Indexation, Expectations, and Stability

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The recent surge of price movement, coupled with a fairly substantial recession has interested economists in alternative adjustments to chronic price inflation. By traditional neo-classical theory, increases in excess capacity would prove sufficient to dampen excess demand and reduce the rate of change in observed prices (perhaps about its expected path). This remedy, however, has proved to be both costly, as measured by the lost output during transition, and time consuming, as evidenced by the rather slow decrease in price movement over the recent slowdown. Consequently, there has been increased interest in alternative ways of "living with" inflation rather than ridding the system of it. One of these ways, proposed by Milton Friedman [1974a, 1974b], is a general system of indexation of nominal contracts to actual inflation rates. It is believed that, by so doing, the pains of inflation, associated with imprecise estimates incorporated in contract negotiations, would be diminished and bargaining could center around real changes such as productivity or relative price movements. Therefore, the burden of inflation is reduced and the economy more rapidly attains its new equilibrium after a disturbance.

However, Friedman's reintroduction of indexation into the economic arena has not met with unequivocal support. Some have contended that the proposed remedy need not be universally optimal, while others question whether it may even be undesirable. Representing the first group, Gray [15] questions the general optimality of total indexation in a recent article. There she demonstrates that Friedman's prescription to reduce the adjustment time will succeed only for monetary disturbances, but, indeed, exacerbates the disequilibrium for real shocks. This does not, of course, nullify the usefulness of the indexation method, as Gray demonstrates, but rather makes its widespread institution dependent upon the empirically observable sources of disturbance in the economy.

The second line of attack upon the general desirability of indexation is more fundamental. To some, general indexation is, in some sense, inflationary. The argument here is that an initial burst of inflation in one sector would spread rapidly, via indexation, to other sectors. The price movements in these other sectors would, in turn, provoke additional price movements elsewhere, including the initial inflationary sectors. Hence, indexation would fuel a self-generating inflation. What is at stake here is the general property of stability of the macroeconomic system. If opponents of indexation, for example Joint Economic Committee [34], Weidenbaum [35], Wendel [36], are correct, the general stability of the economy may be disturbed and lead to inflationary or deflationary explosions. Heller [18] offers perhaps the clearest statement of this position. "Indexation would oil the gears and speed the process of inflation."

Interestingly, very little analytic investigation of this stability issue has been attempted in the current literature. This is partially due to the vagaries of the particular forms that indexation may take. To analyze the stability of a macroeconomic system, one must be able to outline the exact form of the equations determining price movements. This requires knowledge of how indexation will be implemented and what elements of subjective implementation, through expectations of price change, will be supplanted. It requires knowledge of the role of expectations on stability and the dynamics of inflation both with and without indexation. This is the task of the present study.

This paper reports the results of investigating the stability of a series of macroeconomic models with price and wage equations dependent upon, in varying degrees, actual wage and price movements, either through a price expecations mechanism or through direct indexation. For each model the problem of stability with expectations and/or indexation will be discussed and necessary and sufficient conditions for local stability indicated. The results of the study may be outlined in advance. The analysis concludes that indexation can aid economic stability of a macro system. It is useful, on a macro level in any case, only if it is presumed that this institutionalization of expectations is preferred to the less formal scheme. Further, implementation must be carefully considered. The choice of index is important, as is the question of what sectors will be indexed. The local conditions prove more tenuous with some indexation than without it.

1. Indexation - Defining the Term

The term indexation has generally been referred to, in the discussion surrounding its implementation, as the automatic adjustment of individual prices in commodity, labor and capital markets for movements in an overall price index. Which sectors should be indexed and what price index should be used is often omitted from consideration. If the present discussion abstracts from the capital market, three alternatives appear plausible. First, wages may be indexed to commodity prices. Second, price movements may be linked to wage movement. And, finally, both may be a function of some composite index of general prices in society. Below, each of these possibilities will be considered. It will be shown below that the indexation regime chosen, and the degree of indexation adjustment will affect the conditions that must be satisfied for the economy to be able to return to equilibrium after an exogenous disturbance.

Surely in all cases, monetary expansion and the rate of return on money are both non-indexed. The rate of change in the money supply will be the limiting or steady state conditions of the economy. However, the economy's ability to obtain this steady state from a disequilibrium condition is open to question. Essentially the "wrong" indexation scheme could increase price movements away from the new equilibrium rather than toward the steady state solution. This is the focal point of the analysis of stability in a macro system.

2. Stability and A Deterministic Dynamic System

A. The Formal Procedure

The notion of stability of a macroeconomic system centers around the model's ability to converge to a state of equilibrium with excess demand in all markets equal to zero. Formally, following Samuelson [28] and Bellman [4] the analysis will focus on the necessary and sufficient conditions for local stability of a deterministic system.[1] The analysis will use the Hurwitz criterion on the characteristic matrix to indicate the conditions that must obtain to satisfy Lyapunov's Theorum on stable systems. The economic content of the analysis rests in the conditions necessary for convergence.

