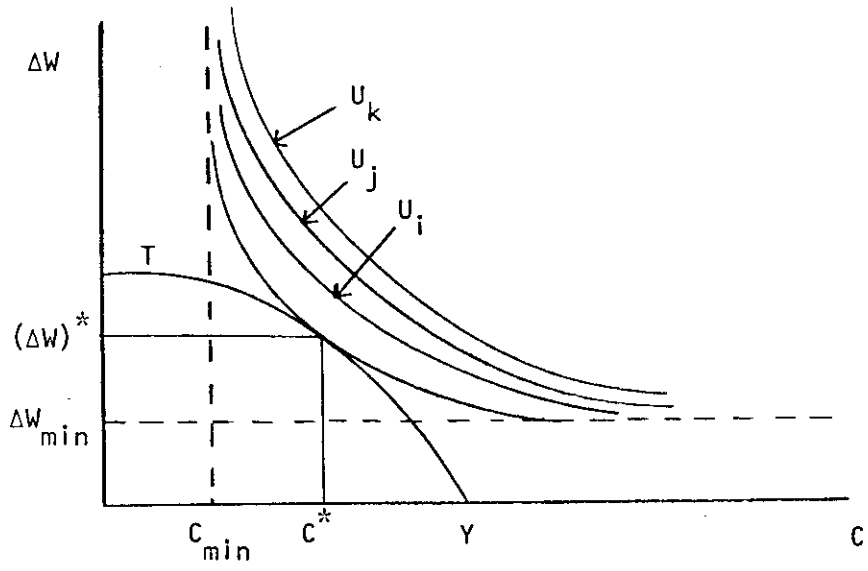
Optimal Saving for an Economy

A government which attempts to increase national welfare through policies designed to affect aggregate (including direct governmental) saving and investment will have wider choices than an individual consumer and will face a somewhat different optimization problem. The objective function envisaged by such a government will presumably utilize some kind of average rate of time preference in establishing tradeoffs between present and future consumption and will have a longer time horizon than an individual, since the government will take somewhat more responsibility for the interests of generations as yet unborn. The primary motivation for rearranging consumption over time will not be to protect living standards against an anticipated decline in earnings but rather to generate substantially larger consumption over an extended time period through postponement of consumption.

For the economy, unlike the individual, the rate at which consumption in one period can be exchanged for consumption in another period depends on the quantity which is to be exchanged. Furthermore, for the economy the relevant rate is not a relatively risk-free lending and borrowing rate, but the rate of return on real investment, which will be inversely related to the volume of such investment in any given time period and must be considered a random variable rather than a known quantity. Transfers of consumption from the current to future periods should occur only to the extent that the marginal return on real assets is large enough to compensate adequately for risk as well as for time preference.

Because of diversification, the overall risk associated with investment in real assets may be relatively small for the economy as compared with the individual, especially if we consider a sufficiently long time span. But the risk of arbitrary redistribution of income remains (reflecting the variance of return among individual enterprises) and this risk perhaps has a place in the national objective function. The government may, of course, act to reduce the risk for individual investors in real assets by such devices as loss carry-back and carry-forward, accelerated amortization, or commodity price supports. Or the government may decide to nationalize both investment decisions and the associated risk.

The single period national optimum is shown in the accompanying graph, where the variables are measured per capita.




Abstracting from the possibility of national dissaving, the maximum potential consumption is net national product, Y, and for every dollar

by which consumption falls short of this, a dollar of net investment, I, occurs.[1] $\Delta W$, on the vertical axis, is defined as

$$\Delta W = \frac{1}{\bar{r}}\int_o^I \rho(I)dI = \frac{\bar{\rho}(I)}{\bar{r}}I$$

Here $\bar{r}$ represents the rate of return (however determined) which is judged to compensate the economy both for the risk of investment in real assets and for time preference, $\rho(I)$ is the expected value of the marginal rate of return on net investment computed net of depreciation, and $\bar{\rho}(I)$ is the average rate of return on net investment. Thus $\Delta W$ is the expected value, discounted to the present, of the perpetual stream of returns generated by current net investment.

The indifference curves $U_i$, $U_j$, $U_k$ are of the form $U = f(C, W_o + \Delta W) = F(C,\Delta W)$ for given $W_o$. The F functions will, of course, shift gradually over time as the per capita stock of capital rises, providing for higher streams of future per capita consumption. The higher the future consumption is, based on the accumulated capital stock, the less society will forego in terms of current consumption to further increase the future stream. In particular, slopes in the neighborhood of $C_{min}$ will become increasingly steep. The indifference curves are drawn asymptotic to $C = C_{min}$ and to $\Delta W = \Delta W_{min}$, where $C_{min}$ is either the subsistence level for the existing population or some higher

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[1] The implicit assumption here is that the government will not permit employment to fall as a result of a reduction in consumption, but will undertake increased governmental investment if private investment fails to increase sufficiently.

socially determined minimum standard of living and $\Delta W_{min}$ may be taken as the amount of net investment per period which will provide sufficient capital goods to accomodate new entrants to the labor force in this period without reduction in the marginal productivity of labor.[1]

The transformation possibilities curve T is closely related to the marginal efficiency of investment $\rho(I)$. In particular the slope of T at any given value of C is $-\frac{\rho(Y-C)}{\bar{r}}$. Since the function $\rho(I)$ shifts over time both in shape and position, so will the function T. Marginal efficiency in period t, $\rho_t(I)$, depends in part on autonomously generated investment opportunities. For developing countries autonomous shifts in the marginal efficiency function depend less on the rate of technological advance than on factors encouraging conversion from less efficient to more efficient configurations of existing technologies (e.g., development of the transportation network and power-generating facilities, increase in labor and entrepreneurial skills, growth of markets) or external factors increasing export demand (e.g. income growth of trade partners, relative increases in the prices of competitive producers, improved marketing arrangements).

The marginal efficiency function also depends on the growth rate of demand for domestically produced consumption and investment goods. If the growth rate of consumption declines due to policies that encourage saving, then certain investments that would otherwise have

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[1]The determination of these asymptotes is discussed by A.K. Sen, "On Optimizing the Rate of Saving," Economic Journal, Vol. LXX1, No. 283, September 1961, pp. 491 ff.

been profitable now become worthless - i.e., those that would have been required to meet consumer demand at the higher growth rate, but are not required at the lower, or those that require domestic market growth to justify the introduction of low cost techniques. Thus an increase in the propensity to save not only frees resources for private and/or governmental investment, but also leads to a change in the investment mix that favors autonomous investment at the expense of induced investment in consumer goods industries. The latter raises productivity only in those cases where domestic market growth leads to the introduction of improved production techniques.

From a practical point of view there are three crucial elements in estimating a national savings optimum. One is to determine the appropriate social discount rate, which really reflects the relative weight attached to the welfare of the current vis-a-vis future generations. The second is to determine how much investment under existing conditions can be expected to generate rates of return, in terms of direct cash flows, in excess of the social discount rate. The third and most difficult is to determine the impact on future marginal efficiency curves of current decisions. To what extent will certain kinds of current government investment today generate new profitable private investment opportunities tomorrow? To what extent will a diminution of the current growth rate of consumption postpone the day when potential economies of scale in the production of consumer goods can be realized? The impact on future investment opportunities may be far more significant to the nation than the direct cash flows attributable to current investment, but traditional theory provides very little guidance for the case in which investment opportunities are not independent over time.

If we are willing to assume that investment in future years is determined independently of the current investment decision, say at $\bar{I}_2$, $\bar{I}_3$,.... and if the marginal efficiency curve is known in terms of the direct cash flows that may be expected to result from investment opportunities currently available, then a simple numerical determination of optimum saving can be demonstrated under the assumption of an aggregate per capita utility function of the logarithmic type:

$$U = \alpha \sum_{t=1}^{\infty} \frac{\log C_t}{(1+\bar{r})^{t-1}} \quad .$$

For simplicity we assume constant population and labor supply. We further assume that existing wealth $W_o$ will, in combination with the constant labor supply, generate a perpetual income stream $Y_o$, in per capita terms. We then have

$$dU = \alpha \sum_{t=1}^{\infty} \frac{dC_t}{(1+\bar{r})^{t-1} C_t}$$

and we wish to find the value $I_1^* = Y - C_1^*$ that equates dU to zero. Consumption in future periods, $C_t$, depends on $I_1$ in the following way

$$C_t = Y_o + \sum_{j=2}^{t-1} \bar{I}_j \bar{\rho}_j - \bar{I}_t + \int_o^{I_1} \rho(I_1) dI_1, \quad t>1,$$

where $\bar{\rho}_j$ is the average rate of return on $\bar{I}_j$, the predetermined investment in period j. Then

$$dC_t = \rho(I_1) dI_1,$$

while from the income constraint

$$dC_1 = - dI_1.$$

Setting $dU/dI_1$ equal to zero and writing the $C_t$ as functions of $I_1$, we find $I_1^*$ as the solution of

$$\frac{1}{Y_o - I_1^*} = \sum_{t=2}^{\infty} \frac{\rho(I_1^*)}{(1+r)^{t-1} \left[Y_o + \sum_{j=2}^{t-1} \bar{I}_j \bar{\rho}_j - \bar{I}_t + I_1^* \bar{\rho}(I_1^*)\right]}$$

where $\bar{\rho}(I_1^*)$ is the <u>average</u> return on current investment of the amount $I_1^*$. We note that the larger $(Y_o + \sum_{j=2}^{t-1} \bar{I}_j \bar{\rho}_j - \bar{I}_t)$, the smaller the contribution to utility of any given increment in $C_t$ due to current investment.

If we were to assume zero investment in future periods, then $C_t$ would be constant over time at a level depending on $I_1^*$; and the optimization condition (7) would simplify to

$$\frac{1}{Y_o - I_1^*} = \frac{\rho(I_1^*)}{Y_o + I_1^* \bar{\rho}(I_1^*)} \sum_{2}^{\infty} \frac{1}{(1+\bar{r})^{t-1}}$$

or

$$\rho(I_1^*) = \bar{r} \frac{Y_o + I_1^* \bar{\rho}(I_1^*)}{Y_o - I_1^*} . \qquad (8)$$

Since realistically we expect future investment to occur, leading to a gradual increase of future consumption above $(Y_o + I_1^{*-}\bar{\rho}(I_1^*))$, the condition (8) gives a value for the marginal return on investment below that at the true optimum and so provides only an upper limit for current investment.

Laissez-Faire Optimum with Risky Assets

We return now to the determination of individual saving decisions, but relax the assumption that the rate of return on assets is known with certainty. Each consumer in pursuing his own optimum now selects an appropriate mix of relatively risk-free debt assets and of real assets, the latter being held either directly or (where capital markets are well-developed) through ownership of corporate stock.[1] The respective weights of the two asset types will reflect the individual's risk aversion as well as the rates of return available on debt and equity. These rates will in turn reflect the marginal efficiency of investment in real assets and the distribution of risk aversion over the population.

The single-period consumption decision should now maximize the sum of the utility of current consumption, $u(C)$, plus the expected value of the utility of end-of-period wealth, $E[v(\tilde{W})]$. The latter serves as proxy for the expected utility of those additions to future consumption that would result from an optimal allocation of current wealth, given the terminal wealth requirement.

