Spectral Analysis of Security Market Prices and Yields in Fundamental Time

by

John R. Percival

Working Paper No. 10-76

RODNEY L. WHITE CENTER
FOR FINANCIAL RESEARCH

University of Pennsylvania

The Wharton School

Philadelphia, Pa. 19174

The literature of Economics and Finance is replete with time series analyses of security markets data, which are performed using spectral and cross-spectral methods.[1] These studies tend to differ from each other only in that they analyze slightly different time periods or slightly different yield and price data.[2] However, in spite of the fundamental similarity in method, the conclusions often differ markedly.

The stated goals of such studies are usually akin to: the discovery of harmonics (cycles) in security market data and the discovery of lead-lag relationships between series. In order to achieve these goals, the typical study engages in what could be called "peak hunting" and "coherence-phase finding."

Peak hunting involves the process of estimating the power spectrum for each of the series to be analyzed and then finding those frequencies which appear to contain more power than others. A cycle corresponding to the frequency containing the greatest power is then often ascribed to the series.

Coherence - phase finding involves the process of first, estimating the cross-spectrum of two time series and then finding those frequencies which appear to exhibit more coherence than others.[3] A cycle corresponding to the frequency containing the greatest coherence is then often ascribed to both series. Then the phase angle at that frequency is examined to determine lead-lag relationships.[4]

Even though very similar data is being analyzed, the resulting peaks, coherences and phase angles seem to differ substantially from spectral study to spectral study. However, it is really not surprising that these substantially different results have been achieved when one considers the fact that significance testing is essentially non-esistent in spectral analysis and that, more often than not, a strong a priori hypothesis does not exist in these studies.

The general problems associated with not having a strong a priori hypothesis to test when using spectral methods have been developed elsewhere.[5] However, in spectral studies of security markets, the implicit a priori hypothesis must be that cycles of undetermined length exist in security market prices and yields. In order for spectral methods to be appropriate, the cycles must be thought to be recurrent, and regular with respect to chronological time (e.g. a recurrent cycle of exactly 52 weeks in length in weekly data).

There is a well accepted body of literature on the character of security markets which suggest that they are efficient. In an efficient market, after allowing for transactions costs, no such regular, recurrent cycles in chronological time could exist. If they did exist, arbitragers would attempt to profit from a knowledge of their existence and in so doing, insure that such cycles could not continue to exist. Given this perceived weakness in the a priori hypothesis and the absence of signifance testing in spectral analysis, one might conclude that previous spectral studies of security markets have found frequencies that contain more, but insignificantly more, power than others in the estimated power spectra. Furthermore, in the absence of a significant cycle in two series at a given frequency, any subsequent analysis at the coherence at that frequency is irrelevant.

In other words, in the absence of an accepted significance test it is only when the data appears to confirm a strong a priori hypothesis that one should accept that there are cycles in the data. Given the literature on efficient markets it is hard to accept that an *a priori* hypothesis suggesting the existence of regular cycles in chronological time is a strong one.

Spectral Analysis in Fundamental Time

As an alternative procedure to that outlined above, this paper will propose a strong *a priori* hypothesis involving the existence of cycles in security market prices and yields, in fundamental time rather than chronological or calendar time. This hypothesis will be tested using spectral methods in a way that overcomes the objections specified above. The hypothesis will be found to be confirmed for certain price and yield series.

The concept of fundamental time to be used in this paper involves the assumption that because of market efficiency the simple passage of calendar time is largely irrelevant in security markets. It is instead the rate of expansion and contraction of economic activity as a whole that determines the rate of expansion and contraction of security market prices and yields. Thus, cycles in security market prices will be related to cycles in macroeconomic activity and these cycles in macroeconomic activity are irregular in chronological time.

The idea of measuring time by the rate of occurence of fundamental events, in order to uncover properties of a time series that may be hidden when the series is sampled at regular chronological intervals has been more fully developed elsewhere.[6] Recently the concept of "operational", "transaction", or fundamental time has been used in empirical studies to demonstrate subordinated processes which may go unnoticed if a time series is sampled in calendar time.[7]

The general hypothesis to be tested in this paper is that certain security market price and yield series are related to National Bureau of Economic Research (NBER) cycles. Some of the series are assumed to be essentially coincident with NBER cycles and others are assumed to demonstrate lead-lag relationships with each other. However, since an examination of Table 1 reveals that NBER cycles are not regular with respect to calendar time, it should not be expected that cycles in security market prices and yields are regular with respect to calendar time.

This paper will present a new methodology which uses spectral analysis in order to establish the existence of cycles in time series and in order to demonstrate the existence of lead-lag relationships. However, the methodology presented here will be shown to be more consistent with the nature of cycles in economic time series than the methodology that has been used in previous studies.

NBER Cycles

The National Bureau of Economic Research has defined five complete (from peak to peak) business cycles that have occurred in the U.S. economy in the post World War II era. The dates of these cycles, designated I through V, are contained in Table I.

Table I

Cycle	Peak	Trough	Peak
I	November 1948	October 1949	July 1953
II	July 1953	August 1954	July 1957
III	July 1957	April 1958	May 1960
IV	May 1960	February 1961	November 1969
V	November 1969	November 1970	November 1973*

*November 1973 is actually not the NBER peak for Cycle V. At the time that this study was begun, the peak for Cycle V had not been determined. November 1973 was chosen due to data availability. However the data does not differ markedly from most early estimates of the cycle peak.

The length, in months, of the contraction and expansion in each of these cycles is given in Table II.

Table II

Cycle	Contraction	Expansion
I	11 months	45 months
II	13 months	35 months
III	9 months	25 months
IV	9 months	105 months
V	12 months	36 months

Methodology

The basic, underlying method of analysis used in this paper is spectral analysis. In order to adequately estimate a power spectrum, a large number of observations is essential.[8] In order to have sufficient observations it is necessary that data be available at relatively short intervals over a relatively long period. This is a problem inherent in any spectral study.