B. A First Case - The Cagan-Goldman Model with Expectations

The point of departure for the present study is the work of Cagan [5] and its stability characteristics as outlined by Goldman [12]. The original Cagan model addressed itself to how demand for cash balances changes with expectations of price change. The model had two markets, commodities and money, and one price expectation function. By Walras' Law, only one of the market expectations is independent. Therefore, one market condition and the price expectations dynamics fully describe the system. Cagan assumes a semi-log demand function of the form

$$(1)\quad \frac{M^d}{P} = \exp\,(-\alpha\pi^e - \gamma);\ \alpha > 0$$

where π^e denotes expected price change. Price expectations are adaptive in the sense of Muth [24] and adjust to observed discrepancies according to equation (2),

$$(2) \quad \frac{d\pi^e}{dt} = \beta(\pi - \pi^e); \quad \beta > 0.$$

The model assumes continual money market equilibrium,

$$(3) \quad \frac{M}{P} = \frac{M^d}{P},$$

and has been shown by Cagan [5] to be stable if and only if

$$(4) \quad 1 - \beta\alpha > 0.$$

Satisfaction of these conditions implies that $\pi^e \to \mu$ and prices move at their expected rate, which equals the rate of monetary expansion. Violation of the condition expressed as equation (4), on the other hand, implies explosive price movement and no stability of the system. The economic implication of the analysis is that price expectations are basically destabilizing and the more rapid the speed of adjustment of expected price change to observed price movement, the more likely the stability conditions would be violated.

Noting the tender balance of stability, Goldman correctly observes that this is due to the instantaneous equilibrium assumption in the money market, equation (3) above. Replacing it with an adjustment process in log linear form, similar to the traditional excess demand equation,

$$(5) \quad \frac{d\frac{M}{P}}{dt}\frac{1}{M/P} = \psi \left(\log \frac{M^d}{P} - \log \frac{M}{P}\right); \quad \psi > 0,$$

his approach more closely follows the traditional demand theory. The general solution of this revised system of two simultaneous differential equations, equations (2) and (5), can be derived and the system's stability will depend upon the signs of the real parts of the characteristic roots of the system, as noted above. Goldman [12] has shown that this condition may be simplified to,

(6) $\psi + \beta > \psi\alpha\beta.$

As expected, the presence of partial adjustment to excess demand in the money market stabilizes the system, as not all of the excess demand is transferred to the commodity market.

3. Modeling Labor Market Dynamics

To expand the analysis from the Cagan-Goldman treatment to a model consistent with our interest in wage and price dynamics and the role of indexation, it is necessary to extend the theoretical framework to a multi-sector context. Here the extensions will involve explicit consideration of the labor sector, and the dynamic process of adjustment in the commodity market, the money market, and the labor market. Once the stability conditions for this expanded model are examined, a sequence of models of increased complexity will be considered. The analysis will center upon the stability of the model without indexation and the impact of general indexation on the conditions that insure local convergence. This procedure will be employed twice below, once for a model that allows price movements only in the presence of excess demand, and once for a model that allows for expectations itself to generate price variability.

A. Model 1-A Goods, Money, Labor Model Without Indexation

The first model considered is a generalization of the Cagan-Goldman model with the labor market explicitly considered, and the demand functions of the economic units fully specified. For the present analysis, price and wage flexibility are assumed, at a non-instantaneous rate, and equilibrium will be at full employment. Accordingly, the excess demand equations of the system become,

$$(7) \quad C^d(\tfrac{W}{P}, g, \tfrac{M}{P}, \pi^e) - C^s(\tfrac{W}{P}) = 0,$$

$$(8) \quad M^d(\tfrac{W}{P}, g, \tfrac{M}{P}, \pi^e) - \frac{M^s}{P} = 0,$$

$$(9) \quad L^d(\tfrac{W}{P}) - L^s(\tfrac{W}{P}) = 0.$$

A microeconomic foundation to derive these aggregate demand functions can be found in Santomero [29], Sidrausky [30], or Stein [31]. In each case all demand functions are shown to be a positive function of real wage, $\frac{W}{P}$, aggregate profit share, g, and real money balances, $\frac{M}{P}$. Price expectations impact negatively upon money demand, however, as the relative price of savings through money balance accumulation is increased, favoring greater present consumption, i.e. the alternative to money. These expectations will be assumed to follow adaptive adjustment of the general functional form as above,[2]

$$(10) \quad \frac{d\pi^e}{dt} = \beta(\pi - \pi^e)$$

where β may be a non-linear functional form with $\beta' \geq 0$, see Barro [2]. To simplify the analysis, the wealth effects on labor supply, discussed explicitly in Santomero [29] will be neglected, leaving the dynamics of the labor market dependent upon only real wage movements, as in Patinkin [25].

