The Cost of Equity Captial: A Reconsideration

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In the early sixties a number of papers developed stock value models in which the cost of equity capital is a function of the firm's equity investment rate. The conclusions reached, however, have for the most part been rejected in the subsequent literature. Instead, it has become widely accepted both in the theoretical work and in the textbooks on finance that the cost of equity capital is equal to the yield at which a firm's stock is selling, and the latter is independent of the firm's investment rate. In fact, one frequently finds the two terms, share yield and cost of equity capital used interchangeably in the literature. However, contributions to the literature during the last few years provide additional support for and clarification of the alternative theory, and a broad reconsideration of the subject is in order.

Part I below critically reviews the literature of the early sixties on both sides of the subject and certain subsequent papers which contributed to the conclusion that share yield and cost of equity capital are equal and independent of investment policy. Parts II and III examine the recent contributions which have materially strengthened the theoretical basis for the contrary conclusion, that the cost of equity for a firm is an increasing function of its investment rate. Part IV summarizes the conclusions reached, reviews the empirical evidence on the subject, and discusses the areas in which the model requires further development.

Before proceeding, it should be noted that there is no question with regard to the relation between the cost of capital and the investment rate on the macro level. The return investors require as a condition for doing the saving required to finance the aggregate level of investment increases with the aggregate level of investment expressed as a fraction of aggregate earnings.

The issue is whether or not this macro relation also holds on the level of the firm. Having this relation also hold on the level of the firm may have important consequences for national income determination, the efficiency of investment, and other such questions. However, these questions will not be considered here.

I

In the early sixties Gordon [11] [12] and Lintner [16] [17] examined the implications for the cost of capital of the constant expected growth rate stock value model.

$$P = (1-b)Y/(k-br) \tag{1}$$

In this equation

P = Present value of the firm's stock.

Y = Expected value of the firm's earnings in the coming year.

b = Expected value of the firm's investment and retention rate for the indefinite future.

r = Expected value of the return on investment with investment the fraction b of earnings.

k = Required return or yield at which the stock is selling.

The derivation of Eq. (1) is explained in the above references.

In the development of this model the authors made the assumptions necessary to equate investment policy with dividend policy - no stock financing and a constant debt equity ratio.[1] The former assumption will be withdrawn shortly, and the latter certainly does not limit the usefulness of the model.

If r and k are both functions of b, the partial derivative with respect to b is

$$\frac{\partial P}{\partial b} = \frac{Y}{(k-br)^2} \left[-k+br+(1-b)\left(r+b\,\frac{\partial r}{\partial b} - \frac{\partial k}{\partial b}\right)\right]. \tag{2}$$

/1 Gordon [12, p. 39] stated explicitly that "The consequence of these three assumptions is that an investor's estimate of b, a corporation's retention rate, implies an estimate of its investment rate." The third assumption was that in arriving at P investors assume that a corporation will earn r and retain b in every future period.

In the above $r'=r+b\ \partial r/\partial b$ is the marginal rate of return on investment when investment is at the rate b. Setting Eq. (2) equal to zero and solving for r' we find that the value of P is maximized when b is set so that

$$r' = (k-br)/(1-b) + \partial k/\partial b \qquad (3)$$

is satisfied.

