Valuation and the Risk of Ruin

by

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INTRODUCTION

In 1966, Miller and Modigliani (hereafter MM) (1966) attempted to test a valuation model for the electric utility industry and to determine the cost of capital for that industry. This test was based on their proposition that the value of the firm is the present value of the stream of expected operating earnings corrected for the effect on valuation due to the tax deductibility of interest, presented in earlier papers (MM(1958) (1963)).

That study was criticized by Crockett and Friend (1967), Gordon (1967), and Robichek et al (1967). These comments were primarily concerned with the testing procedures and interpretation of empirical results.[1/]

Subsequently, however, substantial questions have been raised concerning the lack of consideration of ruin or bankruptcy.[2/] These studies suggest that by ignoring the risk of ruin, MM's empirical work must be considered flawed. One must question such an assertion, however. Since valuation is a concept about the way the market views the securities of firms, one should be concerned with the way the market perceives the risk of ruin of the firm and then uses that perception in the valuation decision. In formulating their model, MM assumed away the problem. The question of risk of ruin, however, deals not just with the existance of fixed charges from the issuance of debt but also with the stability of income and the levels of reserves liquid enough to meet fixed charges when due. Thus, in making the valuation decision, the market not only is analyzing the earnings stream of the firm but also the likelihood of the firm maintaining that stream. That assessment will be dependent on the level of fixed charges (including the presence of debt), the stability of the income stream, <u>and</u> the level of liquid reserves available to absorb variations in the income stream. Thus an analysis of the role of the risk of ruin in the valuation process must treat each of these concerns. This paper presents such a framework for analysis by developing an alternative model in the MM framework which satisfies the criticisms of the previous MM model. An analysis of a group of vulnerable firms and relatively safe firms is then presented using this alternative model.

THE VALUATION MODEL

In developing their valuation proposition, MM suggest that the value of the firm is the present value of the future stream of after-tax net operating income, assuming that all earnings are paid out to security holders, plus the benefits of debt due to tax deductibility of interest. Thus, the market value of a firm, V_{firm}, would be:

$$V_{firm} - TB = \frac{X_1(1-T)}{1+\rho} + \frac{X_2(1-T)}{(1+\rho)^2} + \frac{X_3(1-T)}{(1+\rho)^3} + \ldots \qquad (1)$$

where: X_t = net operating income before financing charges in period t

ρ = after-tax default-free capitalization rate of an all equity firm in that risk class

T = corporate tax rate

B = market value of debt

If it is assumed that the debt is perpetual, that the distribution of earnings is stationary and independent between periods, and that the distribution of earnings after taxes is identical for firms in a homogeneous risk class,[3] the expected value E(X) can be used to approximate the stream of X_t. Thus equation (1) reduces to the usual MM proposition:

$$V_{firm} - TB = \frac{E(X)(1-T)}{\rho} \qquad (2)$$

It is proposed that when there is a risk of ruin, this is not the correct valuation procedure.[4] If there is a possibility of ruin, it cannot be assumed that the distributions are independent between periods but rather the distribution of earnings is dependent from one period to the next.

The distribution of earnings in period 1 is the usual marginal distribution referred to by MM with mean E(X). However, the probability density function of period 2 would not be the same marginal distribution. If an outcome occurred in period 1 which resulted in ruin for the firm, then $X_2 \leq 0$. A similar argument can be advanced for the third and subsequent periods with the earnings distribution being dependent on the firm's not experiencing ruin in both period 1 <u>and</u> 2 thereby reaching period 3, and so on. Thus the distribution of earnings in each period is not the same; therefore using an identical expectation for all periods is incorrect. What is true is that the <u>conditional</u> distributions are identical. That is, the distribution of earnings in each period would be like the marginal distribution in period 1 <u>given</u> that the firm does not experience ruin prior to that period. So, substituting the identical expectation into equation (1) is not the proper form for the valuation of the firm if there is a non-zero risk of ruin. The first and subsequent terms on the right side of the equation should have this general form:

$$\frac{E(X_t|M=t-1)(1-T)Pr(M=t-1) + E(X_t|M\neq t-1)(1-T_G)Pr(M\neq t-1)}{(1+\rho)^t} \qquad (3)$$

where M represents survival to period t. That is, if the firm survives period t-1, then the investors expect to receive $E(X_t|M=t-1)$ in period t.

If the firm is ruined in period t-1, then investors expect to receive $E(X_t|M\neq t-1)$ in period t. This term represents the salvage value of the firm or its securities less reorganization or bankruptcy costs. If these funds represent a gain they are taxed at the capital gains rate, T_G, otherwise there is no tax. For the purposes of this investigation, it is assumed that any salvage value will be absorbed by the cost of reorganization or liquidation as it allows a tractable solution. This assumption is not too unrealistic, however, because if investors do eventually salvage something, it may be so small and take so long to recognize that its present value can be considered to be zero.[5/] A typical example of this situation can be found in the experience of railroad investors. The term in expression (3) then reduces to:

$$\frac{E(X_t | M=t-1)\,Pr(M=t-1)\,(1-T)}{(1+\rho)^t}$$

and equation (1) becomes:

$$V_{firm} - TB = \frac{E(X_1 | M=0)\,Pr(M=0)\,(1-T)}{1+\rho} + \frac{E(X_2 | M=1)\,Pr(M=1)\,(1-T)}{(1+\rho)^2} + \dots$$

$$+ \frac{E(X_t | M=t-1)\,Pr(M=t-1)\,(1-T)}{(1+\rho)^t} + \dots \qquad (4)$$

Since the conditional distributions of the earnings, $E(X_t | M=t-1)$, are the same in each period, there is a constant, $E(X)$, which can be substituted in such a way that

$$V_{firm} - TB = E(X)(1-T) \sum_{t=1}^{\infty} \frac{Pr(M=t-1)}{(1+\rho)^t} \qquad (5)$$

