Dynamic Estimation of Portfolio Betas

by

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I. Introduction

The purpose of this study is to build and test a statistical model for the dynamic estimation of portfolio Betas. Of particular interest is the quality of Beta estimates obtainable from relatively small samples of daily return data. Also of particular interest is an assessment of the relationship between the quality of these estimates and the degree of portfolio diversification.

For obvious reasons it is desirable for a mutual fund manager to have the best possible estimate of the ongoing (and possibly changing) Betas of competitive funds. These estimates together with estimates of the degree of diversification will allow a portfolio manager to develop investment strategies relative to the expected performance of his own portfolio and his competitors in the market cycle ahead.

These estimates will also allow inferences to be made with respect to the current market outlook of each individual competitor. For example, a gradually increasing fund Beta would indicate a bullish outlook on the part of a particular competitor.

Section II will describe the methodology used in the study, section III will provide the results and section IV discussion and conclusions.

II. Methodology

The time series of daily returns for portfolios of common stocks exhibit substantial positive serial correlation due to what is often referred to as the Fisher[1] (1966) effect. Fisher pointed out that if a stock does not trade during the latter part of one day, the underlying return is attributed to the next day (assuming a trade occurs then). This can cause the stock's return on the second day to be partially related to the market's return on the first.

He also showed that this phenomenon implies that the prices of different securities do not all adjust simultaneously to common information. Cohen, Maier, Schwartz and Whitcomb (1977) support this conclusion by demonstrating that, even if capital markets are efficient and generate quotation returns that have zero autocorrelation and zero serial cross correlation, transaction returns for a market index based on transactions prices will be positively serially correlated, due to the existence of bid-ask spreads and because transactions do not occur simultaneously with all demand shifts.

The existence of this autocorrelation effect complicates the estimation problem considerably. Since Beta estimation by the traditional method of ordinary least squares will be inefficient and yield unduly large sampling variance, the methods of Box and Jenkins (1970) will be employed here to deal with this anticipated problem.

Box and Jenkins transfer function model building techniques are ideally suited for this type of estimation problem. A portfolio Beta can be thought of as a measure of the relationship between a change in the market portfolio and a (contemporaneous and/or delayed) response in the fund portfolio. In other words, Beta represents the impulse response function between the market and the fund. Ex ante, we know that this process is buried in noise and that when using daily data there is a correlation structure in both the input (market) and output (fund) series. Box and Jenkins deal with this problem by first prewhitening or, removing the structure in the input series. The estimated cross correlation function between the prewhitened input series and the prewhitened output series will then be proportional to the impulse response function.

Portfolio Betas will be estimated in this manner for 20 well known mutual funds using sample sizes of 70 daily returns.[2] The samples will be obtained for the 70 trading days immediately prior to 8 known market cycles: 4 down cycles and 4 up cycles. The known market cycle and the estimated fund Beta will then be used to predict fund return for that particular market cycle. Prediction errors will be measured and estimates of fund diversification (measured by the portfolio correlation with the market) will be made. The value of timely Beta estimates will be assessed by comparison with estimates made with the full sample of daily return observations.

Test data included daily returns for 20 mutual funds (listed in the Appendix), daily returns for the New York Stock Exchange Composite Index, and daily values of the risk free rate for the period from January 2, 1969 to March 10, 1976. Each series contains 1814 observations.

Figures 1 and 2 give the estimated autocorrelation and partial autocorrelation function[3] for daily returns of the New York Composite Index in excess of the risk free rate over the seven year period. These functions indicate the expected serial correlation. This series is reasonably well fitted by the model given in equation (1):

$$X_t = (1 + .344\ B)a_t \qquad (1)$$

$\pm .045$, 95% confidence interval

where X_t is (the return on the Composite Index on day t minus the risk free rate for day t) times 100,

a_t is white noise, and

B is the back operator, such that $BX_t = X_{t-1}$.

The adequacy of fit of this model is evidenced by reference to figure 3 which gives the estimated autocorrelation function of the generated noise series of the model. The Q statistic for this function is 54 and when compared with a X^2 distribution with 2 degrees of freedom indicates that while the model has greatly reduced the structure in the series it still has some additional structure. Inspection of the autocorrelation function, however, does not reveal an obvious factor to add to the model.[4]

The model in equation (1) is used as the prewhitening model in all subsequent estimation. A series Y_t is computed for each fund, giving the daily return in the fund in excess of the risk free rate.

The prewhitening transformation is applied to both the X_t and Y_t series for a particular sample period prior to a known market cycle. The estimated cross correlation function between the prewhitened X_t and Y_t series should then be proportional to the impulse response function between X_t and Y_t for the sample period. The constant of proportionality is the ratio of the standard deviations of the X_t and Y_t series. The estimated impulse response function (or Beta) for each fund is the sum of the first "n" significant cross correlation terms, each adjusted for sign and multiplied by the ratio of the standard errors of the two series. The criterion for inclusion of a lag term is positive correlation greater than two standard errors.[5]

For example, the estimated cross correlation function for fund #1 prior to cycle #6 is given in figure 4. It can be seen that in this case the only significant correlation is at lag 0. The transfer function can therefore be estimated directly by multiplying the cross correlation at lag zero by the ratio of the standard errors. This is done below. The estimated of Beta is .284.

$$Y_t = .815\left(\frac{.528}{1.517}\right) X_t + a_t \qquad (2)$$

$$Y_t = .284\ X_t + a_t$$

By way of comparison it is interesting to look at the estimated cross correlation function between the two unprewhitened series, shown in figure 5. It can be seen that the correlation at lag 1 also appears significant. Estimates based on this function (as would be done in the normal regression approach) would in this case be much higher. The transfer function estimate would be

$$Y_t = .835\left(\frac{.543}{1.521}\right) X_t + .333\left(\frac{.543}{1.521}\right) X_{t-1} + a_t \qquad (3)$$

$$Y_t = .298\ X_t + .119\ X_{t-1} + a_t$$

and the Beta estimate would be the sum of the X coefficients or .417.

Fund diversification is defined as the correlation between returns in the fund and returns in the market. This correlation is estimated using the equation below:

$$\rho_{x,y} = \beta_y \frac{\sigma_x}{\sigma_y} \qquad (4)$$

where β_y is the estimated Beta for the fund

σ_x is the estimated standard deviation of market returns

σ_y is the estimated standard deviation of fund returns.