The amount of net investment which will ensue under strict laissez-faire clearly depends on the distribution of income according to

[1] We abstract here from possible short-term differences in the rate of return on corporate stock and the rate of profit earned on the underlying real assets.

both time preference and risk aversion, as well as on the efficiency of the capital market in making the opportunities for risky investment widely available to potential investors, in diversifying risk economically and in arranging the transfer of funds from the highly risk averse to those with sufficiently low risk aversion that they are willing to borrow in order to invest in real assets.

Formally, the laissez-faire equilibrium is determined as follows, given income and initial and terminal wealth. Each consumer-investor maximizes the expected value of utility, expressed as a function of current consumption, C, and the stochastic variable $\tilde{W}$, representing wealth at the end of the current period. The maximand may be written

$$E(U) = u(C) + \int v(\tilde{W})\Theta(\alpha, W_o, \tilde{W})dW$$

where $\alpha$ is the proportion of $W_o$ held in risk-free assets and $\Theta$ is the subjective probability distribution (dependent on $W_o$ and $\alpha$) of end-of-period wealth.

Let each consumer have a fixed terminal wealth target, $W_L$, and assume for convenience that he borrows against future earnings (which are treated as certain) an amount equal to the present value of future earnings less terminal wealth. He then begins the period with a fund $W_o^*$ to be allocated between present and future consumption. If $R^*$ is the comparatively risk-free return relative on debt assets and $\tilde{R}$ a random drawing from the distribution of return relatives for initial real assets, then

$$W_o^* = W_o + \sum_{i=1}^{L} Y_i/(R^*)^i - W_L/(R^*)^L$$

and the random variable $\tilde{W}$ is defined as

$$\tilde{W} = [\alpha R^* + (1-\alpha)\tilde{R}_j] \quad (W_o^* - C).$$

The expected value of utility, which depends on $R^*$ and the probability distribution of $\tilde{R}$, is maximized with respect to C and $\alpha$ for each consumer.

It has been shown by Samuelson[1] that for utility functions of the type considered here, the optimum value of $\alpha$, when $R^*$ and the probability distribution of $\tilde{R}$ are known and constant, can be determined independently of the pattern of consumption. Let $\alpha^*$ be this optimum value for a particular consumer. Then return on wealth in period t is

$$1+\tilde{r}_t = \alpha^* R^* + (1-\alpha^*)\tilde{R}_t$$

and its probability distribution is known and constant.

For purposes of example we assume a utility function of the constant elasticity type

$$U = -\alpha_1 C^{1-\gamma} - \alpha_w \tilde{W}^{1-\gamma} + B, \ \gamma>1,$$

where B is the utility of terminal wealth. We wish to maximize

$$E(U) = -\alpha_1 C^{1-\gamma} - \alpha_w (W_o^* - C)^{1-\gamma} E(1+\tilde{r})^{1-\gamma} + B.$$

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[1]Paul A. Samuelson, "Lifetime Portfolio Selection by Dynamic Stochastic Programming," Review of Economics and Statistics, August 1969.

Setting the derivative with respect to C equal to zero, we obtain

$$C = \frac{mW_o^*}{1+m}$$

where

$$m = \left[\frac{\alpha_w}{\alpha_1} E(1+\tilde{r})^{1-\gamma}\right]^{-1/\gamma} = \left[\frac{\alpha_w}{\alpha_1}\right]^{-1/\gamma} E[\alpha^* R^* + (1-\alpha^*)\tilde{R}_t]^{\frac{\gamma-1}{\gamma}}.$$

As in the earlier case where asset yield was assumed to be certain, we observe that consumption is linear in normal income, initial net worth ($W_o$), and the terminal wealth target, which is expected to be a function of normal income. The coefficients differ among individuals, with the more risk averse having relatively high values of $\gamma$ and $\alpha^*$.

Aggregate consumption, in conjunction with current income, determines aggregate saving. There is no reason to believe the latter will precisely exhaust the volume of real investment opportunities with expected rate of return exceeding that on initial real assets. If aggregate saving is inadequate to finance all such opportunities, both $R^*$ and $E(\tilde{R})$ will rise, generating increased saving in subsequent periods and moving the aggregate variables toward equilibrium. In the meantime investment will be constrained below the optimum level. Since saving does not appear to be highly sensitive to asset yields in the short run, a disequilibrium of this kind may be long lasting and provides a strong case for governmental initiatives to stimulate saving.

Other Motives for Holding Assets

So far we have considered only two of the several motives for holding assets: reallocation of consumption over time and the estate motive. There are at least two others of comparable importance:

1. The least expensive way of obtaining certain kinds of consumer services of the desired quality may be to purchase a capital good which produces those services. For example, public transit provides cheaper transportation services than one's own automobile but they are considerably less convenient, while taxis are substantially more expensive if use is frequent. Thus automobile ownership may be the optimal means for obtaining these services. Similar arguments may be made with respect to ownership of home and household durables in order to obtain the particular services desired at minimum cost.

2. Once account is taken of uncertainty as to the future income stream, as well as the possibility of large unforeseen expenditures, a further motive for holding assets comes into play. This may be called the contingency or insurance motive and essentially protects the standard of living against unfavorable future drawings from various relevant probability distributions. We believe this motive to be highly important for individuals, though relatively minor in a national objective function. To the extent that it is both feasible and cheaper to do so, the individual will presumably buy insurance services, such as term life insurance. But against certain contingencies it will be efficient to self-insure, primarily through the accumulation of liquid assets. Assets held for contingency purposes can be considered to generate a continuing service each period, much as an owner-occupied house does. For analytical purposes such assets can be treated in the

same way as a consumer durable and equity in an owned home, though it is probable that the services provided have a higher income elasticity.

If we consider the services rendered to be proportional to the value of the asset from which they derive -- i.e. a $40,000 house generates twice as much housing service as a $20,000 house, considering comfort, convenience, esthetic appeal and status as well as basic shelter -- then the income elasticity of demand for the assets will be less than, equal to, or greater than unity depending on the income elasticity of the services generated. The marginal utility $\frac{\partial U}{\partial A^s}$ of a dollar of service-generating assets purchased in the current period will equal the sum of the utilities of the incremental services, $\Sigma \frac{\partial U}{\partial C^s} \cdot \frac{\partial C^s}{\partial A^s}$, provided in the current and subsequent time periods plus the utility of the increment in terminal asset values. We note that utility is now a function of both $C^s_j$, the services of owned assets in period j, and $C_j$, the consumption of purchased goods and services, for all periods within the planning horizon. It is likely that marginal utility decreases more rapidly when either component is increased holding the other constant, than when both components are increased simultaneously, so that the levels of each component, and not just their sum, should enter the utility function.

For simplicity we again consider a logarithmic utility function and we assume that saving for a retirement fund and for bequests has been predetermined and deducted from current earnings, to establish the earnings available for current expenditures on consumption goods and service-generating assets. We further assume that these adjusted

earnings are constant over time at the level $\bar{Y}$. Then we wish to maximize

$$U_{1,L} = \sum_{j=1}^{L} (\alpha_j \log C_j + \alpha_j' \log C_j^s) + \alpha_{L+1} \log A_L^s. \qquad (9)$$

If services generated are proportional to the value of the asset from which they derive and if, as in the case of contingency balances, assets neither depreciate nor are financed by borrowing, then

$$C_j^s = K A_{j-1}^s = K(A_o^s + \sum_{i=1}^{j-1} I_i)$$

where $A_o^s$ is the initial stock of service-generating assets, and $I_i$ represents acquisition of such assets in period i. We may therefore rewrite (9) as follows

$$U_{1,L} = \sum_{j=1}^{L} \alpha_j \log C_j + \sum_{j=1}^{L} \alpha_j' \log K(A_o^s + \sum_{i=1}^{j-1} I_i) + \alpha_{L+1} \log(A_o^s + \sum_{i=1}^{L} I_i).$$

We observe not only that current investment affects all subsequent levels of asset services, $C_j^s$, but also that the effect on utility of its contribution to each $C_j^s$ depends on the investment levels, $I_i$, in intervening periods, i=2,...,j-1. These levels in turn are affected by $I_1$, with resultant repercussions upon the future consumption of purchased goods and services, $C_j$.

The mathematical complexities resulting from these interdependencies can be avoided if we assume that the consumer-investor suboptimizes by neglecting the impact of future investment on the

contributions to utility of current investment.[1] For present purposes this is equivalent to assuming zero levels of future investment. Then

$$\frac{\partial U_{1,L}}{\partial C_j^s} \cdot \frac{\partial C_j^s}{\partial I_1} = \frac{\alpha_j'}{A_o^s + I_1}, \quad j=2,3,\ldots L.$$

$$\frac{\partial U_{1,L}}{\partial C_j} \cdot \frac{\partial C_j}{\partial I_1} = \begin{cases} -\alpha_1/(\bar{Y}-I_1), & j=1 \\ 0, & j=2,3,\ldots L \end{cases}$$

and we obtain the following first order condition determining $I_1^*$:

$$I_1^* = \frac{\bar{Y}\left(\sum_{j=2}^{L} \alpha_j' + \alpha_{L+1}\right) - \alpha_1 A_o^s}{\alpha_1 + \sum_{j=2}^{L} \alpha_j' + \alpha_{L+1}} . \qquad (10)$$

Note that, as a result of investment undertaken in period 1, the asset stock, and hence the level of asset services $C_j^s$, has been increased for all subsequent periods. The marginal utility attributable to further increments in the $C_j^s$ is thus reduced and this suggests that the consumer-investor, if he continues to apply the same suboptimization rule, will undertake a smaller level of investment in period 2 than in period 1. We have

$$\frac{\partial U_{2L}}{\partial C_j^s} \cdot \frac{\partial C_j^s}{\partial I_2} = \frac{\alpha_j'}{A_1^s+I_2} < \frac{\alpha_j'}{A_o^s+I_1} = \frac{\partial U_{1,L}}{\partial C_j^s} \frac{\partial C_j^s}{\partial I_1}$$

for any given level of $I_1 = I_2 > 0$, where

$$U_{2,L} = \sum_{j=2}^{L} (\alpha_j \log C_j + \alpha_j' \log C_j^s) + \alpha_{L+1} \log A_L^s.$$

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[1] This is analogous to the assumption underlying equation (8). It is considerably more plausible for the household than for a governmental planning agency and for descriptive than for normative purposes.

However, if L is small and $\alpha_2$ substantially less than $\alpha_1$ (as we expect when the rate of time preference is high), the tendency for $I^*$ to fall over time on this account may be offset by other considerations.

In any given time period, investment will be zero if the initial asset stock is sufficiently large. In particular, if in (10) we have

$$A_o^s = \frac{1}{\alpha_1} \bar{Y}\left( \sum_{j=2}^{L} \alpha_j' + \alpha_{L+1} \right),$$

then $I_1^* = 0$, and the household may be considered to have already attained its optimal stock of service-generating assets, $A^{s*}$. We may rewrite (10) in terms of $A^{s*}$

$$I_1^* = \frac{\alpha_1}{\alpha_1 + \sum_{j=2}^{L} \alpha_j' + \alpha_{L+1}} (A^{s*} - A_o^s). \qquad (11)$$

Thus for a logarithmic utility function we find that saving for contingencies, like saving to accumulate a retirement fund, is proportional to the gap between a target asset stock and the stock held at the beginning of the period.

