Price-quantity Adjustments in a Macro-disequilibrium Model

by

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One studies in this paper the dynamic properties of a simple macro-economic model where exchange takes place outside equilibrium on both the product and labor markets without opportunity to recontract. The non-realization of planned supply on one market affects the demand on the other very much in the manner shown in the pioneering work of Clower (1965), Patinkin (1965), Leijonhufvud (1968) and Barro and Grossman (1971). Here however firms and consumers form expectations on sales and income which are based, quite logically, on past market experiences. As Leijonhufvud (1973) points out, the collapsing of realized and expected income is a feature of both Clower's and Barro and Grossman's framework; it precludes the analysis of the recursive interaction process which characterizes the dynamic multiplier.In addition, the fixed-price method (also used in the more formidable technical apparatus of Benassy (1975) and Drèze (1975)) is abandonned; this allows a study of the interplay between price and quantity adjustments.The money wage and output price vary directly with the excess demand signals transmitted to the labor and output markets.Our criterion of selection for the " perceived " market signals is simply based on this principle: what is the single type of quantity information most generally observable on competitive markets in and outside equilibrium?

Money is introduced into the theoretical framework as the unique standard of value, means of payment and store of value for the short (predictable) and longer (unpredictable) term; A specific transaction structure is adopted: Expanding Clower's (1967) original idea to the disequilibrium case, the households are assumed to recognize that they will in general be unable to make use of the uncertain proceeds of their sales of labor services (and ownership of capital services) in any period t when paying for period t purchases. In this model, the consumers' intertemporal decision can be reduced to the choice between current consumption and holdings of liquid assets subject to a liquidity and an expected monetary income constraint .

The model presented in this paper can then be viewed as part of a reappraisal of Neo-Keynesian macroeconomics where 1) the choice theoretic framework on both the consumers' and firms' side, and the informational aspects of disequilibrium are directly tackled in a dynamic set up , 2) the role of expectations and money is formally recognized and shown to be critical in the derivation of Keynesian results when prices and wages are flexible. These results are generally defined, not anymore in terms of existence of unemployment or underemployment equilibria, but of cumulative and lasting contractions of output and employment. It is shown however that one does not need a very far-fetched expectation mechanism to obtain a true Keynesian unemployment "equilibrium" in a decentralized monetary economy with price flexibility.

Section 1 describes the firms' and consumers' optimal behaviors outside equilibrium in an environment where information is limited. Section 2 analyzes the global stability properties of the dynamic system. Section 3 investigates the possible adjustment paths for real income, prices and wages when an initial full employment notional equilibrium is disturbed by a once and for all shock in the money supply or technological conditions. "Business cycles" are shown to be generated.

The interesting behavior of the real wage, the role played by the hoarding of money in "multiplying", the short-run effect of deflation on real income and the need to reinterpret the role of liquid assets as an essential determinant and integral part in the decision to consume are explicitly emphasized.[1]

I

Consider an economy with two decentralized markets (labor and output)where labor services and consumption goods are respectively traded against money. These markets are inhabited by independent auctioneers whose role is to post at the beginning of each period a money wage and output price on the basis of the demand and supply signals received in the previous period (a discrete time model is for the moment considered, although it will later be converted, for mathematical convenience, into continuous time).

The firms choose at the beginning of period t (going from time t to t+1) their output supply signal y_t^s . I shall identify y_t^s as the amount of (perishable) output the businesses offer for period t , i.e.the amount of output they will produce and hope to sell during this time interval; this is clearly the most generally observable supply signal on the commodity market and usually differs from the notional one if a product sales constraint is perceived or there exists a limit on available labor. The level of employment L_t is directly related to y_t^s via the production function :

(1) $$y_t^s = f(L_t)$$

where $f' > 0$, $f'' < 0$. As a fixed labor supply $\bar{L}$ will be postulated, "full employment" output is easily defined by $\bar{y} = f(\bar{L})$; the firms are presumed to know this limit on productive capacity. At less than full employment, the labor demand signal L_t^d , the amount of labor the firms order, coincides with the employment level L_t. When $L_t = \bar{L}$, the relevant labor demand signal will become the notional one; this will lead to a competitive bidding up of money wages when an excess demand for labor develops. In our monetarized economy, the firms pay the labor services and the ownership of capital at the end of period t using the monetary proceeds acquired from their product sales; there are no retained earnings and hence the entire money stock should be in the households' hands at time t+1. The entrepreneurs' market signals are derived from short-run profit maximization. But when exchange takes place outside equilibrium on all markets, the price system must cease to be viewed as conveying all of the relevant information. If a sales constraint on output is operative, the notional labor demand and output supply are not the relevant market signals. As the entrepreneurs cannot possibly know at the very beginning of period t what will be the demand for their product, they have to form expectations y_{t+1}^e on the amount of potentially realizable sales. Hence, to forge their decision, the firms will generally use not only the available information on the posted money wage and output price for period t, but also the amount of sales y_t which came to completion at the end of period t-1 (time t), the known limit on available labor, and, perhaps, the past price p_{t-1}. It is here, one should notice, that the model finds its "Neo-Keynesian" (or Patinkinesque?) flavor: In his explanation of the Principle of Effective Demand [2] , Keynes indeed implicitly assumes that the firms

(or the auctioneer?) know the households'- generally income constrained - demand function; the current market clearing output price can then instantaneously be determined [3].

Formally, the firms' objective is to choose y_t^s and L_t which maximize the expected short run profits:

$$(2) \qquad p_t \, y_t^s \; - \; w_t \, L_t$$

subject to

$$(3.1) \qquad y_t^s = f(L_t)$$

$$(3.2) \qquad y_t^s \leqq y_{t+1}^e$$

$$(3.3) \qquad L_t \leqq \overline{L}$$

Notice that in (2) and (3.2), what should be called "planned" sales, have directly and legitimately been identified with the offer y_t^s [4]. Expressions (3.2) and (3.3) are the expected commodity sales and labor availability constraints. Given the properties imposed on the production function, the above maximization problem must clearly provide the following optimal signals:

$$(4) \quad L_t = f^{-1}(y_t^s) \text{ with } y_t^s = \text{Min} \{ y_{t+1}^e, \, f(L^d(w_t/p_t)), \, \overline{y} \}$$

where $L^d(w_t/p_t)$, the notional labor demand, is derived from the familiar equality between the marginal productivity of labor and the real wage : $f'(L_t^d) = w_t/p_t$. The labor demand signal is $L_t^d = L_t$ if $y_t^s < \overline{y}$ and

$L_t^d = L^d(w_t/p_t)$ if $y_t^s = \bar{y}$ where in all circumstances y_t^s is given by (4). Notice that when (4) provides $y_t^s = \bar{y}$, one must have $L_t^d = L^d(w_t/p_t) \geqq \bar{L}$. It is quite possible however for $L_t^d = L^d(w_t/p_t)$, when $y_t^s < \bar{y}$, to be itself a notional value.

The manner in which the firms form their expectations will often play a major role in the dynamic properties of the model. As the entrepeneurs simply do not know the demand function for their product, one has no alternative but to specify a reasonable "ad hoc" expectation formation mechanism: The known limit on capacity and sales being $\bar{y}$, I shall postulate that the firms' expectation on the amount of sales currently realizable is given by the following simple stock adjustment formula:

$$y_{t+1}^e - y_t = \alpha(\bar{y} - y_t) \tag{5}$$

where $0 \leqq \alpha < 1$, and $y_t \leqq \bar{y}$ is the amount of total sales (real income) effectively realized during the previous period. Expression (5) implies that the firms never expect the current potential limit on sales to be worse than last period effective level of economic activity, although their expectations are naturally affected by the past realizations. If $\alpha > 0$, such an expectational mechanism translates, I shall argue, a "fundamental belief" in the (ultimately) equilibrating forces of competitive price adjustments. Indeed, assume that the short side of the market is always realized: Then, if an excess supply of output prevailed during t-1, y_t is equal to the demand and (5) may imply that the firms believe that a fall of prices will allow them to improve their sales; if on the other hand an excess demand prevailed, y_t is equal to the supply and (5) may imply that the firms believe that <u>even</u> at higher prices they can increase their sales (as long as $y_t < \bar{y}$). Naturally, if $y_t = \bar{y}$, (5) always provides $y^e_{t+1} = \bar{y}$. The particular case $\alpha = 0$ corresponds in this model to the most pessimistic behavior; a similar assumption is implicitly made in macroeconomic textbooks when presenting the Neo-Keynesian dynamic multiplier and does indeed seem most legitimate when used in connection with the assumption of rigid prices.

