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A Determination of the Risk of Ruin

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I. INTRODUCTION

Recently, there has been an increased interest in the role which bankruptcy or ruin plays in the valuation process. Several authors have discussed this subject (Gordon [1971], Quirk [1961], and Smith [1972]) and some have constructed theoretical models attempting to show how the probability or risk of ruin introduces an element of risk into valuation (for example, Bierman [1975], Borch [1969], Tinsely [1970]). The question of corporate survival is, therefore, central to the financial considerations of the firm. None, however, have attempted empirical tests of the role of such a probability in valuation.

One of the problems in empirical investigations of the role of the risk of ruin has been that an adequate way to measure that risk has not been developed. Obviously, unless the risk can be measured, a determination of ruin probabilities and its role in valuation cannot be estimated. One of the earliest attempts was to classify firms based on combinations of various ratios using discriminant analysis. Work by Altman (1969) and others has led to early-warning systems such as those developed by Sinkey (1975)(1977). Yet the essentially static nature of these devices remains their fundamental flaw. Rather than obtaining evidence concerning the firm's likely exposure to failure in its operations, these ratios question the ability of a firm to avoid present failure with its present asset and liability characteristics. If one is concerned with the risk exposure in the future, it is essential to consider the dynamics of a firm's operation and, based upon this, its likely ability to sustain continued functions of the firm. This paper presents such a method for determining the risk of ruin for a firm.

First, the deficiency of the only currently viable alternative, first passage time or equivalently gambler's ruin is investigated followed by the development of a theoretical model to allow the determination of the risk of ruin. Certain applications of the model are then demonstrated. Finally, the risk of ruin measure developed here is compared with other typical risk measures.

II. DEFICIENCY OF FIRST PASSAGE TIME

Previously, the first passage time has been used to determine the risk of ruin for a firm. Bawa [1972], for example, has developed a financial planning model where valuation is based on expected operating income given that ruin does not occur. To determine the probable life of the firm, he uses first passage time. Likewise, Wilcox (1976) uses first passage time as the measure of firm failure. It is now demonstrated that such an approach is not appropriate for most business firms and a more robust technique is needed.

The concept of first passage time arises in the literature from the consideration of the Gambler's Ruin problem of finite Markov chains. If it is assumed that a player plays a sequence of independent games in which he wins \$1 with probability p or loses \$1 with probability $q = 1 - p$, then the sequence $X_0, X_1, X_2 \ldots$ represents his capital after the 0th, 1st, 2nd, etc. trials. If it is assumed that he starts playing with initial capital of \$k, the game ceases when his capital drops below zero. If it is assumed <u>for the moment</u> that an upper barrier, N, exists which limits the values of X_t between 0 and N and which if reached means that the player can not be ruined, the game is a stationary Markov process with state space (0, 1, 2, ... N). It can be shown that if he starts at k and $t_k = \Pr(X_n = 0 \text{ for some } n | X_0 = k)$ is defined as the probability of ruin, then it can be shown[1] that:

$$t_k = \frac{(q/p)^k - (q/p)^N}{1 - (q/p)^N} \qquad p \neq q \tag{1}$$

or

$$t_k = 1 - \frac{k}{N} \qquad p = q \tag{2}$$

It can also be shown that the existance of an upper bound means that ultimately $t_k = 1$, ruin is certain (see for example Parzen [1962] p. 231). Since the probability of ruin is one (ruin is certain), the only quantity of interest is the value of <u>n</u> or time to first ruin. However, since <u>n</u> is a random variable, one must talk about expected time to ruin, commonly called first passage time. Other moments for the distribution can be calculated to finish the specification. It has been shown in several sources (Segerdahl (1955) and Seal (1969) for example) that when ruin is certain, the distribution of first times to ruin is asymptotically Normal in which case the mean and variance specify the distribution. Expressions for these moments have been developed in the references cited.

Now, however, remove the barrier ($N \to \infty$) and consider biased as well as fair games. For negatively biased games ($p < q$), it can be seen from equation 1 that ruin is certain:

$$\lim_{N \to \infty} t_k = 1 \qquad (p < q)$$

Likewise, from equation 2 ruin is certain for fair games, ($p = q$):

$$\lim_{N \to \infty} t_k = 1 \qquad (p = q)$$

If, however, one has a positively biased game ($p > q$), a different result obtains from equation 1

$$\lim_{N \to \infty} t_k = (q/p)^k \qquad (p > q)$$

That is, there is now a non-zero probability of never being ruined ($1 - t_k > 0$), with the deviation from zero depending on how much p is greater than q. This is a stochastic process with a positive drift. The concept of first passage time becomes inappropriate when there is a significant positive probability that first time to ruin is infinity. Even if the mean of the distribution is finite, it makes little sense to compare distributions with infinite variance. A ranking system based on such a situation is thus inappropriate.[2]

If instead of jump sizes of +1 and -1, one allows any number of jump sizes, the problem is more involved but has similar outcomes. Depending on the sizes of the distribution, the process is again positively-biased, negatively-biased or a fair game. With no barrier present, a positively-biased game will again show a significant positive (non-zero) probability that the firm will never be ruined. So, again a first passage time measurement of the risk of ruin is inappropriate.

If there is a barrier present, Definetti (1957) has shown that ruin is certain as in the previous case. However, Gerber (1974) has demonstrated that there is a significant positive (non-zero) probability that the firm will never be ruined even with a barrier as long as the barrier is moving. As previously shown, as long as there is a non-zero probability of never being ruined, the use of first passage time to describe the risk is inappropriate. Thus, if two firms have non-zero probabilities of never being ruined, one cannot use first passage time to differentiate between these two firms with regard to their risk of ruin. Since it is assumed that firms are expected to be profit making enterprises thus characterized by a positive-drift process, the use of first passage time to describe their risk of ruin is thus inappropriate.[3] Therefore, a method of determining the risk of ruin which does not suffer from these deficiencies is needed. Such a model is now developed.