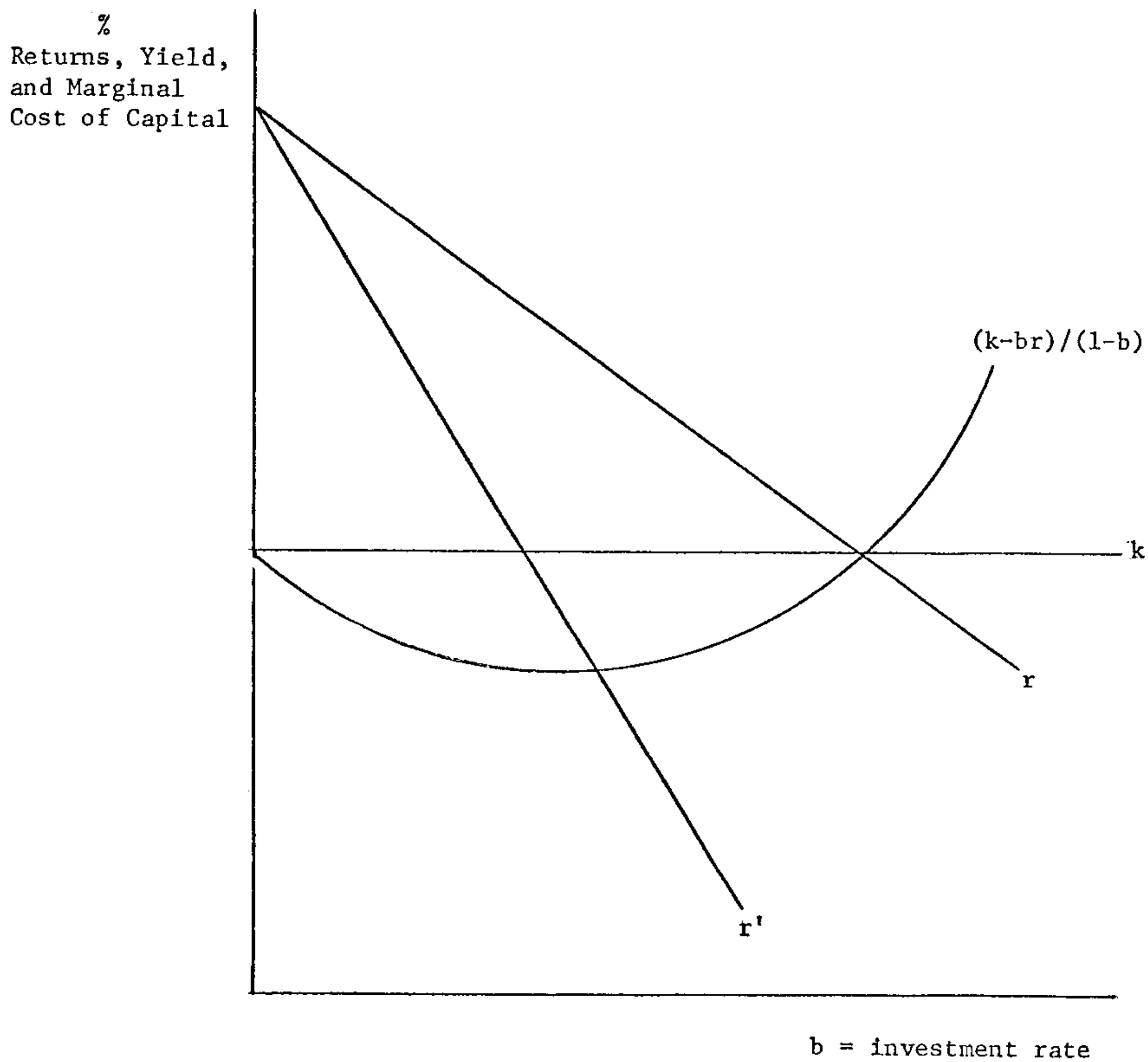
The left hand side of Eq. (3), the marginal rate of return on investment, is a decreasing function of b, the firm's investment rate. The right hand side of Eq. (3) may properly be called the firm's cost of capital, since the term cost of capital is a short-hand expression for the discount rate a firm should use in deciding whether or not undertaking an investment will raise the value of a firm's stock. If the value of r' at some investment rate is above the right hand side of Eq. (3), undertaking the next most profitable investment will raise the value of the firm's stock and vice versa. P is maximized at the value of b which satisfies Eq. (3).