In this paper, the 25 years from November 1948 to November 1973 will be studied. However, in addition to analyzing observations chosen at daily, weekly or monthly intervals throughout and testing for cycles in calendar time, this study uses observations chosen at intervals which vary according to the length of the underlying NBER cycle.

Fifty observations per NBER cycle, twenty-five per contraction and 25 per expansion, are used.[9] Thus, 250 observations per series (five cycles multiplied by fifty observations per cycle) are used in the fundamental time analysis of this study. Table III shows the time interval (in integer number of weeks) per observation in each contraction or expansion which is to be used. While this sampling procedure may appear to be arbitrary, it is important to realize that once one makes the assumption that markets are efficient, this procedure is one step more formal than is sampling at constant calendar time intervals throughout the series. In order to make the procedure still more formal, one would need to be able to determine different rates of change in economic activity within the expansion and contraction of the cycle.

Table III

Cycle	Contraction	Expansion
I	11/25 months = 2 weeks	45/25 months = 8 weeks
II	13/25 months = 2 weeks	35/25 months = 6 weeks
III	9/25 months = 2 weeks	25/25 months = 4 weeks
IV	9/25 months = 2 weeks	105/25 months =18 weeks
V	12/25 months = 2 weeks	36/25 months = 6 weeks

When the power spectrum is estimated for a time series with observations spaced as in Table III, a peak at a frequency of .02 cycles per observation or a cycle length of 50 observations, must be accepted as consistent with the hypothesis that the time series in question contains cycles corresponding to NBER business cycles.

Spectral and Autocovariance Analysis

The typical shapes for the autocovariance function and power spectrum of an economic time series appear in Figures 1 and 2.[10] The shape of the autocovariance function indicates monotonically decreasing autocovariance as the length of the lag increases. The shape of the power spectrum indicates that virtually all of the power is concentrated at the lowest frequency. This spectrum indicates that a cycle longer than the time span covered by the data produces the series of perhaps more likely, the series is dominated by a time trend.

When trend in mean is eliminated from the series, generally the shape of the autocovariance function remains as in Figure 1 with only a change in scale. However, the shape of the power spectrum may change to something more like Figure 3. Figure 3 indicates varying levels of power for different frequencies. However, what is difficult to determine from Figure 3 is whether frequencies containing more power than others, contain significantly more. If a strong *a priori* hypothesis exists, a peak which appears to dominate other peaks and is consistent with the hypothesis might be interpreted as confirmation of that hypothesis.

Government Bond Yields

Weekly data on long term Government bond yields and Treasury bill yields was collected from the *Federal Reserve Bulletin* for the time period covering November 1948 through November 1973.[11] Initially, every fourth observation was then chosen to give 327 "monthly" (every four weeks) observations for each series. The autocovariance function and power spectrum were then estimated for each of the two series.[12] Both autocovariance functions had the general shape illustrated in Figure 1 and both power spectra had the general shape illustrated in Figure 2. As explained above, these shapes might be expected given the dominance of the upward trend in interest rates during the time period covered by the data.

The detrended autocovariance function and power spectra were then estimated for both series.[13] The power spectra for both conformed to the shape given in Figure 3. The power spectrum for the long term Government bond series did not appear to have any clearly dominant peaks. However, the power spectrum for the Treasury bills series did appear to have a dominant peak corresponding to a frequency of .017 cycles per month or a cycle length of about 60 months. There does not seem to be any obvious explanation for the existance of a cycle of 60 months or 5 years length in Treasury bill yields. In the absence of an a priori reason for such a cycle to exist, one is hard pressed to assess its significance.

Having established the calendar time relationships that exist in this Government bond and Treasury bill yield data an attempt was then made to isolate the fundamental time relationships. There exist a number of theories regarding the relationship between interest rates and business cycles. For example, it has been hypothesized that interest rates rise during an economic expansion with increasing demand for credit, relative to supply, and tend to decline during a contraction or recession. In order to find the relationship between NBER cycles and long term Government bond yields and Treasury bill yields, the procedure specified above for spectral analysis in fundamental time was used. From the weekly interest rate data, observations were chosen at the intervals specified in Table III. This sampling produced 250 unequally spaced (in calendar time) observations. The autocovariance function and power spectrum were then again estimated for each of the two time series, but now using 50 lags.

Given the sampling procedure specified above, if a time series contained a cycle corresponding to NBER cycles, one would expect the autocovariance function to appear as in Figure 4 and the power spectrum to appear as illustrated in Figure 5. The autocovariance function in Figure

4 reveals relatively highly negative autocovariance at a lag of 25 observations or ½ NBER cycle. It also reveals relatively high autocovariance at a lag of 50 observations or one complete NBER cycle. The power spectrum illustrated in Figure 5 contains a relatively large peak at a frequency of .02 cycles per observation or a cycle length of 50 observations.

However, the autocovariance functions of the Government bond and Treasury bill yield series with 250 unequally spaced observations were found to conform in shape to Figure 1. In addition the shapes of their power spectra were found to conform to Figure 2. It should be remembered that the underlying time series contain significant trend in mean. Therefore, the detrended autocovariance functions and power spectra were estimated for the unequally spaced Government bond and Treasury bill yield series.

Both of the detrended autocovariance functions in fundamental time were found to conform to the shape illustrated in Figure 4. The power spectrum for the detrended Treasury bill series in fundamental time was found to conform to the shape given in Figure 5 with a definite dominant peak at a frequency of .02 cycles per observation. The power spectrum for the Government bond series was also found to generally conform to the shape given in Figure 5 but the definite dominant peak was found to occur at a frequency of .01 cycles per observation or a cycle length of 100 observations (i.e. two NBER cycles). This result may indicate that the long term yields exhibit a tendency to "skip" a cycle in their conformation to NBER cycles. Thus, while data points which are 50 observations removed from each other are related, there is a tendency for data points which are 100 observations removed from each other to be even more closely related.

