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Capital Budgeting and Portfolio Theory

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Many capital budgeting analyses focus on single projects. These analyses ignore the interaction between the project and the rest of the firm. Analyses that include the interactions between the assets of a firm use portfolio theory and rely primarily on rates of return and may include a market equilibrium. This paper utilizes portfolio theory and includes the level of investment as a variable so that a project can be considered while allowing for economies and diseconomies of scale and for the interaction between the project and the firm.[1] After developing the model and after considering a typical project within the context of the model, it is shown that the model can accommodate changes in a firm's present risk, expected return position; its desired or target risk, expected return position; the interaction between the new project and the firm; and the project's size.

II. Approaches to Capital Budgeting

Capital budgeting analyses are useful in worlds without equilibriums or worlds where new projects are not impounded immediately into the current market equilibrium. In such environments some degree of disequilibrium is possible and a project can have an excess expected return or a positive net present value so that it will be accepted and developed by a firm. The several types of analysis of new projects range from Net Present Value and Internal Rate of Return approaches to mathematical programming and portfolio analysis approaches. These alternative methods coexist because no single method is fully capable of solving the broad set of capital budgeting problems faced by the firm.

The popular Net Present Value and Internal Rate of Return methods and their risk adjusted variants account for the timing of cash flows but are best suited to analyzing single asset firms. These approaches may be inadequate when a multi-asset firm considers a risky project. If these methods are to lead to proper capital budgeting decisions for multi-asset firms, then the expected cash flows must be defined broadly to include the impact of the project on assets currently held by the firm. The risk associated with a new project must also be defined broadly to include the relation between the project and the other assets of the firm. However, if portfolio theory is ignored, there is no generally accepted way to include these interactions in capital budgeting analysis. For example, it is not clear how to calculate the risk premium for a risk adjusted discount rate analysis of a project or how to estimate certainty equivalents for such an analysis. Even when two parameters are used to summarize a project, (2), the emphasis is on the interaction between flows to the project over time rather than on the interaction between the project and other assets.

The portfolio approach to capital budgeting solves some of these problems. Its major contributions include procedures to consider a project and to measure risk formally. In the portfolio model the decision to invest in an asset is partially a function of the asset's impact on the expected return of the firm. The investment decision is also a function of the variance of the expected return of the potential investment and the covariance between that expected return and the expected returns to the other assets of the firm. The portfolio approach to capital budgeting also has some major difficulties. For example, the model typically focuses on rates of return and the risks associated with rates of return so that the lumpiness of projects and economies of scale are not considered in investment decisions.[2] Second, the necessity of making explicit measurements of the interaction between assets makes application of the model difficult. Third, because the one period model generally is used in combination with the assumption of good markets, no transactions costs and no unique opportunities, it is currently applied more easily and realistically in investment market analysis than in capital budgeting analyses where there may be poor markets, high transactions costs, unique opportunities and indivisible projects.

One popular extension of portfolio theory, the capital asset pricing model, is an equilibrium model. However, if this investments market model is extended to cover capital budgeting problems, the necessity of having an equilibrium without market segmentation in markets where it is applied would mean that all new projects would immediately be priced properly. This would make the firm indifferent to any new project except insofar as it wants a different risk, expected return combination in equilibrium. The firm can make the same adjustments via sale or purchase of the market portfolio with corresponding lending or borrowing at the riskfree rate. Even if the equilibrium is restricted to the financial markets and the financial markets are somehow separated from the project market, the capital asset pricing model approach may not be particularly useful in project analysis. In such a case it is not clear that returns and risk in the financial market equilibrium can be compared on a one to one basis with returns and risk in the less well developed project markets.[3]

The model developed here emphasizes a non-equilibrium portfolio framework for considering projects in absolute terms. This framework has several advantages. First, it provides additional flexibility as it partially replaces the cost of capital problem with the firm's target risk, expected return (RER) combination. The essential question in this analysis is whether or not a proposed project will enable the firm to move from its current RER combination (a function of its present asset portfolio) to its possibly different target RER combination. Second, since the model does not rely on an equilibrium, it is possible to consider opportunities unique to the firm and to include special features of the firm. Finally, economies of scale are included as the full range of feasible sizes and alternative production processes can be considered for any one project. In this case the firm may accept the project at some levels of investment, but reject it at other levels.