It is clear that the cost of capital given by Eq. (3) is not independent of b. If k is independent of b, the cost of capital is $(k-br)/(1-b)$, and it first falls and then rises as b increases. Also, P is maximized when $(k-br)/(1-b)$, the earnings yield, is minimized. The relations among the variables r, r', k, $(k-br)/(1-b)$ and b are illustrated in Figure I. Notice that at the optimal investment rate $r'<k$ and $r>k$.

The values of k and $(k-br)/(1-b)$ in Figure I assume that $\partial k/\partial b=o$. With $\partial k/\partial b>o$, the cost of capital may well rise continuously with b because $\partial k/\partial b>o$ and because k in $(k-br)/(1-b)$ is rising with b. This subject will be considered further in Part II.

Figure I

Returns, Share Yield, and the Marginal Cost of Capital



Before proceeding, it should be noted that the model is easily generalized to allowing stock as well as retention financing of investment. Let g = the expected rate of growth in the dividend to a current share, in which case Eq. (1) becomes

$$P = (1-b)Y/(k-g) \tag{4}$$

Miller and Modigliani [19, p. 423] have shown that with s the stock financed investment expressed as a fraction of earnings,

$$g = [(b+s)r(1-b)-sk]/(1-b-s) \tag{5}$$

Substituting this expression for g in Eq. (4) and simplifying results in

$$P = (1-q)Y/(k-qr) \tag{6}$$

where $q = b+s$.

Taking the derivative $\partial P/\partial b$ or $\partial P/\partial s$ holding q constant reveals that they are each equal to zero. Dividend policy per se has no influence on share value and the cost of capital.[1] However, taking the derivative $\partial P/\partial q$ produces Eq. (3) with the investment decision replacing the retention decision.

The Miller and Modigliani [19] paper on dividend policy is the primary theoretical basis for the widespread acceptance of the proposition that the cost of equity capital is equal to the yield at which the company's stock is selling. What this paper established is that with

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[1] This is clearly demonstrated in Fewings [9][10], but it is implicit in the analysis of stock financing in Gordon [11, Ch. 9].

no transaction costs, no taxes and complete certainty the value of a firm is independent of the allocation of its financing between retention and the sale of stock.[1] We agree that with no taxes, no transaction costs and no informational content to the dividend, the choice between retention and the sale of stock to finance a given investment plan is of no consequence to a firm.

However, a firm still has the problem of arriving at its investment plan, and the Miller-Modigliani paper had absolutely nothing to say on this subject. The analysis of that problem by other writers was disposed of with the statement, "...the argument clearly suffers fundamentally from the typical confounding of dividend policy with investment policy that so frequently accompanies use of the internal financing model." [19, p. 425]. What should have been clear is that the Miller-Modigliani paper and the previously cited papers by Gordon and Lintner were concerned with two different problems. Miller and Modigliani were concerned with choice between the two sources of equity funds under assumptions which clearly made the choice of no consequence.[2] Gordon and Lintner were concerned with what the investment plan should be under the same assumptions - no transaction costs, no personal income tax and no informational content to the dividend. In their papers these assumptions were made explicitly or implicitly to abstract from issues that were not

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[1] Complete certainty eliminates the problem. It would make more sense to only assume that there is no informational content to the dividend decision.

[2] The rigorous proof in their paper that the choice is of no consequence is nonetheless of some value insofar as it provides a reference base for going forward to investigate the real problems - transaction costs, differential taxation, information content of the dividend and most of all investment policy.

relevant to the non-trivial problem under consideration, the optimal investment decision.

Let us turn now to the more substantial criticisms of the Gordon-Lintner theory of the cost of equity capital. Looking back at Eq. (3) we see that with k independent of b but r the function of b that gives rise to Eq. (3), the cost of retention capital is (k-br)/(1-b), a function of b. For (k-br)/(1-b) to be the cost of equity capital, the return on investment expressed as a function of b in every future period must be the same and independent of b, which means that the return as a function of the level of investment shifts upward at a constant rate that increases with the value of br.[1] In a comment on one of Lintner's papers Miller [18] argued that this return on investment function has little if any empirical relevance. We will examine Miller's objections to this property of the return on investment function in the next section.