Using these results as an indication that Government bond yields and Treasury bill yields are systematically related to NBER cycles, one is then justified in examining the relationship between the two series for leads and lags in their conformation to NBER cycles. Therefore, the (cross) covariance function and cross spectrum were estimated for the unequally spaced (fundamental time), detrended Government bond and Treasury bill yield series.

If the two time series are coincident in their relationship to NBER cycles, one would expect a graph of their (cross) covariance function to appear as in Figure 6. Figure 6 reveals relatively high covariance at a lag of zero (coincident). In addition, it reveals relatively high negative covariance at lags of + 25 and - 25 observations (½ NBER cycle) and relatively high covariance at lags of + 50 and - 50 observations (one complete NBER cycle removed from each other.) If leads and lags exist between the two series in their conformation to NBER cycles, one would expect the peaks and troughs to occur at lags of other than -50, -25, 0 + 25, and +50 observations. However, assessing the existence of, and length of, leads or lags from the (cross) covariance function is difficult and subjective.

A preferable way to assess leads and lags between the two series in their conformation to NBER cycles would be to examine the measure of coherence and the phase of the transfer function, at the appropriate frequency, which are by-products of the estimation of the cross spectrum. The coherence is analogous to the coefficient of determination and measures the closeness of the relationship between the two series at a particular frequency. The coherence can be examined to help to confirm the existence of a cycle of a particular frequency in both series. The phase of the transfer function then measures the lead or lag at that frequency.

The (cross) covariance function of the unequally spaced (fundamental time) detrended series for Government bonds and Treasury bills conformed generally to the shape illustrated in Figure 6. The coherence at a frequency of .02 cycles per observation was found to be .89, indicating a strong relationship at that frequency. The phase angle indicated a lead of between 2 and 3 observations (remember these observations are not equally spaced in calendar time) for the Treasury bill yield series.

Thus, the empirical results for Government bond and Treasury bill yield series indicate that:

(1) there is some evidence of a fairly long cycle (about 5 years) in calendar time in Treasury bill yields.

(2) both long term Government bond yields and Treasury bill yields appear to conform to changing business cycle conditions.

(3) there is evidence that long term Government bond yields conform more closely to NBER cycles every other cycle.

(4) there is evidence that Treasury bill yields conform to changing business cycle conditions before long term Government bond yields are.

Term Structure Effects

The methodology developed in this paper could also be used to test for a relationship between the term structure of interest rates as reflected in the shape of the yield curve, and busines cycles. As a preliminary step and for a comparison of calendar time relationships with fundamental time relationships, the term structure data could also be examined for cycles in calendar time. The term structure variable which was

tested was created by substracting the Treasury bill yield from the long term Government bond yield at each observation point from the weekly series described above.
Every fourth observation was then selected which gave 325 "monthly" observations covering November 1948 to November 1973.

The autocovariance function and power spectrum were then estimated for this "shape of the yield curve" series using 60 lags. The autocovariance function for the undetrended data (trend in mean should not be as significant a problem here since one interest rate series is being subtracted from another) did not conform to the shape given in Figure 1. Instead, there was evidence of peaks in autocovariance at lags of 24 months and 48 months. The power spectrum did generally conform to the shape given in Figure 2, but there was also evidence of a peak at a frequency of .083 cycles per month or a cycle length of about 12 months.

The detrended autocovariance function and power spectrum were also estimated. The autocovariance function again showed evidence of peaks at lags of 24 months and 48 months. The power spectrum showed evidence of peaks at frequency of .017 and .083 cycles per month or cycle lengths of about 60 months and 12 months. Thus, both the autocovariance function and power spectrum seem to suggest the existence of some sort of weak annual component in the determination of the shape of the yield curve. This might be explained by the seasonality in timing of Treasury offerings. In addition, there appears to be some evidence of a longer cycle which is consistent with the 5 year cycle in Treasury bill yields suggested above.

In order to test for the relationship between the term structure of interest rates and NBER cycles, observations were chosen from the weekly term structure data at the intervals specified in Table III. This gave 250 unequally spaced (in calendar time) observations. The autocovariance function and power spectrum were again estimated using 50 lags. The autocovariance function was found to generally conform to the shape given in Figure 4 and the power spectrum was found to generally conform to the shape given in Figure 5, with a peak at a frequency of .02 cycles per observation or a cycle length of 50 observations. When the detrended autocovariance and power spectrum were estimated for the fundamental time, term structure data, the results were essentially the same as the undetrended results.

Thus, a time series analysis of the term structure data reveals some tendency for an annual component to exist when the data is analyzed in calendar time. When the data is analyzed in fundamental time as (determined by NBER cycles) it appears that the yield difference between long term Government bonds and Treasury bills conforms to NBER cycles. An examination of the unequally spaced (in calendar time) data reveals that this conformation takes the form of a definite tendency for this yield difference to be relatively small near the peak of an NBER cycle and relatively large at the cycle trough.

Corporate Bond Risk Premiums

It has been popularly theorized that corporate bond risk premium should be business cycle related phenomena.[14] The theory is that during an expansion, when the economic outlook is favorable, investors are more willing to invest in lower rated, riskier bonds than they are when the economic outlook is gloomy. This behavior should result in an increase in demand for lower rated, relative to higher rated bonds during the expansion. Therefore, the yield spread between lower rated and higher rated bonds should narrow during the expansion. Conversely, during the contraction, the demand for higher rated bonds should be relatively large and the yield spread should widen.