The standard deviations are estimated using

$$\sigma^2 = \sigma_{11} + 2 \sum_i \sigma_{1i} \tag{5}$$

The Fisher effect will cause σ_{11} to be an underestimate of variance, and equation (5) removes this bias.

The predicted return for a fund for a given market cycle is computed as below

$$\hat{r}_y = \hat{\beta}_y X_c + i \tag{6}$$

where i is the risk free rate

X_c is the return in the market for a given cycle minus the risk rate, and

$\hat{r}_y$ is the predicted return for a particular fund for a particular cycle.

The market cycles tested are listed in Table 1. Samples are drawn to include the 70 trading days <u>prior</u> to each cycle.

III. RESULTS

Table 2 presents the estimated Betas for the 20 funds for the 8 cycles. The funds are ranked on Beta for each cycle. It can be seen that the Beta estimates vary considerably from cycle to cycle. There is some evidence of stability, however. For example fund #20 is ranked in the top three in every period; fund #1 is ranked in the bottom six every period.

Table 3 presents the estimated diversification for each fund. It is seen that there is less stability in the ranking of the correlation estimates over time. This probably reflects the difficulty in measuring standard errors because of the autocorrelation in each series. It can be seen that in five instances the estimate of correlation is obviously in error since it is greater than one.

Table 4 presents the prediction error for each fund over each cycle. The average absolute prediction error is computed for each fund over all 8 cycles and each cycle over all funds. As expected there is a strong tendency for the average absolute error to be related to the size of the market change. Further, prediction errors tend to be mostly of one sign or the other depending on the specific market cycle being predicted. Most prediction errors were negative in cycles 2 and 6 and positive in the remaining cycles. In 6 out of 8 cycles therefore most funds performed worse than predicted. In these cases Beta increased in down cycles and decreased in up cycles. This same phenomenon has been reported by Fisher Black (1976, pg. 3).

There also appears to be a strong relationship, as anticipated between prediction error and diversification. In summary then there appear to be two dominate factors effecting the absolute value of prediction error: the size of the market cycle and the degree of diversification. In addition, other unidentified cycle specific characteristics exert a strong influence on whether the prediction error is positive or negative.[6]

IV. Discussion and Conclusions

The average absolute prediction error over all funds and all cycles was 3.3 percentage points for an average market move of 17.5 percentage points. On an absolute basis these predictions appear to be reasonably accurate.

In addition these results indicate that most of the fund Betas change rather substantially over time. An important question is whether this result is due to trading activity in the fund, inherent nonstationarities of portfolio Betas, or simply estimation error, exacerbated by the use of a small sample size in these tests.

One way in which to address this issue is to recompute predicted returns using Betas estimated over the full seven year sample period. If these prediction errors are significantly greater, this would indicate that the 70 day samples give a more relevant estimate of Beta because they are more up to date with respect to portfolio changes. These results are shown in Table 5 for 5 of the 20 funds. It can be seen that the much larger sample improves results. Average prediction errors over all cycles are smaller for three out of five funds. The grand average absolute error is 2.3 percentage points for the full sample estimates versus 3.1 percentage points for the small sample estimates.

A good way of demonstrating the added power of the large sample estimate is to compare the estimated cross correlation function for a particular fund for both small and large samples. A small sample cross correlation estimate for fund #1 was given in figure 4. A full sample (1814 observations) estimate for this same fund is given in figure 6. It can be seen that the correlations in figure 6 are all very close to zero, except for lags 0 and 1. Interpreting the function in figure 6 is much easier than the one in figure 4.

The poorer performance of the small sample estimates seems to indicate that if one has to choose between a timely small sample and a less timely large one, the latter is preferred. An alternative explaination is possible however. It could be that the small timely samples are actually more accurate but that trading activity within the funds rather quickly eliminates this advantage over the ensuing market cycle.

Further insight on this question is given in Table 6. This table compares small and large sample prediction errors over the sample periods used to estimate Beta rather than over the subsequent market cycles. Therefore the possibility for error due to subsequent portfolio adjustments is eliminated. These data indicate that the quality of the estimates from the small and full samples is virtually identical. Thus, apparently the timeliness of the small sample is just about balanced by the increased information content of the full sample. As we try to predict beyond the 70 day sample in the subsequent market cycle, however, the full sample predictions are superior because (1) the benefit of currency is reduced by portfolio adjustments and of course, (2) the full sample has the "look ahead" advantage of containing the market cycle being predicted.

In conclusion, therefore it should first be noted that both small and full sample estimates of portfolio returns for a known market cycle appear reasonably good. This assessment obviously depends, however, upon the use of the results and can only be determined by the ultimate user. Secondly, although the results indicate rather substantial shifts in the fund Betas over time, this result could be spurious and due simply to the inaccuracy in estimating Beta using a 70 day sample. If the Betas are shifting over time this technique is not able to discerne these shifts.

Footnotes

[1] The effect is named after Professor Larry Fisher, of the University of Chicago.

[2] The choice of a sample size of 70 is somewhat arbitrary but represents a tradeoff between the amount of information and its currency. Box-Jenkins techniques require at least 50 observations. As the sample size increases, the chance that the composition of the portfolio has changed increases also. Some preliminary testing was done with 140 observations but these samples offered only slight improvement over 70 observations.

[3] The partial autocorrelation function can be thought of as the inverse of the autocorrelation function and is very helpful in the identification stage of the analysis. For example, an autoregressive process will have an autocorrelation function which will look very much like the partial autocorrelation function of a comparable moving average process, and vice versa. Furthermore, the order of the autoregressive process will be indicated by the number of significant correlations in the partial autocorrelation function, and vice versa.

[4] The Q Statistic computed here is the sum of the first 30 correlations in the estimated autocorrelation function of the residuals times the number of residuals. Box and Pierce (1970) have shown that if the fitted model is appropriate this statistic will be approximately distributed as a X^2 with 28 degrees of freedom. Values of Q in the upper 10% of the X^2 distribution are an indication that there is still structure present in the series, while this is the case here there is no obvious factor to add to the model which would remove this structure We are comforted, however, by the fact that the Q statistic has been reduced from 224 with 30 degrees of freedom for the differenced series without the moving average factor.