Next we consider investments in real assets, such as a home, which are largely financed by credit and are subject to depreciation. We must now take account of the reduction in future consumption of purchased goods and services resulting from future debt service requirements. Thus if assets are purchased in the amount $I_1$ in period 1, we have

$$\frac{\partial U_{1,L}}{\partial I_1} = K \sum_{j=1}^{L} (1-\delta)^{j-1} \frac{\partial U_{1,L}}{\partial C_j^s} - m \sum_{j=1}^{n} \frac{\partial U_{1,L}}{\partial C_j} + (1-\delta) \frac{\partial U_{1,L}}{\partial A_L^s}$$

where $\delta$ is the depreciation rate and $mI_1$ is the debt service payment, assumed constant for n years. Ignoring the effect of future investment

upon the contribution to utility of current investment and assuming the logarithmic utility function (9), we obtain

$$I_1^* = \frac{\bar{Y}\left(\sum_{j=1}^{L} \alpha_j' + \alpha_{L+1}\right) - m A_o^s \sum_{j=1}^{n} \alpha_j}{m\left(\sum_{j=1}^{n} \alpha_j + \sum_{j=1}^{L} \alpha_j' + \alpha_{L+1}\right)} . \qquad (12)$$

Then the asset stock $A^{s*}$ that generates zero investment is

$$A^{s*} = \frac{\bar{Y}\left(\sum_{j=1}^{L} \alpha_j' + \alpha_{L+1}\right)}{m \sum_{j=1}^{n} \alpha_j},$$

and we may again write optimal investment in the current period as proportional to the gap between $A^{s*}$ and $A_o^s$:

$$I_1^* = \frac{\sum_{j=1}^{n} \alpha_j}{\sum_{j=1}^{n} \alpha_j + \sum_{j=1}^{L} \alpha_j' + \alpha_{L+1}} (A^{s*} - A_o^s). \qquad (13)$$

Effect of Unexpected Current Income

Such deviations of current from average future earnings as are expected on the basis of typical life cycle patterns have already been discussed. Theory suggests that they are entirely saved so long as future earnings are treated as certain and initial and desired terminal wealth are given. Once we abandon certainty about earnings, however, a question must be raised as to whether future earnings growth--which, while relatively certain in the aggregate, is highly uncertain for the individual--has as strong an effect on current consumption as does current income. If not, then the negative deviation of current earnings

from the expected lifetime average that is typical of young households in many occupations may impact consumption as well as saving.

We must also consider the results of unexpected, non-systematic deviations from normal income. To what extent, if at all, will current consumption be affected? If no changes in expectations for the future are induced--i.e., if the deviation is considered to be purely temporary --then the response of consumption would appear to depend on such factors as how promptly the deviation is perceived; whether it is positive or negative and--if negative--whether liquid contingency balances have been built up; whether it coincides with transitory elements in desired consumption; and whether it is large enough to justify an immediate recomputation of the entire consumption path in order to spread its impact in an optimal way.

A plausible argument can be made that both anticipated and unanticipated deviations of current earnings from the expected value of future earnings may have some influence on consumption. It is also plausible that the effect will be less than that of normal income. These are matters that can only be resolved through careful and repeated empirical analyses.

Summary

In summary the demand for assets desired to provide a stream of specific consumer services not otherwise available at the same cost, including specifically the service of liquid assets in protecting against uncertainty of future income and other contingencies, is derived from the demand for the consumer services generated. The latter demand like that for other elements of consumption depends on current and expected normal income and (once we introduce income uncertainty)

probably on current transitory income as well, with the income elasticity of the demand for assets depending on that of the services.

We hypothesize that the desired stock of service-generating assets, like desired wealth at retirement, $W_L$, depends primarily on normal income, $A_s^* = \Phi'(Y_N)$, with transitory income having only a minor impact (though affecting the speed with which the desired stock is acquired). As with $W_L$ the single period optimization condition that the marginal utility of current consumption equal that of wealth at the end of the period[1] will ensure that the desired stock is acquired gradually over an individual's working life. But the rate of acquisition will be faster than for $W_L$ since the current services of the desired asset are sacrificed by postponement of its acquisition, whereas no consumption is financed by $W_L$ until reitrement. For this reason we would expect acquisition of $A^s$ to dominate saving in the early years of the working life and acquisition of assets for retirement purposes, W, to dominate the later years. To some extent this may be tested empirically, since purchases of consumer durables and life insurance, as well as down-payment on a home, obviously contribute to $A^s$. However, a clear distinction cannot be made between financial assets held for protection against contingencies and those which represent a gradual building up of $W_o(1+r)^L$ to $W_L$, although the former are more likely to take the form of savings deposits and the latter of long term bonds and low risk stocks. Actually liquid assets which protect against contingencies constitute a high risk component of $W_L$--high risk in the sense that if needed they will be used up in advance of retirement--but they contribute modestly to the expected value of $W_L$.

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[1] Considered, as throughout this paper, as a proxy for the marginal utility of the future consumption permitted by the marginal dollar of end-of-period wealth.

We have seen that under highly simplified assumptions the individual's optimal time path for the accumulation of $W_L - W_o(1+r)^L$ implies constant consumption over time, with saving varying only to the extent that current income varies from normal income (See equation (4)). However, the accumulation of $A^{s*}$ will probably occur at a declining rate, accompanied by a gradual increase in purchased consumption items, so that the marginal utility of the latter will decline in step with the declining marginal utility of $A^s$ as the volume of $C^s$ is built up. This follows from the single period optimization condition

$$\frac{\partial U}{\partial C_t} = \frac{\partial U}{\partial A_t^s} = \frac{\partial U}{\partial (W_t)}$$

and the assumption of declining marginal utility for both C and $C^s$.

Optimal saving for an individual in period t, with future earnings certain and current earnings equal to expectations, may now be approximated by

$$S_t = Y_t - Y_N + f(r,L)[\Phi(Y_N) - W_t(1+r)^L] + f'(r,L)[\Phi'(Y_N) - A_t^s].$$

Here f' may well be roughly constant and is expected to be large relative to f(r,L) except for those close to retirement. If future earnings are not treated as certain or if unexpected transitory earnings occur in the current period, the coefficient of transitory income is not necessarily 1. Then we have

$$S_t = \beta(Y_t - Y_N) + f(r,L)[\Phi(Y_N) - (1+r)^L W_t] + f'(r,L)[\Phi'(Y_N - A_t^s]. \quad (14)$$

The implications of our theoretical development for empirical analysis of consumer behavior are (1) that planned saving will be a fraction, rising with age, of the gap between actual and desired wealth, with the latter hypothesized to depend on normal income; (2)

that accumulation of service generating assets will tend to precede accumulation of assets for retirement and estate purposes; (3) that deviations of current from normal income will tend to be saved to a greater extent than normal income itself.

## 2. Prior Empirical Studies

While there are relatively few useful empirical results quantifying the impact of saving and wealth on economic growth, especially for developing nations,[1] the empirical literature on the determinants of the saving ratio is much more voluminous for countries at all stages of economic development. The empirical analysis to date has not provided definitive answers to the relative importance of the different factors affecting the saving-income or consumption-income ratio either in developing or developed economies, but it has provided valuable insights in both cases.[2] Since we are primarily interested in this paper in estimating and explaining the propensity to save in different

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[1] A summary treatment of such studies appears in M. Ishaq Nadiri, "International Studies of Factors Inputs and Total Factor Productivity: A Brief Survey," The Review of Income and Wealth, June 1972.

[2] Recent reviews of the relevant literature appear in Raymond F. Mikesell and James E. Zinsen, "The Nature of the Savings Function in Developing Countries: A Survey of the Theoretical and Empirical Literature," Journal of Economic Literature, March 1973; Thomas Mayer, Permanent Income, Wealth, and Consumption, University of California Press, 1972; and Robert Ferber, "Consumer Economics, A Survey," Journal of Economic Literature, December, 1973.

countries, we shall summarize the results obtained from earlier research and then present some new results. However, before doing so, it may be useful to highlight the deficiencies of the available data, both to indicate the difficulties in obtaining definitive answers from these data and to point to procedures for improving the results. We shall also present a simplified theoretical framework for integrating the empirical results.

## Deficiencies of available data

There are essentially three types of data which can be used to analyze consumption or saving propensities: These are (1) aggregate time series data which are usually part of the national income accounts and typically available on an annual or quarterly basis; (2) household or business cross-section data for a particular period of time, customarily a year or quarter; and (3) household or business continuous cross-section or panel data compiled from each sample member for each of a series of periods, again usually annually or quarterly. All three bodies of data are subject to substantial measurement errors, and raise other significant problems for deriving reliable consumption or saving propensities. Each has different strengths and weaknessess, and combined can give more satisfactory estimates of these propensities than they can singly. For developing countries, the aggregate time-series data tend to be less satisfactory than for the developed nations, though even for the latter the savings data are among the least reliable in the national accounts. However, there does not seem to be much systematic difference between the developing and developed countries in the quality of household survey data on consumption and saving. Unlike the general

availability of aggregate time-series and at least occasional household survey data for most countries, continuous cross-section or panel data for households are generally in short supply.[1]

Several key factors tend to bias estimates of the marginal propensities to consume or save and the corresponding income elasticities derived from any of these different bodies of data, and such effects must be neutralized if relatively unbiased estimates are to be obtained. These factors include the commingling of the effects of transitory and normal or "permanent" income (so long as the former is assumed to have smaller effects than the latter for at least some categories of consumption); the correlation of income with omitted variables relevant to consumption; and random or systematic measurement errors in income.[2] Abstracting momentarily from the last two problems, it can be shown that the use of measured income as an argument of the consumption function leads to an estimated income effect which is a weighted average of the effects of transitory and normal income. The weights given these two components of income depend on the proportion of the variance in measured income contributed by each, so that if the two are uncorrelated, the result is an upward biased estimate of one effect and

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[1]Partly at the urging of the one of the authors, India has collected a considerable amount of such data which have not yet been made available publicly. The United States has collected a more restricted amount of such data.

[2]Systematic understatement of income biases elasticity estimates derived from linear relationships but not those derived from logarithmic relationships, if the understatement is proportional to income.

a downward biased estimate of the other.[1] Under reasonable assumptions the tendency of measurement errors in income is to bias downward the estimate of the income parameter,[2] while the bias resulting from correlation of income with omitted variables is, of course, uncertain in direction.

For aggregate time series data, measurement errors -- even if not small -- are probably relatively stable among observations and it is, of course, the variance of the error term and not its magnitude which biases the estimate of the income parameter. Furthermore, over long time periods the variance of aggregate transitory income, while not zero, is likely to be small relative to that of normal income (especially if years of extreme cyclical aberration are omitted), so that something fairly close to the normal income effect may be expected.[3]

The comparative disadvantage of time series lies in the correlation of income with omitted variables relevant to consumption. This problem is not eliminated and may not even be greatly mitigated by the use of methods of estimation somewhat more sophisticated than ordinary least squares, including methods which allow for time lags in the

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[1] See Jean Crockett, "Technical Note on Biases in Estimating Income Expenditure Relationships from Cross-Section Data," in Consumption and Saving, edited by Irwin Friend and Robert Jones, University of Pennsylvania, 1960, Vol. 11, pp. 213-4.