In order to test for the existence of these relationships, weekly data was collected from Standard and Poors Bond Guide on various yield indices covering the November 1948 to November 1973 time period. In particular, the following six yield indices were collected: AAA Industrial Bonds, BBB Industrial Bonds, AAA Railroad Bonds, BBB Railroad Bonds, AAA Utility Bonds and BBB Utility Bonds. From these six yield series, three risk premium series were constructed by substracting the AAA yield from the BBB yield at each observation point for each classification (i.e. Industrial, Railroad and Utility.)

Observations were selected from these three risk premium series at intervals of 4 weeks, which gave 325 observations per series. The autocovariance function and power spectrum were estimated for each of the three "monthly" risk premium series using 60 lags. All three of the autocovariance functions were found to conform to Figure 1 and all three of the power spectra conformed to Figure 2. The three detrended autocovariance functions and power spectra were then estimated. All three of the autocovariance functions were again found to conform to Figure 1 and all three of the power spectra again conformed to Figure 2. Thus, the analysis of the risk premium series in calendar time revealed no evidence of the existence of cycles in calendar time.

Three unequally spaced (in calendar time) series were created by selecting observations from the three weekly risk premium series at the intervals given in Table III. This selection process yielded 250 observations in fundamental time for each of the three risk premium series. The autocovariance functions and power spectra were estimated for these three series using 50 lags. For the Industrial Bond risk premium series, the autocovariance function conformed generally to Figure 1 and the power spectrum to Figure 2. For the Railroad Bond risk premium series, the autocovariance function appeard U-shaped as in Figure 4, while the power spectrum appeared as in Figure 2. For the Utility Bond risk premium series, the autocovariance function appeared to conform to Figure 1 and the power spectrum appeared to conform to Figure 2.

The detrended autocovariance functions and power spectra were estimated for the three risk premium series. The autocovariance functions for all three series appeared to be U-shaped as in Figure 4. This was markedly so for the Railroad series and modestly so for the other two series. The power spectra for all three series appear to conform to Figure 5. However, the peak in the power spectrum occurs at a frequency of .02 cycles per observation only for the Railroad series and at .01 cycles per observation for the Industrial and Utility series. This difference indicates that the three risk premium series to tend to conform to NBER cycles but that sometimes the Industrial and Utility series tend to skip a cycle.

Given this evidence of the conformation of the three detrended risk premium series to NBER cycles, the detrended (cross) covariance functions and cross spectra were estimated for the fundamental time series taken in pairs. All three of the (cross) covariance functions conformed to the general shape given in Figure 6. However, this conformation was clearly the closest for the (cross) covariance function of the Industrial risk premium series with the Railroad risk premium series. The coherences from the cross spectral analysis were .51 for the Industrial and Railroad series, .65 for the Industrial and Utility series, and .21 for the Utility and Railroad series.

From the cross spectral results, an analysis of the phase angles at a frequency of .02 cycles per observation revealed a tendency for the Railroad series to lead the Industrial series by two observations, the Industrial series to lead the Utility series by 4 to 5 observations, and the Railroad series to lead the Utility series by 4 to 5 observations.

Thus there does appear to be some conformation of corporate bond risk premium to NBER business cycles although that conformation seems to be stricter for Railroads than for Industrial or Utility bonds. Examination of the data confirms that the premiums tend to be larger at the trough than they are at the peak which is the form that one would expect, *a priori*, that such conformation would take. There also seem to be lead and lag effects which produce the (stronger) effect of business cycles on Railroad risk premiums before the (milder) effect on Industrial and Utility risk premiums occurs.

Common Stock Prices

The literature of Economic and Finance is replete with studies that present a plausible link between business cycles and common stock prices. One might assume that the value of a share of stock is the present value of the cash flow that the investor expects to receive. Then, if an economic expansion causes expected corporate earnings to rise faster than required rates of return of shareholders, one would expect stock prices to be relatively high during economic expansions and relatively low during contractions.

As a preliminary step to testing for the existence of this assumed relationship between stock prices and business cycles in fundamental time, and for purposes of comparison, the underlying stock price data could be tested for the existence of cycles in calendar time. As mentioned above the efficient markets theory would suggest that no such cycles in calendar time should exist.

For the purpose of these tests, weekly data (week ending closing prices) on the Dow Jones Industrial Average was collected for the November 1948 to November 1973 time period. While the Dow-Jones Industrial Average is technically not the best available indicator of overall stock price levels, it is nevertheless the most widely quoted measure of stock market conditions. From this weekly data, observations were chosen at intervals of four weeks which gave 325 "monthly" observations. The autocovariance function and power spectrum were estimated for this monthly series using 60 lags.

The autocovariance function was found to conform to Figure 1 and the power spectrum conformed to Figure 2. The detrended autocovariance function and power spectrum were then estimated. Once again, the autocovariance function conformed to Figure 1 but the power spectrum was found to conform to Figure 3. From these results there is no evidence of the existence of cycles in calendar time in the Dow Jones Industrial Average as an indicator of overall market price levels.

Observations were drawn at the intervals given in Table III from the Dow Jones Industrial Average weekly data. This sample gave 250 unequally spaced (in calendar time) observations. The autocovariance function and power spectrum were estimated for this fundamental time series using 50 lags. The autocovariance function was again found to conform to Figure 1 and the power spectrum conformed to Figure 2.

The detrended autocovariance function and power spectrum were then estimated for the fundamental time series. The autocovariance function conformed to Figure 4 with a trough at a lag of 25 observations and a peak at a lag of 50 observations. The power spectrum was found to conform to Figure 5 with a definite peak at a frequency of .02 cycles per observation or a cycle length of 50 observations. Thus, there is evidence from the procedure developed in this paper that the Dow Jones Industrial Average also tends to conform to NBER business cycles.