III. PROCESS DESCRIPTION

Before deriving a mathematical model for determining the risk of ruin, it is necessary to describe the process which is being modeled. Earnings come to a firm from revenue obtained by selling a product less the costs incurred in producing that product. There are two types of costs to be considered: variable, which change according to the stochastic nature of the revenue sources, and fixed, which do not vary with revenue but are a function only of the period. So, revenue less variable costs assocated with the generation of that revenue can be defined as variable profit, which would be a stochastic variable. Revenues and variable costs are stochastic due to stochastic sales quantities and (for a multiproduct firm) stochastic prices and costs per unit. Thus for the i^{th} arrival, the variable profit, Z_i, is:

$$Z_i = R_i - VC_i$$

where:

R_i = Revenue for the i^{th} arrival

VC_i = Variable cost for the i^{th} arrival

Now consider the following process. Assume that a firm has some pool of resources at time = 0 of some size, U_o, which are available to prevent ruin.[4] Further assume that all variable profits received by the firm increase the size of this pool and that all payments of fixed costs decrease the size of the pool. Since fixed costs are assumed to be incurred at a steady rate over time, these costs represent a steady drain on the pool. If the size of a variable profit and the arrival pattern of these variable profits were uniform, the size of this pool of resources could be easily determined at any time. If these variable profits can take on different sizes, however, then the size of this pool can no longer be easily ascertained. Since the various sizes of these variable profits describe a frequency distribution, the size of this pool will depend on the size of a variable profit and its occurrence. So the value of this fund, U_t, at some time, t, would be the result of a stochastic process involving the steady drain of fixed costs over time, the value of fund inflows, which would be random, and the starting point, U_o. If U_t is less than zero, ruin occurs because no funds are available to meet fixed costs. In the context of the stochastic process to be used here, risk of ruin is defined as the probability that this pool of funds has been depleted at some time, t, or $Pr(U_t < 0)$.

It should be emphasized that ruin as defined here does not mean bankruptcy. Ruin is defined as that state where the firm cannot be self-sustaining in the sense of requiring an infusion of outside funds to maintain operations. Several options are open to a firm which has been "ruined" according to this definition. One option is to go to the stockholders and request an infusion of capital. If the stockholders can be convinced that the E [PV (future dividend stream)]>Funds needed, bankruptcy will not occur. In fact as Borch [1969] suggests, if this is a viable alternative, one does not need to keep reserves to prevent ruin. However, failing to convince stockholders to invest more funds does not mean bankruptcy must ensue. Another choice is to negotiate with creditors to <u>effectively</u> infuse funds by extending debts beyond the time of payment. Another approach is to go to some third party such as a governmental agency and request funds to continue operations. Such a process which is essentially institutional in nature does not lend itself to this type of mathematical modeling.

Firms realize the problems associated with uncertainty so they maintain a pool of resources which can be called on to provide funds to meet fixed expenses if earnings at a given point in time are insufficient to meet these expenses. The size of these resources will depend on the expected fluctuations. Therefore, the risk of ruin is the risk that available resources needed to be a self sustaining firm will be depleted. Defining what reserves are available to prevent ruin is treated later.

IV. THE MODEL: DERIVING THE RISK OF RUIN

To determine the risk of ruin at any point in time, it is necessary to ascertain the level of the pool of funds, U_t, so as to determine $Pr(U_t < 0)$. The stochastic characteristics of U_t will have the stochastic properties of the variable profit term. The total variable profit of a firm is a random variable which is a function of two other random variables; the number of variable profit arrivals from time 0 to t times the size or value of each variable profit.

If it is assumed that in some interval (0,t], the number of variable profit arrivals has independent increments, that these increments are stationary (i.e., for two points $t>s\geq 0$ and any $h>0$, the random variables $k(t)-k(s)$ and $k(t+h) - k(s+h)$ are identically distributed), that there is a positive probability that in any interval immaterial of length an arrival will occur and that in sufficiently small intervals simultaneous events are not possible, it can be shown that the variable profit arrival follows a homogeneous Poisson process.[5] Likewise if Z is the revenue less the associated variable cost, then VP(Z) represents its distribution function--i.e., VP(Z) is the conditional distribution of Z given that a variable profit arrived.

It is now desired to determine the value, x, of the total variable profit given that k arrivals have occurred where:

$$x = Z_1 + Z_2 + \dots + Z_k.$$

The distribution function of total amount of variable profit, x, of k arrivals is is denoted by $VP^{k*}(x)$ and is known as the k-fold convolution of VP(Z).[6] If x(n) is the total number of arrivals in some period and $\bar{n}$ is the arrival rate, then $F(x,\bar{n})$ is the distribution function. The function $F(x,\bar{n})$ equals the distribution function $VP^{k*}(x)$ given that k arrivals have occurred, summed over all k; $F(x,\bar{n}) = \sum_{k=0}^{\infty} P_{k(t)} VP^{k*}(x)$. Since the number of arrivals has a Poisson distribution,

$$F(x,\bar{n}) = \sum_{k=o}^{\infty} \frac{e^{-\bar{n}}\,\bar{n}^{k}}{k!} VP^{k*}(x).$$

Thus, the total variable profit, x, for the interval (0,t] has a compound or convolution-mixed Poisson distribution[7] where,

$$x = \sum_{i=1}^{k(t)} Z_i \tag{3}$$

Thus the level of the pool of funds at any point, t, can now be identified as:

$$U_t = U_o + \sum_{i=1}^{k(t)} Z_i - FC \cdot t \tag{4}$$

where:

FC = fixed costs per unit period,[8]

t = period = 1, 2, 3, ...

This result is analogous to the negative risk sums or annuity process discussed in the literature of collective risk theory.[9] Using the definition of ruin previously developed, it is necessary to determine the risk or probability that $U_t < 0$, ε, as:

$$Pr(U_t < 0) = \varepsilon \tag{5}$$

Substituting equation (4) into (5),

$$Pr\ (U_o + \sum_{i=1}^{k(t)} Z_i - FC\cdot t < 0) = \varepsilon \qquad \text{or}$$

$$P_r\ (\sum_{i=1}^{k(t)} Z_i < FC\cdot t - U_o) = \varepsilon \tag{6}$$

By applying the central limit theorem an approximation of this probability is possible.

To apply the central limit theorem, however, one must have the moments of the distribution of the random variable, i.e. the mean $E(x,\bar{n})$ and variance σ^2. These are now obtained.