[2] In cases where consumption estimates are derived from income data, this does not necessarily follow since measurement errors in the two variables will then be highly correlated.

[3] See Friedman's discussion of the effects on income elasticity estimates of the length and nature of the period covered. (Milton Friedman, A Theory of the Consumption Function, Princeton University Press, 1957, pp. 125-9).

adjustment of consumption to income through a gradual income-generated shift in tastes.[1] There is a strong time trend in income in most developed and many developing countries, and the effects of this cannot be adequately distinguished from those of (1) time trends in distributional variables: (2) the increasing availability of consumption opportunities through the introduction of new products, improved distribution of existing ones, and expanded credit facilities; and (3) such institutional factors as growth of social insurance and the expansion of communications media, which may have a profound impact on the consumer's preferred distribution of disposable income. In time-series analysis, all of these are impounded into the income coefficient -- primarily, in the models utilizing a lagged dependent variable, into the estimated impact of long-run income, since the lagged dependent variable probably picks up these effects in large degree.[2]

The independent effects of income may of course, be isolated by regressing the deviations of consumption from its time trend against the deviations of income from its time trend. This is essentially what is

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[1]E. G., see Marc Nerlove, "Distributed Lags and Estimation of Long-Run Supply and Demand Elasticities: Theoretical Considerations," Journal of Farm Economics, May 1958; and H. S. Houthakker and L. D. Taylor, Consumer Demand in the United States, 1929-1970, Harvard University Press, 1966. It may be noted that methods such as these, which utilize the estimated coefficient of the lagged dependent variable in arriving at long-run income effects, are subject to bias whenever serial correlation is present in the residuals -- a common occurrence in time-series, especially when quarterly data are used -- unless an appropriate correction for auto-correlation is introduced.

[2]A related problem may arise from variation over time in the extent of asset disequilibrium -- i.e., the magnitude of the deviation of actual household assets from some optimum based on normal income, age distribution and tastes -- since this deviation may be correlated with normal income, though there is no strong a priori reason for expecting it to be so if the growth rate of normal income is fairly steady.

accomplished if we define normal income as the time trend of measured income[1] and transitory income as the deviation from this time trend, treating consumption as a function of both. A reasonably pure estimate of the transitory income effect is then obtained, if problems of simultaneous equations bias are avoided by appropriate estimation procedures. However, if any substantial time trend exists in non-income factors affecting consumption, it does not seem to us feasible to obtain an adequate estimate of the normal income effect from time-series data.

In cross-sectional analysis with individual households as observations, simultaneous equations bias in the estimates of consumption and saving propensities is largely avoided and it is possible to hold constant or estimate the effects of a large number of non-income socio-demographic variables. Such data, unlike the time-series, provide a large number of independent observations, avoid problems of serial correlation, and make possible the measurement of any shifts in the consumption (or saving) function. However, single cross-sections do not permit an adequate analysis to be made of relative prices and other time-dependent effects on consumption and saving. Moreover, while the variance of normal income is likely to be larger than for time-series (unless the latter covers an extremely long time period), the variance of transitory income is very much larger, since the positive transitory income of some households is offset against the negative transitory income of others whenever aggregation occurs. The expected value of

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[1] In general this is probably not an unreasonable approximation to the expected value of anticipated income but in such a period as the middle or late 1930's it would be open to considerable question.

transitory income for individual households is presumably close to zero except in years of recession or of unusally high prosperity. Because the variance of transitory income is relatively higher in the cross-section, we expect the cross-sectional elasticity with respect to measured income to be weighted fairly heavily in the direction of the transitory income effect.

The variance of transitory income may be reduced by considering groups of households rather than individual households as observations, so long as the grouping variable is not itself substantially correlated with transitory income.[1] To the extent that permanent income is correlated with the grouping variable, the variance of permanent income, while falling absolutely, will rise relative to that of transitory income. However, even if the groups are very large, we cannot expect transitory income to vanish or even to be constant over groups since, for example, we cannot expect a recession to have equal impact on all groups, at least for the socio-demographic characteristics which present themselves as the obvious grouping criteria. Certainly some areas, some occupations, some age groups, and some racial groups will be more strongly affected than others.[2]

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[1]For analyses utilizing this approach see Irwin Friend and Irving Kravis, "Consumption Patterns and Permanent Income," American Economic Review, May 1957; Robert Eisner, "The Permanent Income Hypothesis: Comment," American Economic Review, December 1958; Franco Modigliani and Albert Ando, "The 'Permanent Income' and the 'Life Cycle' Hypothesis of Saving Behaviour - Comparison and Tests," in Consumption and Saving, op. cit., Vol. II; and Irwin Friend, "The Propensity to Save in India," Economic Development: Issues and Policies, Dr. P.S. Lokanathan Commemoration Volume, Vora & Co., Bombay, 1966.

[2]Furthermore, when a relatively small number of groups is used, problems of simultaneous equations bias are reintroduced, since disturbances in the consumption of one group may affect significantly the income of that group.

More important, tastes may vary among groups either because the grouping variable exerts an independent influence on consumption and saving or because it is correlated with some other variable which has such an influence. If those groups where consumption tends to be high due to the effects of the grouping or other variable are also characterized by relatively high (or low) levels of normal income, the taste effects will be commingled with the income effect and impounded in the income coefficient. Thus it is necessary to select grouping variables which minimize the taste effects. For this reason it is more sensible to use socio-demographic characteristics as a basis for grouping than expenditures on some class of consumption.

The extent of asset disequilibrium may also vary among cross-sectional groups, though this seems more likely to be correlated with recent values of transitory income or transitory consumption or with the growth rate of normal income than with the current level of normal income. Asset disequilibrium cannot be accurately estimated even if reliable data on group asset levels are available since tastes for the holding of assets may vary sharply. This is particularly true for individual households but may also hold for socio-demographic groups.

Finally, measurement errors in the income of individual households reported in cross-sectional surveys are surely both substantial and variable and, unless the groups which serve as cross-sectional observations are large, will remain a significant source of bias. On all these counts, income propensities or elasticities obtained from single cross sections may be inadequate as an estimate of the normal income effect.

When changes between two cross-sectional years in the consumption and income of individual households or socio-demographic groups are used as observations, the effects of tastes and the grouping variables (if

additive) are held constant and perhaps those of other strongly correlated variables as well.[1] If a given group is composed of the same households in both years, all taste factors are controlled except to the extent that the time trends in tastes discussed previously are operative. If these trends affect all groups in much the same way, then changes in tastes do not bias the estimates of the saving propensity or income elasticity of saving.

However, the variance of the normal income component becomes quite small in first difference relationships, since growth rates of income vary much less among individual households or socio-demographic groups than do income levels. If the variance of transitory income could be entirely eliminated by the grouping process, this would not matter so much, but we have seen that this is not the case especially if the groups are fairly small. Furthermore, with small groups, the variance in measurement error becomes an important component of the variance of changes in measured income. The longer the period of time which has elapsed between the two cross-sections, the larger the variance of normal income change should be, if persistent differences in growth rates of income exist among the groups considered. However, over relatively long periods taste trends are likely to be significant and, to the extent that they differ among groups in a way that is correlated with growth, bias may arise from this source. For small groups and short periods of time, variance in transitory income and in measurement error may dominate the total variance in measured income change. The

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[1]For analyses utilizing this approach see Irwin Friend and Paul Taubman, "The Aggregate Propensity to Save: Some Concepts and Their Application to International Data," The Review of Economics and Statistics, May 1966; and Jean Crockett, Consumer Expenditures and Incomes in Greece, Center of Planning and Economic Research, Athens 1967.

resulting marginal propensity or elasticity estimate will then be strongly downward biased as a measure of the normal income parameter (in cases where normal income has a larger effect than transitory) and perhaps even downward biased as a measure of the transitory income parameter. It can be treated as a lower bound for the former and becomes more useful in this role as we increase the length of time over which the first differences are computed and the sample size for each group.

We might make three observations to end this recital of the limitations of each of the different types of data which can be used to estimate consumption and saving propensities and elasticities. First, much more confidence can be placed on results obtained from more than one body of data and more than one mode of analysis. Second, grouped data (i.e., classified by socio-demographic characteristics), which are relatively infrequently used in the analysis of consumption and saving in individual developing nations, can contribute significantly to such research. Third, continuous cross-section or panel data are especially promising for providing further insights into household (and business) consumption and saving behavior.

## Simplified theoretical framework for interpreting empirical results

Theoretical considerations suggest that the saving of a household should be a function of its income -- both normal and transitory --its initial assets and tastes, where tastes in turn would reflect among other things the socio-demographic characteristics of the household. The interest rate or more accurately the anticipated real rate of return on assets might also be expected to affect household saving, but neither theory nor the evidence from studies within (time-series) and across

(cross-section) different countries clearly indicates that the interest rate plays much of a direct role in influencing aggregate houshold saving.[1] However, interest rate changes might be expected to influence consumption and saving by affecting the value of assets, and there is evidence both in the MPS model of the U.S. economy and in a recent cross-section analysis of the impact of stock market returns on consumption[2] that this indirect influence might be substantial.

Theory has frequently been used to specify the values of the income and asset coefficients in the household consumption and saving functions, but too often in our view this has been associated with

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[1]Much of the empirical evidence is flawed either because nominal rather than real interest rates are used or because the effect of real interest rates on aggregate saving (or consumptin) is confounded with their effect on investment. However, the MPS model of the U.S. economy, which does not suffer from these deficiencies, finds no direct interest rate effect on aggregate consumption (Albert Ando and Robert Rasche, Equations in the MIT-Penn-SSRC Econometric Model of the United States, Mimeo, October 1971). In this connection it is interesting to note that a recent study suggests that the expected real rate of interest on a "risk-free" one month U.S. Treasury bill has been surprisingly close to zero on an annual basis over the past two decades (Eugene Fama, "Short-Term Interest Rates as Predictors of Inflation," The American Economic Review, June 1975).

[2]Irwin Friend and Charles Lieberman, "Short-Run Asset Effects on Household Saving and Consumption: The Cross-Section Evidence," The American Economic Review, September 1975.

unduly restrictive assumptions on what constitutes rational economic behavior.[1] It seems to us a more satisfactory approach is to insert the income and wealth variables suggested by theory, derive the parameters from all the data available, and then spell out their implications for the underlying theory.

The simplest household saving function which makes sense may be written as $S = a + bY_N + cY_T + dA_{-1} + eX + f_i$ where S is saving, $Y_N$ is normal income, $Y_T$ is transitory income or the difference between current income and normal income, $A_{-1}$ is initial net worth, X is capital gains, $f_i$ is a tastes variable, and all variables are in real (measured by consumption prices) per equivalent household terms.[2] It is normally expected that b and c are positive with b<c, d and e are negative with

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[1] E.g., in the well-known "permanent income" theory of Milton Friedman and life-cycle theory of Franco Modigliani, Richard Brumberg, and Albert Ando, which effectively assume a unitary (or close to unity) "permanent" income elasticity of consumption -- and a zero (or close to zero) propensity to consume out of transitory income; or in the more recent work of W. H. Somermeyer and R. Bannink which arbitrarily assumes that households optimize a log-additive multi-period utility function of consumption subject to a lifetime budget constraint (A Consumption-Savings Model and its Applications, North-Holland Publishing Co., 1973). It may be noted that recent research seems to suggest that the utility function of wealth is characterized by a Pratt measure of proportional risk aversion substantially higher than the value of 1.0 for the frequently assumed log utility function. (Irwin Friend, and Marshall Blume, "The Demand for Risky Assets," op. cit.).