Business Cycle Sensitivity in Industries

The technique developed in this paper could also be used to test for the relationship between the performance of various industries and NBER cycles. In addition, through (cross) covariance and cross spectral analysis lead-lag relationships between business cycle sensitive industries could be identified.

Data on industry production and earnings are not available at sufficiently short intervals (weekly) to be used in the procedure developed in this paper. However, industry indices of stock prices are available weekly. For example, Standard and Poors publishes weekly industry indices of stock prices based on week ending closing prices.

At one level of aggregation Standard and Poor's has stock price indices for the "capital goods" and"consumer goods" industries. [15] Weekly data for these two industries was collected for the November 1948 to November 1973 time period. From these series observations were selected at four week intervals. This sampling gave two series containing 325 "monthly" observations each. The autocovariance functions and power spectra were estimated for these series using 60 lags. Both autocovariance functions were found to conform to Figure 1 and both power spectra conformed to Figure 2.

The detrended autocovariance functions and power spectra were estimated for these series. Unexpectedly, the autocovariance functions were found to be U-shaped and conformed generally to Figure 4. The capital goods series seemed to have a trough at a lag of about 28 months indicating negative autocovariance there, and a peak at a lag of about 54 months. The consumer goods series had a trough at a lag of about 27 months with negative autocovariance at the trough, and a peak at a lag of about 49 months. The power spectrum for the capital goods series conformed to Figure 3 but had a definite dominant peak at a frequency of .0167 cycles per month or a cycle length of 60 months.

The consumer goods power spectrum also conformed to Figure 3 and also had a definite dominant peak at a frequency of .0167 cycles per month. Thus, the detrended autocovariance functions and power spectra for the capital goods and consumer goods stock price series indicate the existence of a four to five year cycle in both series.

Observations were then chosen from the weekly capital goods and consumer goods stock price series at the intervals given in Table III. This procedure gave 250 unequally spaced (in calendar time) observations. The autocovariance functions and power spectra were then estimated using 50 lags. The autocovariance functions both generally conformed to Figure 1 and the power spectra both generally conformed to Figure 2.

The detrended covariance functions and power spectra were estimated for the two fundamental time series. The autocovariance functions conformed to the shape illustrated in Figure 4. The trough occurred at a lag of 25 observations for the capital goods series with negative autocovariance at the trough. The trough occurred at a lag of 25 observations for the consumer goods series also but there was small positive autocovariance at that lag. The peak occurred at a lag of 50 observations for the capital goods series as might have been expected, but occurred at 43 observations for the consumer goods series.

The power spectrum for the capital goods series conformed to Figure 5 with a definite peak at a frequency of .02 cycles per observations. However, the power spectrum for the consumer goods series conformed generally to Figure 3 with no dominant peak at any frequency. Thus, the evidence from

from the analysis of the capital goods and consumer goods stock price series in fundamental time seems to indicate a definite conformation to NBER cycles for the capital goods series but relative independence from NBER cycle for the consumer goods series.

The procedure developed in this paper was also used to test for business cycle sensitivity in industries at a more finely determined level of aggregation. The industries analyzed are given in Table IV. Standard and Poors publishes weekly data on indices of stock prices for each of the industries given in Table IV. This data was collected for the November 1948 to November 1973 time period.

Table IV

Group I	Group II
Automobiles	Chemicals
Building Materials	Food
Machinery	Railroads
Oil	Textiles
Retail	Utilities
Steel	

For the industries listed in Group I of Table IV observations were selected at 4 week intervals giving 325 "monthly" observations. The autocovariance function and power spectrum were then estimated for each series. In all cases, the autocovariance functions conformed to Figure 1 and the power spectra conformed to Figure 2. The six series were detrended and the autocovariance functions and power spectra were estimated again. Once again, in all cases the autocovariance functions conformed generally to Figure 1. In all cases except for the Oil stock price series the power spectra conformed to Figure 3 with no dominant peaks. However, the Oil series did appear to have a dominant peak at a frequency of .008 cycles per month or a cycle length of about 125 months. Thus, for five of the six series, there was no evidence of cycles in calendar time. For the Oil stock price series there is some evidence of a relatively long (approximately 10 years) cycle.

Observations were also selected at the intervals given in Table III for all eleven series in Group I and Group II of Table IV. This gave 250 unequally spaced (in calendar time) observations for each series. The autocovariance function and power spectrum were then estimated for each of these eleven series in fundamental time using 50 lags. Of the eleven autocovariance functions, nine conformed generally to the shape of Figure 1. The other two, Chemicals and Railroads, were somewhat U-Shaped as in Figure 4 with troughs at a lag of about 30 observations and peaks at a lag of about 47 observations in each. All eleven of the power spectra conformed to Figure 2.

The detrended autocovariance functions and power spectra were then estimated for the eleven fundamental time series. The results of the autocovariance analysis appear in Table V. These results indicate a tendency for six of the eleven series to be sensitive to NBER cycles.

Seven of the eleven power spectra were found to conform to Figure 3 with no dominant peak. Three of the eleven, Automobiles, Building Materials and Railroads, were found to conform to Figure 5 with a definite dominant peak at a frequency of .02 cycles per observation. One series, Oil, was found to have a definite dominant peak at a frequency of .01 cycles per observation. These finding indicate that four of the eleven series tend to conform to NBER cycles. There seems to be some tendency for one of the four, Oil, to "skip" a cycle.

Table V

Detrended Industry Autocovariance Functions in Fundamental Time

Series	Conforms to	Trough at lag of *	Peak at lag of *
Automobiles	Figure 4	27	47
Building Materials	Figure 4	18	33
Machinery	Figure 4	25	44
Oil	Figure 4	29	49
Retail	Figure 1	--	--
Steel	Figure 1	--	--
Chemicals	Figure 4	28	48
Food	Figure 1	--	--
Railroads	Figure 4	27	49
Textiles	Figure 1	--	--
Utilities	Figure 1	--	--

*Measured in number of observations in fundamental time.