As a full employment equilibrium condition, output is determined by labor market equilibration, which implies that $\frac{W}{P}L + g = Y$ where Y equals output at steady state. Profits then are dependent upon $\frac{W}{P}$ given that

$$(11) \quad g = Y - \frac{W}{P} L.$$

Changes in $\frac{W}{P}$ will, therefore, affect g, evaluated at equilibrium by reducing profit flows, i.e., $\frac{dg}{d\frac{W}{P}} < 0$.[3]

The model has three markets, viz., money, commodities and labor. By Walras' Law, only two of the three markets of the economy are independent. Here the analysis will center upon the money and labor markets. Prices are assumed to respond to excess supply of money, while nominal wages respond to excess labor demand. Assuming a finite speed of adjustment in each market, the dynamic equations of the system may be written as,

$$(12)\quad \frac{dP}{dt}\frac{1}{P} = \psi\left(\frac{M^s}{P} - M^d\left(\frac{W}{P}, g, \frac{M}{P}, \pi^e\right)\right),$$

and

$$(13)\quad \frac{dW}{dt}\frac{1}{W} = \lambda\left(L^d\left(\frac{W}{P}\right) - L^s\left(\frac{W}{P}\right)\right).$$

Such partial adjustment is assumed to exist either because of uncertainty and/or transaction costs associated with price changes, as in Barro [3] or because of finite contract lengths, as in Grossman [16] or Gray [14].

Steady state equilibrium for the model is obtained where the labor market clears at full employment, real money balances are constant, and price expectations equal actual price movements. This may be written in terms of equations (10) through (13), the dynamic equations of the system, as

$$(14)\quad \frac{d\frac{W}{P}}{dt} = \frac{W}{P}\left[\lambda\left[L^d\left(\frac{W}{P}\right) - L^s\left(\frac{W}{P}\right)\right] - \psi\left[\frac{M}{P} - M^d\left(\frac{W}{P}, g, \frac{M}{P}, \pi^e\right)\right]\right] = 0$$

$$(15)\quad \frac{d\pi^e}{dt} = \beta(\pi - \pi^e) = \beta\left\{\psi\left[\frac{M}{P} - M^d\left(\frac{W}{P}, g, \frac{M}{P}, \pi^e\right)\right] - \pi^e\right\} = 0$$

$$(16)\quad \frac{d\frac{M}{P}}{dt} = \frac{M}{P}\left\{\mu - \psi\left[\frac{M}{P} - M^d\left(\frac{W}{P}, g, \frac{M}{P}, \pi^e\right)\right]\right\} = 0$$

Expanding these equations around the equilibrium values, $\frac{W}{P}^*$, $\pi^{e*} = \pi^*$, $\frac{M}{P}^*$, by Taylor expansion and evaluating the resultant differential equations results in a time path of the endogenous variables of the system.

The stability of the system depends upon the sign of the latent roots of the characteristic equation given as $|A-Ir| = 0$ where

$$
(17)\qquad A = \begin{vmatrix} (\frac{W}{P})^{*}\left[\lambda'(L^{d'}-L^{s'}) + \psi'(M_1-M_2g')\right] & (\frac{W}{P})^{*}\psi' M_4 & (\frac{W}{P})^{*}\psi'(M_3-1) \\ -\beta\left[\psi'(M_1+M_2g')\right] & -\beta(\psi' M_4+1) & -\beta\psi'(M_3-1) \\ (\frac{M}{P})^{*}\psi'(M_1+M_2g') & (\frac{M}{P})^{*}\psi' M_4 & (\frac{M}{P})^{*}\psi'(M_3-1) \end{vmatrix}
$$

The necessary and sufficient conditions for this system to be stable can be simplified to [4]

(18 A) $\quad M_3 < 1$,

(18 B) $\quad -\psi' M_4 < 1$,

(18 C) $\quad \lambda'(L^{d'}-L^{s'})(1+\psi' M_4) + \psi'(M_1+M_2g') < 0$.

The usual normality of all goods in the aggregate implies $0 < M_3 < 1$ so that condition A is satisfied, see Tobin [33] or Hendershott [17]. The second condition for stability is the counterpart of that derived by Cagan where $-M_4 = \alpha$ in Cagan's explicit semilog demand function for money, see equation (1) above.

The third condition for stability is the result of the addition of a labor market to the analysis. From (18B) the first term is unambiguously negative. The second term, however, is unclear. As wages rise, the demand for money increases, $M_1 > 0$, for workers, but falls for the equity holders experiencing decreases in profit flows $M_2g' < 0$. If these are viewed as completely offsetting, condition (18B) is satisfied, as is the case if $M_1 < |M_2g'|$. If the total demand for money increases with $\frac{W}{P}$, i.e., $M_1 > |M_2g'|$, then the speed of adjustment in the labor market must be large enough, or the speed of adjustment in the money market, ψ', must be small enough to offset the destabilizing elements in the money market. Interestingly, increases in the adjustment speed of nominal wages to disequilibrium, λ, adds stability to the system as the real wages converges more quickly to equilibrium.

The role of real wages on profit, and the relative magnitude of demand shifts due to alterations in wages and profits, then, becomes crucial in this model, as opposed to those models which exclude labor market dynamics completely. An additional constraint is added to the traditional pure expectations constraint. The latter limits the impact of wage variation on aggregate money demand within bounds. Violation of this condition implies that real wage movement could destabilize the economy that was heretofore thought stable.