It is contended that this equation represents a more realistic valuation procedure if the possibility of ruin exists.[6/]

ESTIMATION OF THE RISK OF RUIN--
THE CASE FOR MAXIMUM RISK EXPOSURE

It is obvious from equation (5) that the likelihood of survival or not being ruined for each period must be determined. Bawa (1972), Wilcox (1971a) (1971b), and others argue that the probability of ruin is a function of first passage time or expected time to ruin. This suggestion assumes that ruin is certain, so the only quantity of interest is expected time to ruin which would define when the truncation of the series is expected to take place. However, it can be shown that if the stochastic process under consideration is a positive drift process either without a barrier or, as is more likely, with a moving barrier, there is a non-zero probability of never being ruined; thus ruin is not certain.[7/] The distribution of first times to ruin has an infinite variance; therefore, making it an inappropriate measure for the risk of ruin.[8/] Since firms are assumed to be profit making enterprises, it can be shown that they will follow a positive drift stochastic process. Thus a different measure for the risk of ruin is needed.

Such a measure is provided using collective risk theory as developed by Seal (1969), Segerdahl (1955), Beard et al (1969), and Cramer (1955). Briefly, if some pool of resources, U_0, are available to prevent ruin, then ruin occurs if at some time these reserves and accumulated variable profit (revenue less variable costs) have been depleted by fixed costs. Since variable profit is a random variable, which is a function of two other random variables (the number of variable profit arrivals and the size or value of each arrival), the value of this pool of resources at any point in time, U_t, is the result of a stochastic process. Thus, the probability or risk of ruin, ε, is $\Pr(U_t<0) = \varepsilon$.

It should be emphasized that ruin as defined here does not mean bankruptcy. Ruin is defined as that state where the firm cannot be self-sustaining in the sense of needing an infusion of outside funds to maintain operations. Several options are open to a firm "ruined" according to this definition. New equity may be sold (if a buyer can be found), fixed assets liquidated, requests for debt extensions made, or some third party such as a governmental agency may be approached for funds. Such a process is essentially institutional in nature and does not lend itself to this type of mathematical modeling.

From a result in Beard et al (1969, p. 52-53), it can be shown that the risk of ruin at some point in time, t, can be determined as:

$$\Pr(U_o + \sum_{i=1}^{N(t)} VP_i - FC \cdot t) < \epsilon \tag{6}$$

where: VP_i = Variable profit for the i^{th} arrival

FC = Fixed cost rate per period including fixed financing charges

$N(t)$ = Number of arrivals in time t

Rearranging and standardizing using the central limit theorem:

$$\Pr\left(\frac{\sum_{i=1}^{N(t)} VP_i - \bar{n}\alpha_1 t}{\sqrt{\bar{n}\alpha_2 t}} < \frac{-U_o + FC \cdot t - \bar{n}\alpha_1 t}{\sqrt{\bar{n}\alpha_2 t}}\right) = \epsilon \tag{7}$$

where: $\bar{n}$ = expected variable profit arrival rate

α_1 = mean of the variable profit jump size distribution

α_2 = second moment about the origin of the variable profit jump size distribution

U_o = pool of resources available to prevent ruin

$\bar{n}\alpha_1 t$ = expected variable profit in (0,t)

$\bar{n}\alpha_2 t$ = variance of the variable profit in (0,t)[9]

If Y_ε is a standard normal variate and the solution of equation:

$$\epsilon = \phi(-Y_\epsilon)$$

then $Y_\epsilon = 2.326$ would be a probability of ruin = 0.01 and a value of $Y_\epsilon = 3.090$ would be a probability of ruin = 0.001. Increasing values of Y_ϵ indicate a decreasing risk of ruin. Thus, Y_ϵ is a safety index and is a measure of the risk of ruin. In the general case, the value of the safety index should be skewness of the distribution using the Cornish-Fisher (Normal Power) expansion as shown in Kauzzi and Ojantaken (1971) and Berger (1972) [3].