The mean is determined as follows:

$$E(x,\bar{n}) = \int_o^\infty x dF(x,\bar{n}) = \sum_{k=o}^{\infty} \frac{e^{-\bar{n}}\ \bar{n}^k}{k!} \int_o^\infty x\ dVP^{k*}\ (x). \tag{7}$$

The integral $\int_o^\infty x\ dVP(x)$ is the mean of $VP(x)$, the distribution of the variable per arrival, and is denoted by α_1. Since there are k convolutions, it can be shown that the mean of the convoluted distribution is:

$$\int_o^\infty x\ dVP^{k*}(x) = k\alpha_1.$$

Thus the previous equation reduces to:

$$E(x,\bar{n}) = \sum_{k=o}^{\infty} \frac{e^{-\bar{n}}\ \bar{n}^k}{k!}\ (k\ \alpha_1) = \alpha_1 \sum_{k=o}^{\infty} \frac{ke^{-\bar{n}}\ \bar{n}^k}{k!} \tag{8}$$

Since $\sum_{k=o}^{\infty} \frac{ke^{-\bar{n}}\ \bar{n}^k}{k!}$ is the mean of the Poisson distribution equal to the expected arrival rate, $\bar{n}$,

$$E(x,\bar{n}) = \bar{n}\alpha_1. \tag{9}$$

The derivation of the variance, σ^2, follows a similar procedure. Let the second moment of the VP/arrival distribution about the origin be denoted as α_2. So

$$\sigma^2 = E(x, \bar{n})^2 - (E(x, \bar{n}))^2 \text{ and } E(X, \bar{n})^2 = \int_0^\infty x^2 \, dF(x) = \bar{n}\,\alpha_2 + \bar{n}^2\,\alpha_1^{\,2}.$$

Thus,

$$\sigma^2 = \bar{n}\,\alpha_2 + \bar{n}^2\,\alpha_1^{\,2} - \bar{n}^2\,\alpha_1^{\,2} = \bar{n}\alpha_2 \tag{10}$$

Thus, the expected variable profit is equal to the expected arrival rate times the expected size of the variable profit per arrival. The variance of the variable profit distribution is equal to the expected arrival rate times the second moment about the origin of the variable profit per arrival. Standardizing the random variable in equation (6) produces:

$$\Pr\left[\frac{\sum_{i=1}^{k(t)} Z_i - \bar{n}\,\alpha_1\,t}{\sqrt{\bar{n}\,\alpha_2\,t}} < \frac{-U_o + FC\,t - \bar{n}\,\alpha_1\,t}{\sqrt{\bar{n}\,\alpha_2\,t}}\right] = \varepsilon \tag{11}$$

If $Y\varepsilon$ is a standard normal variate and ε is the solution to the equation:

$$\varepsilon = \phi\,(Y_\varepsilon),\ \phi' < 0 \tag{12}$$

then $Y_\varepsilon = 2.326$ would be a probability of ruin = 0.01 and a value of Y_ε = 3.090 would be a probability of ruin = 0.001. Increasing values of Y_ε indicate a decreasing risk of ruin. Thus Y_ε is a safety index and is a measure of the risk of ruin. However, this approximation is useful only for nonskewed distributions. In the general case, the value of the safety index must be adjusted for the skewness of the distribution. This is accomplished using the Cornish-Fisher (Normal Power) expansion:[10]

$$\frac{FC \cdot t - U_o - \bar{n}\,\alpha_1\, t}{\sqrt{\bar{n}\,\alpha_2\, t}} = -Y_\varepsilon - \frac{1}{6}\,\gamma_1\,(Y_\varepsilon^2 - 1) \tag{13}$$

where:

γ_1 = skewness factor of the total variable profit distribution

$$= \frac{t\bar{n}\alpha_3}{(t\bar{n}\alpha_2)^{3/2}}$$

α_3 = the third moment about the origin of the variable profit per bill distribution

It should be noted that for symmetrical distributions, the second term on the right side of equation (13) would disappear. It is also assumed that the kurtosis of these distributions does not exert any influence.[11]

Rearranging equation (13), the following is obtained:

$$U_o = +Y_\varepsilon \sqrt{\bar{n}\,\alpha_2\, t} + \frac{\alpha_3}{6\,\alpha_2}\,(Y_\varepsilon^2 - 1) - (\bar{n}\cdot\alpha_1 - FC)t \tag{14}$$

By solving the quadratic expression for the positive root of Y_ε, a safety index at any time, t, can be determined:

$$Y_\varepsilon = \left[\left(\bar{n}\alpha_2 t + \frac{2\alpha_3}{3\alpha_2}\,(U_o + (\bar{n}\alpha_1 - FC)t + \frac{\alpha_3}{6\alpha_2})\right)^{\frac{1}{2}} - (\bar{n}\alpha_2 t)^{\frac{1}{2}}\right]\left(\frac{\alpha_3}{3\alpha_2}\right)^{-1} \tag{15}$$

So if the initial level of the pool of funds, U_o, is given, along with the expected values of the variable profit arrival rate and the moments of the variable profit per arrival distribution, then the safety index, Y_ε, can be determined for any time period, t. Consulting a table of normal distribution values, the probability of ruin, ε, can be derived.

V. WHERE RUIN IS NOT CERTAIN--THE CASE FOR DEFINING MAXIMUM RISK EXPOSURE

The model developed in the previous section has several superior properties as compared to the first passage time model. The present model allows the determination of the risk of ruin at any time rather than as a function of the expected time to ruin. More importantly, while both models are effective for fair and negatively-biased games when ruin is certain, as previously shown, the present model does not break down for positively-biased games when ruin is not certain.[12] Since it is assumed that business firms will operate as positively-biased or positive-drift games, this aspect is critically important. However, if a positively-biased game is assumed, a problem arises in that one must choose the time at which to determine the risk of ruin.

One way to analyze the risk of ruin for a positive-drift process is to determine the probability of never being ruined; that is the value of Y_ε for $t \to \infty$. A complete discussion of this determination can be found in Seal [1969, pp. 119-126] or Beard et al [1969, pp. 137-159]. Bohman [1970] illustrates the estimation technique for this probability.

However, it is not uncommon for firms characterized by positive-drift stochastic processes to not only have non-zero probabilities of never being ruined but in fact to have these probabilities essentially one. If two firms both have very high probabilities of never being ruined, does this mean that investors are indifferent between these two firms with regard to the risk of ruin? It is suggested that investors, creditors, financial managers and others would not be indifferent between them but rather are interested in knowing how close each firm comes to the absorbing barrier and the time when the closest approach occurs. That is, one wants to determine

$$\frac{d\int_{o}^{k(t)} U_t dt}{dt} = 0;$$

i.e., the point of maximum ruin exposure. Neither the first passage time or ultimate ruin models provide this information.