B. Model 1 With Indexation

Indexing the present model may take many forms, as noted in section one. For the present purposes, four distinct cases of indexation will be analyzed specifically. In each case, interest will center upon a fully indexed economy where indexation is continuous. Accordingly, nominal wage and price dynamics are a function of excess demand or supply and the index regime assumed operative. The regimes considered are the following:

(i) indexing nominal wages by a fraction I_P, where $0<I_P\leq 1$, of the movement in the commodity price level;

(ii) indexing nominal wages by a fraction I_P, of the movement in the commodity price level, where $0<I_P\leq 1$, and indexing commodity price levels to the movement of nominal wages by the fraction I_W, of wage changes, where $0<I_W\leq 1$;

(iii) indexing both wages and prices to commodity price change by escalating both nominal prices by a fraction I_p, of price level movements, where $0<I_P\leq 1$, and

(iv) indexing both wages and prices to some composite or weighted price index, with weights I_P and I_W respectively, where both wage change and price change enter the index factor.

To present the implications of these schemes on the stability of the system described above, the exposition will be carried out as follows. Table 1 indicates the impact of indexation of a particular form on the dynamic equations of the economy. Local conditions for stability are presented for each in that same tableau. The text will be reserved for a discussion of the differences that result from the implementation of indexation, relative to the non-indexed case, and the economics of the results.

(i) Indexing nominal wages by the commodity price change rate.

In this most often considered case of indexation the wage dynamics of the model are augmented by the movements in commodity prices. Wages now move due to both excess demand for labor and commodity price inflation. The implication of this adjustment to the dynamics of the model on the conditions required for stability can be seen in section (i) of Table 1.

Indexation effects only condition (C) of the original model. It will be noted, however, that its inclusion renders the conditions less binding in that it reduces the potential destabilizing impact of money market behavior on the system. Essentially, the presence of indexation returns the real wage to its equilibrium value and therefore decreases the instability caused by incorrect relative prices as compared with the equilibrium values.

In the limit with complete indexation, condition (C) is unambiguously satisfied,

$$\lim_{I_P \to 1}\left\{\lambda'(L^{d'}-L^{s'})(1+\psi' M_4) + \psi'(M_1+M_2 g')(1-I_P)\right\} < 0 \;\forall\; M_1, M_2, \psi' .$$

Even without complete indexation, its presence is beneficial. For any I_P greater than zero, condition (C) is more likely to be satisfied for any values of the remaining functions. In fact, stability can be guaranteed for a value of I_P just sufficient to insure this condition is met. A value of I_P

TABLE 1
Necessary and Sufficient Conditions for Stability
Model 1 and Various Index Forms

Dynamic Equations of the Model	Necessary and Sufficient Conditions for Stability: A. The Impact of Wealth Changes on Aggregate Demand	B. The Impact of Expected Inflation on Aggregate Demand	C. The Movement of the Real Wage	D. Feedback Constraints
$\frac{dP}{dt}\frac{1}{P} = \psi(\frac{M^S}{P} - M^D)$ $\frac{dW}{dt}\frac{1}{W} = \lambda(L^d - L^s)$	$M_3 < 1$	$-\psi' M_4 < 1$	$\lambda'(L^{d\prime} - L^{s\prime})(1+\psi' M_4) +$ $\psi'(M_1+M_2g') < 0$	
$\frac{dP}{dt}\frac{1}{P} = \psi(\frac{M^S}{P} - M^D)$ $\frac{dW}{dt}\frac{1}{W} = \lambda(L^d - L^s) + I_P[\frac{dP}{dt}\frac{1}{P}]$	$M_3 < 1$	$-\psi' M_4 < 1$	$\lambda'(L^{d\prime} - L^{s\prime})(1+\psi' M_4) +$ $\psi'(M_1+M_2g')(1-I_P) < 0$	
$\frac{dP}{dt}\frac{1}{P} = \psi(\frac{M^S}{P} - M^D) + I_W[\frac{dW}{dt}\frac{1}{W}]$ $\frac{dW}{dt}\frac{1}{W} = \lambda(L^d - L^s) + I_P[\frac{dP}{dt}\frac{1}{P}]$	$M_3 < 1$	$-\frac{\psi' M_4}{1 - I_W I_P} < 1$	$\lambda'(L^{d\prime} - L^{s\prime})(\psi' M_4 + 1 - I_W) +$ $\psi'(M_1+M_2g')(1-I_P) < 0$	$I_W I_P \leq 1$ $0 \leq I_W \leq 1$ $0 \leq I_P \leq 1$
$\frac{dP}{dt}\frac{1}{P} = \psi(\frac{M^S}{P} - M^D) + I_P[\frac{dP}{dt}\frac{1}{P}]$ $\frac{dW}{dt}\frac{1}{W} = \lambda(L^d - L^s) + I_P[\frac{dP}{dt}\frac{1}{P}]$	$M_3 < 1$	$-\frac{\psi M_4}{1 - I_P} < 1$	$\lambda'(L^{d\prime} - L^{s\prime})(\psi' M_4 + 1 - I_P) +$ $\psi'(M_1+M_2g')(1-I_P) < 0$	$0 \leq I_P < 1$
$\frac{dP}{dt}\frac{1}{P} = \psi(\frac{M^S}{P} - M^D) + I_c K$ $\frac{dW}{dt}\frac{1}{W} = \lambda(L^d - L^s) + I_c K$ where $K \equiv V_P[\frac{dP}{dt}\frac{1}{P}] + V_W[\frac{dW}{dt}\frac{1}{W}]$	$M_3 < 1$	$-\frac{\psi' M_4}{1 - I_c} < 1$	$\lambda'(L^{d\prime} - L^{s\prime})(\psi M_4 + 1 - I_c) +$ $\psi'(M_1+M_2g')(1-I_c) < 0$	$0 \leq I_c < 1$