The work of Gordon and Lintner was integrated, and in some respects carried forward by Lerner and Carlton [14]. They developed an investment decision model in which the return on investment function has the properties described above and the required return is an increasing function of the growth rate. However, they provided no new empirical evidence and their theoretical argument seemed to have been no more convincing that the previous efforts. In addition their argument was marred by an error that was commented on by Ben-Shahar and Ascher [2] and Crockett and Friend [6].[2]

[1]This property of the return on investment function was first noted by Bodenhorn [3].

[2]Lerner and Carlton confused the return on the existing assets with the firm's return on investment. A stockholder may use one to estimate the other, but the firm can't make its investment decision on the assumption that they are equal.

The latter comment was the clearest statement of the problem and the issues to date, but the authors were not persuaded there was any merit in the critical assumptions of the theory--the return on investment function and the relation between k and growth.

With regard to the proposition that a share's yield is an increasing function of the firm's investment rate, Gordon [11] argued that a share's yield is a weighted average of the sequence of discount rates, k_t, $t=1\to\infty$ used to discount each future dividend, the risk of a dividend increases with its date in the future, and therefore the k_t might well increase with t. In that event their average, which is k, would increase with the rate of growth in the dividend. Higgins [13] showed that the independence of the value of the firm of its relative use of retention and stock financing under the Miller-Modigliani assumptions holds when the future dividends are discounted at different rates. However, Higgins was concerned with the exact same problem as Miller and Modigliani, the value of a firm when the financing of a given investment plan between retention and the sale of stock is changed.

Higgins' analysis is irrelevant when investment is the decision variable. If the k_t at which the dividend in t is discounted is an increasing function of t, their average is an increasing function of the rate of growth in investment, and the cost of capital is not equal to k. However, a paper by Chen [5] cast serious doubt on the likelihood that the k_t increase with t. He showed that the risk of future dividends must increase at an increasing rate with time for k_t to increase with t, and there seemed to be no a priori basis at the time for believing that dividend risk increased in this way with time.[1]

Lintner [17] also presented a theoretical argument in support of the proposition that risk and k increase with growth. We know of no efforts to disprove the reasoning under which he reached his conclusion. Perhaps the complexity of the argument is why it had so little impact on the subsequent literature.

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[1]We will see shortly that a major limitation of the Chen analysis was its failure to recognize that the risk free interest rate increases with the time in the future of a certain payment.

II

We have seen that either or both of two conditions make the cost of equity capital a function of the firm's investment rate. One condition is dependence of a share's yield on the investment rate. The other condition is dependence of the return on investment function in future periods on the firm's current investment decision.

An ingenious paper by Elton and Gruber [7] models an exceptionally wide range of assumptions with regard to a firm's future rate of return on investment functions, and derives the value of the firm and its cost of capital for each of these functions under the assumption that k is independent of the investment decision. For our purposes the results of this paper may be summarized as follows. First, if the return on investment function in each future period is independent of the firm's current investment decision, regardless of how these return on investment functions shift over time, a firm's cost of capital is k. That is, the firm's investment decision should equate the marginal rate of return on investment with the firm's share yield.

On the other hand, if the return on investment function in period j depends in some way on the investment decisions in period $t<j$, then regardless of what this relation is, a firm's cost of capital depends on its investment decision and is not equal to k, even if k is independent of that investment decision. Let us refer to a firm's return on investment function as 1) the absolute return on investment function when r is expressed as a function of the dollar investment in a period and 2) as the relative return on investment function when r is expressed as a function of q, the investment divided by the earnings in the period.

Eq. (1) is based on the assumption that the relative return on investment function in period j is independent of the investment decision in prior periods. This means that the absolute return on investment function shifts outward at a constant rate over time with the shift rate equal to the product qr. Hence, as has been noted in earlier papers, raising q raises the investment opportunities available in future periods as long as qr rises with q. It therefore pays a firm to set the investment at a level at which $r'<k$.