In this environment the firm's present and target RER combinations may be different from its cost of capital. The cost of capital, obtained from an equilibrium confined to the financial markets, represents the minimum expected rate of return the firm can accept from a project. If the firm presently accepts the cost of capital as its expected rate of return, then its undiversified portfolio will have a greater standard deviation of return than diversified investors utilimately must accept. However, the firm can choose any RER combination as a target as long as the risk and expected return portions of that target will, at a minimum, allow investors to retain ownership in the firm on the current financial markets equilibrium. In this framework the firm may try to capitalize its uniqueness by setting a target RER combination that includes a higher expected return and/or a lower risk than that required in the financial markets today. If the firm reaches this target it will be reflected in the financial markets via a higher corporate valuation in a future market equilibrium.

The firm can not use an arbitrarily difficult target and expect to find acceptable projects. It turns out that as a firm increases the expected return or decreases the acceptable risk portions of its target RER combination the set of acceptable protects gets smaller. If corporate requirements are too stringent, then the firm will have no acceptable projects.

III. Portfolio Theory and Project Decisions

As an intepretation of the two asset case of portfolio theory, this model emphasizes the expected profit of the proposed project and its correlation with the rest of the firm's portfolio. Based on the firm's present RER position, its target RER position, the size of the project and the standard deviation of return of the project, the firm uses expected profits and the correlation between the project and the firm to determine the acceptability of the project.

The expected return for the firm's present n asset portfolio and the standard deviation of the portfolio can be represented by $E(R_B)$ and σ_B respectively. However, the firm's target expected return, $E(R_A)$, and its target portfolio standard deviation of return, σ_A, to be obtained after adding the $n+1^{st}$ asset in the portfolio may be different from current levels. Let the firm add the $n+1^{st}$ asset to its portfolio where the new asset requires P dollars and the current value of the firm is V dollars. In this case, the expected return on the project, $E(R_P)$, the standard deviation of return for the project σ_P , and its correlation with the other assets of the firm, $C_{\pi B}$, must satisfy

$$E(R_A) \leq \frac{V}{V+P} E(R_P) + \frac{P}{V+P} E(R_B) \qquad (1a)$$

and

$$\sigma_A \geq \left(\left(\frac{V}{V+P}\right)^2 \sigma_B^2 + \left(\frac{P}{V+P}\right)^2 \sigma_P^2 + \frac{2\ P\ V}{(V+P)^2} \sigma_B \sigma_P C_{\pi B} \right)^{1/2} \qquad (1b)$$

to meet or surpass the firm's target RER.

Let (1a) be an equality if $E(R_P) = E(R_P^*)$ and let (1b) be an equality if $C_{\pi B} = C^*_{\pi B}$ for a given σ_P. In this case $E(R_P^*)$ is the minimum expected rate of return for the project satisfying (1a) or is the minimum rate of return acceptable to the firm. From (1b) $C^*_{\pi B}$ is the maximum acceptable correlation between the project and the firm which allows the firm to meet its risk target. If the absolute size of the project is included in the risk and expected return considerations then, from (1a), the minimum acceptable expected profit for the investment is

$$E(\pi^*) = PE(R_P^*) = (V+P)\ E(R_A) - V\ E(R_B). \qquad (2a)$$

From (1b) the maximum acceptable correlation between the expected profit for that amount of investment and the firm today is

$$C^*_{\pi B} = ((V+P)^2\ \sigma_A^2 - V^2\ \sigma_B - P^2\ \sigma_P^2)\ /2VP\sigma_B\sigma_P. \qquad (2b)$$

The constraints described in (2a) and (2b) can be summarized into one equation for convenience and for comparative analysis of alternative situations. In this case, the relationship between the maximum acceptable correlation and minimum acceptable expected profit associated with a given project for a firm with a given target RER position is

$$C^{*}_{\pi B} = \left\{\frac{\sigma_A^2 - \sigma_P^2}{2V\sigma_B\,\sigma_P}\;\frac{E(\pi^*) + V(E(R_B) - E(R_A))}{E(R_A)}\right\} + \frac{\sigma_A^2}{\sigma_B\,\sigma_P} + \frac{V(\sigma_A^2 - \sigma_B^2)}{2\;\;\sigma_B\,\sigma_P}\;\frac{E(R_A)}{E(\pi^*) + V(E(R_B) - E(R_A))}\Bigg\}. \quad (3)$$

This equation represents the various maximum correlation, minimum expected profit combinations allowing the firm to move its portfolio from its current RER to its target RER. The graph of equation (3) in Figure 1 covers all sizes of a project where $E(R_A) = E(R_B)$, $\sigma_A = \sigma_B$, $\sigma_P = 1.2221\sigma_B$, $V > 0$.