Indexation in this manner substantially alters the constraints on the system to obtain local conditions for stability. First, the index weights themselves must be limited below one, not only individually but also the product value, condition (D). Accordingly, macroeconomic stability forbids full indexation of either wages or prices when both types of indexation are instituted.

Beyond this, most conditions previously obtained for the non-indexed model become insufficient to assure stability. Essentially, the presence of joint indexation renders the economy less stable than before. Values of elasticities that were previously acceptable for convergence now render the model explosive.

This result may be seen with reference to conditions (B) and (C). The first of these is the model's version of Cagan's α, in general form. With the advent of indexation of this kind, smaller values of M_4 prove destabilizing. Also, the greater is indexation, the greater is the potential for explosive reaction to price movement. Indexation reinforces price movements so that, once disturbed from equilibrium, prices rise (a) due to shifts in money balances, as price expectations adjust, and (b) due to the indexation regime. This may be seen by expressing condition (B) as

(20) $-\psi' M_4 + I_W I_P < 1$,

and is therefore quite consistent with the self-generating argument offered by Heller [18].

The same destabilizing effects can be seen in condition (C). As indexation increases, the second term in this expression increases in absolute size. If $(M_1+M_2 g')$ is positive, the multiplicative factor $(1-I_p)$ increases its size and reinforces the explosive tendencies in the money market. Quite independently, indexation also adds another element of uncertainty. Notice the presence of the additional positive term in condition (C), i.e., $(-I_W \lambda'(L^{d'}-L^{s'}))$.

Indexation of this kind destabilizes the labor market by transferring wage movements into additional price movements and added disequilibrium in the money market.

Note that the cause of the instability induced by indexation of this nature can be isolated to the indexation of prices to wages by I_W. Essentially, this scheme of wage and price adjustment is the sum of case (i) above and the indexation of prices on wages. It was shown above that the former aided stability, yet stability is disturbed by the present regime. It follows, then, that I_W fosters instability in the economy. This interpretation can be demonstrated by a reexamination of conditions (B) and (C) relative to their counterparts in regime (i). The former has a policy induced price effect caused by the wage indexation parameter, I_p, fueling further price effects by the tied relationship between wages and prices I_W. This did not exist in the first case, as continued feedback was ruled out by construction. Further, the present model hampers the stabilizing adjustment of nominal wages by its explicit feedback to the price equation. Therefore, the stability of the labor market which before served to offset the potential instability of price movements is weakened in this structure.

(iii) Indexing both commodity and labor markets by a general commodity price.

A third alternative indexation scheme is to allow all prices, whether they refer to commodity or labor purchases, to automatically increase at a general price index rate. Here one may think of a system where price increases are allowed according to the cost of production materials. This case is presented on line (iii) of Table 1.

As in case (ii) above, this method of indexation destabilizes the system by exacerbating the movement of commodity prices and reinforcing the potential destabilizing behavior of price expectations. This may be seen by rewriting the second condition as

$$(21)\quad -\psi' M_4 + I_p < 1,$$

which is more restrictive than the non-indexed regime.

The result of this regime on condition (C) is likewise detrimental to the underlying stability of the model. The economics of the situation flows rather directly. Increases in prices due to indexation accelerate disequilibrium, while wage adjustment tends to return the real wage toward its equilibrium value. By indexing both by the same quantity, I_P, the labor market's equilibration process is hampered. This may be seen by rewriting condition (C) as

$$(22)\quad \lambda' (L^{d'}-L^{s'})\left[\psi' M_4+1\right]+\psi'(M_1+M_2 g')-I_P\left[\lambda'(L^{d'}-L^{s'})+\psi'(M_1+M_2 g')\right] < 0.$$

The first part of this expression is identical to the non-indexed case which requires that the underlying model satisfy the condition that $\left[\lambda'(L^{d'}-L^{s'})+\psi'(M_1+M_2 g')\right]$ be unambiguously negative. By the addition of this indexation scheme, the model is therefore forced closer to its boundary condition for stability and, indeed, some systems that were previously stable are rendered unstable.