This suggestion is not unusual. Bond rating agencies, for example, claim to look at just such a point. Moody's, for example, states that:

> "Since ratings involve a judgement about the future, on the one hand, and since they are used by investors as a means of protection on the other hand, the effort is made when assigning a rating to look at "worst" potentialities in the "visible" future rather than solely at the past record and the status of the present."[13]

It is now necessary to specify how the probability of ruin at the riskiest point is determined. The aim is to find that time which has the greatest possibility of ruin given the initial level of reserves, U_o. This is consistent with maximizing ε with respect to t which is equivalent to maximization of the exposure to ruin. Using equation (14) and taking the derivative of Y_ε with respect to t:

$$0 = + \frac{dY_\varepsilon}{dt} \; \bar{n} \; \alpha_2 \; t^{\frac{1}{2}} + \frac{(-Y_\varepsilon \;\; \bar{n} \; \alpha_2)}{2t^{\frac{1}{2}}} - \frac{1}{6} \frac{\alpha_3}{\alpha_2} \, 2Y_\varepsilon \frac{dY_\varepsilon}{dt} + FC - \bar{n} \; \alpha_1 \qquad (16)$$

Setting $dY_\varepsilon/dt = 0$, simplifying and solving for t:

$$t = \frac{Y_\varepsilon^2 \; \bar{n} \; \alpha_2}{4(\bar{n}\alpha_1 - FC)^2} \qquad (17)$$

This value of t is the time to the riskiest point and is denoted as t_{rp}.[14] Substituting equation (17) in equation (14) and simplifying, the safety index at the riskiest time is found to be:

$$Y_{\varepsilon rp} = \sqrt{\frac{12\ U_o\ \alpha_2\ (\bar{n}\alpha_1 - FC)\ +\ 2\ \alpha_3(\bar{n}\alpha_1 - FC)}{3\ \bar{n}\ \alpha_2^2 + 2\ \alpha_3(\bar{n}\alpha_1 - FC)}} \tag{18}$$

where:

α_i = ith moment about the origin of the variable profit per arrival term.

From this form, a value of the safety index is calculated from the initial reserves, U_o, the moments of the variable profit per arrival distribution, fixed costs and the expected arrival rate.[15] Then using a table of values for the standard normal distribution, a value for the probability of ruin at the riskiest point, ε_{rp}, can be determined.[16]

VI. RISK OF RUIN FOR ELECTRIC UTILITIES

In order to demonstrate the technique used to determine the maximum risk exposure or the risk of ruin at the riskiest time some firms in the electric utility industry are chosen and data generated. Although the electric utility industry is regulated, it is expected that differences in the risk of ruin can be discerned.[17] As previously mentioned, bond rating agencies state that they attempt to look at the worst potentialities in the visible future. Since the stochastic process looks at a similar point and the bond rating agencies detect large differences among firms in the electric utility industry as witnessed by bond ratings from BA to AAA, it is expected that the stochastic process should also be able to detect these variations. Thus, the electric utility industry should be able to demonstrate the usefulness of this technique.

The formula shown in equation (18) indicates that the risk of ruin at the most dangerous time is a function of five variables: the expected number of variable profit arrivals in a period of time, $\bar{n}$, the first three moments of the variable profit per arrival distribution, α_1, α_2, α_3, the fixed cost rate, and the starting point of the stochastic process, U_o, which are the initial reserves of the firm.

Since the arrival of the variable profit for an electric utility corresponds to the arrival of a bill payment, the terms, α_1, α_2, and α_3, represents the various moments for the variable profit per bill distribution for the firm. This distribution is not available and must be determined from the variable profit distributions of the different classes of service.[18]

Briefly, the billing schedule for each class is broken into service blocks based on use of electricity. The frequency distribution of these service blocks is then determined using the OGIVE[19] method. Applying the rate schedules from the utilities' tariff and the variable cost schedules for the rate class yields a frequency distribution for variable profit per bill.[20] By combining the rate classes, the distribution of the firm's variable profit per bill is determined from which the moments can be generated.[21]

Another element in equation (18) is the expected bill arrival rate $\bar{n}$. These bills are issued by utilities uniformly over time and are paid uniformly over time by customers. Because of these billing practices, it can be assumed that bill arrivals follow the Poisson assumption of a uniform arrival rate. The expected bill arrival rate consists of the total number of customers times the billing frequency.

Next, expected net income, $\bar{n}\ \alpha_1$ - FC must be estabished, which essentially means identifying the rate of fixed costs. All operating costs not classified as variable would be included. Several items need special mention. It is assumed that an amount of funds equal to depreciation is used to generate sufficient assets to maintain the earnings ability of the firm, thus a fixed claim on the arriving variable profit.[22] Likewise, taxes are deducted as less than 2 percent of total taxes vary with income. Finally, interest charges but not preferred or common dividends are included.

The remaining variable is the initial reserves, U_o, which consist of the liquid capital and hidden reserves of the firm. These reserves are available for use until sufficient variable profit to meet fixed expenses has been generated from current earnings. As previously defined, ruin occurs when this reserve and accumulated income has been depleted. Thus, the definition of U_o depends on the definition of what constitutes ruin. As discussed in footnote 2, Steindl [1965] and Borch [1969] have suggested definitions based on total capital but such definitions are not really practical for industrial firms and are hard to defend. In another paper, Borch [1968] argues that the reserves should consist of working capital and that ruin occurs when this working capital is depleted. In a similar vein, Bawa [1972] suggests that ruin is the depletion of working capital. These definitions, however, ignore debt capacity, if available, which must be included as the firm can utilize this source without being forced to confront stockholders, creditors, a third party, or bankruptcy court. For example, most utilities have negative net working capital if unused debt capacity is ignored. In this investigation, a definition of ruin advanced by Bierman et al [1975] is employed which assumes that debt holders or other creditors will force reorganization if a firm is unable to meet contractual obligations because working capital is too low <u>and</u> the firm cannot obtain more debt. Thus U_0 is defined as some level of working capital which can rapidly be converted to liquid funds, free of prior claims, and usable for defending against ruin, and the funds available through borrowing.[23] It is assumed that this borrowing potential is not a function of time, but can be drawn down at time 0 if necessary.[24]

Firms in the sample were chosen from a listing of Class A and B utilities required to report to the Federal Power Commission and obtain at least 90% of their revenues from electric service. The safety index and the risk of ruin for these firms were determined as of December 31, 1971 as this is the latest date for which complete data are available. Firms were chosen on a wide geographical basis, representing each region of the country, and encompassing a large range of sizes so as to not introduce any bias due to size or location. Furthermore, a cross-section of bond ratings from BA to AAA was chosen so as to be able to differentiate the risk of ruin among firms.