I shall describe the consumers'choice theoretic framework along the lines I have developed elsewhere (cfr. Lorie (1976)); particular attention is devoted to a formalization of the fact that money is used both as means of payment and store of value in order to properly diagnose some important form of multiplier processes. The households are assumed to be, beyond the current and immediate future periods, facing a considerable amount of uncertainty. This induces them to attach utility to the holding of assets (here idle money balances) at the end of their short term planning horizon, for a broad "precautionary " motive. Accordingly, the consumers are supposed to maximize the following utility index:

$$U\left(y^d_t, y^d_{t+1}, \frac{M^d_{t+2}}{p^e_{t+2}} \right) \tag{6}$$

where y^d_t and y^d_{t+1} are respectively the current and planned future output demand and M^d_{t+2} are the planned holdings of asset money balances at the end of the horizon [5]. The utility function is assumed to be additive with the partial derivatives $U_1 > 0, U_2 > 0$, $U_3 > 0$; $U_{11} < 0$, $U_{22} < 0$, $U_{33} < 0$. In accordance with the specific payment structure of this model, the housholds receive at the beginning of period t (time t+1) a predetermined money income $m_t = p_{t-1}\ y_t$ in return for the economic activity of the previous period [6]. Recall that the consumers' labor supply signal is assumed to be a constant $\bar{L}$ defining full employment. If a steady state notional equilibrium prevails, the consumers would expect to receive at the end of period t (time t+1) a real income equal to $\bar{y} = f(\bar{L})$. When exchange may take place outside equilibrium, and in particular a sales constraint prevails on the labor market, the households will form expectations on the level of realized money income to be receive at time t+1. These expectations then play a crucial role in the derivation of the consumers' current effective demand [7]. From the above specifications, the consumers' attainable set is defined by the following sequence of budget constraints:

$$p_t\ y_t^d + M_{t+1}^d = M_t + m_t \tag{7.1}$$

$$M_{t+1}^d \geqq 0 \tag{7.2}$$

$$p_t\ y_{t+1}^e = m_{t+1}^d \tag{7.3}$$

$$p_{t+1}^e\ y_{t+1}^d + M_{t+2}^d = M_{t+1}^d + m_{t+1}^d \tag{7.4}$$

$$M_{t+2}^d \geqq 0 \tag{7.5}$$

where M_t is the predetermined initial level of <u>idle</u> money balances kept from period t-1 to t; m_t is the nominal income realized during period t-1 and received at time t; M_{t+1}^d is the current asset demand for idle balances to be held for period t+1 (naturally, this demand can never be negative).The "dichotomized" constraints (7.1)-(7.5) reflect the use of money as means of payment and interestingly differentiate between an "active" and "passive" demand for money [8]. The possible constraint on effective income y^e_{t+1} might limit m^d_{t+1} in (2.3) and adversely affect the households'decision; in particular, their current demand y^d_{t+1}. Interestingly, this feedback effect (the Keynesian multiplier) can only operate in our monetarized economy through a change of the asset demand M^d_{t+1}; this is indeed clearly implied by the dichotomized constraint (7.1) . The above framework adds to Barro and Grossman's presentation in that it suggests that an asset demand for money is often at the origin of multiplier processes [9]; a similar conclusion is hinted at by Folley and Hellwig.

I shall assume an interior solution for M^d_{t+1} and hence that a current asset demand for money always exists [10]. Furthermore, one takes $p^e_{t+1}=p^e_{t+2}=p_t$ [11]. The following current demands are then easily derived from the above maximization problem:

(8.1) $$y^d_t = y^d\left(y^e_{t+1}, \frac{M_t + m_t}{p_t} \right)$$

(8.2) $$\frac{M^d_{t+1}}{p_t} = M^d\left(y^e_{t+1}, \frac{M_t + m_t}{p_t} \right)$$

with

(8.3) $$0 < \frac{\partial y^d_t}{\partial y^e_{t+1}} = \frac{\partial y^d_t}{\partial \frac{M_t+m_t}{p_t}} < 1$$

The function (8.1) is of course a Keynesian consumption function with, however, expected current effective income (to be paid at the end of period t) and real balances replacing the sole effective income as independent variables. The property (8.3) concerning the marginal propensities to consume can easily be derived, given the restrictions on the utility function, by a simple comparative static analysis.

Here again, the households'income expectations will crucially affect the dynamic properties of the model. As it does seem reasonable to assume that the consumers do not drastically differ from the firms in the formation of their expectations, I shall postulate (cfr. (5)) that $y^e_{t+1} = y_t + \alpha (\bar{y} - y_t)$. This expectational mechanism introduces in part a Robertsonian lag in the consumption function (8.1) (recall indeed that y_t refers to the previous period realized income). It will undoubtedly exert a distabilizing influence on the motion of the economic system although the fact that $y^e_{t+1} > y_t$ when $y_t < \bar{y}$ cushions this influence.

Assuming that the short side of the market is always satisfied, the level of sales (and hence real income) effectively realized at the end of period t is :

$$(9) \qquad y_{t+1} = \mathrm{Min} \{ y^d_t, y^s_t \} = \mathrm{Min}\{ y^d(y^e_{t+1}, \frac{M_t+m_t}{p_t}), \ \mathrm{Min} \{y^e_{t+1}, f(L^d(w_t/p_t)), \bar{y}\} \}$$

where $y^e_{t+1} = y_t + \alpha (\bar{y} - y_t)$. Hence, the change of real income from period t-1 to period t is given by:

$$(10) \quad y_{t+1} - y_t = \mathrm{Min} \{y^d(y^e_{t+1}, \frac{M_t+m_t}{p_t}) - y_t, \ \mathrm{Min} \{y^e_{t+1} - y_t, f(L^d(w_t/p_t)) - y_t, \bar{y} - y_t\} \}$$

The following Walrasian price adjustments are now postulated :

$$(11.1) \qquad p_{t+1} - p_t = \lambda\left[y^d(y^e_{t+1}, \frac{M_t+m_t}{p_t}) - \mathrm{Min}\{ y^e_{t+1}, f(L^d(w_t/p_t)), \bar{y} \}\right]$$

$$
\begin{aligned}
&(11.2.a) \qquad w_{t+1} - w_t = \eta \left[f^{-1}(y^s_t) - \overline{L} \right] \\
& \qquad\qquad\qquad\qquad\qquad\qquad \text{if } y^s_t = \operatorname{Min}\{ y^e_{t+1}, f(L^d(w_t/p_t)), \overline{y}\} \begin{cases} < \overline{y} \\ = \overline{y} \end{cases} \\
&(11.2.b) \qquad\qquad\qquad = \eta \left[L^d(w_t/p_t) - \overline{L} \right]
\end{aligned}
$$

where $y^e_{t+1} = \overline{y} + \alpha (\overline{y} - y_t)$; $\lambda > 0$ and $\eta \geqq 0$ are the degrees of response of the output price and money wage to excess demands on the commodity and labor markets.Those are independent price adjustments because in a decentralized monetary economy, one must reject the view that the real wage is directly determined by the wage bargaining process on the labor market (Keynes himself placed great emphasis on this ; it is well understood today that the market structure associated with monetary exchange creates greater informational problems outside equilibrium).

There should be little debate concerning the legitimacy of the demand and supply signals considered in (11.1) and (11.2) : The output demand signal is a "constrained" one when the households do not expect full employment (i.e. $y^e_{t+1} = \overline{y} + \alpha (\overline{y} - y_t) < \overline{y}$); a "notional" one otherwise (recall that y^e_{t+1} is always bounded by $\overline{y}$). The labor demand signal is a constrained or notional one depending not only upon the firms' sales perception, but also upon the real wage. The use of the labor supply $\overline{L}$ presumes however that there are no search costs associated with the transmission of this signal and hence no "discouraged workers" under persistent unemployment. Some reader would perhaps prefer to see in (11.1) the notional signal instead of the current offer y^s_t . I have refrained from adopting this point of view mainly because the notional signal is not generally directly observable on the market. In any case,as one will later observe, the forthcoming analysis is not crucially affected by our specific choice; although the consideration of the notional supply would result in a much greater price variability[12]. Similarly, if the firms face a limit on available labor, their "constrained" and directly observable supply signal is $y_t^s = \overline{y}$; but, in all logic, the expressed competitive labor demand signal when $y_t^s = \overline{y}$ is $L^d(w_t/p_t) \geqq \overline{L}$. Finally, one should notice the absence of any "spillover" effect (on the labor supply) when a positive excess demand for output emerges; the logical consistency of the model however is not endangered[13].

II

The time path for the real income, ouput price and money wage is governed by the system (10)(11.1)(11.2). Clearly, there exists a unique steady state full employment equilibrium real income $\overline{y}$ and money prices $p_t = p$, $w_t = w$ associated with a fixed money stock $S = M_t + m_t$[14]. The notional equilibrium output price p and wage w can be calculated from 1) the level of real wage solving $L^d(w_t/p_t) - \overline{L} = 0$, 2) the output price solving $y^d(\overline{y}, \frac{S}{p_t}) - \overline{y} = 0$. Naturally, the Classical property $w/p = f'(\overline{L})$ holds.

As my primary interest is to analyze not only the stability properties per se, but also the laws of motion of the economy for relatively large deviations from full employment, a global analysis is needed. Given the complexity of the system (10) (11.1)(11.2), some "tour de force" is required. I propose first to take a continuous approximation of the above system by defining $\dot{y}_t \simeq y_{t+1} - y_t$, $\dot{p}_t \simeq p_{t+1} - p_t$, $\dot{w}_t \simeq w_{t+1} - w_t$ (the dot on the variables stands for time derivative) and then to perform a qualitative analysis using the phase diagram technique.[15] Despite its obvious three dimensions, the problem can be coaxed on the plane because of the relatively simple behavior of the money wage.