Using that result, the value of the safety index is determined as:

$$\frac{-U_o + FC\cdot t - \bar{n}\alpha_1 t}{\sqrt{\bar{n}\alpha_2 t}} = - Y_\epsilon - \frac{1}{6}\ \gamma_1(Y_\epsilon^2 - 1) \tag{8}$$

where:

$$\gamma_1 = \text{skewness factor} = \frac{t\ \bar{n}\ \alpha_3}{(t\ \bar{n}\ \alpha_2)^{3/2}}$$

By solving the quadratic expression for the positive root of Y_ϵ, the safety index at anytime, t, can be determined:

$$Y_\epsilon = \left[\left(\bar{n}\alpha_2 t + \frac{2\alpha_3}{3\alpha_2}\left(U_o + (\bar{n}\alpha_1 - FC)t + \frac{\alpha_3}{6\alpha_2}\right)\right)^{\frac{1}{2}} - (\bar{n}\alpha_2 t)^{\frac{1}{2}}\right]\left(\frac{\alpha_3}{3\alpha_2}\right)^{-1} \tag{9}$$

This model is superior to the first passage time method in that while both models are effective when ruin is certain (fair and negatively-biased games), the present model does not breakdown for positively-biased games when ruin is not certain.[10/] Thus evaluating Y_ϵ at any time, t, and consulting a normal probability table to estimate the risk of ruin, ϵ_t, the probability of survival in equation (5) can be determined as $(1-\epsilon_t)$.

However, it is evident that the probability of ruin varies from one period to the next. Thus, in order to employ equation (5) to determine the role of the risk of ruin in valuation some further assumptions are needed to obtain a tractable solution. To do this, one must choose some time or set of times which might be considered in the valuation process. It is suggested that investors are interested in determining the maximum exposure to ruin and when it occurs. That is one wants to determine that point in time where:

$$\frac{d\int_{o}^{N(t)} U_t dt}{dt} = 0$$

This suggestion is not unusual. Bond rating agencies claim to look at just such a point. Moody's for example states that:

> Since ratings involve a judgment about the future, on the one hand, and since they are used by investors as a means of protection on the other hand, the effort is made when assigning a rating to look at the 'worst' potentialities in the 'visible' future rather than solely at the past record and the status of the present.[11]

Thus, such an assumption about the valuation process seems appropriate.[12]

Determining the probability of ruin at the riskiest point is consistent with maximizing ϵ with respect to t. Using equation (9), taking the derivative of the safety index with respect to t and setting the derivative to zero gives the time of maximum exposure to ruin as:[13]

$$t_{rp} = \frac{Y_{\epsilon}^{2}\bar{n}\alpha_2}{4(\bar{n}\alpha_1 - FC)^2} \qquad (10)$$

Determining the time to the most dangerous point is important if the probability of survival at the riskiest point is used to adjust the earnings variable in each period. If this point occurs at some distance from the present, an event that occurs far in the future and is not heavily weighted would be used to adjust earlier earnings thus understating the quality of the expected earnings stream.

Substituting this result into equation (9) gives the probability of ruin at the riskiest point or the maximum exposure to ruin:

$$Y_{\varepsilon_{rp}} = \sqrt{\frac{12U_o(\bar{n}\alpha_1 - FC) + 2\alpha_3(\bar{n}\alpha_1 - FC)}{3\bar{n}\alpha_2^2 + 2\alpha_3(\bar{n}\alpha_1 - FC)}} \tag{11}$$

From this equation, the maximum probability of ruin, ε_{rp}, or minimum probability of survival, $(1-\varepsilon_{rp})$, is determined. It is suggested that since investors not only assess the risk of ruin based on stability of the income stream and reserves available to avert ruin but also that they attempt to assess the maximum exposure to ruin, equation (11) should closely approximate investors' *ex ante* perceptions of the risk of ruin. The extent to which that is true is an empirical question to be investigated later.

In equation (5), therefore, the probability of survival in each period can be approximated by $(1-\varepsilon_{rp})$. That is, $Pr(M = t-1) = (1-\varepsilon_{rp})$ for all t. Since for a particular firm, $(1-\varepsilon_{rp})$ is a constant, equation (5) reduces to:

$$V_{firm} - TB = \frac{E(X)(1-T)(1-\varepsilon_{rp})}{\rho} \tag{12}$$

Thus, it is proposed that this model can be used to describe the valuation of a firm and to determine the way the market perceives the risk of ruin in making its valuation decision.

It also satisfies the criticisms leveled at the MM model concerning the implications for capital structure decisions in that it no longer is beneficial to issue unlimited debt as the original MM model would. While the firm receives benefits of the tax deductibility feature of interest, additional debt can increase the risk of ruin thereby offsetting these benefits. MM provided an *ad hoc* condition for determining the amount of debt in the capital structure by allowing debt up to a limit set by institutional factors. With this model, however, the impact of debt issuance can be determined depending on the level of available reserves and the stability of the income stream.

Now that the valuation model has been determined, it is necessary to investigate the risk of ruin in the valuation process. The model developed in this section will be used to investigate the way the market views the risk of ruin in the valuation process. However, equation (12) will not be used directly for the empirical testing. In testing their model, MM had to introduce a scale factor based on asset size because the error terms of the regression were heteroscedastic. To avoid a similar problem, the log form of equation (12) will be used:

$$\log(V_{firm} - TB) = - \log \rho + a.\log(E(X)(1-T)) + b \bullet \log(1-\epsilon_{rp}) + v \qquad (13)$$

where: a and b are constant coefficients

v = residual

The coefficient a can be interpreted as a measure of the relationship between the market determination of the expected value of the earnings distribution and the measure used to estimate the expected value based on past data. In such a fashion, it can be interpreted as the growth possibility for the firm as viewed by the investors as defined by MM. If investors view the firm as static (no growth or contraction) the value of a will be one. Values other than one indicate that investors view the firm as having investment possibilities greater than or less than the cost of capital depending on the direction of deviation from one.[14] Likewise, the value of coefficient b indicates the role of the risk of ruin in valuation as perceived by investors. Thus, equation (14) is used for the empirical investigation pursued here.