[5] This procedure may appear "ad hoc" and to some extent it is. Time series analysis in general appears "ad hoc" to many. The two standard error criterion is fairly standard, however.

[6] This effect could well be explained with the addition of a second input variable (analagous to the two factor model). There is no theoretical reason why this cannot be done with the Box-Jenkins methodology. The programming for this, however, has just become available.

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Figure 1

Estimated Autocorrelation Function - NYSE Composite Index
Daily Returns minus Risk Free Rate

1814 Observations
Standard Error ≈ 0.03

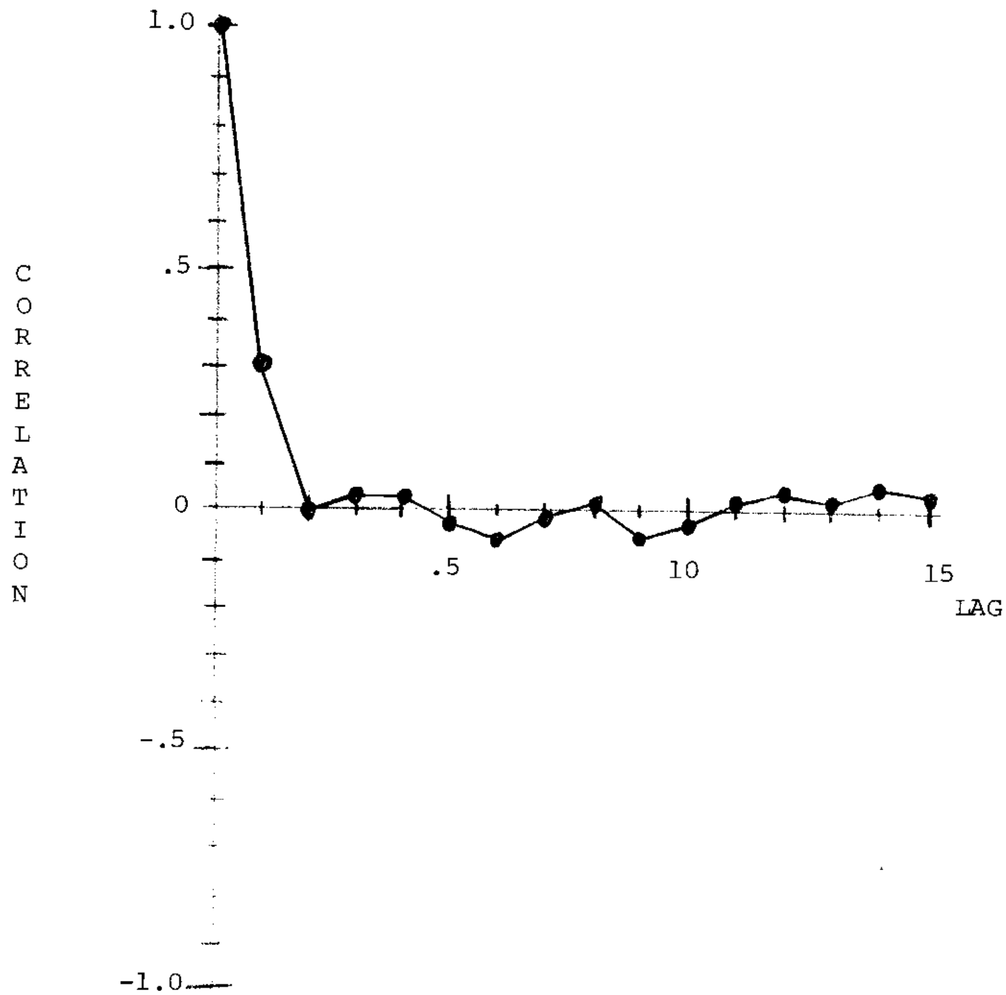

Figure 2

Estimated Partial Autocorrelation Function - NYSE Composite Index
Daily Returns minus Risk Free Rate