It would appear, therefore, that allowing wages to be indexed to prices and prices to be indexed to a similar price index, perhaps because of some input cost rationalization, would render the economy less stable than the non-indexed case.

(iv) Indexing commodity and labor markets by a composite index

The last case to be considered is a general or composite index scheme. Here, one may consider the impact of allowing for general indexation of both wages and prices using an index including both wage rates and commodity prices. The weights of each will remain in general form, but must sum to one.[7] This index then may be written as,

$$(23)\quad K = V_P\left[\frac{dP}{dt}\frac{1}{P}\right] + V_W\left[\frac{dW}{dt}\frac{1}{W}\right].$$

Indexing to this composite index, K, may be complete or partial and may be treated analytically as increasing prices and wages by $I_C[K]$ where $0 < I_C \leq 1$ in Table 1.

As in the previous models with commodity price indexation, the second condition is more likely to be violated in this regime than in a regime with no indexation at all, as is the case for condition (C). In fact, conditions (A), (B), and (C) are identical to those of model (iii) above with the index I_C replacing I_p in all equations. The interpretation is also indentical with the constraints on expectations and labor market adjustment more serious than the non-indexed case.

(v) Summary of model one results

The impact of indexation on model one may now be summarized, taking into account the results obtained above.

1. In all cases where commodity prices had been indexed, the model tended to be less stable. Expectations were joined by indexation to make explosive price movements more likely for any value of the underlying functions.

2. Wage indexation of whatever form reduced the severity of the stability conditions by reducing real wage movement that may have been destabilizing to the overall economy.

3. The nature of the index clearly affected the results, and the constraints upon the system's stability.

C. Model 2 - A Goods, Money, Labor With Expectations

In model one, expectations affected economic behavior of the micro units by altering the consumption-money balance trade-off. Effectively, following Cagan, expected inflation was viewed as a negative return from money balances, and shifts in that return altered economic behavior. Indexation did not effect this situation as money balances are not indexed and therefore the economic agent's money balance behavior is unaffected by widespread contract adjustment.

Model two attempts to expand the role of expectations in the economy by adding expectations to the dynamic equations determining wages and prices. The economic rationalization for its inclusion in a general equilibrium model was developed in the neo-classical growth literature, see Johnson [19], Allen [1] or Stein [31], for prices, and the Phillips Curve literature for wages, see Phelps [27], Mortensen [23], Lucas and Rapping [20]. Briefly it may be described as follows. For a steady state equilibrium in model one above, real wages are constant, with their nominal values increasing at the rate of monetary expansion. Prices, too, increase at rate μ which must equal the expected rate π^e. From the dynamic equation for prices, however, this implies that excess demand must continually exist at this equilibrium point so that prices may be forced up at their equilibrium rate. This situation will soon become expected. Prices will rise because of excess demand, as before, and also because of the expected price movement. In equilibrium $\pi = \pi^e$ and excess demand goes to zero. The labor market reacts similarly. The presence of constant price increases indicates that the real value of nominal wage settlements is diminished. In equilibrium, a steady real wage requires that $\frac{dW}{dt}\frac{1}{W} = \pi = \pi^e$. It is reasonable to expect that these expectations form in the labor market directly, so that no explicit disequilibrium be required to have nominal wages escalate at the expected price change rate.

Some have contended that this argument is overspecified and that only a portion of expectations is incorporated into the dynamics of wage and price adjustment, see Gordon [13], Perry [26]. To accomodate both positions, the present model will consider the impact of price expectations on both wage and price markets but allow for various degrees of adjustment to these expectations. In the limits, therefore, the model will approach model one above, as the impact of expectations goes to zero, or approach the fully incorporated expectations as the impact goes to one. The model is also constructed to allow for asymmetry between the wage and price markets by specifying the impact separately for each with no presumption of equality.

Following the procedure above, the dynamic equations of the model will be specified and sufficiency conditions for stability will be discussed. These conditions will then be compared using Table 2 to these same conditions when this model is indexed in any of the four ways outlined above.

The basic dynamic equations of model two are

$$(24)\quad \frac{dP}{dt}\frac{1}{P} = \psi\left(\frac{M}{P} - M^d\left(\frac{W}{P}, g, \frac{M}{P}, \pi^e\right)\right) + k(\pi^e); \quad k' > 0,$$

$$(25)\quad \frac{dW}{dt}\frac{1}{W} = \lambda\left(L^d\left(\frac{W}{P}\right) - L^s\left(\frac{W}{P}\right)\right) + \phi(\pi^e); \quad \phi' > 0.$$

Following the procedures outlined above to obtain the stability conditions, results in line 0 of Table 2.

As expected, expectations tend to destabilize the money market in that increases in π cause shifts in π^e and both wage and price movements. Accordingly, the presence of k' in condition (B) decreases the stable range of the economy by decreasing the permissible values for $\psi' M_4$.[8]

At the same time, however, the presence of expectations in the wage equation increases the economy's tendency toward stability. The greater the value of ϕ', for any given value of k' and ψ', the more likely the sufficiency conditions will be satisfied.