For the purposes of this study, a group of firms which would be expected to be relatively safe, electric utilities, are analyzed as well as a group of firms which would be considered riskier. Each group is analyzed as of December 31, 1971. There is no reason to expect any bias due to a particular date or set of dates.

RISK OF RUIN AND THE VALUATION OF ELECTRIC UTILITIES

First to be considered is a group of electric utilities similar to that used by MM (1966). Equation (13) must be modified, however. Gordon (1967) observed that since regulatory agencies set returns to equity, there is no beneficial effect of the tax-deductibility of interest. Thus, a post-tax model is needed.[15/] The valuation of a firm in a regulated industry is therefore:

$$V_{firm} = \frac{E(X)(1-\varepsilon_{rp})}{\rho} \tag{14}$$

Thus, equation (13) reduces to

$$\log(V_{firm}) = -\log\rho + a\cdot\log(E(X)) + b\cdot\log(1-\varepsilon_{rp}) + v \tag{15}$$

and is the form used in the estimation.

In defining the expected value of the earnings distribution, E(X), MM in their 1966 study had to estimate the expected earnings using two stage least squares with its attendent problems of multicollinearity. Here, however, the expected value of the earnings distribution per period will be assessed as determined for use in equation (7):

$$E(X) = \bar{n}\alpha - FC$$

In this case, however, fixed costs, FC, do not include financing costs. Since similar terms appear in both independent variables, it will be critically important to analyze for multicollinearity.

Of all the variables, the probability of survival is the most complicated. As previously described, it is not the probability of survival which is calculated, but rather its complement, the probability or risk of ruin. Beard et al (1969) demonstrates a method of measuring the risk of ruin based on the expected number of payments or bills received by the firm in a period of time, $\bar{n}$, the first three moments of the variable profit per bill distribution about the origin, the expected net income for a period, and the initial reserves available to the firm. For electric utilities the distribution of variable profit per bill for a class of service is first determined from a distribution of KWHR/bill for that class of service and applying the rate schedule appropriate for that class. This distribution is adjusted for the variable cost per bill for a class. The distributions for the classes of service are then combined to get the moments of the firm's distribution α_1, α_2, α_3 (the mean and the second and third moments about the origin). Expected net income is determined to be net operating income after interest but excluding preferred and common stock dividends. Finally, the initial reserves, U_0, are calculated according to the method outline in Beard et al (1969, p. 52-53) and Bierman et al (1975).

The value of the firm is defined as the market value of the firm's securities, i.e., the debt, preferred stock and common stock. Each of these quantities is determined as of December 31, 1971.

The value of the common and preferred stock is found by multiplying the number of shares of stock outstanding by the market price. An average of

December and January weekly closing prices is used to remove purely short-term randomness in price quotations.[16]

The market value of debt is similarly determined by taking the book value of each series of debt and multiplying by the market price, expressed as a percentage of $100. The quotations of market value were also obtained from Moody's Public Utility Manual.[17]

It can be seen that the measurement for the firm is probably subject to error. There is no reason, however, to believe that these errors are not purely random and will give a systematic bias to the estimates.

Firms in the sample were chosen from a listing of class A and B utilities required to report to the Federal Power Commission and obtain at least 90% of their revenues from electric services. Firms were chosen on a wide geographical basis, representing each region of the country, and encompassing a large range of sizes so as to not introduce any bias due to size or location.

Table 1 presents a listing of the firms, the safety index and probability of survival at the time of maximum exposure to ruin as well as the time to this maximum exposure for the firms in the sample.[18] Since most firms exhibit essentially no risk of ruin using this method, it is necessary to use a dummy variable for analyzing the role of the risk of ruin in the valuation of electric utilities. The dummy variable to be used is the safety index which does provide a measure of the risk of ruin for the firm. The regression model is as follows:

$$\log V_{firm} = - \log \rho + a \log(E(X)) + b \log(Y_{\varepsilon_{rp}}) + v.$$

Table 1. The Safety Index, Probability of Survival and Time to the Most Dangerous Point for Electric Utilities--By Firm

Firm	Safety Index, $y_{\varepsilon_{rp}}$	Probability of Survival, ε_{rp}	Time to the Most Dangerous Point, Year
1. Duquesne Light	626	1.0	5.4
2. Kansas Gas and Electric	621	1.0	24.0
3. Southwestern Public Service	398	1.0	0.26
4. Ohio Edison	374	1.0	5.9
5. Utah Power and Light	237	1.0	2.8
6. Detroit Edison	147	1.0	0.97
7. Ottertail Power	119	1.0	2.7
8. Kentucky Utilities	115	1.0	2.76
9. Central Maine Power	109	1.0	1.8
10. Southern California Edison	72	1.0	0.26
11. Hawaiian Electric	63	1.0	0.62
12. Boston Edison	61	1.0	3.8
13. Tampa Electric	60	1.0	0.52
14. Portland General Electric	45	1.0	0.50
15. Public Service Co. of New Hampshire	43	1.0	0.8
16. Puget Sound Power and Light	29	1.0	0.05
17. Carolina Power and Light	20	1.0	0.08
18. Upper Peninsula Power	12.2	1.0	1.07
19. Edison Sault Electric	1.119	0.87	0.49
20. Newport Electric	0.1009	0.54	0.005