Thus, analysis of the autocovariance functions suggests business cycle sensitivity in six of the eleven industry series. The power spectra confirm such sensitivity in four of the six. Those four series, in addition to the capital goods series which was found to be busines cycle sensitive above, were also analyzed for lead-lag relationships in their conformation to NBER cycles by estimating the relevant (cross) covariance functions and cross spectra.

All of the (cross) covariance functions were found to conform to Figure 6. The peaks and troughs in the (cross) covariance functions are given in Table VI.

The coherences and suggested lead-lag relationships from the estimated phase angles at a frequency of .02 cycles per observation are given in Table VII.

The results from Table VII tend to indicate that the order in which these five stock price series are affected by changing business cycle conditions is:

Table VI

Peaks* and Troughs* in Cross Covariance of Detrended Fundamental Time Industry Stock Price Series

Base Series	Capital Goods	Automobiles	Bldg. Materials	Railroads	Oil
Capital Goods:					
Peaks		-50,-7,+47	-46,0,+33	-50,-4,+48	-50,-1,+48
Troughs		-30,+19	-22,+19	-27,+19	-28,+19
Automobiles :					
Peaks	-47,+7,+50		-50,0,+43	-47,0,+49	-47,0,+47
Troughs	-19,+30		-19,+28	-25,+28	-27,+27
Building Materials :					
Peaks	-33,0,+46	-43,0,+50		-41,0,+50	-38,-11,+48
Troughs	-19,+22	-28,+19		-18,+19	-20,+30
Railroads:					
Peaks	-48,+4,+50	-49,0,+47	-50,0,+41		-47,0,+50
Troughs	-19,+27	-28,+25	-19,+18		-24,+25
Oil :					
Peaks	-48,+1,+50	-47,0,+47	-48,+11,+38	-50,0,+47	
Troughs	-19,+28	-27,+27	-30,+20	-25,+24	

*in number of observations lead or lag of other series on base series

Table VII

Coherence and Suggested Lead-Lag* Relationships at Frequency of .02 Cycles per Observation in Fundamental Time

Base Series	Capital Goods	Automobiles	Bldg. Materials	Railroads	Oil
Capital Goods:					
Coherence		.68	.14	.78	.64
Lead		-5	- 1	-3	-3
Automobiles:					
Coherence	.68		.11	.78	.81
Lead	+5		+5	coincident	+1
Bldg. Materials :					
Coherence	.14	.11		.19	.04
Lead	+1	-5		-3	-6
Railroads:					
Coherence	.78	.78	.19		.68
Lead	++3	coincident	+3		coincident
Oil:					
Coherence	.64	.81	.04	.68	
Lead	+3	-1	+6	coincident	

*in number of observations; positive means base series leads, negative means base series lags

1. Automobiles

2 and 3. Railroads and Oil

4. Building Materials

5. Capital Goods

This order suggests that consumer durables are affected first by changing business cycle conditions. Oil and transportation appear to be affected next, followed by building and housing and finally followed by capital goods. There is no evidence that building is countercyclical to the other series as has often been suggested. There is evidence however that capital investment lags increased demand for goods and services. Of course all of these suggested relationships rest upon the assumption that stock prices reflect underlying economic conditions in the industry.

Summary

This paper presents a methodology for using spectral analysis in fundamental time. This procedure overcomes some very important objections to traditional spectral analysis. The procedure was used to test some widely held beliefs about the behavior of security market prices and yields. The results indicate that:

1. long term and short term Government security yields are influenced by changing business cycle conditions

2. short terms yields are affected before long term yields

3. rates tend to be high at the cycle peak and low at the trough

4. the shape of the yield curve is influenced by business cycle conditions with the difference between long term yields and short term yields tending to be relatively small near the peak of a cycle and relatively large near the cycle trough

5. Corporate bond risk premiums are influenced by business cycle conditions, tending to be small at the cycle peak and large at the cycle trough

6. the Dow Jones Industrial Average is influenced by changing business cycle conditions, tending to be high at the cycle peak and low at the cycle through

7. certain industry stock price indices are influenced by business cycle conditions and exhibit lead lag relationships in their sensitivity to changing economic conditions.