In the other models explored by Elton and Gruber in which the future return on investment functions depend on current investment, the cost of capital is also not equal to k. It varies with the investment rate but not with the product qr, and whether or not the cost of capital at the optimal q is above or below k depends on the model chosen. Therefore, a necessary condition for a firm's cost of capital to be equal to k is that a firm's absolute return on investment function in each future period is independent of its prior investment decisions.

Let us now take up the criticism by Miller of the plausibility of the independence of the relative return on investment function assumed in Eq. (1). Miller's criticism of the model began with a reference to the "IBM paradox." "That is, if the current earnings yield on IBM is, say 2 per cent, then some student can invariably be found who will seriously suggest that the company ought to undertake any investment which promised to yield at least 2 per cent plus epsilon." [18, p. 314].

This objection to the model is probably the major reason for the decline of interest in it. After all, in the early sixties we had just established that the yield at which a share sells is $k=(D/P)+g$ and not the earnings yield. To immediately turn around and argue that nonetheless the earnings yield should be used as the cut-off rate in capital budgeting undoubtedly seemed a little embarrassing particularly when it produced such bizarre results. We will see shortly that a complete model of share valuation does not produce such bizarre results.

However, Miller did not use the "IBM paradox" to reject the model. He acknowledged that undertaking investments with a 2 per cent return is justified when "...under Lintner's dependence assumption, they also shift the investment-productivity schedule in every future period upward and to the right..." [18, p. 315].

Miller went on to refer to this capital budgeting criterion as "a verbal trick resulting from his use of the term 'marginal rate of return on investment' to refer only to the direct return of the low-yielding, 'loss-leader' projects." [18, p. 315]. It is true that if we include the excess returns on future investment projects made possible by the current investment project in the latter's cash flow, the cut-off rate should be k and not the earnings yield on the share. It is also true as Miller implicitly concedes that if the excess returns on future projects generated by the current investment are not included, the cut-off rate is the earnings yield, continuing to assume that k is independent of the investment decision. What would be incorrect is to use k as the cut-off rate and only include the cash flows directly attributable to the current project in arriving at its internal rate of return.

The choice between the two correct approaches, therefore is a matter of convenience and not semantics or trickery. When confronted with two projects I and II with the return on I low, the return on II high and realizing the high return on II dependent on undertaking I, it is convenient to arrive at the correct decision by evaluating both I and I+II. With the time dependency under consideration, it may well be more convenient to calculate the direct return on the current project and use the earnings yield as the cut-off rate.[1]

The fundamental issue, therefore, is whether or not the absolute return on investment function in a period is independent of, or shifts upward with, the size of the firm's prior investment decisions. Miller concludes his discussion with a rejection of dependence, using an appeal to common sense to justify the conclusion. He wrote "The dependence assumption, for example, requires one to believe that, even though IBM would earn only 2 per cent on a new factory to produce obsolete 650's the construction of such a plant would automatically make it possible to expand the capacity for 7090's on which the company earns 40 per cent!" [18, p. 315]. On the other hand, the position that the investment opportunities available to IBM, General Motors or any other firm are independent of their prior investment decisions means that IBM and General Motors would have the same investment opportunities today if they had undertaken no investment over the prior fifty years. What could be less true?