1814 Observations
Standard Error ≈ .03

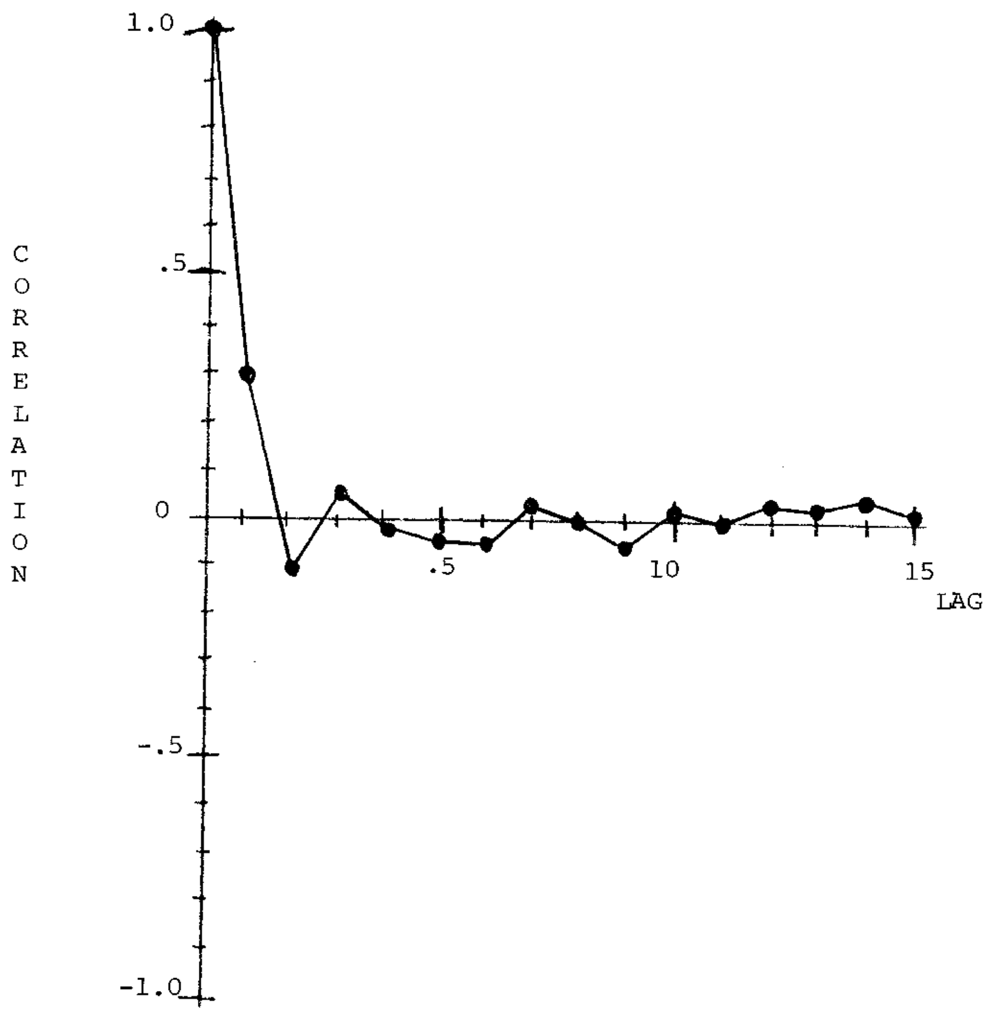

Figure 3

Estimated Autocorrelation Function - Residuals to Equation (1)
1814 Observations
Standard error $\approx$ .03

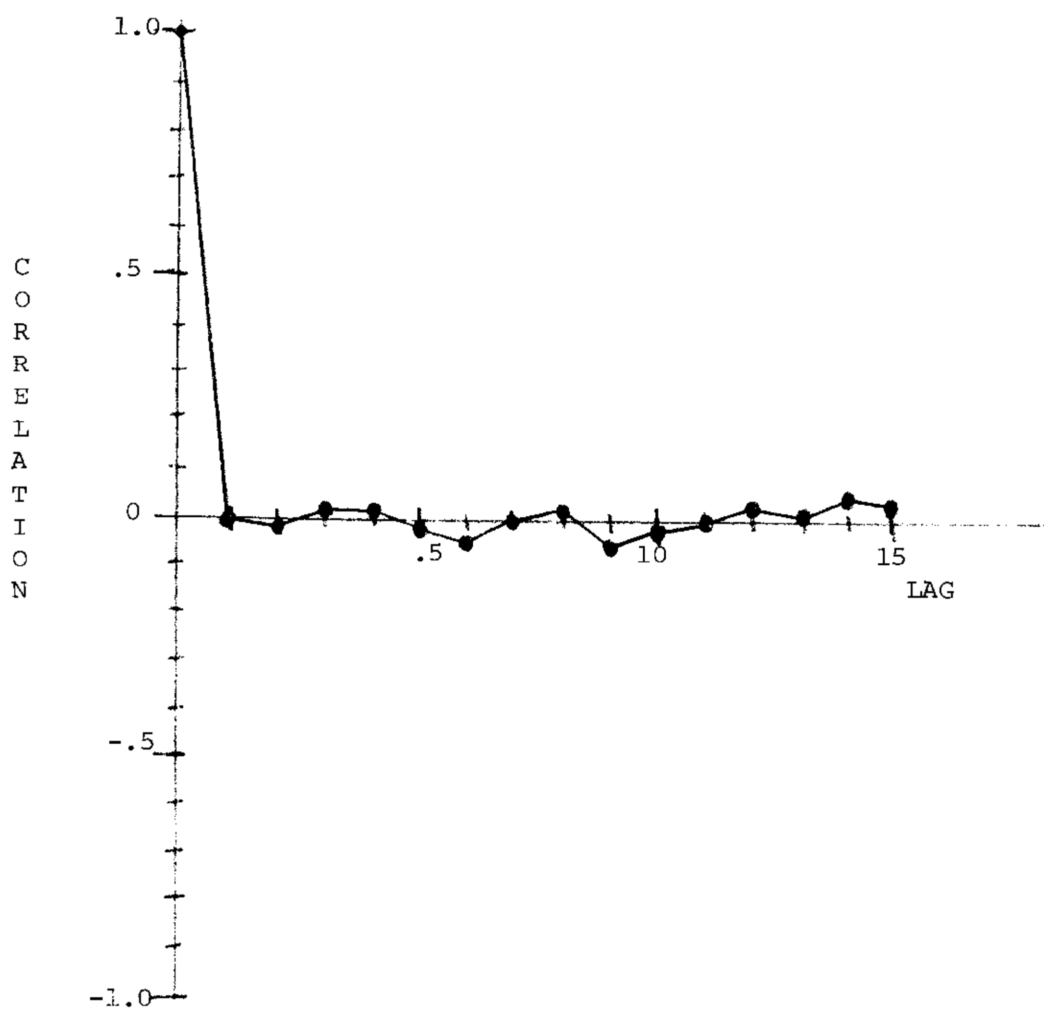

Figure 4

Estimated Cross Correlation Function
Prewhitened NYSE Composite-Daily Returns minus Risk Free Rate
Prewhitened Fund #1 - Daily Returns minus Risk Free Rate
Cycle #6
70 Observations
Standard Error ≈ .12