It should be noted that these results follow quite closely the results obtained from indexing the previous model. This is to be expected. Both expectations and indexation result in price and wage movement in light of actual price change. Accordingly, both have the same qualitative results on the stability issue.

D. Model 2 With Indexation

In order to analyze the impact of indexation on this economy, it must be recognized that to consider the role of indexation on stability, indexation, in whatever form, must be dominant. By this, it should be understood that the regime of indexing must either dominate price expectations so that prices move according to the indexing rule or be dominated by expectations, in which case no meaningful indexation exists. It is in the first sense that one must consider the stability of indexation, for if it is viewed as a political device, price expectations will dominate, with perhaps additional intersector problems as indexation begins to fall from favor. In the present analysis, the imposition of an indexation regime is assumed to supplant the k and ϕ functions in their respective markets. Indexation, therefore, becomes the dominant force affecting these markets.

(i) Indexing nominal wages by the commodity price change rate.

This method of indexation replaces the function in the wage market with the indexation term I_p. The resultant dynamic equations are listed under line (i) of Table 2. The conditions for local stability indicated by the Hurwitz conditions are identical to those obtained for the basic model, with the exception of the I_p rather than ϕ' in condition (C). By the dominance condition,

(26) $I_p \geq \phi'$

so that indexation of nominal wages unambiguously increases the stability of the economy.

The economics of this result is identical to that outlined above. The fundamental benefit of indexation of the labor market is its ability to reduce labor market disequilibrium. If the index regime is dominant, it implies that wage movements will move more closely with price movements in the indexed world. Therefore, labor market disequilibrium will be reduced and the economy inherently more stable. On the other hand, if expectations are as efficient as indexation, implying that equation (26) holds with equality, no benefit results from indexing wages. Labor's return is effectively indexed through expectations and a formal implementation adds nothing to the overall stability of the system.

TABLE 2
Necessary and Sufficient Conditions for Stability
Model 2 and Various Index Forms

Index Form	Dynamic Equations of the Model	Necessary and Sufficient Conditions for Stability: A. The Impact of Wealth Changes on Aggregate Demand	B. The Impact of Expected Inflation on Aggregate Demand	C. The Movement of the Real Wage	D. Dominance Condition
(0)	$\frac{dP}{dt}\frac{1}{P} = \psi(\frac{M}{P} - M^D) + K(\pi^e)$ $\frac{dW}{dt}\frac{1}{W} = \lambda(L^d - L^s) + \phi(\pi^e)$	$M_3 < 1$	$-\psi' M_4 + K' < 1$	$\lambda'(L^{d\prime} - L^{s\prime})(1 + \psi' M_4 - K') +$ $\psi'(M_1 + M_2 g')(1 - \phi') < 0$	
(i)	$\frac{dP}{dt}\frac{1}{P} = \psi(\frac{M}{P} - M^D) + K(\pi^e)$ $\frac{dW}{dt}\frac{1}{W} = \lambda(L^d - L^s) + I_P[\frac{dP}{dt}\frac{1}{P}]$	$M_3 < 1$	$-\psi' M_4 + K' < 1$	$\lambda'(L^{d\prime} - L^{s\prime})(1 + \psi' M_4 - K') +$ $\psi'(M_1 + M_2 g')(1 - I_P) < 0$	$I_P \geq \phi'$
(ii)	see Model I(ii) of Table 1				$I_W I_P < K'$ $I_W > \phi'$ $I_W < K'$
(iii)	see Model II(iii) of Table 1				$I_P < K'$ $I_P < \phi'$
(iv)	see Model II(iii) of this Table				

(ii) Indexing wages by price change and prices by wage change.

The indexation of the economy according to this method results in a model already treated as model I(ii). At that time, the dynamic equations were indicated and the sufficiency conditions outlined. Comparing this regime with the stability conditions with expectations effecting price movements but no indexation, results in the following inequality constraints for indexation to be preferred to the non-indexed regime,

(27A) $I_W I_P < k'$

(27B) $I_P \geq \phi'$

(27C) $I_W \leq k'$

Condition (A) requires that the net effect of the cross indexation be less than the non-indexed pure expectations effect in the price equation. This is required to limit the feedback between sectors in the indexed regime and to constrain the instability caused by price expectations. The second condition requires the labor market adjustment be more rapid in the indexed economy than the non-indexed world, the same condition that resulted in model (i) above. The third condition on the movement of inflation in the face of movements in wages changes is ruled out by the dominance rule outlined above. It is impossible to effectively determine the movement of prices if the institutional arrangement allows for less adjustment than expectations would imply. The empirical importance of this condition is, however, ambiguous without empirical estimates of k' and ϕ'. Note some limiting conditions associated with potential indexation. As $I_p \rightarrow 1$ the labor market constraint becomes less binding as seen by (27B). However, given the feedback relationship between wage and price movements, their product increases so that condition (27A) becomes a more serious constraint. At the other extreme, as $I_p \rightarrow 0$, the economy becomes dominated by ϕ' and the importance of indexation declines. On the other hand, as the price adjustment index connecting wage movements to automatic price increases rises, $I_W \rightarrow 1$, if $I_W > k'$, the dynamics of the system will be determined by I_W. Unfortunately this dominance will accelerate price movements and render the overall economy less stable. This behavior is consistent with the previous analysis which indicated that automatic adjustment, via indexation or expectations, increases instability in the money or commodity market, but decreases the instability of the labor market by increasing the speed of wage rate adjustment.