In fact we may go on to note that a sufficient if not a necessary condition for perfect competition is independence of investment

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[1]Notice that in this model there is no depreciation so that each project has an infinite life. With projects that have a finite life the return is the discount rate which equates the cash flows on the infinite sequence of replacements with the initial outlay. What we have here is something different, the additional benefits to the firm due to the growth opportunities made possible by undertaking the project.

opportunities of prior investment decisions - or more generally of an individual's or firm's history. Clearly such independence exists with regard to the investment in publicly traded shares by individuals. However, having it exist in general would imply that everyone born in the United States has the same opportunity to become the president of the country, not as an abstract political right but as a technological possibility. Clearly our investment and our employment opportunities depend on our history.[1]

It does not follow that the exact form of the dependence assumed in Eq. (1) is correct. As Elton and Gruber [7] have shown, a wide range of possibilities with regard to the functional form of the dependence exist. However, the assumption that the investment opportunities grow at the rate qr is the most plausible, simple, and analytically tractable among those considered by Elton and Gruber.[2] This only suggests that it be given serious consideration, with its ability to describe and produce correct decisions empirically as the further test to be satisfied.

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[1]The same conclusion is reached by Arrow [1].

[2]The alternative explored by them is to have the shift in the return function a function of r. Since r is an inverse function of q, either can be taken as the independent variable. It would appear more reasonable to make the independent variable the produce qr, since the shift in the investment opportunities function is favorable as long as qr increases with q, and it becomes unfavorable when the decline in r offsets the rise in q.

III

With regard to the relation between share yield and a firm's investment rate, a major break through in knowledge was accomplished by Fewings [8][9][10].[1] In [10] he showed that with the end of period value of k uncertain, but with the firm's rate of return on investment certain, the risk of the holding period return on a share is an increasing function of its investment rate. The argument can be stated quite briefly as follows.

One plus the holding period return on a share during t is

$$R_t = (D_t + P_t)/P_{t-1} \tag{7}$$

with D_t the dividend during t, and P_{t-1} and P_t the share's price at the start and end of t. With g_t the rate of growth in the dividend, k_t the share's yield at the end of t, and $P_t = D_{t+1}/(k_t - g_t)$, Eq. (7) may be written as follows:

$$R_t = \quad D_t + \frac{D_{t+1}}{k_t - g_t} \qquad \frac{k_{t-1} - g_{t-1}}{D_t} \tag{8}$$

Eq. (5) with the variables which may change over time subscripted is

$$g_t = \frac{qr(1-b) - sk_t}{1-q} \tag{5a}$$

Note that at the start of t, $D_{t+1} = D_t(1+\sigma_t)$, r is certain, k_{t-1} is known, and q, b and s are decision variables. k_t and g_t are uncertain, the latter because k_t enters into the determination of g_t. Substituting Eq. (5a) for g_{t-1} and g_t in Eq. (8) and simplifying results in

/1 Haugen and Wichern [26] and Boquist, Racette and Schlarbaum [25] have shown that the risk of a bond increases with its duration or growth. Fewings went beyond this conclusion to establish the interrelations among risk, growth, investment rate and financing policy for a firm's common equity. His work, therefore, leads to conclusions on the influence of growth and investment on a firm's cost of capital.

$$R_t = 1+k_{t-1} + \frac{(k_{t-1}-k_t)(1+qr)}{k_t-qr} \tag{9}$$

The only random variable in this expression is k_t, and the value of Beta for the share at th- start of t is

$$\beta_j = \frac{Cov_{t-1}(R_{jt},R_{mt})}{Var_{t-1}(R_{mt})} = \frac{(1+q_jr_j)Cov_{t-1}\left(\frac{k_{jt-1}-k_{jt}}{k_{jt}-q_jr_j}\right), \left(\frac{k_{mt-1}-k_{mt}}{k_{mt}-q_mr_m}\right)}{(1+q_mr_m)Var_{t-1}\left(\frac{k_{mt-1}-k_{mt}}{k_{mt}-q_mr_m}\right)} \tag{10}$$

It is clear that if k_{jt} and k_{mt} are correlated $Cov_{t-1}(R_{jt},R_{mt})$ is positive, and the risk of the share increases with q_j, the firms investment rate. The presence of q, outside and inside the covariance expression both contribute to the increase in β_j with q_j.