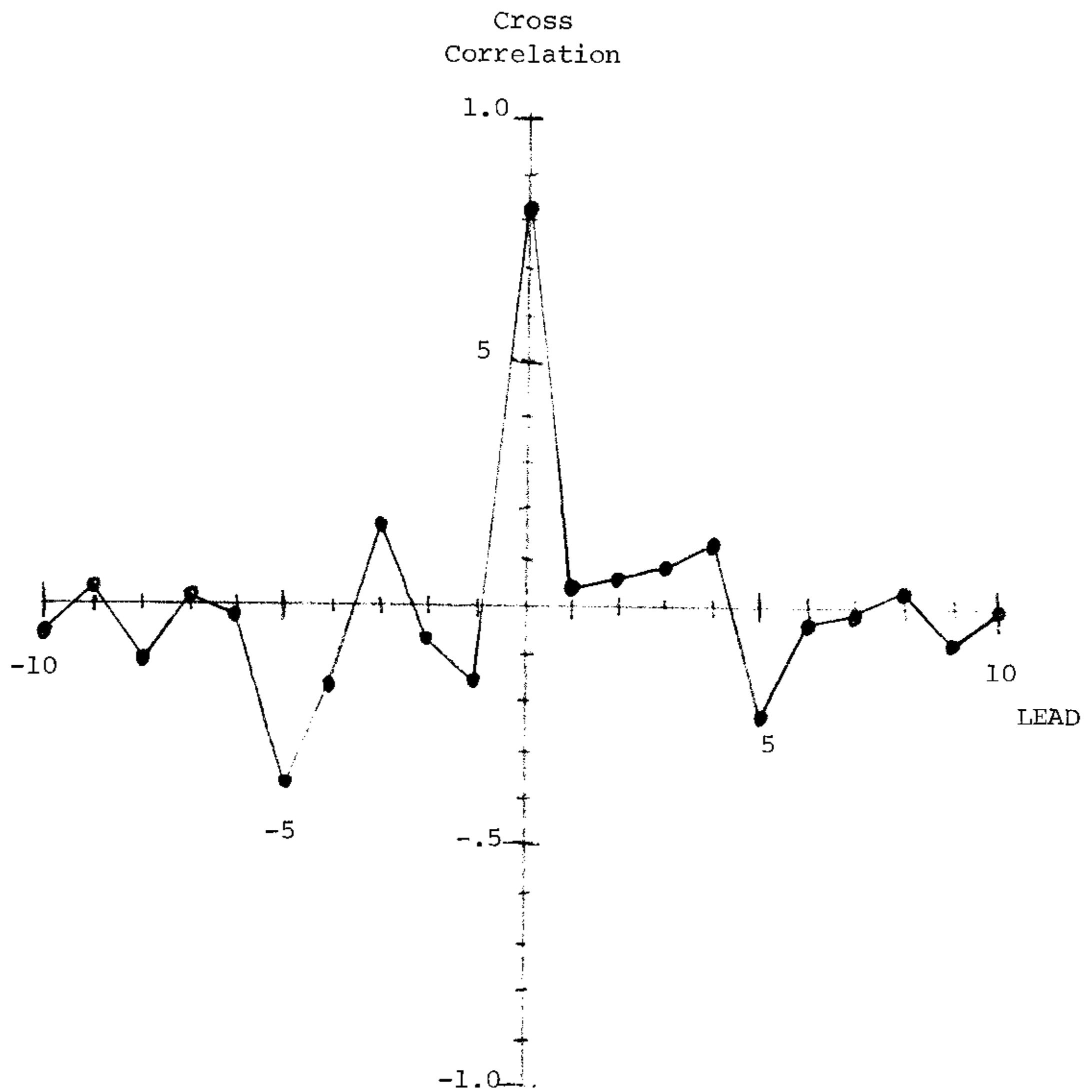

Figure 5

Estimate Cross Correlation Function
NYSE Composite Index-Daily Returns minus Risk Free Rate
Fund #1 - Daily Returns minus Risk Free Rate
Cycle #6
70 Observations
Standard Error ≈ .12

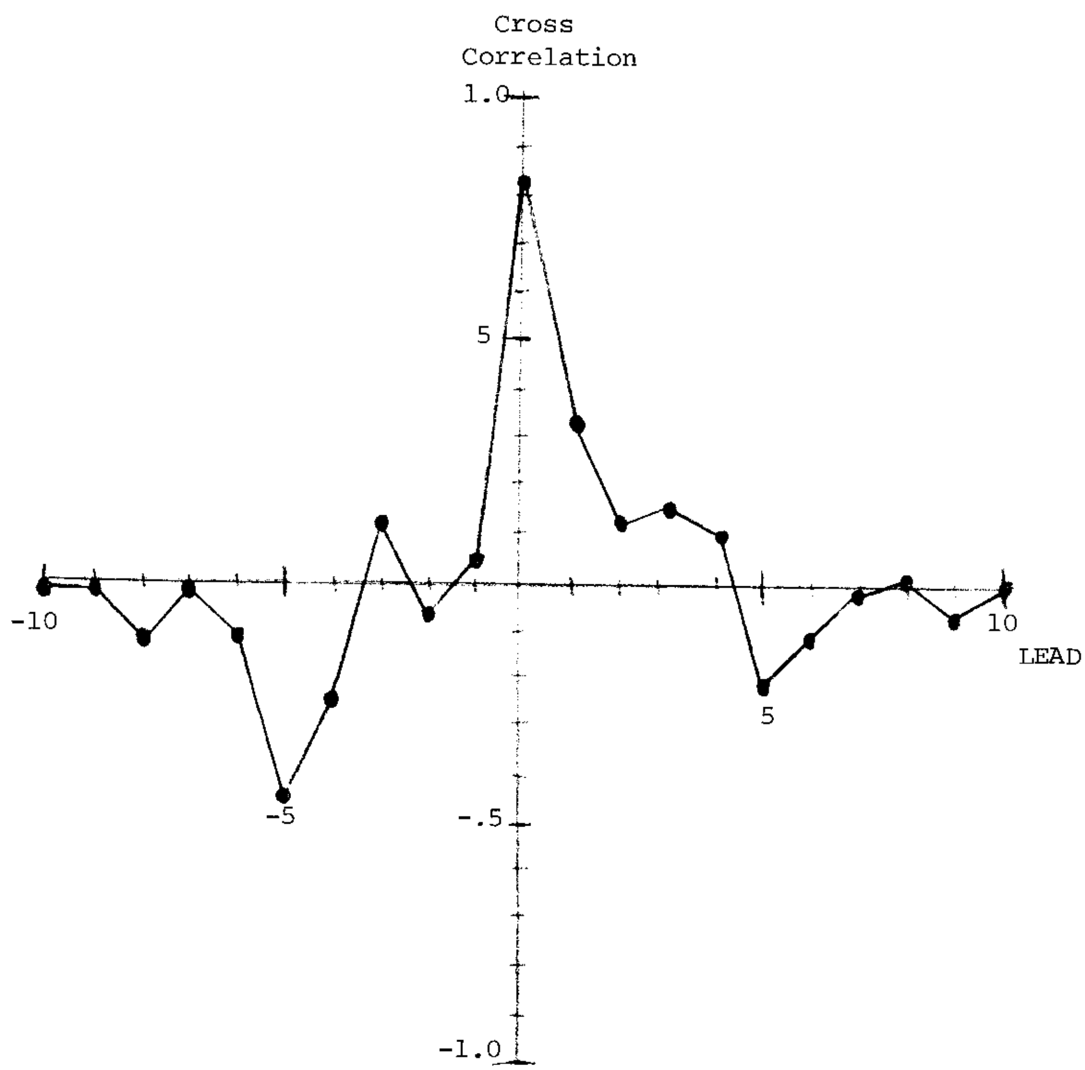

Figure 6

Estimated Cross Correlation Function
Prewhitened NYSE Composite Index-Daily Returns minus Risk Free Rate
Prewhitened Fund #1 - Daily Returns minus Risk Free Rate
Cycle #6
1814 Observations
Standard Error ≈ .03