[2] Normal income may be defined as the exected value of annual household income over whatever time span is most relevant to the consumption decisions and asset goals of the household. The length of the time span will vary among households and may depend on the specific decision to be made. The span which is most relevant to the average family for most decisions is an open question. In the present state of our knowledge it may be not much longer than a single year or as long as the entire working life of the household head. The theoretical treatment followed in this section is similar to that presented in other papers by one or both of the present authors.

d > e in absolute value, and a is either negative or zero.[1] The relationship has been written in linear form for convenience, and should be regarded only as an approximation to the true form. The reason for distinguishing between $Y_N$ and $Y_T$ is that the household is considered to react differently to temporary or short-run changes in income than to longer-run trends to which it has geared its living standards and its customary disposition of resources among competing uses. Some theorists believe that c is close to one whereas b is relatively small. However, only empirical analysis can determine the relative magnitude of these two coefficients.

The easiest way of indicating the theoretical rationale of an initial assets or net worth variable in the saving function is to note that desired assets for any point of time is likely to be closely related to normal income, i.e. $A_t^* = f(Y_{Nt})$. For convenience we shall first assume that transitory income does not affect the desired relation of assets to normal income. Assets of course are desired for several major purposes: to provide a continuous flow of services; to provide a lump sum at a future date for some specific contingency such as retirement or for an estate; to provide protection against uncertainty; and to maximize long-run income. The time span for achieving desired assets or the time period for which such assets are planned will vary with the purpose of the assets. Those desired to build up an estate might be accumulated at a more or less even rate throughout the saver's total or working lifetime. Desired assets to provide a continuous flow of service (such as from a home) or to provide protection against un-

[1] A more complete discussion of the expected relation among these coefficients appears in "The Aggregate Propensity to Save: Some Concepts and Their Application to International Data," op. cit.

certainty would presumably be built up to the desired relation with normal income over a much shorter time period. Assets accumulated for retirement purposes would be built up during the working lifetime and used up thereafter so that the quantity of assets desired for this objective would depend on the stage in the life cycle and on the retirement age as well as on income.[1]

A consideration of these saving objectives does not permit us to prescribe the precise functional relation between $A^*$ and $Y_N$. However, for simplicity we shall write $A^* = k_0 + k_1 Y_N$ though only empirical investigation can determine whether the relation is in fact linear and the intercept significantly different from zero. Now, assuming no transitory income and no capital gains, actual assets at the end of any period t may be written as $A_t = A_{t-1} + \alpha(A_t^* - A_{t-1})$ where $\alpha(A_t^* - A_{t-1})$ is planned saving.[2] $A^*$, planned assets, depends only on $Y_{Nt}^{t-1}$ or normal income for t estimated at t-1, and $\alpha$ is the speed of adjustment or the fraction of the difference between $A_t^*$ and $A_{t-1}$ which is made up in period t. The speed of adjustment like the asset-income relation is an average of different values for different saving objectives and for different stages in the life cycle. The speed

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[1] For the retired it is not clear that the desired stock of assets at the end of any year depends on normal income, but it is more likely to depend on the actual stock at the beginning of the year, the expected retirement span and the desired size of bequest. Of course actual assets at the beginning of the retirement span should be highly correlated with normal income during the working life.

[2] In this formulation, it is assumed that there is no difference between $Y_{Nt}$ and $Y_{Nt}^{t-1}$ or normal income for t estimated at t-1.

of adjustment, which is assumed here to be a constant, may be a function of the difference between $A_t^*$ and $A_{t-1}$ and of $Y_{Nt}^{t-1}$. Its value obviously depends also on relative tastes for consumption and assets, which will influence the speed with which any disparity between $A_t^*$ and $A_{t-1}$ is made up, and on the length of period over which saving is measured. In the saving equation presented above, initial assets affects saving, with the coefficient equal in magnitude but opposite in sign to the speed of adjustment. The normal income coefficient is the product of the desired assets-normal income ratio and the speed of adjustment. The omission of assets, which is frequently necessary in empirical analysis for lack of data, would bias the income coefficient downward in view of the positive correlation between income and assets.

To allow for transitory income, $Y_T$, we have to decide whether it is to be regarded as completely unforeseen, which obviously depends on the definitions used to distinguish $Y_N$ and $Y_T$. If $Y_T$ is completely unforeseen and does not enter into planning for the period, we have $A_t = A_{t-1} + k_2 Y_{Tt} + \alpha(A_t^* - A_{t-1})$ and $S_t = \alpha k_0 + \alpha k_1 Y_{Nt}^{t-1} + k_2 Y_{Tt} - \alpha A_{t-1}$ where $k_2$ is the proportion of $Y_T$ saved. Part of unforeseen and abnormal income may be consumed, though under certain versions of the permanent income hypothesis $k_2 = 1$. The value of $k_2$ also depends on the length of the period. Under the unlikely circumstances that $Y_T$ is defined in such a manner as to be completely foreseen at the beginning of the period, the relevant equations become

$$A_t = A_{t-1} + k_2 Y_{Tt} + \alpha(A_t^* - k_2 Y_{Tt} - A_{t-1}) \text{ and}$$
$$S_t = \alpha k_0 + \alpha k_1 Y_{Nt}^{t-1} + k_2(1-\alpha) Y_{Tt} - \alpha A_{t-1}.$$

With the introduction of $Y_T$ the truth now presumably lies somewhere between the last two $S_t$ equations. If $Y_{Tt}$ is partly unforeseen, then

$S_t$ will not be zero even if planned saving is zero. The omission of assets would again be expected to bias downward the normal income coefficient (and to a lesser degree the transitory income coefficient).

Obviously, desired assets $A_t^*$ can be written as a different function of $Y_{Nt}$.[1] Presumably $A_t^*$ would not normally be a function of $\Sigma_t Y_{Tt}$ since this would in most circumstances be expected to approach zero, but $A_t^*$ might depend on the variance of $Y_{Nt}$ as well as its level.

The above discussion indicates the rationale for the introduction of $Y_N$, $Y_T$ and $A_{-1}$ in the basic saving relation presented at the beginning of this section. It is easy to show that X, or real capital gains per household, would be expected to have a coefficient between zero (if X is unforeseen) and the same negative coefficient as $A_{-1}$ (if X is foreseen). (Satisfactory data for estimating X are not generally available in developing countries, but it is not believed that this gap introduces a serious bias in estimating the other coefficients in the basic saving relation). The tastes variable $f_i$ is introduced in this somewhat

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[1] Data on net worth and income for a sample of United States families (Federal Reserve Bulletin, March 1964) suggest that the relation between A and Y may be logarithmic rather than linear so that it may be more legitimate to write $\log A^* = \log k + b \log Y_N$ or $A^* = kY_N^b$ (rather than $A^* = kY_N$). If S may still be written as $\alpha(A^* - A_{-1})$, then $S = \alpha k Y_N^b - \alpha A_{-1}$ (rather than $S = \alpha k Y_N - \alpha A_{-1}$). If instead the asset adjustment process is written in logarithmic form, i.e., $\frac{A}{A_{-1}} = (\frac{A^*}{A_{-1}})^c$, then $S = d \frac{Y_N^e}{A_{-1}^f} - A_{-1}$ (minus an adjustment factor for any capital gains) where $d = k^c$, $e = bc$, and $f = c-1$. However, these data which relate A to Y, rather than $A^*$ to $Y_N$, are suggestive rather than conclusive for family saving functions.

unrealistic manner simply to highlight the fact that even for given values of the more objective economic variables affecting saving, there are wide differences in saving tastes which may have a major impact on the amount of saving desired or realized. It might be noted that the omission of a tastes variable in ordinary cross-section analysis would be expected to introduce a downward bias in the absolute value of the assets coefficient in our basic theoretical construct and probably less so in the income coefficients. In such analysis it is sometimes possible to use a saving ratio for a prior period as an approximate technique for holding tastes constant.

Our theoretical discussion of the determinants of saving has suggested that cross-section analysis which omits data on assets and tastes will tend to introduce some downward bias in the income coefficients we derive.[1] There is a statistical limitation of cross-section data on saving which may tend to accentuate this bias. The average propensities to save based on the usual survey definition and measurement of saving (before conceptual and statistical adjustment) systematically understate saving.[2] As a result, the estimated marginal propensities to save derived from most cross-section studies may be biased downward, though the conceptual and statistical limitations of

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[1]Data for the years 1960 and 1961 from the U.S. Survey of Consumer Finances suggest that this bias is rather small. See Jean Crockett and Irwin Friend, "Consumer Investment Behavior," Conference on Investment Behavior, Universities-National Bureau Committee for Economic Research, Princeton University Press, 1967.

[2]See Irwin Friend and Stanley Schor, "Who Saves?" The Review of Economics and Statistics, May 1959.

survey data would not be expected to affect the derivation of marginal saving as much as average saving propensities.

In analyzing bodies of data where assets are not available but where it is reasonable to assume that asset disequilibria are relatively small (e.g., studies of saving behavior in normal periods using countries as the unit of observation), our basic saving regression becomes $S = a + b\Delta Y_N + cY_T + f_i$.[1] In this relationship b can be construed as the desired assets-income coefficient (or ratio if the constant term in the original saving regression or in the regression of desired assets on normal income is zero). The marginal propensity to save out of normal income would then be the product of the desired asset-income coefficient or ratio by the speed of adjustment (to the desired relationship) which is the asset coefficient in the original saving regression. It should be noted that in empirical work the utilization of a $\Delta Y_N$ term involves some of the problems discussed earlier in connection with the application of continuous cross-section techniques.

Results of earlier studies

All empirical studies show both for developing and developed nations that the rate of increase in household saving is fully as high as that in disposable income. However, the basic question of whether the marginal rate of saving out of disposable normal income is in excess of the average rate has not been answered conclusively for either developing or developed nations. Our own interpretation of the extremely

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[1] Luis Landau, "Saving Functions in Latin America," in Hollis Chenery, ed., Studies in Development Planning, Harvard University Press, 1971 suggests that while $\Delta Y_N$ may be more relevant for the advanced economies, $Y_N$ is more relevant for underdeveloped countries.

extensive analysis of this question in the United States[1] leads us to the tentative conclusion that the marginal rate of saving out of normal income has been in excess of the average rate, or in other words that the propensity to save is greater for high than for low incomes. This we feel has been true even though the long-run time-series data for the U.S. show no significant upward movement in the ratio of saving to income over a period of fairly steadily rising ncome, a result which in our view reflects a number of offsetting institutional (and distributional) changes tending to decrease saving for given income.

Looking at the same question across countries at different stages of economic development again leads to no definitive conclusion about the relationship between the household saving-income ratio and per capita real income, but again to some suggestion of a positive if weak relationship between these two variables. Kuznets found a positive relationship between the saving-income ratio and per capita real income, but Houthakker found no relationship or more precisely that per capita saving rose proportionally, rather than more than proportionally, with per capita disposable income.[2] Landau for 20 Latin American countries

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[1] E.g., see "Consumption Patterns and Permanent Income," and "Consumer Investment Behavior," op, cit.; and Nissan Liviatan, "Estimates of Distributed Lag Consumption Function from Cross-Section Data," and Paul Taubman, "Permanent and Transitory Income Effects," both in The Review of Economics and Statistics, February 1965.