Each point on the curve in Figure 1 corresponds to some unique level of the project and indicates the minimum acceptable expected profit and maximum acceptable correlation that can be associated with that size project. If the actual expected profit from the project is greater than the target requirement $\left[E(\pi) > E(\pi^*)\right]$ and if the actual correlation between the project is less than the target requirement $\left[\bar{\rho}_{\pi B} < \rho^{*}_{\pi B}\right]$, then it is acceptable. The point representing the acceptable project, $\bar{E}(\pi)$, $\bar{\rho}_{\pi B}$, falls below and to the right of the point on the curve $E(\pi^*)$, $C^{*}_{\pi B}$ or in the acceptance area, T, bounded by and including the solid lines. If the project meets exactly the requirements described by the curve in Figure 1, then it should be accepted if the firm's target RER is different from its current RER.

In this form the combination of capital budgeting and portfolio theory generates a range of acceptable levels of investment for a project as well as a specific accept-reject decision for any given amount of investment so the firm can pick the project size best suiting its needs. Alternatively, the reason(s) for project rejection are easily recognizable. In turn, the firm may recast the project to suit its needs or adjust its specifications to accept the project.

This form also simplifies the portfolio theory approach to capital budgeting in that it is not necessary to estimate the correlation between the project and the firm immediately and fully. Instead the project and the firm can be analyzed separately to obtain the input for the right side of (3). In turn, this generates the maximum acceptable correlation between the project and the firm. If $C^*_{\pi B}$ is greater than 1, then the project is acceptable regardless of the actual correlation between it and the firm. If $C^*_{\pi B}$ is less than -1, then the project should be rejected. If $-1 \leq C^*_{\pi B} \leq 1$, the actual correlation between the project and the firm needs only to estimated sufficiently to determine its size relative to $C^*_{\pi B}$.

IV. Extensions of the Model

The model can be extended so the firm can consider alternative target RERs and portfolio revision and disinvestment.

A. Alternative Corporate Targets

The example in Figure 1 is based on the simplifying assumption that the firm's present and target RERs are identical. Consider the firm that will accept a project meeting either one of two alternative RER targets. The first target ($E(R_A) = E(R_B)$ and $\sigma_A = \sigma_B$) was described earlier. The second target is assumed to be $E(R_A) > E(R\ )(= 1.005E(R_B))$ and $\sigma_A \ \ \sigma_B$ $(=1.01\ \sigma_B)$ or the firm will accept increased risk coincident with increased expectations. If $V > 0$ and $\sigma_P = 1.2221\sigma_B$, then the first target generates a curve identical to that in Figure 1. However, the second curve has a different form. Both curves are shown in Figure 2. If the firm wants a profit $E(\pi^*_1)$ and accepts the first target, then it accepts projects with correlations and expected profits falling on or below and to the right of the boundary formed by points $E(\pi^*_1)$, A_1 and B_1. If the second target is preferred, then projects are accepted if their correlations and expected profits fall on or below and to the right of the boundary formed by points $E(\pi^*_1)$, A_1 and B_2.

Although it appears that there are more acceptable projects when using the second target instead of the first target, it turns out that points A_1 and A_2 represent different amounts of investment. This can be shown be considering the total differential of (2a) or

$$dE(\pi^*) = (V+P)\ dE(R_A) + E(R_A)\ dP. \qquad (4)$$

If $dE(\pi^*) = 0$ and if $dE(R_A) > 0$ as the firm moves vertically in Figure 2 from target 1 to target 2, then dP must be less than zero $(V,P,E(R_A) > 0)$ so that A_2 represents less investment than A_1.

If a firm wants to invest P dollars and considers alternative target RERs, then dP=0 in (4). Since $dE(R_A) > 0$ as the firm shifts from target 1 to target 2, $dE(\pi^*)$ must be greater than zero. Resultingly, a curve representing a fixed level of investment over varying corporate targets will look like curve PP in Figure 2.[4] From the figure, if the firm wants to invest P dollars and accepts target 2, then the project must have a combination of expected profits and correlation falling on or below and to the right of the boundary formed by points B_4, A_3 and B_3 so that there is a change in acceptable projects coincident with the change in corporate targets. For example, projects falling within the rectangle formed by the points $E(\pi^*_1)$, A_1, B_5 and B_4 are acceptable using the first target, but are rejected using the second target. However, projects falling on or below and to the right of the boundary bounded by points B_4, B_5 and B_1 are acceptable regardless of the target in this case.