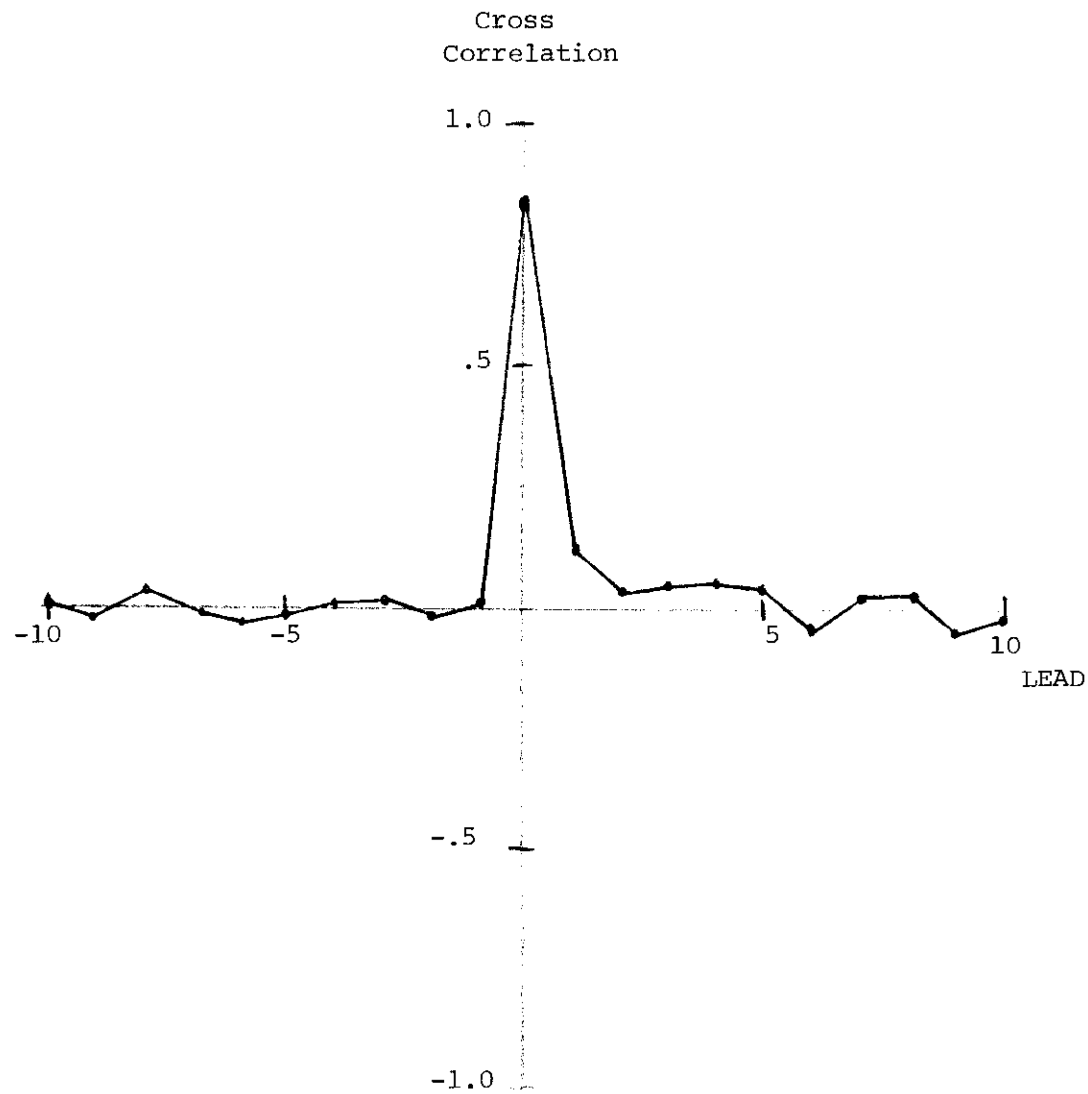

Table 1

Market Cycles Tested

Cycle	Market Change (%)	Date	Duration (Trading Days)
1	-24.6	3/3/70 - 5/26/70	59
2	+30.7	11/18/70 - 4/28/71	110
3	-10.7	9/8/71 - 11/23/71	54
4	+12.7	10/16/72 - 1/11/73	58
5	-13.7	3/13/74 - 5/29/74	53
6	+21.4	10/3/74 - 11/11/74	27
7	-14.3	7/15/75 - 9/16/75	44
8	+11.8	9/16/75 - 11/17/75	44

TABLE 2

Estimated Beta and Ranking on Beta for twenty mutual funds for eight, seventy day sample periods.