[2] Simon Kuznets, "Quantitative Aspects of Economic Growth of Nations, V: Capital Formation Proportions: International Comparisons for Recent Years," Economic Development and Cultural Change, July 1960, Part II; and H.S. Houthakker, "On Some Determinants of Saving in Developed and Underdeveloped Countries," in E.A.G. Robinson, ed., Problems in Economic Development, Macmillan, 1965.

and Singh for 70 countries found marginal savings propensities in excess of average propensities for per capita gross and to a lesser extent for net domestic saving, or in other words found that countries with higher per capita income tended to save a somewhat larger proportion of their income.[1]

It should be noted that even if on the average individuals and countries with relatively high per capita incomes tend to have a higher than average propensity to save, the differences in the saving-income ratio associated with the level of income may be smaller than those associated with differences in tastes. The extremely high personal savings rates of Japan and Germany over the post-World War II period appear to be a case in point, though at least part of such "abnormal" saving was probably induced by the unusually high growth rates of their economies and, at least initially, by the need to build up assets destroyed during the War.

Empirical studies indicate that the propensity to save varies substantially for different types of income. Virtually all time-series and cross-section studies of the marginal propensity to save out of transitory income in the U.S. indicate that it is higher than the corresponding propensity for normal income, even though most indicate that

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[1] "Saving Functions in Latin America," op. cit.; and S. K. Singh, "The Determinants of Aggregate Saving," International Bank for Reconstruction and Development, Mimeo, April 1971.

it is well below the value of one suggested by "permanent" income theory.[1] Most of these studies also suggest that the normal income elasticity of total saving is substantially higher than one. It is interesting to note that while the influence of transitory income on total saving in any year -- particularly on investment in durables and changes in liquid assets -- seems to be greater than that of normal income, the reverse is true for contractual saving.[2].

Confining our attention to normal income, as noted earlier most of the evidence -- though not all one-sided -- suggests that the richer households save a higher proportion of their income than the poor. Pretty universally, it is found that for given normal income entrepreneurs --i.e. owners of business firms -- have a higher propensity to save than other individuals. As a result, it has been found that entrepreneurs account for a high proportion of total house-

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[1]E.g., see Jean Crockett, "Income and Asset Effects on Consumption: Aggregate and Cross-Section," Studies in Income and Wealth, Vol. 28, 1964; and "Consumer Investment Behavior," "Estimates of Distributed Lag Consumption Function from Cross-Section Data," and "Permanent and Transitory Income Effects," op. cit. Normal income in these studies has been defined in a number of different ways, including a time trend of measured incomes, simple and weighted averages of current and past measured incomes, and the average of incomes for groups of households classified by characteristics uncorrelated with transitory income.

[2]See H. S. Houthakker and John Haldi, "Household Investment in Automobiles: An Intemporal Cross-Section Analysis," in Consumption and Saving, op. cit., Vol. 1; and "Consumer Investment Behavior," op. cit.

hold saving both in developed and developing economies, with some studies suggesting that saving out of employment income is close to zero in many countries.[1] Private corporations, like owners of unincorporated businesses, also have a very high marginal and average propensity to save, with their savings propensities very much larger than those for the household sector.[2]

The evidence on differences in the propensities to save in urban and rural areas is rather conflicting and may reflect deficiencies in the measurement of both income and saving. In the U.S., the survey data suggest that farmers save more than the rest of the population for given income, while in India the reverse seems true.[3]

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[1] Irwin Friend and Irving Kravis, "Entrepreneurial Income, Saving and Investment," American Economic Review, June 1957; "Who Saves?" op. cit.; H.S. Houthakker, "An International Comparison of Personal Savings," Bulletin of the International Statistics Institute, 1961; and J. G. Williamson, "Personal Saving in Developing Nations: An Inter-Temporal Cross-Section from Asia," Economic Record, June 1968.

[2] "Entrepreneurial Income, Saving and Investment," supra.; National Council of Applied Economic Research, Saving in India, April 1961; Irwin Friend, Individuals' Saving: Volume and Composition, John Wiley & Sons, 1954; and I. Little, T. Scitovsky, and M. Scott, Industry and Trade in Some Developing Countries: A Comparative Study, Organization for Economic Cooperation and Development, 1970. It should be noted that saving by private corporations, which of course is more important in the developed than in the developing countries, is at least in part offset by a reduction in saving by households as a result of capital gains to the household sector associated with corporate retention of earnings. Most of these capital gains accrue of course to upper income households so that survey data can be regarded as understating the direct and indirect saving by such households.

[3] "Entrepreneurial Income, Saving and Investment," and Individuals' Saving: Volume and Composition, op. cit.; National Council of Applied Economic Research, All India Household Survey of Income, Saving and Consumer Expenditures, 1972; and K. L. Gupta, "On Some Determinants of Rural and Urban Household Saving Behavior," Economic Record, December 1970. It might be noted that the urban marginal saving propensity of .34 derived in the NCAER 1972 study seems much too high and probably reflects the absence of any adjustment of measured incomes for transitory components.

While there are a multiplicity of estimates of the asset effect on saving for the U.S. both from time-series and cross-section estimates, comparable data do not exist for the developing countries. For the U.S., the MPS time-series model now suggests a long-run annual deterrent effect of assets on saving of .054 per dollar of assets, with the full effect reached within two years of new asset accumulation.[1] Household cross-section studies in the U.S. have suggested somewhat smaller negative effects,[2] while other time-series analyses in the U.S. point to values ranging from zero to considerably larger negative effects.[3] When our basic saving regression is converted into change form, regressing $\Delta S$ on $\Delta Y_N$, $\Delta Y_T$, and $S_{-1}$ as a crude proxy for $\Delta A_{-1}$, a cross-section analysis of 22 different developed and developing countries over the period 1953-60 obtained asset effects ranging from -.07 to -.10.[4] Clearly, we cannot say much about the value of the asset effect on saving, except that it appears to be negative in accordance with theory, and that it is likely to be less than -.10 in absolute value and probably is very much under that figure in the U.S.

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[1]For non-stock assets the full effect is reached within one quarter. For stock it takes two years. The earlier MPS model, Equations in the MIT-Penn-SSRC Econometric Model of the United States, op. cit., pointed to a somewhat different result.

[2]"Short-Run Asset Effects on Household Saving and Consumption: The Cross-Section Evidence," and "Consumer Investment Behavior," op. cit.

[3]Michael Evans, "The Importance of Wealth in the Consumption Function," Journal of Political Economy, August 1967; and Franco Modigliani and Albert Ando, "The Life Cycle Hypothesis of Saving: Aggregate Implications and Tests," American Economic Review, March 1963 and March 1964.

[4]"The Aggregate Propensity to Save: Some Concepts and Their Application to International Data," op. cit.

When S is regressed on $\Delta Y_N$ and $Y_T$ in another cross-section analysis of these same 22 countries and period, on the assumption that asset disequilibria were not important for these observations, the $\Delta Y_N$ coefficient ranges from 2.1 to 2.3, pointing to a desired asset-income ratio of somewhat over two.[1] If the asset coefficient is construed as a speed of adjustment to the desired level of holdings, as it should be according to our analysis, then it is possible to multiply the desired asset-income ratio by the speed of adjustment to derive the marginal propensity to save out of normal income.

While without continuous cross-section data or at least data on saving or consumption and income for more than one period, it is not possible to hold tastes constant, information is usually available in most cross-section studies on a number of socio-demographic characteristics which are related to savings behavior for given values of the key economic variables. Of these, probably the two most important (apart from the distinction between entrepreneurs and other households) are size of family and age of head. In household survey data, size of family is found to be negatively correlated with saving (positively with consumption) for given values of household income, while for age of head the results are mixed.[2] For the U. S., Japan, and

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[1] Supra. The United States ratio of personal assets to disposable income was considerably higher, but we do not have corresponding ratios for the other countries.

[2] For U. S. household survey results covering both age of head and size of family effects, see Jean Crockett and Irwin Friend, "A Complete Set of Consumer Demand Relationships," Consumption and Saving, op. cit., Part 1, pp. 54 and 173-4; and Individuals' Saving: Volume and Composition, op. cit., pp. 130-1. For Japan, India, and Indonesia, see Toshiyuki Mizoguchi, Personal Savings and Consumption in Postwar Japan, 1970; National Council of Applied Economic Research, Urban Income and Saving (in India), 1962; and A. C. Kelley and J. G. Williamson, "Household Saving Behavior in Developing Economies: The Indonesian Case," Economic Development and Cultural Change, April 1968.

Indonesia, age of head seems positively correlated with saving for given values of household income; for India the correlation is negative.[1] One investigator on the basis of a cross-section analysis of 74 developing and developed countries found not only that dependency ratios (the proportion of the population below 15 years of age or above 65) were negatively related to the overall and per capita gross saving-income ratio, but also that the relatively high proportion of the population below 15 years of age was of particular importance in explaining the relatively low saving ratios of the less developed countries.[2]

The above brief summary of the results of prior empirical studies dealing with saving (or consumption) propensities related to total saving (or consumption) only. For economic planning purposes, which make use of estimates of the aggregate marginal propensity to save and assess the desirability and feasibility of raising that propensity, concern with the propensity to save in particular asset forms stems either from an interest (1) in improving the quality of national saving and investment or (2) in using information on the substitutability of specific components of saving with other forms of saving as contrasted with consumption. In this paper, we are interested mainly in the second of these concerns. From this point of view, if all forms of saving are

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[1] The relative saving propensities of households headed by older people will depend on the proportions of retired and non-retired heads included in this age group.

[2] N.H. Leff, "Dependency Rates and Savings Rates," American Economic Review, December 1969. For criticism and further discussion of these results, see comments by K. L. Gupta, N. A. Adams and Leff in the American Economic Review, June 1971.

highly substitutable with each other, clearly a government will not be successful if it attempts to increase the nation's saving by providing tax incentives for purchases of government bonds or development funds. The increase in one form of saving will be largely offset by a decrease in another.

We are familiar with only one comprehensive published study of the marginal rates of substitution between specific components of saving and all other forms of saving combined for given levels of income -- a study of U. S. saving behavior based on both cross-section and time-series data.[1] The two most interesting conclusions of that study are that expenditures on consumer durable goods as a whole seem to be more substitutable for or competitive with the total of personal saving (exclusive of durables) than with the total of expenditures on non-durable goods and services, and that of all the major components of saving the component least substitutable with other forms of saving is life insurance. In other words, an increase in expenditures on consumer durables for given income is likely to be largely reflected in a decrease in saving rather than in consumption, while in contrast an increase in life insurance equity is likely to be associated to a substantial extent with a decrease in consumption. Similar findings were obtained in an analysis of life insurance in India.[2] There are also several time-series and cross-section (area) studies for the

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[1] Irwin Friend and Robert Jones, "The Concept of Saving," in Consumption and Saving, op. cit., Vol. II. See also comments by Edward F. Denison, Arthur M. Okun, and Friend and Jones.

[2] National Council of Applied Economic Research, Contractual Saving in India, 1963.