Clearly, from (11.2), $\dot{w}_t < 0$ whenever $y_t < \bar{y}$ (then indeed $y^s_t \leqq y^e_{t+1} < \bar{y}$; moreover, if $y_t = \bar{y}$, $\dot{w}_t \gtreqless 0$ whenever $w_t/p_t \lesseqgtr w/p$ where w/p is the notional equilibrium real wage.

For any given w_t, a locus $\dot{y}_t = 0$ is determined in the plane p_t, y_t by equating the righthand side of (10) to zero. To represent the graph of $\dot{y}_t = 0$, one proceeds as follows: a) First assume that $y^d((1-\alpha)y_t + \bar{y}, \frac{S}{p_t}) - y_t$ was always the relevant minimal element in the right-hand side of (10), then d_1 (a potential locus $\dot{y}_t = 0$) is derived from solving for p_t the equation $y^d((1-\alpha)y_t + \bar{y}, \frac{S}{p_t}) - y_t = 0$. When properly interpreted, the locus d_1 is a Keynesian demand price function (i.e. a locus of equilibrium points on the commodity market) and is represented in figures 1.a and 1.b . The slope of d_1 is :

(12)
$$\theta_1 = \frac{\partial p_t}{\partial y_t} = \frac{-(1 - (1-\alpha) \partial y^d_t / \partial y^e_{t+1})}{\frac{S}{p_t^2} \frac{\partial y^d_t}{\partial \frac{S}{p_t}}} < 0$$

as $0 \leqq \alpha < 1$ and $0 < \partial y^d_t / \partial y^e_{t+1} < 1$. Above (below) d_1, $y^d((1-\alpha)y_t + \bar{y}, S/p_t) - y_t$ is negative (positive). Notice that d_1 does not depend upon the money wage w_t; this is essentially due to the use of an aggregate consumption function with identical marginal propensities to consume out of labor income and profits [16].

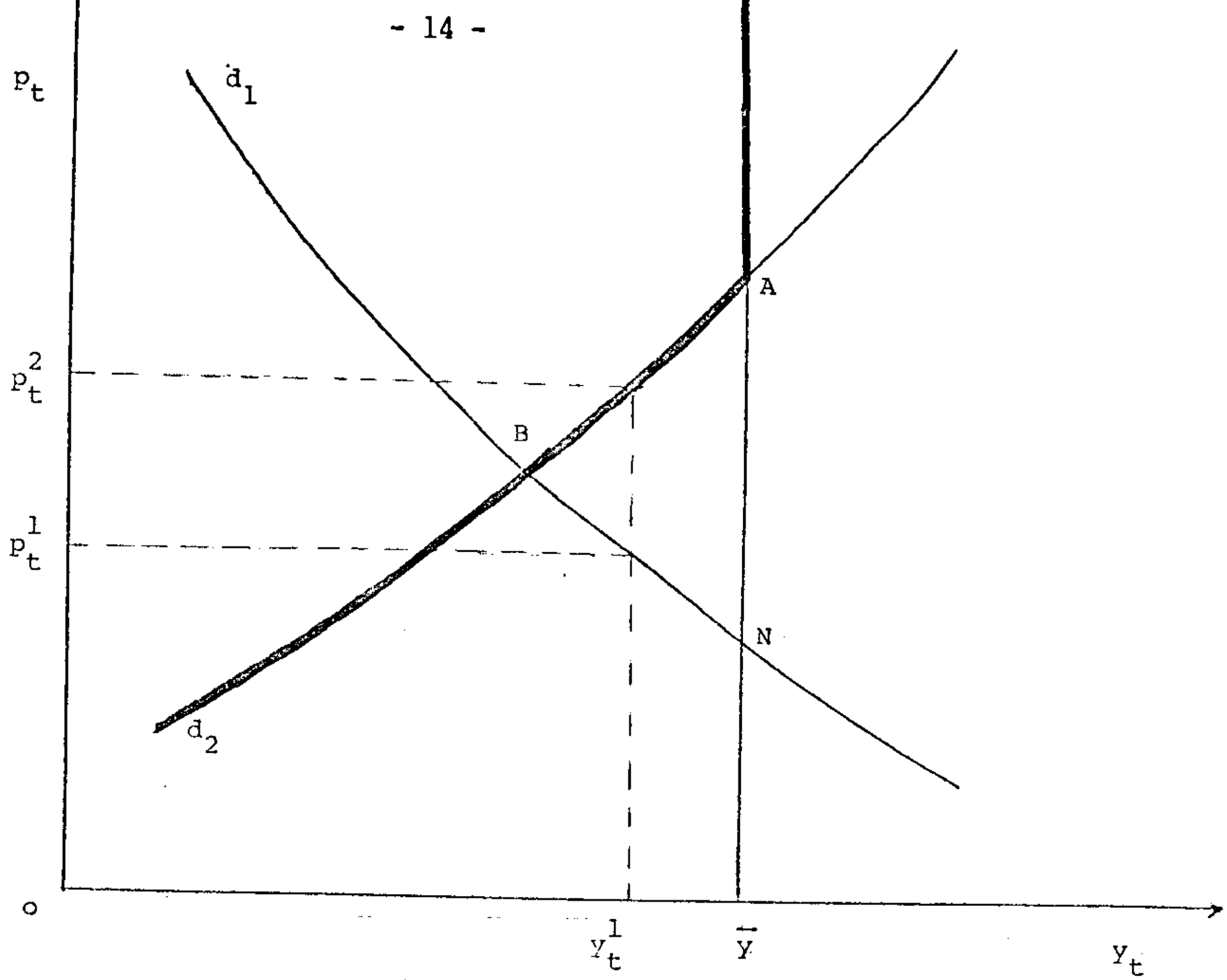

figure 1.a

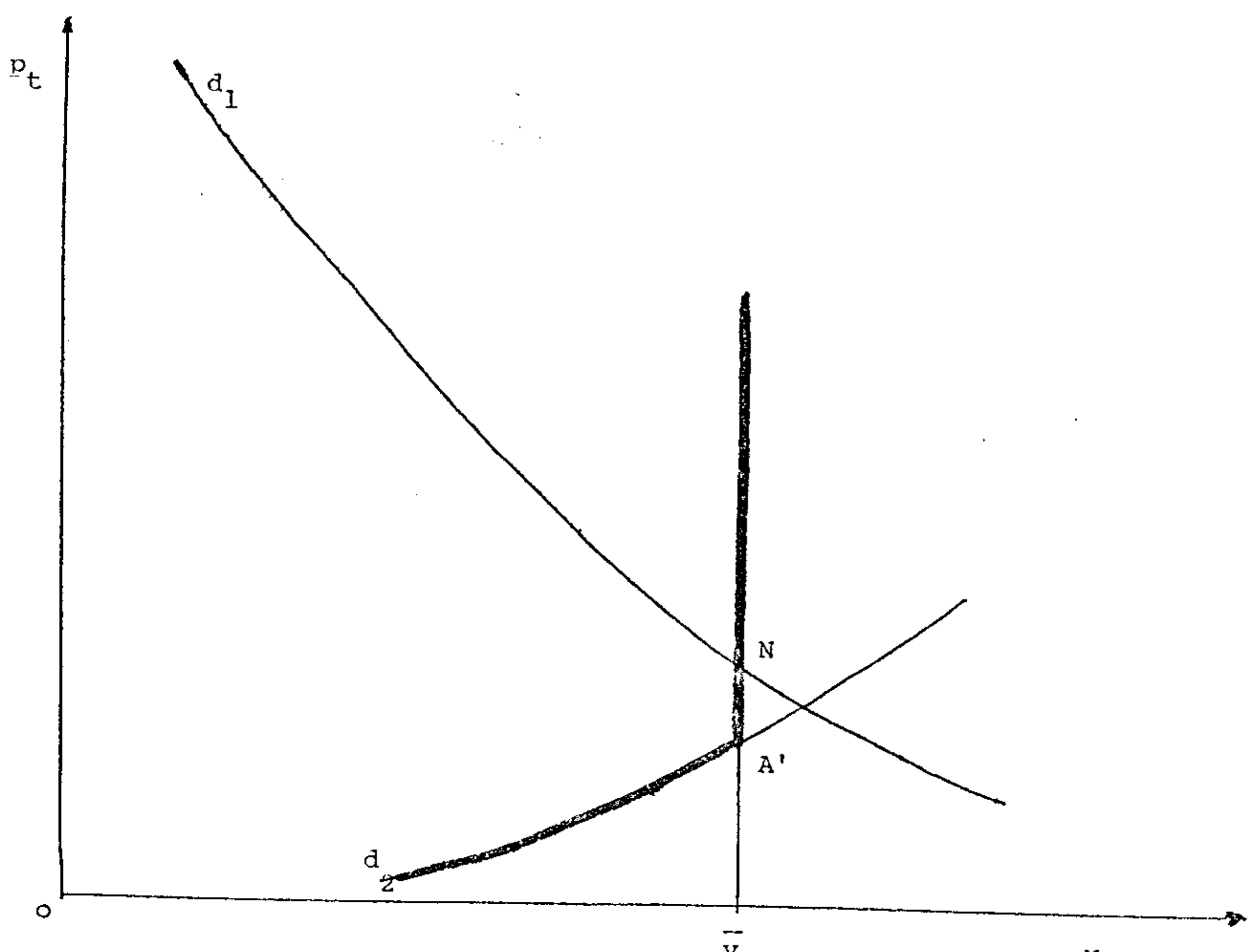

figure 1.b

b) Secondly, assume that Min $\{ (1-\alpha)y_t+\alpha\bar{y}\ ,\ f(L^d(w_t/p_t)) - y_t,\ \bar{y} - y_t \}$ was always the relevant minimal element in the right-hand side of (10); to describe the potential locus $\dot{y}_t = 0$ in this case, one must consider the vertical line elevated at the value $y_t = \bar{y}$ (which annuls the first and third elements of the above expression [17]) and the locus d_2 derived from solving for p_t the equation $f(L^d(w_t/p_t)) - y_t = 0$. The slope of d_2 is