(iii) Indexing both commodity and labor markets by a general commodity price index.

This model is also treated above as model I(iii), with the sufficiency conditions indicated in Table 1. Comparison of these conditions to obtain inequality constraints that, if satisfied, would render the economy more stable, results in consistent restrictions, viz.,

(28A) $k' > I_p$,

(28B) $\phi' < I_p$.

The first of these is clearly violated by the dominance condition outlined above. If indexation is to be effective, it must increase prices by more than expectations would have in its absence. If not, expectations will continue to determine price movements. If, however, $I_p > k'$ the conditions for money market stability become more restrictive. Indexation has effectively increased the movement of prices, given a disturbance, and may result in instability. The second constraint, on the other hand, is unambiguously binding by the dominance criterion.

The net effect on stability, therefore, is destabilizing in the money market and stabilizing in the labor market. The net effect is ambiguous, as in case (ii) above, but will depend upon these to countervailing forces on the system.

(iv) Indexing commodity and labor markets by a composite index.

This case of indexation, too, has been analyzed above. The composite indexation of model one results in identical dynamic equations as results from composite indexation of the expanded model here. Comparison of the sufficiency conditions results in inequality constraints for greater stability that are identical to (28) above. As before, the first of these conditions is violated by the dominance condition but the second is satisfied.

(v) Summary of model one results.

The net results of indexation of the expectations model may be summarized with reference to the conditions obtained above. Generally, we conclude that all results obtained in the earlier model remain valid. Specifically, commodity market indexation of whatever type destabilizes the economy, and labor market indexation increases its stability. Further, indexing both markets by adminstrative fiat has ambiguous, offsetting, results. In any case, these complex schemes obscure the impact of indexation on stability and are clearly dominated by simple wage adjustment. More specifically, three points warrant mention.

(1) Indexation of an expectations model can only be meaningful if it dominates expectations, in the sense that the index adjustment becomes the relevant one for wage and price dynamics. Any attempt to partially index at a rate below prevailing expectations will make any indexation scheme inoperative.

(2) A dominant system of indexation is preferred to simple expectations in the labor market as it reduces disequilibrium more quickly than would be the case without this institutional setup. However, for it to be the dominant force in the price equation it must add instability to the economy and render the system inherently less stable than its non-indexed counterpart.

(3) The expectations model is equivalent to the indexed model in the case where expectations approximate the index scheme. If indexation is complete and expectations are rational in the sense of Lucas [21], this condition for equivalence would be satisfied. It may also be satisfied for a less-than-complete

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[1]The present analysis centers upon a deterministic system rather than one subject to stochastic disturbances. However, it can be shown that the stability conditions obtained in the text are relevant for the stochastic system as well. Specifically the deterministic system's conditions become both necessary and sufficient for this more general class of models. This proof is available to the reader on request.

[2]All that is needed for the analysis to preceed is to assume that price expectations are a function of observed price movements, i.e., $\pi^e = f(\pi)$ with $f'>0$. The use of adaptive expectations in (10) is in no way substantive.

[3]Given an aggregate production function $Y = F(K,L)$ with the properties of neoclassical theory and holding capital stock constant equation (11) becomes

$$g = F(L^d(W/P)) - \frac{W}{P} L^d(\frac{W}{P})$$

with

$$\frac{dg}{d\frac{W}{P}} = F'L^d - \frac{W}{P}\frac{L^{d'}}{L^d} - L^d$$

where

$$-\frac{W}{P}\frac{L^{d'}}{L^d} = \eta.$$

Therefore as long as $\eta \leq 1$ $\frac{dg}{d\frac{W}{P}} < 0$ or equivalently $g = g(\frac{W}{P})$ with $g'<0$.

[4]The complete mathematical appendix obtaining these results is available from the authors on request.

[5]It will be assumed throughout the paper that $(M_1+M_2g')>0$. Only the conditions relating the stability conditions to this view of the system will be presented in the body of the text. The alternative is intrinsically less interesting as then no problem of labor market instability exists and stability is assumed. See the mathematical appendix for proof of this assertion as well as the counterparts to Table 1 for this world.

[6]For $(M_1+M_2g)<0$ this equation (19) implies overindexation. However, evaluating the system at that point results in a trivial economic equilibrium at zero output. See the mathematical appendix.

[7]Violation of this summation constraint releases the discussion from an economic to a political arena, as, it is reported, in Brazil.

[8]A variant of this case is treated by Goldman [12].

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