The results of the regression are shown with the corresponding t-statistics:

$$\log(V_{firm}) = \underset{(6.10)}{-\,2.7081} + \underset{(38.6)}{1.01}\log(E(X)) + \underset{(0.40)}{0.01}\log(Y_{\varepsilon_{rp}})$$

The model explains the value of the firm very well with the standard error of the regression only 0.67% of the mean. Before reviewing the results of the regression, the error terms must be checked for homoscedasticity. Using the technique of Glejser (1969), the residuals were found to be homoscedastic, normally distributed, and uncorrelated with the independent variables verifying that there is no remaining systematic unexplained variation. These results also show that virtually all of the variation in the dependent variable is explained by the independent variables, leaving essentially no unexplained variation. Likewise an analysis of the correlation matrix reveals insignificant multicollinearity.

Examining each of the coefficients shows that the value of a and the constant are significantly different from zero at the 1 percent significance level. They hypothesis that a is equal to one cannot be rejected at the 1 percent level of significance. Although the coefficient b has a positive value, the hypothesis that it is equal to zero cannot be rejected at the 1 percent significance level. The hypothesis that the constant term is zero can also be rejected at the 1 percent confidence level. The value of the antilog of the constant term is 6.67 percent and can be taken as an estimate of the default-free cost of capital for the electric utility industry. Since the risk of ruin does not enter the valuation process, $\hat{\rho}$ is also an estimate of the overall cost of capital.

The results of the preceding analysis lead to several interesting conclusions. One of the most important findings is that all of the variation in value except for a very small amount of random disturbance is explained by variation in current net operating income as suggested by MM. For regulated utilities, there is no other systematic influence on the valuation of these securities. Besides supporting the MM hypothesis such a result also supports the requirement that the market for the securities of regulated utilities be an efficient market.[19/]

These results also demonstrate that several important effects result from the regulatory process. One is to eliminate from the market as of the date investigated any perceived risk of ruin, contrary to the beliefs of regulatory and security evaluation authorities. Furthermore, finding the coefficient of the log(E(X)) term equal to one shows that investors view regulated utilities as static with stationary earnings distributions as previously discussed. This result verifies evidence of Gordon's that investors do not see growth in the industry resulting from projects returning more than the cost of capital.[20/]

The regression performed here also resulted in an earnings capitalization rate of 6.67 percent. This value falls within the bounds expected. The lower bound would be the prevailing level of long-term Treasury securities which the market looks at as being default free. According to the Bond Market Monthly Review published by Salomon Brothers, the yield on representative long-term Treasury securities outstanding on December 31, 1971, was 6.00 percent. Investors required a very small risk premium of only 67 basis points (.67 percent) to compensate for earnings fluctuations as well as the fact that electric utility securities are not as easily marketable as Treasury securities.

The upper bound is a function of regulation. If the regulatory commissions do not allow more than a set return on rate base, which is nearly the same as total assets, then no investor would capitalize earnings of the industry higher than that number. The average return on rate based earned by the firms in the sample for 1971 is 7.3 percent.[21/] Thus, the cost of capital would be less than or equal to 7.3 percent adjusting slightly downward for differences between rate base and total assets. For example, if the rate base were 90 percent of total assets, the two returns (7.3 percent vs. 6.7 percent) would be the same. This result implies that regulatory commissions need set rates such that the return on total investment is slightly above the prevailing rate on long-term Treasury securities to insure that the electric utilities can raise capital.

RISK OF RUIN AND THE VALUATION OF RISKY FIRMS

Since electric utilities exhibit virtually no risk of ruin, it is not surprising that the risk of ruin does not enter into the valuation process. Although the results are consistent with the model developed here, they do not verify the usefullness of such an approach. Thus, a sample of more risky firms is needed.

A group of firms in such areas as airlines, oil drilling, and the like were assembled to analyze the impact of the risk of ruin on valuation. Unlike electric utilities, however, these firms do not publish nor even collect the necessary data to estimate the moments of the variable profit distribution using the method outlined in the previous section. Another method of estimating this distribution is therefore needed.

Before estimating the moments of the variable profit distribution, it is necessary to obtain measures of the variable profits for each firm using a method developed by Girod <u>et al</u> (1972). First, using ten year's data from the COMPUSTAT

Quarterly Tapes, dollar sales for four adjacent quarters are plotted against profits before interest and taxes for the same quarters. A simple linear regression is used to fit a least-squares line through the intersecting points. The slope of the line obtained represents the average variable profit rate for the firm; i.e., the incremental profit contribution expressed as a percentage of sales. The point intersecting the vertical axis can be taken as an estimate of average fixed operating costs.[22/]

Because there are only four observations for each regression, the results are strongly influenced by the position of each observation--particularly by the fourth quarter when a certain amount of "window dressing" is practiced. To correct the data to a more normal long-term level, the logarithms of the intercepts for each regression are plotted. A least-squares line is fit to obtain the long-run growth function of fixed costs. These fixed costs are then added to actual profit before interest and taxes to obtain variable profit.[23/]