$$(13) \qquad \theta_2 = \frac{\partial p_t}{\partial y_t} = \frac{1}{-f' \dfrac{\partial L^d}{\partial \frac{w_t}{p_t}} \dfrac{w_t}{p_t^2}} > 0$$

One can easily verify that d_2 shifts to the right (left) as w_t decreases (increases). On the right (left) of d_2, $f(L^d(w_t/p_t)) - y_t$ is negative (positive). The figures 1.a and 1.b illustrate the two polar cases where the money wage is so high (low) as to have d_2 intersecting d_1 on the left (right) of $\bar{y}$. Naturally, for $w_t = w$, d_2 would pass through the point N. The graph of the potential locus $\dot{y}_t = 0$ given under b) by the equation Min $\{ (1-\alpha)y_t + \alpha\bar{y} - y_t,\ f(L^d(w_t/p_t)) - y_t,$ $\bar{y} - y_t \} = 0$ is defined by the portion of d_2 on the left of $\bar{y}$ and by the vertical line above d_2 when $\bar{y}$ is reached (cfr. the thicker drawing in figures 1.a and 1.b). This is obvious for $y_t < \bar{y}$. For $y_t = \bar{y}$, only the values of p_t greater than $\bar{y}$ A (or $\bar{y}$ A') solve the above equation; for $p_t < \bar{y}$ A (or $\bar{y}$ A'), the minimal element in the above equation would indeed be $f(L^d(w_t/p_t)) - \bar{y} < 0$.

The locus $\dot{y}_t = 0$ associated with (10) can nowbe easily derived: For $0 < \alpha < 1$, it is entirely defined by the broken line $d_1 B\ d_2$ in figure 1.a or d_1 N A' d_2 in figure 1.b; any other point on the potential locuses would not pass the ultimate test of equating the right-hand side of (10) to zero. Take for instance $y_t = y_t^1 < \bar{y}$ in figure 1.a : if the output price is p_t^1 one has $y^d((1-\alpha)\, y_t^1 + \alpha\,\bar{y},\ S/p_t^1) - y_t^1 = 0$, but $f(L^d(w_t/p_t^1) - y_t^1 < 0$ (one is on the right of d_2) which must imply (cfr. (10)) $\dot{y}_t < 0$ at p_t^1, y_t^1 ; if the output price is p_t^2 one has $\mathrm{Min}\{\,(1-\alpha)\, y_t^1 + \alpha\, y_t^1 - \bar{y},\ f(L^d(w_t/p_t^2) - y_t^1\ ,\ \bar{y} - y_t^1\} = 0$, but $y^d(\,(1-\alpha)y_t^1 + \alpha\,\bar{y}\ ,\ S/p_t^2\,) - y_t^1 < 0$ (one is above d_1) and hence (cfr. (10)) $\dot{y}_t < 0$ at p_t^2, y_t^1 ... The reader will then easily check that $\dot{y}_t < 0$ everywhere outside the region encompassed by d_1 B d_2 in figure 1.a or d_1 N A' d_2 in figure 1.b , and $\dot{y}_t > 0$ inside.

In the particular case where $\alpha = 0$,it follows from the above analysis that the locus $\dot{y}_t = 0$ is now defined by the <u>entire</u> area encompassed by d_1 B d_2 or d_1 N A' d_2; indeed, for any point inside that area one has (cfr. (10)) $\dot{y}_t = \mathrm{Min}\{\,0,\ f(L^d(w_t/p_t)) - y_t,\ 0\} = 0$ (as on the left of d_2, $f(L^d(w_t/p_t)) - y_t > 0$).

For any <u>given</u> value w_t, a locus $\dot{p}_t = 0$ is determined in the plane p_t, y_t by equating the left-hand side of (11.1) to zero. To analyze the graph of the locus $\dot{p}_t = 0$, one again proceeds as follows: a) First assume that $(1-\alpha)\, y_t + \alpha\,\bar{y}$ was always the relevant minimal element in $\mathrm{Min}\{\,(1-\alpha)\, y_t + \alpha\,\bar{y},\ f(L^d(w_t/p_t)),\ \bar{y}\,\}$; then d_3, a potential locus $\dot{p}_t = 0$, is derived from solving for p_t the equation $y^d(\,(1-\alpha)\, y_t + \alpha\,\bar{y},\ S/p_t) - [(1-\alpha)\, y_t + \alpha\,\bar{y}] = 0$ and is illustrated in figure 2 [18]; it does not depend upon w_t .The slope of d_3 is given by:

$$\text{(14)} \qquad \theta_3 = \frac{\partial p_t}{\partial y_t} = \frac{-(1-\alpha)(1-\partial y_t^d/\partial y_{t+1}^e)}{\dfrac{S}{p_t^2}\ \dfrac{\partial y_t^d}{\partial \frac{S}{p_t}}} < 0$$

Comparing (12) and (14), one concludes that $\theta_3 > \theta_1$ everywhere for

$0 < \alpha < 1$. Clearly, d_1 and d_3 uniquely intersect at $y_t = \bar{y}$ if $0 < \alpha < 1$ and are identical if $\alpha = 0$. Above (below) d_3, $y^d((1-\alpha)\, y_t + \alpha\, \bar{y}, S/p_t) - \left[(1-\alpha) y_t + \bar{y}\right]$ is negative (positive).

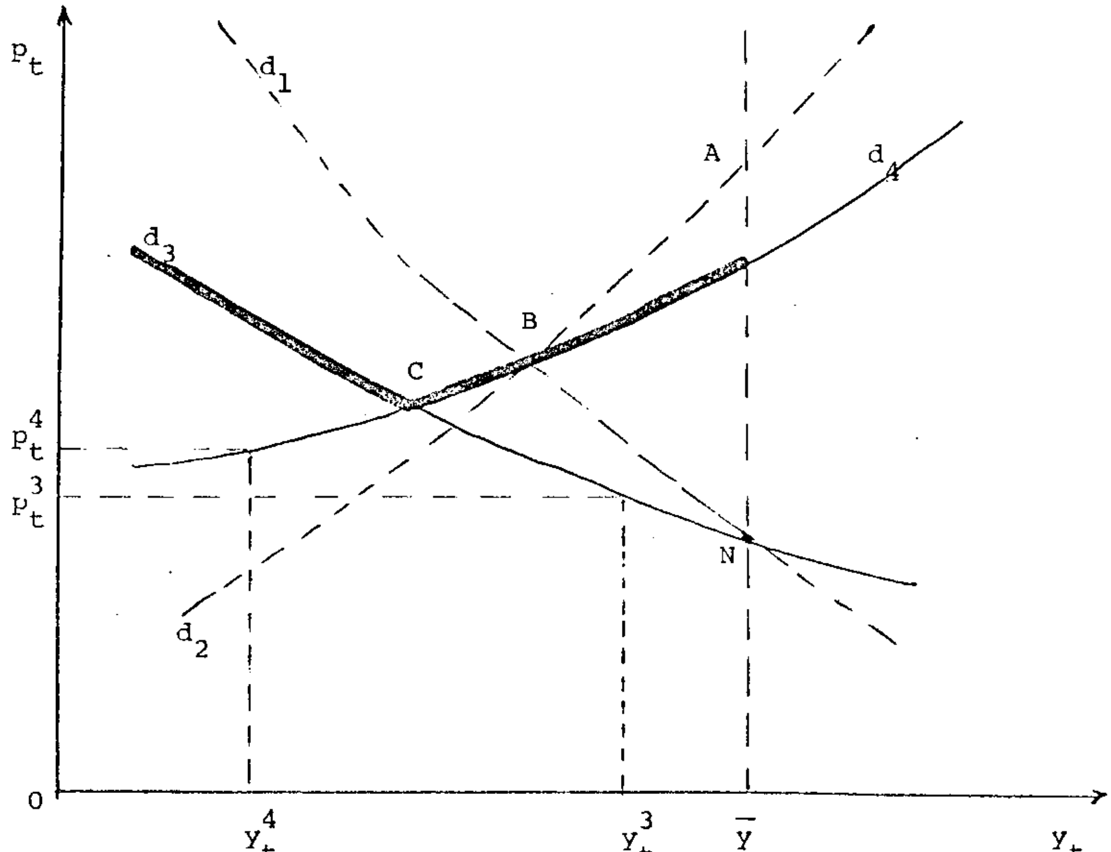

Figure 2

b) Secondly, assume that $f(L^d(w_t/p_t))$ was always the relevant minimal element in Min$\{ (1-\alpha) y_t + \alpha\, \bar{y},\ f(L^d(w_t/p_t)),\ \bar{y}\}$; then d_4, a potential locus $\dot{p}_t = 0$, is derived from solving for p_t the equation $y^d((1-\alpha) y_t + \alpha\, \bar{y}\ , S/p_t) - f(L^d(w_t/p_t)) = 0$ and is also illustrated in figure 2. The slope of d_4 is given by:

$$\text{(15)} \qquad \Theta_4 = \frac{\partial p_t}{\partial y_t} = \frac{(1-\alpha)\ \partial y_t^d / \partial y_{t+1}^e}{\frac{S}{p_t^2} \frac{\partial y_t^d}{\partial \frac{S}{p_t}} - f' \frac{\partial L^d}{\partial \frac{w_t}{p_t}} \frac{w_t}{p_t^2}} > 0$$