In [8] Fewings allowed r_t as well as k_t to be random variables. The above conclusion is reached both with r_t equal to the realized value in ther period just ended and an exponential average of the past realized values of the rate of return. The conclusion holds with q=b+s applied to actual earnings during the period and under the more realistic assumption that the dividend rate is applied to expected earnings at the start of each period.[1]

Let us now turn our attention back to the complete expression for the optimal investment decision

$$r' = \frac{k-qr}{1-q} + \frac{\partial k}{\partial q} \tag{11}$$

[1] In other words, he made a start in recognizing the influence of the informational content of the dividend on firm behavior.

with q the investment decision and not merely the retention decision. With k an increasing function of qr, the right hand side of Eq. (11) lies above the earnings yield, both because $\partial k/\partial q$ is positive and because k in $(k-qr)/(1-q)$ increases with qr.

To use this expression, however, we must specify how k varies with q. To test the conflicting hypothesis with regard to the relation between k and g=qr Brigham and Gordon [4] ran the regression[1]

$$D/P = \alpha_o + \alpha_1 g + \ldots \qquad (12)$$

It was shown in [4] that if k is independent of growth, α_o is an estimate of k and we should find $\alpha_1=-1$. Instead we found α_o less than k by all reasonable standards and $\alpha_1>-1$ with a very high degree of statistical significance.

An alternative model is

$$Y(1-q)/P = \alpha_o(1+qr)^{\alpha_1} \qquad (13)$$

Eq. (13) simply states that the price investors are willing to pay for a share increases with the growth in total dividends in a manner that makes $Y(1-q)/P$ asymptotically approach zero. A variant of Eq. (13) was tested by Brigham and Gordon [4] and Gordon [13], and estimates of the coefficients were obtained.[2]

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[1]The regression model included additional risk variables for leverage, pre tax earnings instability, and percentage electricity sales of total sales.

[2]The model tested did not include stock financing, and therefore was

$$Y(1-b)/P = \alpha_o(1+br)^{\alpha_1}$$

with the additional risk variables described previously. Empirically the difference between q and b is negligible for purposes of parameter estimation.

Adding qr to both sides of Eq. (13) results in

$$Y(1-q)/P + qr = k = \alpha_o(1+qr)^{\alpha_1} + qr \tag{14}$$

Taking the derivative

$$\frac{\partial k}{\partial q} = \alpha_1\alpha_o(1+qr)^{\alpha_1-1} r' + r' \tag{15}$$

Substituting this expression for $\partial k/\partial q$ in Eq. (11) and simplifying results in

$$r' = \frac{k-qr}{1-q} \quad \frac{1}{-\alpha_1\alpha_0(1+qr)^{\alpha_1-1}} \quad , \tag{16}$$

and noting that $k-qr = \alpha_0(1+qr)^{\alpha_1}$ we obtain

$$r' = \frac{1+qr}{-\alpha_1(1-q)} \tag{17}$$

Possible ranges for the variables in Eq. (17) are $8 < -\alpha_1 < 25$, and $.02 < qr < .15$.

We have resolved the so-called IBM paradox. Assuming that k is independent of q resulted in an unreasonably low cut-off rate for IBM's equity investment rate in the early sixties. Recognizing that k is an increasing function of q substantially raises the cut-off rate. Assume that $-\alpha_1=20$. If the price of IBM stock was maximized at q=.7 with qr=.21, the cut-off rate for that company was determined by Eq. (17) was about a 20% return on investment.

IV

The previous pages have developed a model of stock valuation for arriving at the equity financing and investment decision which maximizes the value of the existing common equity. The central issues in the development of the model were (1) the relation between the yield on a share and the firm's investment rate, and (2) the relation between a firm's return on investment function in future periods and the firm's current investment decision.