Once the observations of variable profits on a quarterly basis are obtained, the model generating the series must be determined so as to obtain the moments of the variable profit distribution. The sample autocorrelations for each firm are first examined to determine the appropriate model for that series.[24/] The criterion of model selection followed in this study has been the representation of each series in the most parsimonious form that is consistent with its stochastic structure. Parameters included in the models are those for which estimates are significant or which are required to eliminate serial correlation in the residuals. Thus, the procedure has not been to minimize the variance of prediction errors but to obtain the simplest adequate representation. The χ^2

value for the appropriate degrees of freedom is determined at the 5% level of significance under the null hypothesis of zero serial correlations in the residuals of the model. The values of the Q statistic support the hypothesis that the individual series have been reduced to white noise series in each case.[25/]

In this manner, the distribution of variable profits is obtained. The mean, α_1, can be determined from the parameters of the fitted model and the other moments are a function of the moments of the residual distribution. Box and Jenkins (1970, ch. 3) demonstrate how to obtain the moments of the process' distribution from the model parameters and the moments of the residuals. Once these moments are generated, equation (11) can be used to determine the risk of ruin for a given firm at the riskiest point for that firm.[26/] Table 2 gives the safety index, probability of survival, and time to the riskiest point for the firms in the sample. Again, the riskiest point for virtually all of the sample is close to the present.

In a similar manner, the expected value of the earnings distribution is determined by fitting a suitable model to the time series of earnings before interest and taxes so as to obtain a white noise series.[27/] The parameters of the fitted model are also used to obtain the expected value of the earnings distribution, E(X). It is expected that this method not only more closely approximates the formation of expectations in the market but also avoids the multicollinearity problem in the two stage least squares method which plagued MM.

The value of the firm is determined in a similar fashion to that described for electric utilities. As before, there is no reason to believe that any measurement errors are not purely random.

Table 2. The Safety Index, Probability of Survival and Time to the Most Dangerous Point for a Selected Group of Firms--By Firm

	Firm	Safety Index, $Y_{\varepsilon_{rp}}$	Probability of Survival, ε_{rp}	Time to the Most Dangerous Point, Year
1.	Fairchild Camera	0.350	.637	5.2
2.	Trans World Airlines	0.670	.748	0.28
3.	Sanders Assoc.	0.789	.785	5.2
4.	Allied Supermarkets	0.90	.816	0.75
5.	Bowmar Instruments	1.00	.841	0.64
6.	World Airways	1.13	.871	0.31
7.	Gordon Jewelry	1.14	.873	0.51
8.	National Airlines	1.27	.898	0.23
9.	Milgo Electric	1.50	.933	1.3
10.	Dixilyn Co.	1.68	.953	2.0
11.	Sprague Electric	1.76	.960	0.8
12.	Xtra Inc.	1.96	.975	0.3
13.	North American Coal Co.	2.44	.9927	0.9
14.	High Voltage Engineering Co.	2.52	.9941	4.2
15.	Reading and Bates Co.	2.55	.9946	0.08
16.	Western Airlines	2.60	.9963	0.31
17.	Tandy Corp.	2.696	.9965	0.69
18.	Cenco Inc.	3.22	.9994	0.79
19.	Alaska Interstate	3.29	.9995	3.46
20.	Grolier Inc.	4.73	.99995	5.2

The results of the regression are shown with the corresponding t-statistics:

$$\log(V_{firm} - TB) = -\ 0.576 + \underset{(5.98)}{0.664} \log(E(X(1-T))) + \underset{(2.2)}{2.07} \log(1-\varepsilon_{rp})$$

The model explains the value of the firm well, although not as well as the utility sample, with a coefficient of determination of 0.67. Since this group of firms is not a homogenous industry, such a result is not surprising. Again, using Glejser's test (1969) the residuals were found to be homoscedastic, normally distributed, with no serial correlation. Likewise an analysis of the correlation matrix reveals insignificant multicollinearity.28/

Examing each of the coefficients shows that the value of a is significantly less than one but greater than zero. If this group of firms were an industry, investors would view it as a contracting industry; i.e. one in which the long-run expected earnings are less than the expected value of the current earnings distribution. Such a result is not surprising given the high risk nature of the group (in fact two members of the sample have disappeared as separate entities since 1971).

More importantly, the coefficient b is significantly different from zero, unlike that obtained for the electric utility sample. While the estimated value of b is somewhat higher than the predicted value of 1, it is not significantly different from 1 at the 95% confidence level. This result suggests that not only is the risk of ruin measure developed here an adequate measure of the ex ante risk of ruin perceived in the market but also the risk of ruin at the riskiest point is a useful proxy to the actual risk of ruin determined by market participants.29/

Finally, the value of the antilog of the constant term is 0.56 which can be taken as an estimate of ρ. This number is quite a bit higher than for electric utilities but seems reasonable. The average β (the usual capital asset pricing model measure of market risk) for the electric utilities in this sample as of December 31, 1971, was 0.31 while the average β for the sample of risky firms as of the same date was 2.07. Thus, the firms in the risky sample not only have higher risk of ruin but also higher market risk which would correspond to the higher $\hat{\rho}$ observed.