One can easily verify that d_4 shifts to the right (left) as w_t decreases (increases). Notice from (13) and (15) that $\Theta_4 < \Theta_2$. Moreover, given any money wage w_t, d_4 and d_2 uniquely intersect on d_1; this is so because on the locus d_1, $y^d((1-\alpha)y_t + \alpha\bar{y}, S/p_t) = y_t$. On the left (right) of d_4, $y^d((1-\alpha)y_t + \alpha\bar{y}, S/p_t) - f(L^d(w_t/p_t))$ is negative (positive).

The locus $\dot{p}_t = 0$ associated with (11.1) consists of the broken line d_3 C d_4 (cfr. the thicker drawing in figure 2). Any other point on the potential locuses would not pass the ultimate test of equating (11.1) to zero. Take for instance $y_t = y_t^3 < \bar{y}$: if the output price is p_t^3, one has $y^d((1-\alpha)y_t^3 + \alpha\bar{y}, S/p_t^3) - [(1-\alpha)y_t^3 + \alpha\bar{y}] = 0$, but $y^d((1-\alpha)y_t^3 + \alpha\bar{y}, S/p_t^3) - f(L^d(w_t/p_t^3)) > 0$ (one is on the right of d_4) and hence $f(L^d(w_t/p_t)) < (1-\alpha)y_t^3 + \alpha\bar{y}$ which must imply (cfr. (11.1)) $\dot{p}_t > 0$ at p_t^3, y_t^3. Next, consider $y_t = y_t^4 < \bar{y}$: if the output price is p_t^4 one has $y^d((1-\alpha)y_t^4 + \alpha\bar{y}, S/p_t^4) - f(L^d(w_t/p_t^4)) = 0$, but $y^d((1-\alpha)y_t^4 + \alpha\bar{y}, S/p_t^4) - [(1-\alpha)y_t^4 + \alpha\bar{y}] > 0$ (one is below d_3) and hence $(1-\alpha)y_t^4 + \alpha\bar{y} < f(L^d(w_t/p_t^4))$ which must imply (cfr. (11.1) $\dot{p}_t > 0$ at p_t^4, y_t^4. One then easily verifies that $\dot{p}_t > 0$ everywhere below d_3 C d_4 and $\dot{p}_t < 0$ above.

Combining the locuses $\dot{y}_t = 0$ and $\dot{p}_t = 0$ just analyzed, the phase diagrams 3.a and 3.b are derived for $0 < \alpha < 1$. The arrows indicate the laws of motion of p_t and y_t on the left of the absorbing full employment barrier $\bar{y}$ (as $y_t^s \leq \bar{y}$, y_t is always bounded by $\bar{y}$). Figure 3.a (3.b) corresponds to the case where the money wage w_t is above (below) the notional equilibrium level w ; if $w_t = w$, B and A' would coincide with N.