On the first issue, the recent work of Fewings provides the theoretical basis for the conclusion that the required yield on a share increases with the investment rate.[1] There is no lack of empirical evidence in support of this conclusion. Brigham and Gordon [4], Gordon [13] and others have found very strong inverse correlation between dividend yield and expected dividend growth. Brennan and Sharpe and Sosin [24] have found high inverse correlation between dividend yield and Beta.[2] Putting these two pieces of evidence together, we have positive correlation between growth and Beta or risk. Direct evidence on the correlation between the Beta Measure of risk of a firm's stock and its investment rate is reported in Gordon [13,

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/1 The Fewings proof assumed the Gordon-Lintner growth model for share valuation. Whether or not the conclusion holds under more general assumptions remains to be seen.

/2 Brennan's results are described by Jensen [23]. Since the personal income tax creates a tax preference in favor of high retention and growth stocks, the inverse relation between dividend yield and Beta is quite remarkable. It means that the total pre-tax return investors require on a share increases with the fraction recevied in the form of capital gains due to the increased risk notwithstanding the tax consequences.

pp. 122-8], Fewings [9] and Bar-Yosef and Kolodny [20].[1]

On the second issue, we saw from the work of Elton and Gruber that one cannot arrive at the influence of a firm's investment decision on the value of its stock without making some assumption about how the investment decision will influence the profitability of future investments. The two alternatives here are that a firm's absolute return on investment function is independent of its current investment decision, and that the absolute return function is dependent on the current investment decision. It has been shown that between these two assumptions only the latter is at all plausible, in which case the cost of equity capital depends on the firm's investment decision apart from the previously discussed risk influence.

However, the Gordon-Lintner growth model makes the strong assumption that the form of the dependency is such that a firm's relative return on investment function is independent of its prior investment decisions. We know that this assumption is wide of the mark for some firms and is not strictly true for any firm. Nonetheless, its intuitive appeal, simplicity, and power compel attention. Furthermore, under a widely respected theory of knowledge, the test of a theory is not the accuracy of its assumptions but its accuracy in explaining and predicting variables of interest. Our model has been found quite useful in measuring share yield, and it has met with some success in explaining investment behavior.[2]

/1 Black and Scholes [21] claim that the data reveals no relation among dividend yield, risk and return. However, their analysis of the data is beyond our comprehension.

/2 The alternatives to $k=(D/P)+br$ or qr are that k is the earnings yield or that k is the dividend yield plus the past rate of growth in price. Under the investment policy model presented in this paper we don't get corner soluations and we don't conclude that firms in general are irrational.

Nonetheless, we would be the last to argue that the investment model described above is completely adequate. A particularly desirable extension is the identification and correct treatment of investment opportunities which do not satisfy the assumption that a firm's relative return on investment function is independent of prior investment decisions.

However, this is not the only limitation of the model, and it may not be the most important of the limitations. We have assumed a fixed debt-equity ratio, and insofar as the ratio may be changed, particularly in the short run, the model does not establish the optimal investment and equity financing decision. It was assumed that there is no informational content in a firms dividend decision with regard to current earnings. More generally, it was assumed that there is no problem in communication between the management and the stockholders of a firm. Finally, we have assumed that there is no personal income tax.[1]

It should be added that we have taken as the objective of a firm's management the maximization of the market value of the existing common equity. A management may also have other goals such as the long run survival and growth of the firm, and the management may be able to strike a compromise among these goals. Insofar as that is true, the accuracy of our model in explaining the price of a share is not impaired, but the accuracy of the model in predicting the firm's investment decision is impaired.

/1 The consequences of withdrawing this assumption are established in Gordon and Gould [12].

Finally, we conclude with a brief comment on growth. It is well known that corporate managements make a satisfactory rate of growth one of the seemingly independent objectives of the firm. This has led some writers to develop models of the firm in which management's goal is the maximization of the rate of growth in sales, subject to a profit constraint. We now see that such caricatures of corporate managements are not necessary to explain their behavior. Concern with growth may arise from the psychic and other rewards to management of a firm which offers opportunities for advance ment to subordinates and themselves. However, concern with growth, not maximum growth, is also a consequence of a relative return on investment function that is independent of prior investment decisions.

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