The results obtained here lead to several interesting conclusions. One of the most important findings is that even with a set of risky firms, the variation in value is explained by variations in the expected net operating income adjusted for the probability of survival, as suggested by MM.[30] Since not all of the total variation in value is explained, some unsystematic variation is associated with each firm. However, it is shown that there appears to be no other systematic influence on value.

Another conclusion of the analysis is that, while the MM hypothesis is supported, the market is not indifferent to the levels of fixed charges.[31] Thus, the value of two firms with identical fixed operating costs can be influenced by the debt/equity ratio beyond the point where the fixed charges associated with certain levels of debt lead to a significant risk of ruin as discussed by Wrightsman and Horrigan (1975).[32] It can, therefore, be seen that although the MM theoretical analysis is sound, they indeed chose the sample well for their empirical work. However, using the modification shown here, their analysis still is useful in describing the valuation process of the firm when there is nonnegligible risk of ruin.

SUMMARY AND CONCLUSIONS

This paper has investigated the role of the risk of ruin in the valuation process. A modification of the MM valuation model is developed which incorporates the risk of ruin directly in the model. This risk of ruin is determined using collective risk theory where it is a function not only of fixed charges from debt issues but also the stability of the income stream and the reserves available to prevent ruin. Using this model it is possible to investigate investors' perceptions of the risk of ruin in the valuation of a firm's securities. An analysis of electric utilities in 1971 determined that investors did not anticipate any risk of ruin in the firms and as a result valued the securities of electric utilities based only on net operating income, as MM suggested, as well as to suggest that criticism of the MM empirical work for failing to take default risk into account is not warranted.

An analysis of a group of risky firms, on the other hand, shows that investors systematically take the risk of ruin into account when the risk of ruin is significant. However, even in the presence of significant default risk, the MM valuation proposition can obtain when adjusted for this risk of ruin.

Finally, the method of estimating the risk of ruin used here is found to be a useful measure of the risk of ruin as determined by market participants. Obtaining such a measure allows further research into the role of various financial management decisions on security valuation through the impact on the risk of ruin.

FOOTNOTES

*Assistant Professor of Finance, The Wharton School, University of Pennsylvania. The author would like to acknowledge the helpful suggestions of Irwin Friend, Marshall Blume, and Michael Adler as well as the cogent comments of an anonymous referee on an earlier draft. Financial support by the Rodney L. White Center for Financial Research, University of Pennsylvania, is also gratefully acknowledged.

[1] Gordon (1965), for example, criticized the method of introducing growth in the MM model and suggested that there is no tax benefit from debt for electric utilities. He also asserted that MM used the wrong estimate of expected earnings as the independent variable.

[2] Stiglitz (1969), for example, examined the MM model and determined that for an investor to replicate the pattern of returns from a firm when the possibility of ruin exists, the individual must negotiate a loan on the same basis as a firm, i.e., limited liability using stock as collateral. Tinsley (1972) and Borch (1968) have also discussed the role the risk of ruin plays in valuation.

[3] MM's arbitrage procedure necessitated the assumption of homogeneous risk classes. Hamada (1969) demonstrates that the MM procedure holds without the assumption of homogeneous risk classes or arbitrage process. Instead a switching operating involving stocks with the same market price of risk is utilized.

[4] In the literature two terms, bankruptcy and ruin, are used interchangeably. Bankruptcy implies reorganization of the firm under the guidance of judicial proceedings and in accordance with Federal bankruptcy laws. In certain cases the firm is not reorganized by a court; instead, the current investors merely do not participate in any earnings of the firm, thereby ruining the current investors but maintaining the existence of the firm. The stream of earnings to current investors can be interrupted from added costs of reorganization due to financial distress even though the judiciary may not be involved. In this paper the term ruin is used to signify any truncation of the infinite series for whatever reason. Bankruptcy is reserved for those situations in which legal bankruptcy is involved. Thus, bankruptcy is an extreme form of ruin, but ruin can occur without declaration of bankruptcy. One way ruin can occur is if the firm defaults on interest payments causing an interruption of the earnings stream to the security holders (Higgins and Schall(1975) provide an in-depth discussion of this concept of bankruptcy).

[5] See, for example, V.S. Bawa (1972).

[6] This result differs from that developed by Scott (1976) who suggests that the risk of ruin enters the valuation process essentially as a default premium added to the default-free rate of interest. The approach used here appears preferable as no assumptions of institutional response to ruin are needed.

[7] See Parzen (1962, p. 118-121) and Gerber (1974) for discussion and proof of this proposition.

[8] Seal (1969, p. 114-115) has shown that the distribution of first times to ruin is asymptotically Normal. He also provides an example of the determination of the "risk of ruin" at some point of time. This determination requires a finite variance, however. With an infinite variance, the result is indeterminate. Likewise, such hypothesis testing as a test of the equality of several means, as described in any mathematical statistics text, cannot be accomplished. This test is for comparing distributions of first times to ruin to determine whether they are the same and cannot be done with an infinite variance.

[9] The moments of the variable profit distribution are derived in Beard <u>et al</u> (1969, p. 22-23) .