Using equation (18), the safety index and the risk of ruin are obtained and shown in Table 1. Likewise, the time to the most dangerous point as determined by equation (17) and the Moody's bond ratings of each firm are also shown.

Examination of two firms in the sample such as Edison Sault Electric and Duquesne Light demonstrate the robustness of the technique developed here. The probability of ultimate survival (never being ruined) for both firms is ≈ 1 using the formula developed by Seal [1969, p. 115]. Since there is <u>a significant</u> non-zero probability of never being ruined, differences in first passage

Table 1. Safety Index, Risk of Ruin, and Time to the Most Dangerous Point--by Firm.

	Firm (Bond Rating[1])	Safety Index $Y_{\epsilon rp}$	Risk of Ruin, ϵ_{rp}	Time to the Most Dangerous Point, Year
1.	Duquesne Light (AA)	626	0.0000	5.4
2.	Kansas Gas and Electric (AA)	621	0.0000	24.0
3.	Southwestern Public Service (AA)	398	0.0000	0.26
4.	Ohio Edison (AAA)	374	0.0000	5.9
5.	Utah Power and Light (A)	237	0.0000	2.8
6.	Detroit Edison (AA)	147	0.0000	0.97
7.	Ottertail Power (Baa)	119	0.0000	2.7
8.	Kentucky Utilities (AA)	115	0.0000	2.76
9.	Central Maine Power (A)	109	0.0000	1.8
10.	Southern California Edison (AA)	72	0.0000	0.26
11.	Hawaiian Electric (A)	63	0.0000	0.62
12.	Boston Edison (AA)	61	0.0000	3.8
13.	Tampa Electric (AA)	60	0.0000	0.52
14.	Portland General Electric (Baa)	45	0.0000	0.50
15.	Public Service Co. of New Hampshire (A)	43	0.0000	0.8
16.	Puget Sound Power and Light (Baa)	29	0.0000	0.05
17.	Carolina Power and Light (A)	20	0.0000	0.08
18.	Upper Peninsula Power (A)	12.2	0.0000	1.07
19.	Edison Sault Electric (Ba)	1.119	0.1300	0.49
20.	Newport Electric (Ba)	0.1009	0.4600	0.005

[1] Moody's Public Utility Manual, 1972, First Mortgage Bonds.

time are an inappropriate measure of the differences in the risk of ruin of these firms. Yet, one would not be indifferent between these firms. Edison Sault has a greater exposure to risk as signified by the safety index than Duquesne Light. Furthermore, one not only can tell the maximum exposure to ruin using the safety index but also _when_ this maximum exposure occurs. Thus, the model developed here provides a method of determining the risk of ruin for a firm when the other methods would break down.

These results also indicate that except for the very smallest utilities, the firms in the sample have enough reserves to defend against ruin. Their safety index is greater that 5 and thus exhibit essentially no risk of ruin. The small sizes of the moments of the variable profit distributions, as compared to annual income, enable them to maintain their reserves. Not only are the earnings steady from one period to the next, but there is little possibility of fluctuation. It would take a major disaster which would wipe out an entire franchise area for an extended period of time to make changes in the pattern of returns substantial enough to change the risk of ruin. This result confirms the fact alluded to in some texts but not before shown empirically that unlike most industrial concerns, utilities do not find it necessary to maintain a large net working capital position.[25]

VII. RISK OF RUIN FOR VULNERABLE FIRMS

Since electric utilities exhibit virtually no risk or ruin, the usefulness of the process developed here cannot be fully appreciated. Although the results are consistent they do not verify the usefulness of such an approach. Thus, a sample of more risky firms is needed.

A group of such potentially vulnerable firms in areas like airlines, oil drilling, and the like were assembled to analyze the risk of ruin. Unlike electric utilities, however, these firms do not publish nor even collect data on unit revenue, costs, or usage. As a result, the method used in the previous section cannot be utilized to estimate the parameters in equation (18). Furthermore, these are multiproduct firms which would make such estimations extremely tedious even if the data were available. Another method of estimating certain of the variables in equation (18) is therefore needed.

Before estimating the moments of the variable profit distribution, it is necessary to obtain measures of the variable profits for each firm. Using ten year's data from the Compustat Quarterly Tapes, forty observations of revenues and income for a firm can be obtained (one per quarter). Cost schedules for the period are not provided so these measures are inappropriate for the determination of variable profits. Instead a method developed by Girod et al (1972) is used to derive the observations of variable profits. Dollar sales for the first four adjacent quarters are plotted against profits before interest and taxes (EBIT) for the same quarters. A simple linear regression is used to fit a least-squares line through the intersecting points. The slope of the line obtained represents the average variable profit rate for the firm; i.e., the incremental profit contribution expressed as a percentage of sales. The point intersecting the vertical axis can be taken as an estimate of average fixed operating costs.[26] Data for the fifth quarter is added and that for the first quarter dropped. The intercept is again estimated by fitting a least-squares line. This process is continued for all quarters.