Figure 1

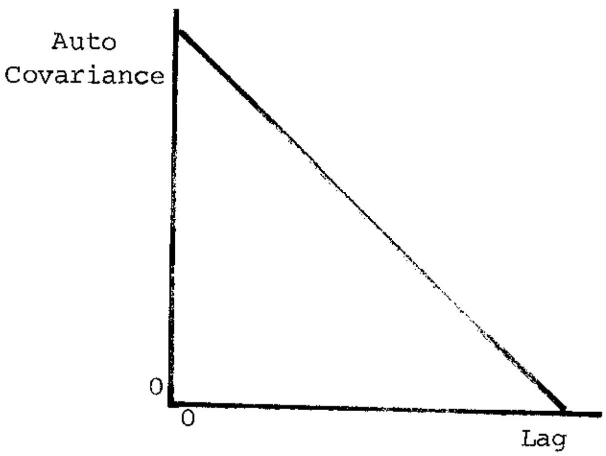


Figure 2

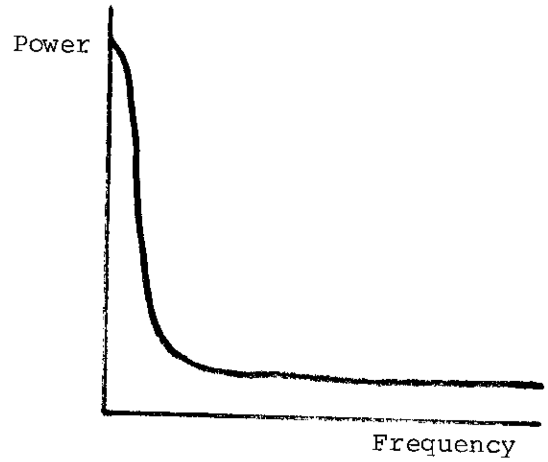


Figure 3

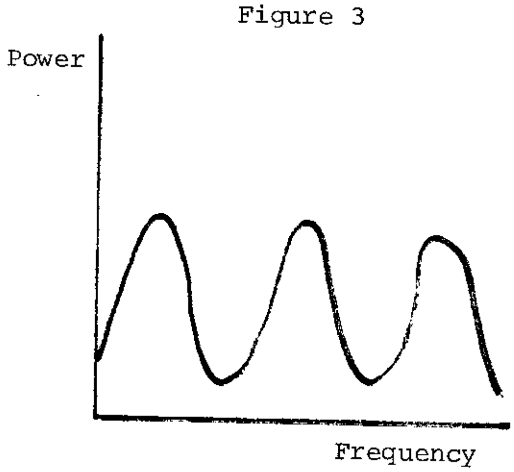


Figure 4

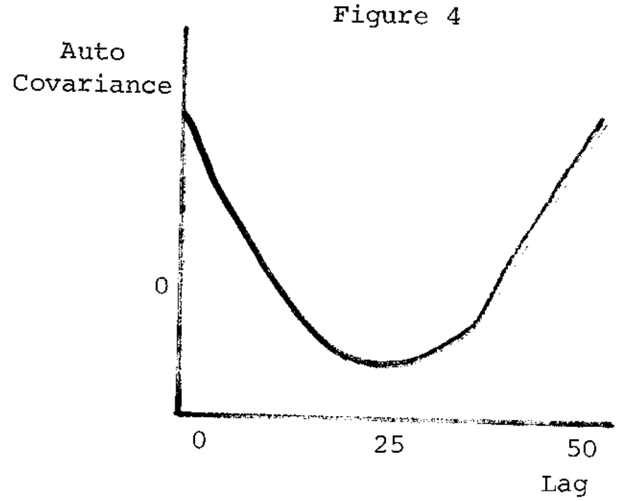


Figure 5

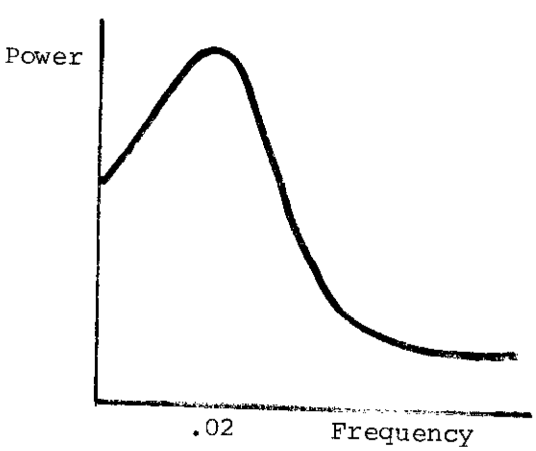
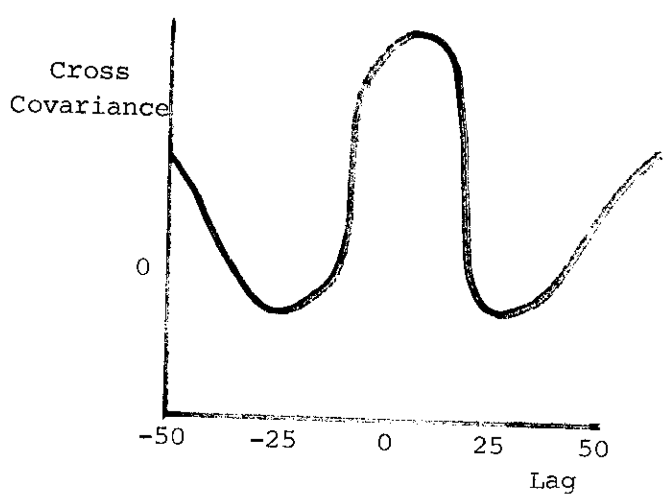


Figure 6



Appendix

Industry Stock Price Indices 1948-1973 (For details see Standard and Poors Trade and Security Statistics)

Capital Goods - approximately 120 companies from the following groups:

Agricultural Machinery	Mining and Smelting
Building Materials	Office and Business Equipment
Chemicals	Printing and Printing Equipment
Copper and Brass	Railroad Equipment
Electrical Equipment	Ship building
Fertilizers	Shipping
Lead and Zinc	Steel and Iron
Machinery	Steel Alloys

Consumer's Goods - approximately 190 stocks from the following groups:

Apparel	Motion Pictures
Automobiles	Paper Containers
Auto Parts	Publishing
Beverages	Radio
Cigar Manufacturers	Rayon and Acetate Yarns
Cigarette Manufacturers	Retail Stores
Confectionary	Shoe
Containers	Soaps
Drugs	Sugar
Floor Covering	Textiles
Food Products	Tires and Rubber
Household Appliances	Vending Machines

Automobiles - approximately 4 stocks

Building Materials - approximately 20 stocks from the following groups:

Air Conditioning	Heating and Plumbing
Cement	Roofing and Wellboard

Chemicals - approximately 12 stocks

Foods - approximately 30 stocks from the following groups:

Biscuit Baking	Dairy Products
Bread and Cake Baking	Meat Packers
Canned Foods	Packaged Foods
Corn Refiners	

Machinery - approximately 25 stocks from the following groups:

Construction and Material Handling	Speciality
Industrial	Steam Generating
Oil Well	

<u>Oil</u> - approximately 30 stocks from the following groups:

Crude Producers
Integrated Domestic
Integrated International

<u>Railroad</u> - approximately 25 stocks

<u>Retail</u> - approximately 30 stocks from the following groups:

Department Stores	Mail Order and General Chains
Food Chains	Variety Chains

<u>Steel</u> - approximately 10 stocks

<u>Textiles</u> - approximately 6 stocks

<u>Utility</u> - approximately 50 stocks from the following groups:

Electric Power	Natural Gas Pipelines
Natural Gas Distributors	Telephone

Footnotes

1 Spectral Analysis is a statistical method which involves the estimation of a power spectrum for a time series. The power spectrum is a representation of a Fourier transform of the autocovariance function. Heuristically, the power spectrum is an autocovariance function in the frequency domain rather than the time domain. There are many excellent references on spectral analysis. For example see: Granger and Hatanaka (1964), and Jenkins and Watts (1968).

2 A partial list of spectral studies of U.S. interest rates include: Fand (1966), Sargent (1968), Melnik and Kraurs (1969) and Smith and Marcis (1972). A partial list of spectral studies of the U.S. stock market includes: Granger and Morgenstern (1963), Godfrey, Granger and, Morgenstern (1964), Granger and Morgahstern (1970), and Fleming (1973).

3 Heuristically, a cross-spectrum is a (cross) covariance function between two time series in the frequency domain rather than the time domain. Coherence measures the similarity between the two series at a particular frequency and can be interpreted in a way not dissimilar to a coefficient of determination (R^2).

4 Given that a cycle of a given frequency exists in each of two time series the phase angle measures how much out of phase one series is with respect to the other in the frequency domain. This phase difference can then be converted into a lead or lag in the time domain.

5 See Percival (1971) and Percival (1975).

6 See Feller (1966) p. 133

7 See Clark (forthcoming) and Westerfield (1973)

8 See Granger and Hatanaka (1964) and Jenkins and Watts (1968)

9 For more details on sufficient observations see Granger and Hatanaka (1964) and Jenkins and Watts (1968).

10 See Granger (1966).

11 All bond yields are yields to maturity.

12 The estimation was done using program BMDo2T, Autocovariance and Power Spectral Analysis, on the IBM 360 Computer at the University of Pennsylvania.

13 The trend in mean in a series was removed by a least squares fitting method as follows:

$$A_{px} = R_{px} - B - \sigma i$$

where $i = o, 1, \ldots, n - 1$

$$\sigma = \sum_{i=o}^{n} x_i \, (2 - n + 1) / (((n - 1)\, n\, (n+1)) /6)$$

$$B = X - \alpha(n - 1)/2$$

A_{px} is the autocovariance of series X after detrending at lag p

R_{px} is the autocovariance of series X at lag p

n is the number of discrete data points

[14] See Sloan (1966) and Van Horne (1970) for example. This theorized relationship is also presumably the basis for the so-called "Barrons Confidence Index" as a leading indicator.

[15] See the Appendix to this paper for a description of the industry stock price indices

References

1. Clark, Peter K., "A Subordinated Stochastic Process Model with Finite Variance for Speculative Prices," Economica (forthcoming).

2. Fand, D., "A Time-Series Analysis of the Bills-Only "Theory of Interest Rates," Review of Economics and Statistics (November 1966), 361-371.

3. Feller, W., An Introduction to Probability Theory and Its Applications, Volume II (New York: John Wiley and Sons, 1966).

4. Fleming, R., "How Risky is the Market?" Journal of Business (July 1973), 404-424.

5. Godfrey M., C.W.J. Granger and O. Morgenstern, "The Random Walk Hypothesis of Stock Market Behavior," Kyklos 17 (1964), 1-30.

6. Granger, C.W.J, "The Typical Spectral Shape of an Economic Variable," Econometrica (January 1966), 150-161.

7. ________________, and M. Hatanaka, Spectral Analysis of Economic Time Series (Princeton, Princeton University Press, 1964).

8. ________________, and O. Morganstern, "Spectral Analysis of New York Stock Market Prices," Kyklos 16 (1963), 1-27.

9. ________________, Predictability of Stock Market Prices (Lexington, Massachusetts: Heath Lexington Books, 1970).

10. Jenkins, G. and D. Watts, Spectral Analysis (San Francisco: Holden-Day, Inc. 1968).

11. Melnik, A. and A. Kraus, "Short-Run Interest Rate Cycles in the U.S.: 1954-1967,"Journal of Financial and Quantitative Analysis (September 1969), 291-299.

12. Percival, J., "A Comment: Short-Run Interest Rate Cycles in the U.S.: 1954-1967," Journal of Financial and Quantitative Analysis (June 1971), 1043-1045.

13. Percival, J. "On the Selective Use of Spectral Analysis," Review of Economics and Statistics (February 1975), 107-109.

14. Sargent, T., "Interest Rates in the Nineteen-fifties,"Review of Economics and Statistics (May 1968), 164-172.

15. Sloane, Peter E., "Determinants of Bond Yield Differentials - 1954-1959," Yale Economic Essays (Spring 1963), 3-56.

16. Smith, V.K., and R. Marcis, "A Time-Series Analysis of Post-Accord Interest Rates," Journal of Finance (June 1972), 598-608.

17. Van Horne, J.C., Function and Analysis of Capital Market Rates (Englewood Cliffs, New Jersey: Prentice Hall, Inc. 1970).

18. Westerfield, R., "The Distribution of Common Stock Price Changes: An Application of Transactions Time and Subordinated Stochastic Models," Working Paper No. 6-73b, Rodney L. White Center for Financial Research, University of Pennsylvania.