[10] It has been demonstrated in Gerber (1964) that this result obtains whether or not an upper barrier is present as long as the barrier, if present, is moving.

[11] Moody's (1972). This does not imply that Moody's or other rating agencies would necessarily be considering the same point as the stochastic process. In establishing bond ratings other factors enter in a non-uniform way such as differences in perceived marketability. Such a consideration is not included in the stochastic process.

[12] Another reason for preferring the risk of ruin at the riskiest point is that it can be shown that the value of Y_ε at discrete points is a function of firm size. The risk of ruin at the riskiest point, however, is independent of firm size and depends only on the distribution of variable profit Beard <u>et al</u> (1969, pp. 149-151) .

[13] It is easily shown that the relevant second-order conditions are also satisfied.

[14] Such a testing method was suggest by MM (1966, p. 348) . They were unable to use such a procedure because of the way growth is introduced into their model. Defining growth as done here allows the use of this method for removing heteroscedasticity as well as using the regression model to determine whether or not growth is perceived to exist by investors.

[15] Because of the unique aspects of utility regulation, Gordon (1967) claims regulated industries receive no tax benefits from debt and thus the after-tax earning, should be the independent variable and value of the firm the dependent variable. For if X^T is too large, the rate structures are adjusted to reduce X^T enough to result in the "proper" or fair rate of return.

After some discussion between Elton and Gruber (1971)(1972) and Gordon and McCallum (1972), it was decided that the post-tax model is correct when the following view of regulation is true:

> ...that regulators have a rate of return they believe is appropriate for the firm and that they adjust prices, so that expected long-run earnings after tax but before interest yield this rate. The regulators do not adjust (these prices) to eliminate period to period variations in earnings whether these changes are caused by changes in exogenous variables or management actions. (Elton and Gruber (1972, p. 1151)).

For the purposes of this paper, the post-tax argument, as suggested by Gordon will be adopted.

[16] If a particular series of quotations was not available, the price was established on the basis of the effective yield of other stocks for the same firm of a similar series. For example, assume three series of preferred stock with stated yields of 3%, 4%, and 5% were issued by a firm, all of which had a par value of $100. If the 3% series had a price of $37.50 per share and the 5% series had a price of $62.50, this would suggest an effective yield of 8%. If the price of the 4% was not available, it would be assumed to be $50 a share.

[17] Values of all debt issues were available for most firms. In those instances where the price of a particular issue was not available, the market value was estimated on the basis of yields reported by Moody's for comparably rated issued of equal maturity. Book value was used for notes payable since they are maturing within one year.

[18] Examination of the time to maximum exposure indicates that for most firms, this time is near enough to the present so as to not bias the results.

[19] Either the switching method of Hamada or the arbitrage method of MM previously discussed require an efficient market for the securities involved. An extensive discussion of the characteristics and evidence of efficient markets can be found in Fama (1970).

[20] Gordon (1967, p. 1277).

[21] Federal Power Commission (1971, pp. 745-47).

[22] It should be noted that these fixed costs do not represent a shut-down minimum level but rather the continuing fixed costs (including the fixed portion of semivariable costs) incurred during normal operating conditions.

[23] Using this methodology, Girod et al (1972) demonstrate the ability to estimate the variable profit structure for General Motors and Ford within 0.1 per cent which is sufficient for the purposes of this study.

[24] Autocorrelation functions and fitted models used here are too voluminous to be included but are available from the author.

[25] Positive drift processes are submartingale in that they are a white noise series around a positive mean. A white noise series is a random sequence which is independent, normally distributed with zero mean and constant variance. See Box and Jenkins (1970, ch. 3) for a discussion of the properties of linear stationary models. The Q-statistic is explained in Box and Jenkins (1970, chapter 8).

[26] It should be noted that since the report of quarterly earnings are filed at somewhat fixed intervals the Poisson assumption is only approximately met. However, there is negligible impact on the estimation procedure.

[27] See Griffin (1975) for a comprehensive treatment of this proposition.

[28] A correlation analysis of the use of NOI (1971) versus the expected value of the earnings distribution, E(X), as determined here shows that value is much more highly correlated with E(X) than NOI (1971) (.982 for E(X) versus .429 for NOI(1971)). This result suggests that the methodology used in constructing E(X) is closer to that used by the market than assuming that net operating income follows a random walk as would be assumed using NOI (1971).

[29] Correlation analysis also shows that the risk of ruin at the riskiest point is highly correlated with another measure of risk, the firms $\underline{\beta}$ as determined from the CAPM.

Such a result tends to also uphold the position that there is a systematic component to the risk of ruin rather than having it considered as purely non-systematic risk as is usually assumed.

[30] Recently Milne (1975) has clarified the Stiglitz conflict with MM to show that operating under the same assumptions, the Stiglitz arguments correspond to those of MM.

[31] While these results conflict with the recent assertion by Miller (1976) that bankruptcy has no impact on valuation, they do suggest that the effect of bankruptcy probability is not a major determinant as previous writers have asserted.

[32] The capital structure can be related to the firms characteristics through equation (11). The optimal debt to equity ratio will that level of interest which results in ε_{rp} essentially zero as viewed by the market. While the specification of a decision rule for optimal capital structure is beyond the scope of this paper, it offers a promising path for future research.

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