If the firm will accept projects qualifying under target 1 or target 2 or under any alternative target between targets 1 and 2, then the firm has still more flexibility and can accept projects requiring P dollars and falling on or below and to the right of the boundary formed by points $E(\pi^*_1)$, A_1, A_3 and B_3 so that the curve becomes part of the boundary. Finally, in this case the firm should accept projects falling on the boundary if $E(R_A) \neq E(R_B)$ and/or $\sigma_A \neq \sigma_B$ and will be indifferent if both relationships are equalities. Alternative targets are more helpful and realistic than the concept that the firm has a single hurdle rate. Here the firm can evaluate projects against a set of acceptable corporate goals or can analyze projects in situations where corporate goals are ill-defined except insofar as it knows it will accept a variety of outcomes. Alternative targets may be helpful in a capital rationing situation. In this situation, the firm can simply reevaluate its prospective projects under progressively more stringent conditions thereby eliminating enough projects so that the available capital can be allocated among only the potentially most beneficial projects.

B. Portfolio Revision and Disinvestment

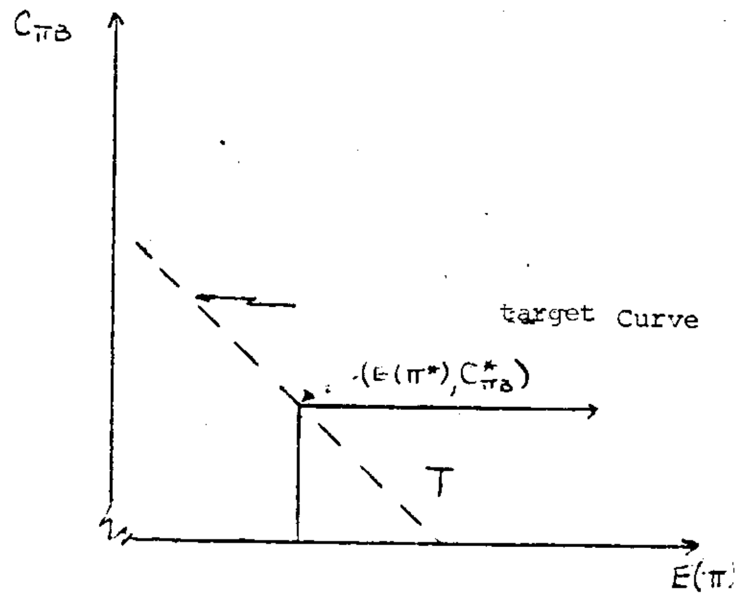
Although this analysis emphasizes the new project or investment schedule and assumes that the first quadrant of each figure is the most relevant portion of that figure, this analysis can also be used for disinvestment problems ($P < 0$) and for portfolio revision (disinvestment, then investment). In each case the firm changes its asset structure rather than merely adds to it. Although minimum acceptable profits to a project could have been negative previously (depending on corporate goals), they are more likely to be negative in this case and any quadrant can contain acceptable expected profit, correlation combinations. For example, in the disinvestment case with no concurrent investment, the firm must typically reduce its minimum acceptable expected profits. Since disinvestment and portfolio revision are often associated with changed corporate goals, this model appears uniquely suited to this case.

Conclusion

Although portfolio theory is generally associated with expected rates of return and the distributions of those rates of return it has been demonstrated that the model can be used for analyses of single projects or entire investment schedules. Moreover, this approach allows the explicit consideration of the corporate risk, expected return goals and changes in those goals, alternative investment and disinvestment policies the interaction between the new project and the firm, the economies and diseconomies of scale of the project and various corporate portfolio management policies.