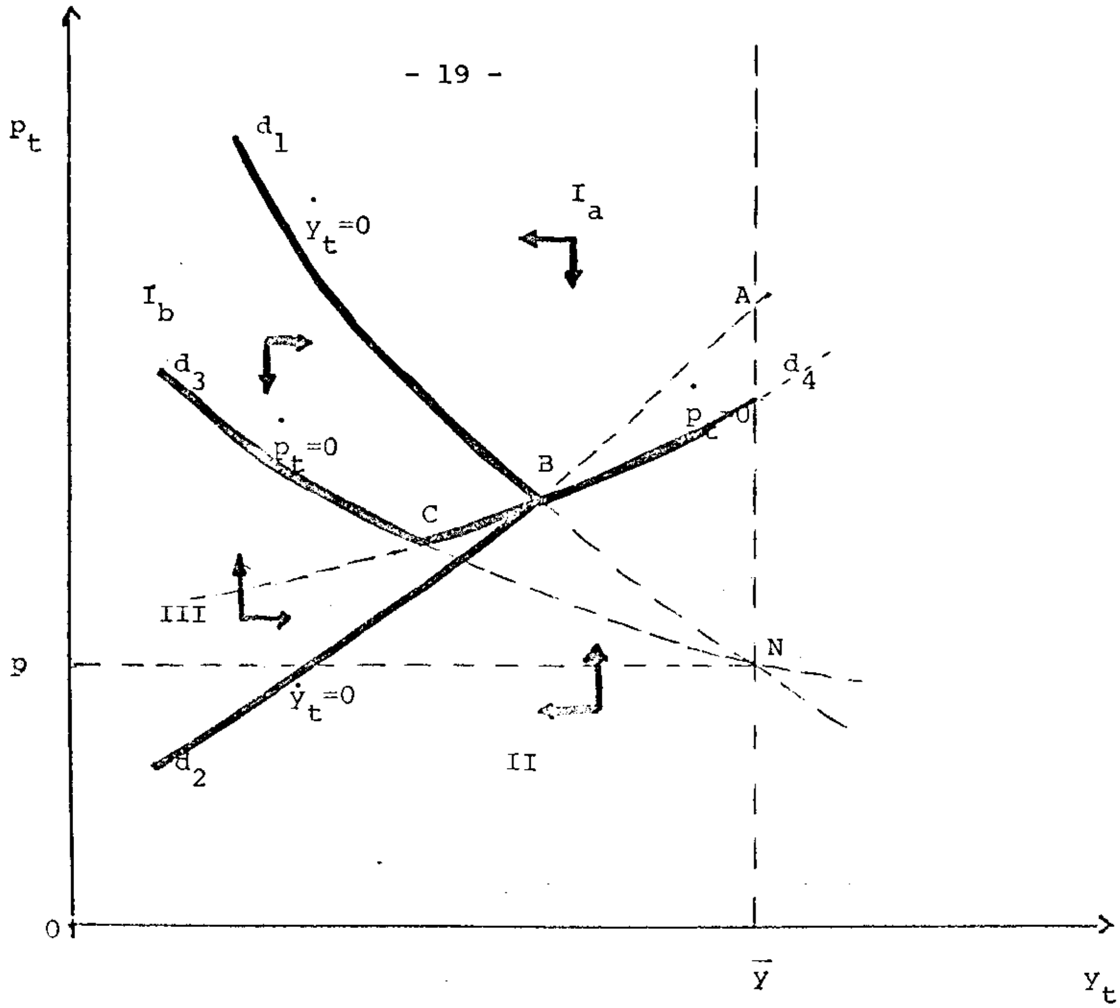

figure 3.a

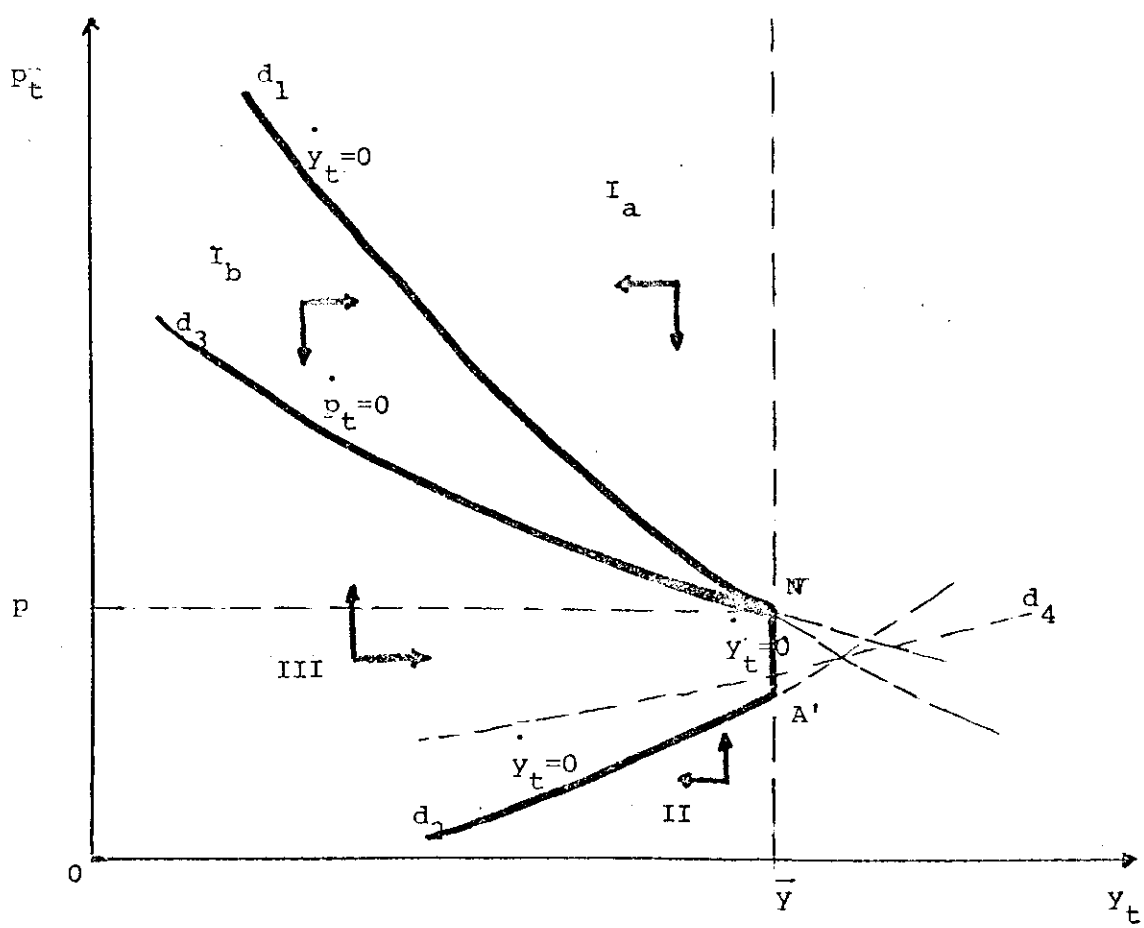

figure 3.b

If the money wage is rigid at whatever initial level (i.e. $\eta = 0$ in (11.2)), the sub-system (10) (11.1) is globally stable [19] : Any path in figure 3.a will reach the "equilibrium" B where $\dot{y}_t = 0$, $\dot{p}_t = 0$. Point B corresponds to the type of Neo-Keynesian unemployment "equilibrium" ultimately obtained as the limit of a price-quantity adjustment process under the restrictive assumption of a rigid money wage; it is actually a purely Keynesian "equilibrium" in the sense that at B (but not generally during the adjustment process !) the real wage equals the marginal productivity of labor (recall that B is a point of both d_2 and d_4).Similarly, any path in figure 3.b will reach N where $\dot{y}_t = 0$, $\dot{p}_t = 0$ and full employment prevails, together with a positive excess demand for labor.

The global stability of the entire system (10) (11.1) (11.2) for $\eta > 0$ and $0 < \alpha < 1$ can be qualitatively established using the above analysis: Following (11.2), w_t falls whenever $y_t < \bar{y}$; hence over time, as long as $y_t < \bar{y}$,the segments d_2 B and C Bd_4 in figure 3.a (or d_2 A' in figure 3.b) move downward along d_1 and d_3 towards N (or further below N in figure 3.b). Neither the configuration of regions I_a to III, nor the laws of motion of p_t and y_t are altered. Any path for the economy ultimately converges to the full employment real income $\bar{y}$. To conclude that the notional equilibrium $\bar{y}$, p, w is globally stable at N, one must also account for what happens to the economic system when $\bar{y}$ is reached without w_t or (and) p_t at their notional values. Two possibilities only can occur : a) Full employment is reached at the point N itself with $p_t = p$, $y_t = \bar{y}$ but $w_t < w$. Then $\dot{p} = 0$, $\dot{y}_t = 0$ and $\dot{w}_t > 0$ as

as $w_t/p_t < w/p$ and there is an excess demand for labor; the money wage increases shifting d_2 A' in figure 3.b upward towards N until the excess demand for labor vanishes at $w_t = w$ (the stability of this process is obvious given the properties of the notional labor demand function). b) Full employment is reached at a point K along A' N with $p_t < p$ and $w_t < w$; there is an excess demand for commodities and hence $\dot{p}_t > 0$. Because K is on the left of d_2 A' (a portion of a curve d_2), $f(L^d(w_t/p_t)) > \bar{y}$ [20] ; hence one must have $w_t/p_t < w/p$ and an excess demand for labor with consequently $\dot{w}_t > 0$. As the price and money wage adjustments (11.1) and (11.2) are entirely independent, there seems to be no guarantee that the subsequent evolution of the money wage and output price will take place at full employment .
In Appendix however I prove that for initial values $y_t = \bar{y}$ and $w_t \leqq w$, $p_t \leqq p$ such that $w_t/p_t \leqq w/p$, the solution to (10) (11.1) (11.2) always implies that p_t, w_t converge to p, w with $w_t/p_t \leqq w/p$, and $y_t^s = y_t = \bar{y}$ for all t . Hence the global stability of the notional equilibrium $\bar{y}$, p, w when $0 < \alpha < 1$ and $\eta > 0$ is qualitatively established.

The particular situation where $\alpha = 0$ is rather interesting and deserves closer attention : Recall that the curves d_1 and d_3 coincide when $\alpha = 0$ and that $\dot{y}_t = 0$ in the entire area encompassed by d_1 B d_2 (cfr. figure 1.a). Figure 4 provides then a relevant phase diagram [21]. Assume first that the money wage is fixed (or $\eta = 0$): any path for y_t and p_t ultimately reaches an "equilibrium" point on the segment d_1 B where $\dot{y}_t = 0$ and $\dot{p}_t = 0$. Any such "unemployment equilibrium" being on the left of d_2 and d_4 corresponds to a situation where the real wage is less than the marginal productivity of labor; such a theoretical result is indeed strongly supported by empirical observations during recessionary periods. Assume next that the money wage is allowed to decrease with the existence of an excess supply of labor (i.e. $\eta > 0$), the curve d_2 B d_4 shifts downward with B towards N; but clearly, the above results remain essentially unaltered. Moreover, once an "unemployment equilibrium" point like Q (where the commodity market clears and the firms have no incentive to change their plans) is obtained, the money wage could keep falling without affecting real output or employment.