Because there are only four observations for each regression, the location of the intercept is strongly influenced by the position of each observation--particularly by the fourth quarter when a certain amount of "window dressing" is practiced. To correct the estimates of fixed costs to a more normal long-term level, a least-squares line is fit to the logarithms of the intercepts to obtain the long-run growth function of fixed costs. The fixed costs obtained from the cost function are then added to actual profit before interest and taxes for each quarter to obtain variable profit.[27]

Once the observations of variable profits on a quarterly basis are obtained, the model generating the series must be determined so as to obtain the moments of the variable profit distribution. The sample autocorrelations for each firm are first examined to determine the appropriate model for that series.[28] The criteria of model selection followed in this study has been the representation of each series in the most parsimonious form that is consistent with its stochastic structure. Parameters included in the models are those for which estimates are significant or which are required to eliminate serial correlation in the residuals. Thus, the procedure has not been to minimize the variance of prediction errors but to obtain the simplest adequate representation. The χ^2 value for the appropriate degrees of freedom is determined at the 5% level of significance under the null hypothesis of zero serial correlations in the residuals of the model. The values of the Q statistic support the hypothesis that the individual series have been reduced to white noise series in each case.[29]

In this manner, the distribution of variable profits is obtained. The mean, α_1, can be determined from the parameters of the fitted model and the other moments are a function of the moments of the residual distribution. Box and Jenkins [1970, ch. 3] demonstrate how to obtain the moments of the process' distribution from the model parameters and the moments of the residuals.

The value for $(\bar{n}\alpha_1\text{-FC})$ is similarly determined. Taking the estimate of variable profit for each quarter and subtracting the estimate of fixed costs derived from the long-run growth function as well as actual interest and taxes, a time series of observations for $(\bar{n}\alpha_1\text{-FC})$ is generated. Filtering this series using the same technique as discussed for the variable profit series provides the estimates of $(\bar{n}\alpha_1 - FC)$ from the parameters of the fitted model.[30] The value of U_o is determined using the same definition as outlined for the electric utilities. Unlike that sample, however, most of these firms had no unused debt capacity as defined here as most had interest coverages of less than 1.0. Finally, the arrival rate of variable profits, $\bar{n}$, is taken as unity since there is one observation of variable profit per quarter.[31]

Table 2 gives the safety index, the risk of ruin, and the time to the most dangerous point for these firms as of December 31, 1971, the results obtained are in contrast to those obtained for the electric utility industry. The firms shown here are very much more risky indicating in that these firms have greater variation in variable profits, fewer resources to prevent ruin or a combination of both. These results are consistent with experience in that two of the firms in the sample have ceased to exist as separate entities since 1972. In contrast there has not been an electric utility disappear as a separate entity for several decades.

VIII. SAFETY INDEX AND OTHER RISK MEASURES

Finally, it would be instructive to compare the risk measure developed here with other risk measures available to the public. Reviewing Tables 1 and 2 reveals that there appears to be a relation between Moody's ratings and the safety index. That is, Moody's index appears to decline as the safety index declines thus one should be able to differentiate between "safe" firms (bond ratings AAA-Baa) and "vulnerable" firms (Ba and below). To measure the strength of the association between the two series the Spearman rank correlation is determined. After adjusting for ties, it is found that the rank correlation is 0.9 which is significant at the 1% level. This result indicates that as the safety index declines so does the bond rating (from AAA to Ba). So the safety index can provide an alternative estimate of the risk of ruin to a Moody's rating.[32]

Table 2. The Safety Index, Probability of Survival and Time to the Most Dangerous Point for a Selected Group of Firms--By Firm

Firm (Bond Rating[1])	Safety Index, $Y_{\varepsilon_{rp}}$	Risk of Ruin ε_{rp}	Time to the Most Dangerous Point, Year
1. Fairchild Camera	0.350	.363	5.2
2. Trans World Airlines (B)	0.670	.252	0.28
3. Sanders Assoc. (B)	0.789	.215	5.2
4. Allied Supermarkets (B)	0.90	.184	0.75
5. Bowmar Instruments	1.00	.159	0.64
6. World Airways	1.13	.129	0.31
7. Gordon Jewelry (Ba)	1.14	.127	0.51
8. National Airlines	1.27	.102	0.23
9. Milgo Electric	1.50	.067	1.3
10. Dixilyn Co. (B)	1.68	.047	2.0
11. Sprague Electric (Ba)	1.76	.040	0.8
12. Xtra Inc.	1.96	.025	0.3
13. North American Coal Co.	2.44	.0073	0.9
14. High Voltage Engineering Co.	2.52	.0059	4.2
15. Reading and Bates Co. (Ba)	2.55	.0054	0.08
16. Western Airlines (Ba)	2.60	.0037	0.31
17. Tandy Corp. (Ba)	2.696	.0035	0.69
18. Cenco Inc. (Ba)	3.22	.0006	0.79
19. Alaska Interstate (Ba)	3.29	.0005	3.46
20. Grolier Inc. (Ba)	4.73	.0005	5.2

[1] Moody's Industrial Manual, 1972, if available.

It would also be interesting to compare the safety index developed here with other quantitative measures of risk. One such measure is the firm's systematic risk, commonly denoted as β, as determined by the capital asset pricing model (CAPM). Using the β's for each firm in both Tables 1 and 2, the Spearman Rank correlation analysis shows that the safety index at the riskiest point is highly correlated with these β (a rank correlation of -0.8203 which is significant at the 1% level). That is, as the safety index declines (the risk of ruin increases) the systematic risk increases. Such a result is not surprising in that the average β for the electric utilities as of the sample date was 0.31 while the average β for the set of risky firms as of the same date was 2.07. Thus, these firms not only have higher risk of ruin but also higher market risk. This result tends to uphold the position that there is a systematic component to the risk of ruin rather than having it considered as purely non-systematic risk as is usually assumed.[33]

Finally, a comparison with other quantitative risk measures would be appropriate. The two most common are Altman's (1969) Z-score and Wilcox's (1976) Gambler's Ruin score. Calculating the Z-score for each of the firm's in the risky group as of the same date, December 31, 1971, shows that at a 95% confidence level, six would fail in the next year, four would be safe and ten wold not be classifiable. Likewise, calculating Wilcox's Gambler's Ruin score as of the same date shows that fifteen of the twenty firms would have been predicted to fail. Taking the average safety index suggests that approximately one out of ten would be expected to fail. While the fact that exactly two firms in the group actually ceased operations as separate entities was coincidental, it suggests that the safety index is a far more accurate measure than Wilcox's Gambler's Ruin score or Altman's Z-score. Furthermore it provides more information than Altman's since half of the firm's could not be classified by Altman's method.[34]

It has thus been shown that the safety index at the point of maximum exposure to ruin can provide a way of ranking firms according to relative safety. Such a ranking can be considered useful in measuring the risk of ruin of a firm besides such other traditional measures as bond ratings, a firm's systematic risk, or other quantitative measures.