FUND	MARKET CYCLE 1		2		3		4		5		6		7		8		Averag	
	$\hat{\beta}$	Rank	$\hat{\beta}$	Rank	$\hat{\beta}$	Rank	$\hat{\beta}$	Rank	$\hat{\beta}$	Rank	$\hat{\beta}$	Rank	$\hat{\beta}$	Rank	$\hat{\beta}$	Rank	$\bar{\beta}$	Ra
20	1.06	3	1.30	2	1.48	1	1.36	2	1.23	2	1.07	2	1.29	1	1.59	1	1.30	[illegible]
8	.84	11	.84	10	1.11	3	1.65	1	1.27	1	1.02	6	1.02	7	1.11	6	1.11	[illegible]
10	.85	9	1.15	4	1.05	5	1.22	4	1.18	3	1.15	1	1.09	5	1.16	5	1.11	[illegible]
11	.89	8	.97	5	1.04	6	.96	12	1.03	8	1.00	7	1.13	2	1.04	9	1.09	[illegible]
6	1.10	2	1.22	3	1.41	2	1.13	9	1.02	9	.71	15	.72	15	1.01	10	1.04	[illegible]
13	.93	6	.88	7	1.02	7	1.27	3	1.14	4	.83	12	1.05	6	1.09	8	1.03	[illegible]
18	.93	6	.83	12	1.11	3	1.18	6	1.05	7	1.05	3	.95	9	1.10	7	1.03	[illegible]
19	.85	9	.92	6	.99	9	1.00	10	1.13	5	1.04	5	1.11	3	1.20	3	1.03	[illegible]
12	1.12	1	.59	17	.82	14	1.14	8	.89	12	.91	9	1.10	4	1.41	2	1.00	[illegible]
4	.81	13	.75	14	1.02	7	1.19	5	1.08	6	.94	8	.96	8	1.01	10	.97	[illegible]
16	.97	4	1.45	1	.96	10	.92	13	.89	12	.70	16	.62	17	1.17	4	.96	[illegible]
3	.82	12	.85	9	.84	13	.98	11	1.00	10	1.05	3	.91	10	.97	12	.93	[illegible]
7	.97	4	.74	15	.92	12	1.18	6	.94	11	.89	10	.88	11	.87	13	.92	[illegible]
15	.71	15	.88	7	.95	11	.88	14	.80	15	.87	11	.84	12	.84	15	.85	[illegible]
9	.73	14	.76	13	.80	15	.80	16	.72	16	.77	14	.78	13	.75	17	.76	[illegible]
17	.41	20	.66	16	.76	16	.86	15	.82	14	.79	13	.72	15	.80	16	.73	[illegible]
14	.66	18	.84	10	.63	20	.69	17	.64	18	.66	17	.75	14	.85	14	.72	[illegible]
2	.71	15	.48	20	.70	18	.66	18	.65	19	.34	19	.48	19	.64	18	.67	[illegible]
5	.53	19	.57	18	.65	19	.62	19	.68	17	.65	18	.65	18	.52	19	.61	[illegible]
1	.71	15	.58	19	.74	17	.51	20	.42	20	.28	20	.27	20	.37	20	.49	[illegible]

TABLE 3

Estimated correlation with the market and ranking on correlation for twenty mutual funds for eight, seventy day sample periods

FUND	MARKET CYCLE 1 ρ	1 Rank	2 ρ	2 Rank	3 ρ	3 Rank	4 ρ	4 Rank	5 ρ	5 Rank	6 ρ	6 Rank	7 ρ	7 Rank	8 ρ	8 Rank	Averag $\bar{\rho}$	Ra
11	.79	9	.96	4	.96	3	.96	1	.97	1	.78	2	.98	1	.96	4	.95	[illegible]
10	.83	4	.93	8	.96	3	.95	3	.95	4	.96	10	.97	4	.96	4	.94	[illegible]
13	.89	2	.94	6	.94	6	.92	8	.94	5	.96	10	.96	5	.96	4	.94	[illegible]
4	.70	11	.95	5	.98	1	.84	16	.93	8	.97	6	.96	5	.96	4	.94	[illegible]
3	.64	13	.93	8	.94	6	.94	5	.85	12	.97	6	.96	5	.97	3	.91	[illegible]
6	.83	4	1.10	1	.92	12	.88	12	.90	10	.90	17	.72	19	.96	4	.90	[illegible]
8	.78	10	.88	14	.92	12	.83	17	.94	5	.98	2	.92	10	.84	18	.90	[illegible]
18	.45	18	.93	8	.98	1	.95	3	.97	1	.98	2	.95	8	.95	9	.90	[illegible]
7	.50	17	.94	6	.96	3	.86	15	.96	3	.99	1	.98	1	.95	9	.90	[illegible]
9	.61	15	.91	12	.93	10	.91	9	.94	5	.98	2	.92	10	.95	9	.89	[illegible]
20	.85	3	.85	17	.91	15	.83	17	.92	9	.96	10	.86	14	.95	9	.89	[illegible]
15	.69	12	.87	15	.93	10	.94	5	.76	16	.97	6	.92	10	.91	15	.89	[illegible]
16	.81	6	1.09	2	.92	12	.82	19	.86	11	.86	18	.68	20	.92	14	.88	12
19	.62	14	.93	8	.94	6	.88	12	.76	16	.96	10	.98	1	1.02	2	.88	12
14	.45	18	1.08	2	.88	17	.91	9	.68	19	.91	16	.92	10	.96	4	.88	12
17	.52	16	.90	13	.94	6	.93	7	.81	15	.97	6	.93	9	1.15	1	.87	15
12	.81	6	.66	19	.81	19	.96	1	.83	13	.92	15	.78	18	.93	13	.87	15
1	.80	8	.82	18	.90	16	.47	20	.83	13	.80	19	.80	17	.87	17	.83	17
5	.43	20	.87	15	.85	18	.89	11	.76	16	.95	4	.86	14	.37	20	.81	18
2	.91	1	.30	20	.80	20	.87	14	.62	20	.73	20	.84	16	.52	19	.75	20

TABLE 4

Prediction error and ranking on prediction error for twenty mutual funds over eight known market cycles.