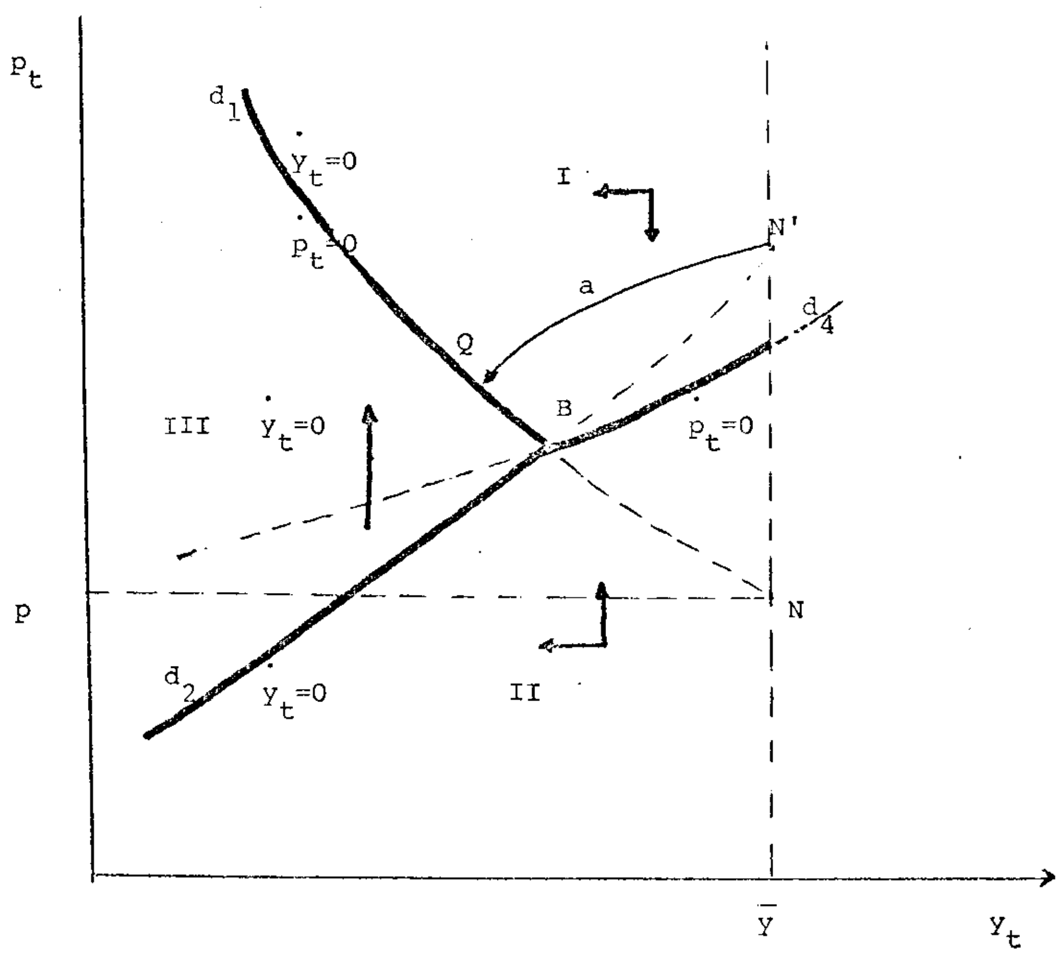

figure 4

If there exists a minimum wage (arbitrarily close to zero) at which the labor supply becomes infinitely elastic (the type of "Keynesian" labor supply so popular in the macroeconomic textbooks), a stable underemployment equilibrium would be obtained despite the money wage and output price flexibility. These conclusions are unaffected by replacing the supply signal considered in (11.1) by the (not generally observed) notional one[22]. Of course, the above results do crucially depend upon the monetized (and hence decentralized) structure of the economy. Notice that the configuration of figure 4 and the initial disequilibrium point N' could emerge when an initial notional equilibrium is disturbed by a once and for all decrease of the money supply. One may venture the interpretation that during the Great Depression(where the extremely pessimistic business environment might have implied a coefficient α close to zero), the cumulative contraction of output, prices and wages followed more closely the type of path indicated by a in figure 4. By the end of 1931, the wholesale price index had pretty much completed its vertiginous fall; it stabilized from 1932 to 1933 together with real output while the money wage kept decreasing because of the huge excess supply of labor. This paradoxical behavior of prices might be explained by the fact that by 1932, the effective excess demand for goods (not the notional one,of course) was extremely low (i.e. the "unemployment equilibrium" was essentially reached). The continual fall of the money wage lead, contrary to the Classical (and Keynesian) theoretical prediction , to a decrease of the real wage from 1932 to 1933 without production picking up.

III

How well does the "automatic" mechanism associated with the system (10) (11.1) (11.2) perform to restore full employment when $0 < \alpha < 1$ and $\eta > 0$? If one defines as "homeostatic" [23] an economic system where <u>any once and for all</u> disturbance away from full employment is <u>directly</u> followed by a monotonic return of real income and employment to the notional equilibrium, our Neo-Keynesian model is clearly <u>not</u> "homeostatic" (observe that one requires monotonicity only for the real income path, not for the product price and wage; otherwise very few dynamic systems indeed would be "homeostatic"). Indeed, any path starting in region I_a or II will temporarily take the economic system further away from the full employment $\bar{y}$. This is an important conclusion: the adjustment path displays Keynesian <u>multiplier</u> effects with a cumulative contraction of output and employment <u>despite</u> the money wage and price flexibility. Keynesian results do not depend upon some assumption of wage rigidity. The above properties constitute, of course, a prima facie argument for occasional exogeneous controls of the aggregate demand to break the (temporarily self-defeating) expectation mechanism of consumers and firms. It is not to say that intervention can never be justified in an "homeostatic" system as the speed at which the economy adjusts might still be a problem.

In this Neo-Keynesian model, the cumulative contraction of output and employment has a very different origin in region I_a and II. An excess supply prevails on both the output and labor markets in I_a, while only on the labor market in II. Contraction proceeds in the latter, not with the existence of unsold production, but solely because of the insuf--ficience of the demand price to cover the supply price (marginal cost); indeed, all points in II are on the right of d_2 where the notional supply for period t is less than the past realized level of sales.

The simplest conceivable shock which would disrupt the economy from a notional equilibrium to a disequilibrium position like A in figure 3.a would be a once and for all decrease of the money supply; such a decrease might have been responsible for the downward shift of d_1 and d_3 leading to d_1 intersecting d_2 (which does not depend upon the level of money stock) at B on the left of $\bar{y}$. The initial fall of the money supply reduces aggregate demand and creates an excess supply of commodities driving the output price downward; the firms (partially) react by cutting down production. The initial fall in the real income also feedsback on the consumers' expected current income and leads to a further cumulative weakening of the aggregate demand, and so on.... Unemployment drives the money wage (and hence the point B) downward enlarging the regions I_a and I_b. Any path in the "extensible" region I_a must eventually enter I_b ; this will happen when the positive real balances effect starts to dominate depressing income expectations in the consumption function. In I_b, the current aggregate demand exceeds the past realized sales. Production and real income increase although, ex post, an excess supply still prevails on the product market, driving the price further down (the money wage decreases as long as $y_t < \bar{y}$). Once it enters I_b, the real income path moves monotonically towards B, itself tending to N , until notional equilibrium is restored [24].

The dynamics of region II could directly become relevant when an initial equilibrium is disrupted by either a technological shift decreasing everywhere the marginal productivity of labor or a one-shot attempt to increase the money wage above the notional equilibrium level. In any case, d_2 and d_4 shift to the left so as to intersect at B on d_1. Starting from N, output and real income decrease because the firms cut down production as the demand price is insufficient to cover the cost. An excess demand for output develops pushing the output price upward.This adjustment process restates, of course, the familiar cost-push inflation phenomenon. Unemployment develops and, if competitive forces are at work (or back at work) on the labor market, the money wage falls shifting d_2 and d_4 back to the right enlarging I_a,I_b and III. Eventually, any path starting in region II will have to enter some region I_a or III: a) If it enters region I_a, the contraction of output continues because of a deficiency of the aggregate demand which is due to an unfavorable income expectation and real balances effect. The return to notional equilibrium then follows the path already studied for the deflationary case. The fact that an initial "once and for all" shock may lead to successive situations of excess demand and supply on the product market is a particularly interesting theoretical finding obviously not devoid of empirical content.
b) If it enters region III, the firms' relevant supply signal (the notional one) becomes greater than the past realized sales. As an excess demand for output still prevails, realized income increases together with the product price. The real income path moves monotonically towards B,

itself tending to N when the wage is flexible, until notional equilibrium is restored. It should be pointed out that the above analysis ignores the possibility that full employment is reached with $w_t < w$ or (and) $p_t < p$. An "ultimate" wage-price adjustment taking place at $y_t = \overline{y}$ might be required (cfr. our global stability analysis).

Although the system (10) (11.1) (11.2) is also operational for the case where an initial notional equilibrium is disrupted by a once and for all increase of the money supply, decrease of the money wage or upward technological shift, the dynamics are far less exotic because of taking place at full employment; they are consequently left to the reader. This assymetric behavior which gives a strong Keynesian flavor to the model is only partially due to the assumption of a fixed labor supply.

Clearly, the real wage is allowed to move independently from the marginal productivity of labor in this Neo-Keynesian model; and the "necessary" inverse relationship [25] between the real wage and output is broken. Indeed, the firms are not constrained anymore to operate on their supply curve. Hence for instance in region I_a, if the adjustment is "mainly" in quantity and the wage is flexible, the contraction of real income could take place together with a decrease of the real wage. In region II, with or without wage flexibility, a decrease of the real wage always accompanies the reduction of real income. Recall also that when $\alpha = 0$ and $\eta > 0$, a fall of the real wage is consistent with $\dot{p}_t \simeq 0$ and $\dot{y}_t \simeq 0$ around the "unemployment equilibrium".

The role played by money in the dynamic analysis can now be appraised: First of all, it is only because money serves as unit of account and means of payment that independent price and wage adjustments can be conceived; otherwise the real wage (barter exchange rate) should respond to an excess demand for labor or output. Secondly, money must ultimately lie at the bottom of the deviation-amplifying process in region I_a. For a more rigorous treatment of this point, I shall go back to the discrete time framework: Assume that the households, while along a steady state $\bar{y}$, p', w' associated with a money stock S', face a decrease of their initial money balances from S' to S at the beginning of period t. The amount of output supplied on the market during priod t is still $\bar{y}$ (cfr. (4) and (5)) and the households still expect to receive at the beginning of period t+1 a full employment level of income $\bar{y}$ (cfr. (5)). The product demand however falls because of the money stock reduction (and the output price did not change from t-1 to t as notional equilibrium prevailed earlier) which means that the sales decrease and that the households receive at time t+1 a nominal income $m_{t+1} = p' \; y_{t+1} < p' \; \bar{y} = m_t$. At the beginning of period t+1, the firms decide to curtail production (cfr. (4) and (5)); similarly, the consumers' expected real income y^e_{t+2} is such that $y_{t+1} < y^e_{t+2} < \bar{y}$. The output demand y^d_{t+1} is provided by (8.1) if, as assumed, an interior solution for M^d_{t+2} prevails. When compared with the derivation of y^d_t, two things have changed at period t+1 : 1) the expected real income has fallen, 2) the real value of the initial money balances has increased as (cfr. (11.1) $p_{t+1} < p_t = p'$. Given (8.3), 1) tends to reduce demand while 2) tends to increase it. If the effect of falling income expectations dominates the Pigou effect, one has $y^d_{t+1} < y^d_t$; and this essentially causes the deviation amplifying process in I_a.