The ability to use this technique to calculate a risk ruin is influenced by two elements. The calculation procedure is dependent on the definition of U_o, the starting point of the stochastic process. It may be possible to agree on the definition of working capital available to defend against ruin, but the correct determination of debt capacity where applicable is still an unsettled question. A consistent and accurate estimate of the starting point for the stochastic process, U_o, also requires a consistent and accurate definition of ruin. Defining ruin as the state where working capital and unused debt capacity are depleted is a very simplistic view of the ruin process albeit a necessary one to allow empirical evaluation. Measures of the risk of ruin will, therefore, be highly influenced by the characterization of the ruin process.

IX. SUMMARY AND CONCLUSIONS

The results of this study show that is possible to quantify the risk of ruin through a safety index determined using a variation of the annuity process of collective risk theory. The determination of this safety index does not rely upon the assumption that ruin is certain which characterizes other models such as first passage time. It is also demonstrated that when ruin is not certain the present model is superior to other models in that the maximum risk exposure can be determined as well as when this maximum exposure occurs. Estimates of the risk of ruin for some safe and vulnerable firms demonstrate the usefulness of this technique. Finally, it is shown that the measure developed is consistent with or superior to other risk measures currently available.

FOOTNOTES

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[1]See, for example, Parzen [1962, p. 233].

[2]See Seal [1969, p. 114-115] for an example of the determination of the "risk of ruin" at some point of time using first passage time. This determination requires a finite variance, however. With an infinite variance, the result is indeterminate. Likewise such hypothesis testing as a test of the equality of several means, as described in any mathematical statistics text, cannot be accomplished. This test for comparing distributions to determine whether they are the same (such as distributions of first times to ruin) cannot be done with an infinite variance.

[3]The extent to which positive drift processes characterize actual firms is an empirical question. Every firm analyzed here follows a positive drift process including several which ultimately were ruined. Likewise, Griffin (1976) demonstrates similar results for 94 firms. Thus, the problem is by no means a trivial one.

[4]Beard et al [1969, p. 160] suggest that the value of U_o should be the initial capital of the firm which can be used to prevent ruin. This would be any capital which is liquid enough to meet fixed expenses if the accumulated profit is not sufficient. It is also suggested, however, that so-called hidden reserves be included. That is, not only the visible liquid capital of the firm is included, but also the capital that the firm can obtain in the market to fend off ruin not for expansion of eanring power. Steindl [1965, p. 76] suggests that U_o should be set at some positive level of debt but this ignores the availability of working capital. Borch [1969, p. 286], on the other hand, suggests that U_o must be a function of shareholder desires. If they are always ready to rescue a company, it is not necessary to keep any capital in the company to prevent ruin. At the end of a period, the owners will either divide up the profits or cover a deficit. Of course, there would still remain the question of when they should cut their losses and liquidate the firm. In this paper the definition of Beard et al. will be used as more nearly approximating the ruin process. Definitions of both terms, initial liquid capital and hidden reserves, are provided in the section on empirical investigation.

[5]See Parzen [1962, p. 118-121] or Kahn [1968, p. 402] for discussion and proof of this proposition.

[6]It should be noted that only the assumption of stationarity in the variable profit distribution is critical to the process described here since the estimation of the total variable profit, x, would, in general, be indeterminate with a nonstationary variable profit distribution. It is shown later, however, that a nonstationary variable profit distribution can usually be converted to a stationary distribution using readily available statistical techniques.

[7]A discussion of the compound or convolution-mixed Poisson distributions can be found in Seal [1969, p. 31], Beard [1969, p. 18-40] or the seminal work by Cramer [1955].

[8]It should be emphasized that the fixed costs considered here should consist of actual capital outlays, not just some bookkeeping entry since, as previously discussed, these costs represent a drain on the pool of resources.

[9]For a complete discussion of the negative or annuity process see Prabhu [1961, p. 759], Saxen [1948], Seal [1969, pp. 116-119] or Cramer [1955].

[10]A discussion of the use of the Cornish-Fisher expansion in collective risk theory can be found in Kauzzi and Ojantakanen [1971]. Berger [1972] and Beard et al [1969, p. 43] develop the approximation of ruin probabilities using the Normal Power approximation.

[11]Should it be desired to include an adjustment for kurtosis, however, a third term is added to the right side of equation containing a factor, γ_2, which is a function of the kurtosis.

[12]The result obtains whether or not a barrier such as the payment of dividends is present as long as the conditions outlined in Gerber [1974] are observed.

[13]Moody's [1969]. This does not imply that Moody's or other rating agencies would necessarily be considering the same point as the stochastic process. In establishing bond ratings, other factors enter in a non-uniform way such as judgement of the rater. For example, one factor which influences ratings is differences in the perceived marketability of securities. Such a consideration is not explicitly included in the stochastic process.

[14]It is easily shown that the relevant second-order conditions are also satisfied.

[15] It should be noted that these results are not functions of firm size. It can easily be shown that with a continuous testing period both t_{rp} and $y_{\varepsilon rp}$ are influenced only by the distribution of the variable profit for each arrival.

[16] The method developed here assumes that ruin can occur only at the testing points and does not consider what happens between testing points. Seal [1969, p. 104-106] has developed an expression for the risk of ruin given that ruin has not occurred any time previously which avoids this assumption. He demonstrates, however, that the integro-differential equation cannot be solved explicitly. It is unlikely, though, that ruin could occur much before the riskiest point so the method developed here is a good approximation for the maximum exposure to ruin.

[17] For the determination of the risk of ruin for another regulated industry, commercial banking, see Santomero and Vinso (1977).

[18] The different classes of service are residential, commercial, industrial, street and highway lighting, other public authorities, and sales for re-sale.

[19] Parkerson [1963].

[20] Since the tariff schedule is given for a particularly utility and it is assumed that the variable cost for producing a unit of electricity is the same immaterial of service block, the stochastic element in the variable profit distribution is the usage of electricity. For the utilities involved in this study, 88 to 95 per cent of all electricity sold is used by the household sector. Investigations of this sector showed that as of December 31, 1971, the usage distributions of this class had not changed over the previous two decades. Although the distributions of the other blocks were also relatively stable and the percentage usage of electricity used by each service block had not materially altered, the stationarity of the overall usage distribution seemed appropriate because of the stability of the household sector. Furthermore, most companies had experienced no more than one tariff adjustment in the previous decade and that adjustment had generally occurred in the previous year or so. Likewise, since variable costs were such a minor component of the cost of providing electricity, assuming that variable costs would not change enough in the future to alter the variable profit distribution seemed appropriate. Thus, the assumption that the distribution of variable profit was stationary appeared reasonable especially for the lengths of time considered here.