FUND	MARKET CYCLE 1		2		3		4		5		6		7		8		Average
	Error	Rank	Error	Rank	Error	Rank	Error	Rank	Error	Rank	Error	Rank	Error	Rank	Error	Rank	AAE
9	3.4	7	-3.3	7	.3	3	.2	3	.2	1	-4.2	16	0	1	- .8	5	1.6
7	-1.5	3	-4.4	9	.3	3	3.0	12	.2	1	-3.0	14	- .4	2	-1.4	8	1.8
10	8.2	15	0	1	.4	5	4.6	15	-3.3	13	-2.5	9	2.9	16	1.4	8	2.9
3	5.2	12	- .3	2	.1	1	- .5	5	-4.8	16	-2.7	10	2.3	13	.1	1	2.0
5	3.9	8	-1.5	3	.7	8	-1.7	8	2.1	9	-4.3	17	-1.8	10	- .7	4	2.1
11	1.5	3	4.8	10	1.2	13	.1	1	- .6	3	2.8	12	-4.1	19	4.0	15	2.4
15	4.9	11	-2.9	6	- .9	9	1.8	9	2.5	11	-3.5	15	2.5	14	- .4	3	2.4
17	10.0	19	-5.4	11	.2	2	.6	6	-1.2	7	-1.5	4	.5	3	1.2	6	2.6
1	1.7	5	-9.3	17	.6	6	- .3	4	5.1	18	-2.7	10	1.6	8	-1.3	7	2.8
18	-1.2	2	-7.1	12	1.5	14	3.2	13	-2.4	10	-1.6	5	1.7	9	3.8	14	2.8
13	- .1	1	-9.7	18	1.1	12	4.6	15	-1.6	8	-2.1	7	- .7	5	4.1	16	3.0
4	4.1	9	-7.5	13	3.1	17	5.5	17	1.0	6	-1.6	5	1.0	7	2.8	11	3.3
14	6.1	13	-1.9	4	4.6	19	-2.9	11	5.9	12	-2.8	12	- .9	6	3.4	13	3.6
8	8.5	16	-8.5	14	- .9	9	5.5	17	- .9	5	- .9	2	1.8	10	3.1	12	3.8
19	7.0	14	-10.2	19	-2.0	15	.1	1	-6.8	20	.2	1	- .5	13	5.5	17	4.0
2	4.3	10	-12.6	20	3.5	18	- .7	7	4.3	14	- 5.4	18	3.4	18	2.5	10	4.6
20	8.5	16	9.1	16	.6	6	5.6	19	-2.9	12	- .9	2	3.3	17	6.7	18	4.7
6	8.7	18	-3.8	8	3.0	16	8.4	20	4.9	17	-10.7	20	2.0	12	- .1	2	5.2
12	2.7	6	- 8.6	15	7.1	20	1.8	9	.8	4	2.3	8	2.7	15	16.3	20	5.3
16	14.9	20	-2.8	5	.9	9	3.3	14	4.7	15	-7.4	19	7.5	20	6.8	19	6.0
AAE	5.3		5.7		1.7		2.7		2.8		3.2		2.1		3.3		3.3

Table 5

Comparison of Prediction errors, for five funds, over eight known market cycles using small and full sample Beta estimates

Fund	Sample	Market Cycles 1	2	3	4	5	6	7	8	AAE
1	Small($\hat{\beta}$ varies)	1.7	-9.3	.6	- .3	5.1	-2.7	1.6	-1.3	2.8
	Full($\hat{\beta}$= 0.51)	6.7	-11.4	3.0	- .3	3.9	2.2	-1.8	.3	3.7
8	Small($\hat{\beta}$ varies)	8.5	-8.5	-.9	5.5	-.9	-.9	1.8	3.1	3.8
	Full($\hat{\beta}$ = 1.06)	3.1	-1.3	-.3	-2.0	2.0	0	1.2	2.5	1.6
10	Small($\hat{\beta}$ varies)	8.2	0	.4	4.6	-3.3	-2.5	2.9	1.4	2.9
	Full($\hat{\beta}$ = 1.10)	2.0	-1.5	-.2	3.1	-2.2	-3.6	2.8	.8	2.0
13	Small($\hat{\beta}$ varies)	-.1	-9.7	1.1	4.6	-1.6	-2.1	-.7	4.1	3.0
	Full($\hat{\beta}$ = .98)	-1.3	-6.6	1.5	.9	.6	1.1	.3	2.8	1.9
18	Small($\hat{\beta}$ varies)	-1.2	-7.1	1.5	3.2	-2.4	-1.6	1.7	3.8	2.8
	Full($\hat{\beta}$ = .97)	-2.2	-2.8	3.0	.5	-1.3	-3.3	1.3	2.2	2.1
AAE	Small	3.9	6.9	.9	3.6	2.7	2.0	1.7	2.7	3.1
	Full	3.1	4.8	1.6	1.4	2.0	2.0	1.5	1.7	2.3

Table 6

Comparison of Prediction errors, for five funds over the sample periods prior to each market cycle, using small and full sample Beta estimates

Fund	Sample	Market Cycle 1	2	3	4	5	6	7	8	AAE
1	Small($\hat{\beta}$ varies)	1.1	-6.5	-2.2	-.3	-5.1	1.2	-4.5	-.9	2.7
	Full($\hat{\beta}$ = .51)	1.9	-6.8	-2.7	-.3	-4.5	-5.0	.6	-2.2	3.0
8	Small($\hat{\beta}$ varies)	4.4	-6.9	-1.5	10.8	7.8	-.6	-.1	0	4.0
	Full($\hat{\beta}$ = 1.06)	3.6	-6.1	-1.7	11.3	6.3	-1.7	.8	.4	4.0
10	Small($\hat{\beta}$ varies)	1.4	-11.0	1.5	3.2	10.0	2.1	5.7	5.6	5.1
	Full($\hat{\beta}$ = 1.10)	.4	-11.1	1.6	3.3	9.4	3.4	5.9	6.2	5.2
13	Small($\hat{\beta}$ varies)	4.9	-5.3	-1.3	6.7	.3	2.7	1.3	-1.1	3.0
	Full($\hat{\beta}$ = .98)	4.7	-5.2	-1.3	6.9	-.9	-1.4	-.2	-.1	2.6
18	Small($\hat{\beta}$ varies)	2.0	-3.3	-1.1	4.6	2.0	-2.1	.2	-.5	2.0
	Full($\hat{\beta}$ = .97)	1.8	-2.8	-1.5	4.7	1.4	.1	.7	.7	1.7
AAE	Small	2.8	6.6	1.5	5.1	5.0	1.7	2.4	1.6	3.3
	Full	2.5	6.4	1.8	5.3	4.5	2.3	1.6	1.9	3.3

APPENDIX

List of Mutual Funds in the Sample

1. Axe-Houghton B
2. Axe-Houghton Stock
3. Chemcial Fund
4. Dreyfus Fund
5. Eaton & Howard Balanced
6. Fairfield Fund
7. Fidelity
8. Keystone (K-2)
9. Zoomis-Sayles Mutual
10. Mass Investors Growth
11. Mass Investors Trust
12. Omega Fund
13. Oppenheimer Fund
14. Pioneer Fund
15. Stein Roe & Farnham Balanced
16. Vance Sanders Special
17. George Putnam Fund
18. Putnam Growth Fund
19. Putnam Investors Fund
20. Putnam Vista Fund