Figure 1

The Acceptable Correlation Between a Project and the Firm as a Function of Expected Profit*

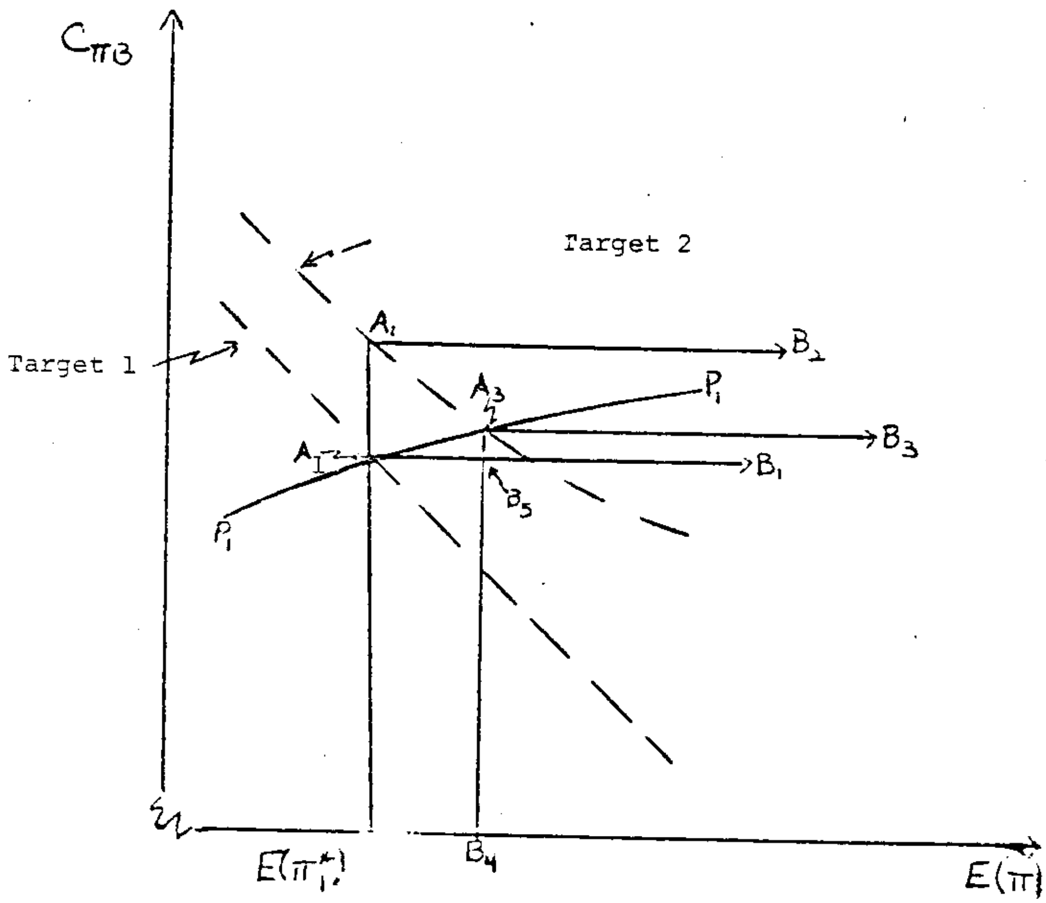


* For this graph $E(R_A) = E(R_B)$, $\sigma_A = \sigma_B$, $\sigma_P = 1.2221\sigma_B$ and $V > 0$ so that the target curve is

$$C^*_{\pi B} = 0.818 - 0.202\, [E(\pi^*) \;\; / V E(R_A)]$$

Figure 2

The Impact of Alternative Targets*



* Target 1 -

$E(R_A) = E(R_B)$; $\sigma_A = \sigma_B$, $\sigma_p = 1.2221\sigma_B$ and $V > 0$

$C^*_{\pi B} = 0.818 - 0.202\,[E(\pi^*) \quad /VE(R_A)]$

Target 2 -

$E(R_A) = 1.005\,E(R_B)$, $\sigma_A = 1.01\sigma_B$, $\sigma_p = 1.2221\sigma_B$ and $V > 0$

$C^*_{\pi B} = 0.835 - 0.193[E(\pi^*) \quad /VE(R_A)] + 0.001V + 0.008[VE(R_A)/(E(\pi^*) \quad -0.005E(R_A$

Footnotes

1. Smith (8) does some work in this area as part of the portfolio revision problem. However, he uses a rate of return approach and focuses on the problems within the context of security portfolio management.

2. Several authors 1 , 3 , 4 , 7) have valued assets on a dollar basis in a portfolio framework. However, the rate of return or efficiency approach to portfolio theory has generally dominated the approach based on levels.

3. If there were a 1 to 1 relationship, then investors would take the most beneficial opportunities from both markets and the segmentation would disappear.

4. The exact shape of PP will depend on the various alternatives considered in this range.

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