[21]The moments for the firm's variable profit per bill distribution can be can be calculated from the individual frequency distributions. The mean, α_1, of the distribution is determined as follows:

$$\alpha_1 = E(VP/bill) = \sum_{j=1}^{a} \left[\sum_{i=1}^{n} Z_{ij} \; f_j \; (Z) \right] g_j$$

where

Z_{ij} = level of variable profit per bill for service block i in class j

i = number of block levels in a given class

j = class

$f_j(Z)$ = frequency distribution of variable profit per bill for class j

n = number of service blocks for class j

a = total number of classes

g_j = frequency with which a given bill will come from a given class j.

The other moments are found in the same way,

$$\alpha_2 = E((VP/bill)^2)firm = \sum_{j=1}^{a} \left[\sum_{i=1}^{n} (Z_i - \overline{VP}_j)^2 \; f_j(Z_i) + (\overline{VP}_j)^2 \right] g_j$$

$$\alpha_3 = E((VP/bill)^3)firm$$

$$= \sum_{j=1}^{a} \left[\sum_{i=1}^{n} (Z_i - \overline{VP}_j)^3 \; f_j(Z_i) + 3 \; (Z_j) \sum_{i=1}^{n} (Z_i - \overline{VP}_j)^2 \; f_j \; (Z_i) + \overline{VP}_j)^3 \right] g_j$$

where: $\overline{VP}_j$ = expected variable profit per bill for class j.

[22]By using depreciation as the estimate of the funds needed to maintain the earnings ability of the firm, it is implicitly assumed that actual cash outlays for fixed assets are relatively uniform while, in fact, these outflows may be large and intermittant. If the pool of resources had been built up in anticipation of the outlay, the actual timing of the outlay should not affect the risk. To the extent that depreciation under - or overstates actual outlays, the estimate of the risk can be affected. Even for gross variations the impact will be minimal however, as the primary influence on the risk of ruin is the α_2 term which is unaffected by this assumption. For example, in the firms analyzed here, there is minimal deviation of actual expenditures from depreciation schedules. However, a 100 percent deviation from depreciation schedules would result in less than a 10 percent deviation in the safety index which would be a trivial change in the actual risk of ruin. Thus, the methodology developed here is insensitive to this assumption.

[23] Of course, U_o can also be defined as net working capital, unused borrowing potential, unused equity potential, and possible governmental subsidiaries. In such a case, ε would be the probability of bankruptcy. This method allows the determination of either probability merely by adjusting the definition of U_o. In fact, one can calculate any intermediate probability by similarly changing the definition of U_o. For example, the probability of needing governmental subsidy to survive is calculated the same way only U_o is now net working capital plus unused borrowing and equity potentials. The definition developed here is based on a desire to determine the probability of not sustaining the firm without approaching the equity holders, the government, or bankruptcy court.

[24] Net working capital is defined as assets easily converted to liquid funds less claims on those assets. Thus, each current account must be investigated to determine the extent of its liquidity or its claim on liquidity. Likewise, unused debt capacity cannot be unambiguously determined since there is no agreement in the literature on an appropriate definition. Since previous writers such as Carleton (1969), Meyers and Pogue (1975) and others use interest coverage as an estimation of this debt capacity, such a measure is also used here. This amount of debt capacity does not depend on the age of the debt structure of the firm because of regulatory process assumed for electric utilities. However, if the limits imposed are relaxed, the value of the starting point of the stochastic process, U_o, would clearly change. If the debt capacity developed here understates actual debt capacity, the risk of ruin at the riskiest point will be over-stated. The converse is likewise true. Since there is no unambiguous method for determining unused debt capacity, the process used here appears reasonable as it fits the institutional framework within which the firms must operate.

[25] See for example Graham, Dodd and Cottle [1962, pp. 275-276].

[26] It should be noted that these fixed costs do not represent a shut-down minimum level but rather the continuing fixed costs (including the fixed portion of semivariable costs) incurred during normal operating conditions.

[27] Using this methodology, Girod et al demonstrate the ability to estimate the variable profit structure for General Motors and Ford within 0.1 percent which is sufficient for the purposes of this study.

[28] Autocorrelation functions and fitted models used here are too voluminous to be included but are available from the author. The process is described in Box and Jenkins (1970) and is similar to that utilized by Griffin (1976).

[29] Positive drift processes are submartingale in that they are a white noise series around a positive mean. A white noise series is a stationary random sequence which is independent, normally distributed with zero mean and constant variance. See Box and Jenkins (1970, ch. 3) for a discussion of the properties of linear stationary models. The Q-statistic is explained in Box and Jenkins (1970, chapter 8). It should be noted that the filtered variable profit series fulfills the requirement of stationary previously discussed in footnote 6.

[30] Since the stochastic element $(\bar{n}\alpha_1)$ is the same, the model specification will be identical as for the variable profit distribution. Only the parameter estimates will change as the removal of the various fixed cost components from $\bar{n}\alpha_1$ merely constitutes an adjustment of the mean.

[31] It should be noted that since the report of quarterly earnings are filed at somewhat fixed intervals the Poisson assumption is only approximately met. However, there is negligible impact on the estimation procedure.

[32] Moody's suggests that their ratings are not designed to only differentiate default risk among firms but also to indicate the investment quality which includes marketability and other factors. Santomero and Vinso (1977), however, demonstrate that discriminant analysis using the variability of the jump size distribution and availability of reserves can differentiate between safe and vulnerable banks.

[33] It also suggests that since there is a strong systematic component to the risk of ruin, attempts to remove the risk of ruin by diversification will be unsuccessful.

[34] It must be noted that none of the methods provide an absolute ranking. For example, one of the firms which failed was Bowmar Instruments. The Gambler's Ruin score showed it was one of the five which were expected to survive and Altman's Z-score placed it as one of the firms in the "safe" category. While the safety index is quite low, it was by no means the riskiest firm in the group. However one would have estimated the riskiness far better using the safety index than by utilizing the alternative measures.

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