United States and cross-section studies across developed and less developed countries which suggest that variables like the ratio of household saving in financial claims to saving in tangible assets or the relative number of offices of financial institutions are positively correlated with the amount of national savings.[1]

Finally, we shall end this summary of prior empirical studies by pointing to the plethora of widely different estimates of the overall marginal propensity to save that are typically obtained for the same country for the same period of time by using different bodies of data or different estimating techniques.[2] To obtain an estimate on which much reliance can be placed, it is essential to use, check against each other and attempt to reconcile the results from all these data sources and analytical techniques. Thus, for India in the early 1960's it was found that estimating the marginal propensity to save out of normal personal disposable income from cross-section data (where normal income was defined as a linear function of the average incomes for each of the socio-demographic classes to which the household belonged) gave a somewhat higher result than that obtained from the time-series data available at the time but turned out to be quite close to that based on the subsequently revised time-series data.[3] This cross-section estimate on the other hand was very much lower than that obtained from

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[1] See for example the interesting analysis by U Tan Wai, Financial Intermediation and National Savings in Developing Countries, Yale University Economic Growth Center Discussion Paper No. 126, 1971. The basic problem in such analyses, of course, is to distinguish which way the causation goes.

[2] E.g., see Irwin Friend, The Propensity to Consume and Save in Argentina, Documento do Trabajo, Instituto Torcuato Di Tella, Centro de Investigaciones Economicos, December 1966, pp. 20-21.

[3] "The Propensity to Save in India," op. cit.

the usual procedures which do not distinguish between normal and transitory income. Even simple techniques for reducing the effects of transitory income on the estimated marginal propensity to save out of normal income, such as omitting the extreme income groups or confining the analysis to households with reasonably stable incomes over a three year period, greatly improved the comparability of results, though they were not as satisfactory in eliminating fully the disparities between the cross-section and revised time-series estimates.

## 3. Analysis of Columbian Household Data, 1967-68

We present here a new analysis of household consumption and saving data for Columbia, collected in four surveys covering four different quarters in the years 1967-68.[1] Over 2,100 households in the four principal cities were included in the sample, which was stratified by income. In some cases the same households were interviewed in two or more quarters, with the results from each such interview included as a separate observation, raising the total number of observations to more than 2,900.

Weighted linear regressions were computed relating alternative savings concepts, as well as various saving components and total consumption, to normal and transitory income, age and family size. Separate regressions were computed for the retired. Capital gains and asset data were generally not available, but the ratio of dividend plus interest income to total income was included as a proxy for the ratio of stocks plus fixed interest obligations to income. If desired holdings of

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[1]We wish to express our appreciation to the Centre de Estudios de Desarrollo of the Universidad de los Andes, Bogota, for access to these data.

these (and other) asset items are roughly proportional to income, then the variable utilized, while reflecting tastes in part, will give some indication of asset disequilibrium.

Normal income was estimated from a regression relating household disposable income to mean disposable income for the occupation-of-head, education-of-head, tenure-of-dwelling-unit, and housing-expense classes into which the household falls. Dummy variables, permitting different constant terms for each of the four cities, were also included in the normal income regression. Occupation and education accounted for 30% of disposable income variance ($\bar{R}^2$), while the remaining variables raised the $\bar{R}^2$ to 36%. All of the income means were highly significant for those in the labor force, though only education proved so for the retired. This is a somewhat higher degree of explanation than was obtained by applying a similar technique to Indian household survey data.[1]

The use of housing expense as a grouping variable is open to some question when group consumption means are regressed against group income means since such procedures may lead to a confusion of taste effects with normal income effects on total consumption -- i.e., those with a strong preference for quality housing may reduce their saving on this account, so that the relatively high total consumption of this group is only partially due to relatively high mean income. The same objection does not apply with equal force to our present use of mean income for a given housing expenditure group as a component of normal income for households in that group because several other components also affect normal income.

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[1] See "The Propensity to Save in India" op. cit.

Mean income for the age-of-head class of the household was not utilized in estimating normal income (though it will be recalled that the retired are analyzed separately). This is consistent with the theory that expected average income over one's earning years is relevant as a determinant of consumption, even in the early years when current income is likely to be below the expected average. The present approach treats this shortfall as a transitory factor, largely reflected in dissaving.

Clearly there are elements of normal income not caught by an estimate based on means for socio-economic groups. To the extent that the household head has work-related abilities above or below the means of the several groups into which he falls, his normal income is likely to be above or below this estimate.

Two components of transitory income were distinguished in the study: one, which was designated $Y_T$, reflected large identifiable windfalls such as lottery winnings or inheritances, while the other was the residual, $Y-Y_N-Y_T$, which remains after $Y_T$ and the estimated value of $Y_N$ are subtracted from current income. The measurement error in this residual, if Y and $Y_T$ are accurately reported, is equal in magnitude but opposite in sign to the error in our estimate of normal income. The biases resulting from these errors are proportional to the difference between the true values of the two income effects involved--negative in the case of the estimated marginal propensity to consume out of normal income and positive for the residual transitory income effect. For normal income, the bias will be a small fraction of the difference between the true income coefficients, if, as we assume, the variance of the deviation of individuals' normal income from the estimated value is small relative to the total variance of their normal income.

Three major concepts of saving were analyzed: (1) financial saving, computed as a residual after eliminating from disposable income all known expenditures other than purchases of financial assets;[1] (2) financial saving plus down payment on house plus social security contributions (life insurance was not available); and (3) the preceding plus purchases of household durables, expenditures for major home improvements and down payment on automobiles (or alternatively value of autos purchased less value of autos sold). Amortization plus interest payments on home and car were treated, alternatively, as being entirely consumption (with the amortization portion considered to roughly approximate depreciation), one-half saving or entirely saving, with very minor differences in the empirical results.

For all households combined, the three income measures were all highly significant regardless of the saving concept used, with the coefficient of $Y_N$ smallest and that of $Y_T$ the largest. (See Table 1). For alternative versions of the first two savings concepts the marginal propensity to save ranged from .25 to .27 with respect to normal income and .52 to .54 for residual income $(Y - Y_N - Y_T)$.[2] For the third concept, which includes expenditures for consumer durables, the marginal propensity varied from .32 to .33 with respect to normal income and

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[1]Unreported expenditures may constitute a significant proportion of financial saving, as here estimated; and results reported must be evaluated with this consideration in mind. It should be noted that when the purchase of a residence or an automobile is financed by borrowing the data presented here do not include the total value of the purchase as saving invested in real assets, which is then largely offset by negative financial saving in the amount of the debt incurred. Rather, the saving in real assets is limited to the down payment plus any amortization occurring within the same time period. Thus there is little negative effect of such a purchase on financial saving.

[2]When all three components of income are combined into a single income variable, the marginal propensity to save is found to be in the neighborhood of .36 to .40.

from .57 to .58 with respect to residual income. With respect to windfall income the marginal propensity was .68 for the first savings concept, .84 for the second and .89 when consumer durables are included.

For the retired, only the residual income was significant, yielding a marginal propensity of .53 for all three savings concepts. However, for consumption both normal income and residual income were highly significant yielding marginal propensities of .93 and .47, respectively, when imputed rent for homeowners in the consumption variable is replaced by one half of amortization plus interest payments on homes. These values rise to .99 and .64 when imputed rent is included in consumption as well as in income. These marginal propensities to consume may be compared with corresponding estimates for all households of .74 and .46 when imputed rent is absent from consumption, and .79 and .54 when imputed rent is included in consumption.

The substantial increases in propensities to consume out of normal and residual income which result from the inclusion of imputed rent in consumption may be spurious, reflecting the addition of the same variable to both consumption and income. It should be noted that income is substantially overstated for homeowners still paying off their mortgages if imputed rent is included in the income figure but interest on the mortgage is not subtracted out. On the other hand, for homeowners who are free of mortgages, consumption is understated when imputed rent is replaced by half of amortization plus interest in the consumption figure. The overstatement of income in the first case, being positively correlated with an element of consumption, leads to upward bias in the estimated marginal propensity to consume.

For all households, normal income elasticities were about 1.75 for financial saving and purchases of household durables, over 1.5 for total saving and for automobile purchases (less trade in), and quite low for increases in equity in residences and for major improvements in residences. The elasticities for total saving and for financial saving agree closely with 1960-61 results for the United States, but those for consumer durables are considerably higher and those for equity in homes and major improvements considerably lower.[1] The elasticity of total consumption with respect to normal income for all households in the Columbian sample was .84 to .88 (depending on the treatment of imputed rent), as compared with 1.07 for the retired (including imputed rent). Amortization plus interest was negligible for the latter group, suggesting that owned homes were free of mortgages.

The ratio of dividends plus interest to total income had a significantly negative effect on financial saving and total saving, both with and without durables expenditures, but negligible effects on other saving components. This suggests that holdings of stocks and fixed interest obligations, given income, tend to discourage financial saving, as would be expected. The effect of household size was also significantly negative for financial and total saving and close to significance for household durables.

Financial saving and total saving excluding durables tended to be significantly higher, given income and the other regression variables, for households with age of head over 65. Expenditures for household durables were highest for households under 35 and declined progressively

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[1] "Consumer Investment Behavior" op. cit. pp. 123-125.

thereafter, with significant differences between the youngest and each of the older groups. This is in accordance with theoretical expectations and consistent with other empirical findings. Auto purchase and increase in equity in home tended to be relatively high for the 35-49 age group, though not significantly so. Thus, the observed age pattern tends to confirm the hypothesis that service-generating assets are accumulated first (household durables predominantly by the age group under 35 and automobiles and equity in home by the age group 35-49) while financial saving comes much later.

The extent of substitutability or complementarity between savings components was tested by regressing various components and aggregates thereof against other components, holding constant income and demographic variables. Financial saving appears to be highly competitive with increase in equity in residences, household durables and especially social security contributions, but complementary with purchases of vehicles and major home improvements.[1] Somewhat surprisingly, all three income coefficients in the financial saving regressions are raised considerably by the inclusion of household durables as an explanatory variable, suggesting substantial multicollinearity among financial saving, durables purchase and income. There is some suggestion that social security contributions are also complementary with purchases of vehicles and major home improvements. Household durables appear to be competitive with (or substitutable for) increase in equity in residences

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[1]This last would be a highly surprising result and quite different from findings of other countries if borrowing to finance the purchase of a vehicle were deducted from financial saving, but debt incurred to finance the purchase of real assets has been netted against saving invested in real assets and so does not affect our estimate of financial saving.

as well as with financial saving. Thus vehicle purchases and major home improvements in these data are found to behave quite differently from other savings components, with a high degree of substitutability occurring within the latter group but not between the latter and the former group.

Finally an attempt was made to determine whether relationships identical with those in Table 1 except that they are non-linear in $Y_N$ effected any improvement in the statistical results. The non-linearity was introduced either by substituting $Y_N^{\alpha}$ for $Y_N$ (where $\alpha$ took on values between 1.1 and 1.5) or by including a $Y_N^2$ term in the regression in addition to $Y_N$. The substitution of $Y_N^{\alpha}$ for $Y_N$ did not improve the fits of the regressions. While the addition of a $Y_N^2$ term did improve the fits somewhat, the coefficients of $Y_N^2$ had the wrong sign (i.e., negative in the saving regressios and positive in the consumption regressions) perhaps as a reflection of multicollinearity with $Y_N$.