Without $M^d_{t+2} > 0$ at optimum, one has $y^d_{t+1} = (M_{t+1} + m_{t+1})/p_{t+1}$; in which case, only 2) is at work and hence $y^d_{t+1} > y^d_t$. The economic system is then for all practical purposes homeostatic.

More generally, in this model, reduced income expectations from one period to the next lead to the current hoarding of non active money balances to span consumption over the short term horizon and at the same time constitute what is seen as a reasonable stock of purchasing power for the unpredictable future. Comparing the expenditure constraint (7.1) "ex post" at period t and t+1, one has: on the right-hand side, $M_t + m_t = M_{t+1} + m_{t+1} = S$; on the left-hand side, $p_{t+1} < p_t$ and $y^d_{t+1} < y^d_t$. Hence one must have $M^d_{t+2} > M^d_{t+1}$. Obviously, it is because the holdings of inactive money balances has increased that the velocity of income at t+ 1 defined by $p_{t+1}\ y_{t+2}/ S$ will be lower than the velocity of income at t defined by $p_t\ y_t / S$.

The diagnosis for sizable deviation-amplifying movements in I_a provided in this paper appears essentially different from Leijonhufvud's suggestion : Here it is the replenishment , not the depletion, of liquid "buffer stocks" in the face of falling income expectations which render those expectations pretty much self-fulfilling. Here the cause of the cumulative process itself does not only lie in the role of money as means of payment but also often as store of value.

Empirical investigations using very short term time series data would be needed to check the theoretical findings. It may be hard, and perhaps impossible , to differentiate inside a period of contraction the time where direct autonomous contractionary forces are at work from the time where contraction proceeds endogeneously. Hoardings of more nominal money balances might be expected to decrease in the initial phase; to increase afterwards. In any case, a simultaneous estimation of the households' behavior concerning consumption and holdings of liquid assets should provide useful information regarding the role played by money in affecting both the amplitude and length of certain recessionary movements.

Finally, it should be noticed that the consumption function (8.1), although almost purely Keynesian, does make consumption depend upon 1) the previous period realized money income, 2) the expected real income to be receive at the end of this period. Given the households' expectation mechanism, the expected real income y^e_{t+1} is always (if $0 < \alpha < 1$) greater than the realized income y_t as long as $y_t < \bar{y}$. Consequently, the consumption function (8.1) is closer to the Permanent Income Hypothesis than it first appears, in the sense that it embodies some form of less than unit elastic expectation mechanism. Such a property also characterizes the firms' expectation mechanism (if $0 < \alpha < 1$); it is as instrumental as the flexibility of wages and prices in guaranteeing the ultimate return to notional equilibrium.

APPENDIX

I shall start with some preliminary results: If at some instant t : $y_t = \bar{y}$, $w_t \leqq \bar{w}$, $p_t \leqq \bar{p}$ and $w_t/p_t \leqq w/p$, one must have : a) $y_t^s = \bar{y}$, $L^d(w_t/p_t) - \bar{L} \geqq 0$ and hence $\dot{w}_t \geqq 0$ (cfr. (11.2.a) (11.2.b)) b) $y^d(\bar{y}, S/p_t) \geqq \bar{y}$ and hence $\dot{p}_t \geqq 0$ (cfr. (11.1)); then also, (10) is satisfied for $\dot{y}_t = 0$.

Assuming for the moment that the entire subsequent price-wage adjustment takes place at full employment; (11.1) (11.2) then become:

(15.1) $$\dot{p}_t = \lambda \left[y^d(\bar{y}, S/p_t) - \bar{y} \right]$$

(15.2) $$\dot{w}_t = \eta \left[L^d(w_t/p_t) - \bar{L} \right]$$

The stability analysis of such a system is illustrated in figure 5.

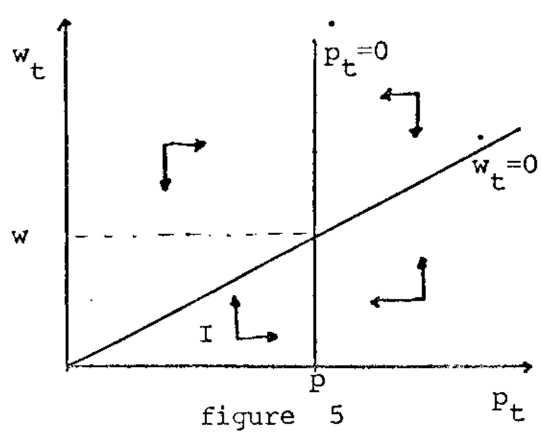

figure 5

Notice that the slope of $\dot{w}_t = 0$ is w/p, the notional equilibrium real wage.

One is only interested in paths starting in region I of figure 5, where $w_t \leqq w$, $p_t \leqq p$ and $w_t/p_t \leqq w/p$. Any such path will converge to w,p without <u>ever</u> being able to <u>leave</u> region I . For all instant t, our preliminary results must then apply and in particular (10) is satisfied for $\dot{y}_t=0$. The path $y_t=\bar{y}$ together with the path for p_t and w_t determined by (15.1) (15.2) is consequently the solution to the system (10) (11.1) (11.2) when initally $y_t=\bar{y}$, $w_t \leqq w$, $p_t \leqq p$ and $w_t/p_t \leqq w/p$.

Footnotes

*The Wharton School, University of Pennsylvania. I am indebted to Professors E.C.H. Veendorp and Anthony Santomero for some very helpful comments and suggestions on an earlier draft.

1 Our interpretation of the importance of liquid assets is similar to the central idea of Folley and Hellwig's (1975) illuminating paper.

2 Cfr. Keynes (1936), Chapter 3.

3 Grossman (1972) correctly interprets the General Theory market framework as one where the product market is always clearing. For a dynamic version of Keynes'paradigm, and its relation to the dynamic multiplier, see Lorie (1976).

4 In this one period model, it would naturally be unoptimal for the firms to offer more than they plan to sell.

5 As time passes by, the horizon moves correspondingly; hence, the planned future decisions will not generally be implemented.

6 $p_{t-1}\, y_t \leq p_{t-1}\, y^s_{t-1}$ is of course the value of the product sales realized during period $t-1$; it is also the sum of the labor income $w_{t-1}\, L_{t-1}$ (where L_{t-1} is the level of employment at $t-1$) and monetary profits $n_t = p_{t-1}\, y_t - w_{t-1}\, L_{t-1}$.

7 Obviously, this is Clower's essential argument: Keynes' consumption function can only be logically understood in such a choice theoretic paradigm.

8 Cfr. Clower, op. cit.

9 Notice that if at some period of time M^d_{t+1} was equal to zero at optimum, y^d_t would be equal to $(M_t + m_t)/p_t$ and hence independent of y^e_{t+1}; but then, a decrease of the expected income y^e_{t+1} does not necessarily reduce y^d_t.

10 Once M^d_{t+2} enters the utility function (6) for the broad precautionary motives associated with the"unpredictable" future, this assumption is legitimate for a fairly large set of possible values for m_t, m^e_{t+1}, M_t; even if the households were to display a preference for present consumption.

11 This assumption, made essentially for mathematical convenience, may or may not be perceived as entirely reasonable. Observe however that for the class of log additive utility functions, the current demands would in any case always turn out to be independent of both p^e_{t+1} and p^e_{t+2}.

12 Our approach espouses Veendorp's (1974) view in his exchange with Grossman (1974). I must acknowledge however that a more complete analysis of the relevant supply signal outside equilibrium is needed.

13 This is partly due to the particular framework considered. A limit on the amount of output available might result in forced savings; but the consumers do not have anything to fear (in terms of achieving, ex post, a suboptimal level of utility in this period) from transmitting to the markets their fixed labor supply and effective output demand signals.

14 Recall that, given our transaction structure, the entire money stock is in the consumers'hands at the beginning of each period.

15 Cfr. Quirk and Ruppert and Olech (1963).

16 Naturally, the position of d_1 does depend upon the level of money stock S ; a decrease (increase) of the money supply would shift d_1 to the left (right).

17 Notice that the value of these elements is always positive for $y_t < \bar{y}$.

18 Purely for reference, the curves d_1 and d_2 are reproduced in figure 2 (d_2 is drawn for $w_t > w$ as in figure 1.a; but this is inessential).

19 They represent "sign stables" cases in the classification given by Quirk and Ruppert, op. cit. footnote 15

20 Notice that if K corresponds to A', $f(L^d(w_t/p_t)) = \bar{y}$ and hence $\dot{w}_t=0$.

21 I have assumed that $w_t > w$; but the argument is essentially similar if $w_t \leqq w$.

22 Then, from (11.1) , the relevant $\dot{p}_t=0$ would be the entire curve d_4.The points along d_4 on the left of B then constitute "unemployment equilibria" where $\dot{y}_t=0$. Notice however that p_t would continue to fall with w_t when $\eta > 0$.

23 Cfr. Leijonhufvud, op. cit.

24 Observe that any path in I_b can cross into region III along C B (possibly more than once when the wage falls); but in all cases $\dot{y}_t = 0$.

25 The "necessity" of this relationship in a purely Classical or Keynesian framework is due to the assumption of a continuously clearing product market.

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