## 4. Methodological and Policy Implications

The new empirical results which we have obtained for the marginal propensity to save out of normal income by the household sector in Columbia, analyzing the cross-section data in a much different manner from that customarily followed, seems to us to be as good as the available cross-section data permit. However, before these results are taken at face value, it would be highly desirable to check them against and attempt to reconcile them with corresponding results obtained from a careful analysis of time-series data.

For further improvements in estimating the marginal propensity to consume and save, it would be necessary to collect either satisfactory

panel data on an annual (or more frequent) basis for household income, consumption and saving over several years, or at least data from a number of single or discrete annual household surveys again extending over a period of several years. As suggested previously, panel data would provide a basis for tracing the response of household saving to changes in income, assets, and other relevant variables as these variables change over time, since the major differences in tastes among households can be held reasonably constant. Such data can also provide a basis for a somewhat more satisfactory separation of income into normal and transitory components. Finally, it should be possible to reduce measurement errors through the continuous collection of data from the same households, or at the very least to insure that measurement errors are reasonably consistent for the same household from one time period to another so that income coefficients might still be relatively unbiased. The availability of a number of single cross-sections or discrete surveys would permit a similar type of analysis, with tastes held constant, though groups of households rather than single households would now be the unit of measurement. Clearly, panel data would provide more information than a series of discrete surveys but even the latter would be highly useful.

Our new empirical results in conjunction with those of earlier studies provide some (though limited) insights into appropriate Government policies if it is desired to encourage and increase saving other than in consumer durables. From the viewpoint of enhancing saving, changes in institutional arrangements which facilitate purchases of consumer durables, such as the liberalization of installment credit, should be discouraged since purchases of consumer durables are likely to

be offset for the most part not by a reduction in consumer expenditure on other goods and services but rather by a reduction in liquid assets or other forms of saving. While data on life insurance were not available for Columbia, studies in other countries suggest that another potentially useful device for increasing saving is for the Government to initiate or expand different types of life insurance programs, since saving in this form seems to be substitutable in large part with consumption rather than with other forms of saving. There is also some evidence that making financial institutions more readily accessible to the general public may increase the overall propensity to save (as well as presumably improving the quality of saving).

It is possible that new forms of saving could be provided which would serve to enhance total saving. Thus the Government could issue specially tailored securities to promote saving, which would provide purchasing power protection, would pay relatively high interest rates, might be bought on a contractual basis, and would not be redeemable except under special circumstances for a substantial period of time. If very high interest rates are required to make these securities attractive, they could be sold only up to a specified maximum amount which is a decreasing proportion of household net worth or income, or to households below a designated income level, to ensure that the income distribution is not affected adversely. It would be important to ensure that households do not get these special incentives to simply change the form of their saving rather than increase its total amount, so that it would be desirable if possible to gear permissible purchases to changes in, rather than to the level of, net worth. In general, if the interest rates on these securities are fixed at a tax-adjusted level lower than

the marginal productivity of investment, such measures would benefit the country as a whole as well as the individual purchaser.

Other changes in institutional arrangements, as well as in the financial instruments available to households, may also promote their saving. While the available data do not permit a definitive judgement, both the evidence and theoretical considerations suggest that making savings institutions more accessible to the general public would increase saving. However, there are obvious costs involved and they may offset the benefits. A different type of institutional change which might have a more favorable cost-benefit ratio in the promotion of saving is the more general adoption of a system of wage payments in private industry, like that in Japan, in which annual bonuses geared to company profitability constitute a substantial part of total wages paid. Though experts disagree on the relative importance to be ascribed to these bonuses in explaining the high saving rate in Japan, most agree it has played a significant role. Bonuses to a substantial extent seem to have been treated as though they constituted transitory income, even though a large share was obviously foreseen. If bonuses could be paid in Government bonds not ordinarily redeemable for a substantial period of time, their impact on saving could be increased. This would be also true of Government lotteries and similar large one-time or infrequent payments.

In view of the apparent central importance of tastes in differentiating the saving propensities of people in different countries, any Government interested in promoting saving on a voluntary basis should not overlook the potential of adopting measures to affect the psychological reactions of its citizens to the desirability and

importance of saving. In the long-run, success in instilling such reactions probably will depend fully as much on the educational process in the public schools as on the more customary forms of Government exhortation.

Obviously, a more direct approach which a Government can adopt to increase aggregate saving is to raise taxes and increase its saving-income ratio to a level above that of the private sector. No one doubts the ability of most Governments to raise taxes but some economists have concluded that increased taxation has led on the average to reduced or at best unchanged national saving because any increased saving by the public sector is offset by decreased saving in the private sector.[1] However, most economists have concluded that higher taxes are normally associated with higher Government saving which is not completely offset by reduced private saving.[2] Moreover, a higher proportion of Government than of private expenditures classified as consumption may represent outlays that increase the value of human capital. In any case, it is perfectly clear that a Government even without using coercive methods can increase national saving by appropriate tax and expenditure measures. Germany and Japan in the period after World War II illustrate that this is not only a theoretical possibility.

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[1] E.g., "Differences in Saving Ratios Among Latin American Countries," op. cit.

[2] E.g., see R.A. Bhatia, "A Note on Consumption, Income and Taxes," International Monetary Fund, Mimeo, November 1967.

One caveat is in order. In view of the basic importance of income in the determination of the level of saving, measures taken to influence private saving without regard to income effects may be self-defeating. Thus in a period marked by a considerable amount of unemployed resources, government measures taken to encourage saving might be expected, in the absence of offsetting measures, to lower income and hence aggregate saving by lowering spending not only on consumption but on investment. Assuming a period marked by full employment and inflationary pressures, a rise in the propensity to save induced by measures which do not correspondingly depress the propensity to invest might be expected to be associated with more aggregate investment and saving rather than with lower real income.

An increase in the national saving-income ratio probably can be accomplished by several combinations of fiscal and monetary policy designed to increase public or private saving and investment. One possible approach would be to encourage private investment (which is much more interest elastic than saving) by relatively easy monetary or credit conditions throughout the cycle, and to encourage consumption during recessionary periods and saving during inflationary periods by appropriate fiscal policy, though such measures could raise a number of political and economic problems including that of income distribution. Another possible approach is to use selective fiscal policy instead of or together with monetary measures to stimulate investment (public as well as private) over the entire cycle, with countercyclical fiscal policy again used to increase saving in the boom and consumption in the recession. In addition to varying the general level of tax and

government expenditures, fiscal policies designed to affect saving and investment would include both consumption taxes[1] and tax concessions for productive saving geared to the level of wealth (with relatively higher concessions to people in lower income groups). If it seems desirable to encourage private saving over the entire cycle, without discouraging investment via higher interest rates, additional help might be provided by institutional changes which would make readily available different saving media geared to the needs and tastes of specific sectors in the population.

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[1]Selective consumption taxes are much easier to administer than generalized consumption taxes and permit a heavier burden to be placed on expenditures which are considered to be socially undesirable.

Table 1: Savings and Consumption Regressions for Columbia, 1967-68*

All Households, Quarterly Totals

| Dependent Variable | Marginal Propensity with Respect to: Normal Income ($Y_N$) | Residual Income ($Y-Y_N-Y_T$) | Windfall Income ($Y_T$) | Elasticity With Respect to Normal Income | Coefficient of: Family Size | Age Dummies: 35-49 | 50-64 | 65 & over | Ratio of Dividends Plus Interest to Total Income | Constant Term | $\bar{R}^2$ | Mean (Tens of pesos) |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Financial Saving | .252 (22.2) | .520 (58.5) | .675 (7.7) | 1.75 | -15.3 (-5.7) | -2.6 (-.2) | 8.2 (.4) | 133.3 (3.7) | -213.5 (-1.8) | .1 | .580 | 119.5 |
| Total Saving I[a/] | .265 (25.2) | .541 (65.9) | .840 (10.3) | 1.53 | -14.9 (-6.0) | 8.5 (.5) | 4.0 (.2) | 129.7 (3.8) | -246.2 (-2.3) | -1.1 | .638 | 135.3 |
| Total Saving II[b/] | .323 (35.5) | .572 (80.2) | .892 (12.6) | 1.54 | -15.4 (-7.1) | 1.3 (0.1) | -28.7 (-1.7) | 85.7 (2.9) | -226.1 (-2.4) | -7.3 | .730 | 163.7 |
| Total Insurance | .007 (5.5) | .009 (9.3) | -.004 (-.4) | 2.30 | .1 (.4) | -1.3 (-.7) | 3.2 (1.3) | -3.7 (-.9) | -1.5 (-.1) | -4.0 | .038 | 2.5 |
| Increase in Equity in Residences | .012 (2.6) | .021 (5.7) | .164 (4.5) | .63 | .4 (.3) | 11.1 (1.5) | -4.2 (-.5) | -3.6 (-.2) | -32.8 (-.7) | -1.2 | .019 | 15.8 |
| Major Improvements in Residences | .003 (2.0) | .006 (5.9) | .014 (1.4) | .44 | .5 (1.6) | 1.6 (.8) | -2.3 (-1.0) | -3.1 (-.8) | -5.1 (-.4) | -1.6 | .014 | 3.8 |
| Household Durables | .048 (16.7) | .025 (11.2) | .009 (.4) | 1.76 | -1.1 (-1.6) | -11.9 (-2.7) | -25.6 (-4.8) | -38.1 (-4.1) | 24.7 (.8) | 1.8 | .125 | 22.6 |
| Net Outlay on Vehicles[c/] | .025 (2.8) | .026 (3.7) | .051 (.7) | 1.58 | -.6 (-.3) | 21.2 (1.6) | -8.7 (-.5) | -5.2 (-.2) | -13.7 (-.1) | -11.0 | .006 | 13.1 |
| Total Consumption I[d/] | .791 (79.7) | .545 (70.1) | .110 (1.4) | .84 | 18.4 (7.8) | 17.5 (1.2) | 38.1 (2.1) | -11.3 (-.4) | 423.3 (4.1) | -11.4 | .799 | 777.8 |
| Total Consumption II[e/] | .734 (55.8) | .464 (45.1) | .187 (1.8) | .88 | 14.1 (4.5) | 8.8 (.4) | -9.7 (-.4) | -124.7 (-3.0) | 259.9 (1.8) | 2.8 | .643 | 698.4 |
| Consumption of Nondurables and Services[f/] | .661 (73.5) | .413 (58.6) | .127 (1.8) | .82 | 15.9 (7.4) | -.5 (0) | 24.6 (1.5) | -81.4 (-2.8) | 238.9 (2.6) | 12.1 | .756 | 662.7 |

* T-tests are shown in parenthesis beneath regression coefficients.

[a/] Financial saving plus social security contribution plus increase in equity in residences (down payment plus half of amortization and interest).

[b/] Financial saving plus social security contribution plus down payments on house and car plus purchases of household durables plus major home improvements.

[c/] Value of vehicles purchased less value of vehicles sold.

[d/] Includes imputed rent on owner occupied homes, expenditures for major home improvements, interest and amortization on vehicles loans; excludes amortization and interest on mortgage loans.

[e/] Includes expenditures for major home improvements, half of amortization and interest payments on mortgage loans and net outlay on vehicles ; excludes imputed rent.

[f/] Includes half of amortization plus interest on mortgage; excludes imputed rent.