The Value of the Firm Under Regulation

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The basic theoretical relationships between the value of the firm and leverage were set forth by Modigliani and Miller (MM). Much work has tested the MM relationships empirically, including studies which used data from regulated industries. Gordon has stated that, because earnings before interest and taxes are not held constant in regulated industries, the MM formula used in empirical work is invalid. However, Elton-Gruber (EG) challenge Gordon's statement. The present paper shows that both the Gordon and the EG formulae hold only under special conditions. Under "normal" conditions of demand, both formulae underestimate the value of the levered firm. We show that there is no *a priori* method of estimating the effect of leverage on the value of a regulated firm without knowledge of specific supply and demand conditions. As researchers do not usually know these conditions, the results of papers testing the MM propositions with data on regulated industries are ambiguous. General formulae for the discount rate and the valuation of a levered firm in a regulated industry are presented.

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I. INTRODUCTION

Over the years, the economics profession has been concerned with the effect of leverage on the value of a firm. The basic theoretical relationships were set forth by Modigliani and Miller (MM). In a classic paper[1] they show that, without taxes, the value of a firm should not be affected by leverage policy. In a later article[2], MM show that, with taxes, an increase in leverage should increase the value of the firm.

Much other work has tested the above relationships empirically.[3] Data on regulated industries were used for some of these papers, including one by Miller-Modigliani themselves.[4] The methodology of these papers implicitly assumes that earnings before interest and taxes (EBIT) of the firm are not affected by a change in the capital structure. This assumption has been questioned for firms in regulated industries.

Recently, Gordon has pointed out that the ratio of the firm's expected earnings after taxes but before interest to total assets is usually held constant in regulated industries.[5] This observation implies that the EBIT are affected by regulation. He argues that, in such a case, the correct valuation formula is precisely the one developed by Modigliani and Miller in the absence of taxes. Gordon's argument may cast doubt on the studies examining regulated industries. However, though Elton-Gruber[6] (EG) accept Gordon's observation of the regulatory constraint; they dispute Gordon's conclusions by arguing that MM's method of valuation with taxation is still appropriate for firms in regulated industries.

The present paper examines the validity of MM's valuation method for a regulated firm and reaches conclusions even more pessimistic than those of previous authors. We show that there is no a priori method of estimating the effect of leverage on the value of a regulated firm without a knowledge of the specific supply and demand conditions of that firm. Researchers do not usually know the supply and demand conditions of an industry. This means, first, that all the papers examining MM's propositions by using data on regulated industries may very well be invalid, or at least their conclusions are ambiguous. Furthermore, it implies that future researchers must be very cautious when using and interpreting data on regulated industries.

The structure of the paper is as follows. In section II, we show that special conditions of demand can be found where either Gordon's or Elton-Gruber's valuation formulae hold. However, it is proved that, under "normal" conditions, both Gordon's and Elton-Gruber's valuation formulae are incorrect, since they underestimate the value of a levered firm.

General formulae for the discount rate and the valuation of a levered firm in a regulated industry are then presented. These formulae, rather than those presented by other researchers, should be used in empirical research.

II. THE HYPOTHESES

Gordon argues that the value of a firm in a regulated industry is:

$$V = \frac{\bar{X}^t}{\rho} = \frac{\bar{X}(1-t) + tI}{\rho} \tag{1}$$

where:

V = Value of Firm

$\bar{X}$ = Expected earnings of firm before interest and taxes (EBIT).

$\bar{X}^t$ = Expected earnings after taxes but before interest.

ρ = Capitalization rate for unlevered firms in a given risk class.

t = Income tax rate for corporations.

I = Interest payments on debt.

Elton-Gruber argue that the value of a firm in a regulated industry is:

$$V = \frac{\bar{X}(1 - t)}{\rho} + tD \tag{2}$$

where:

D = Amount of debt in the capital structure of the firm. For a given rate of interest rate, r, $D = \frac{I}{r}$.

Elton-Gruber show that equation (1) holds if $\frac{\tilde{X}^t}{\bar{X}^t}$ possesses the same distribution for all degrees of leverage. Similarly, they show that equation (2) holds if $\frac{\tilde{X}}{\bar{X}}$ possesses the same distribution for all degrees of leverage.[7] Our paper analyzes these two ratios. Section IIA shows that $\frac{\tilde{X}^t}{\bar{X}^t}$ possesses the same distribution for all degrees of leverage only under certain specific conditions. Section IIB shows that $\frac{\tilde{X}}{\bar{X}}$ possesses the same distribution, for all degrees of leverage, only under other certain specific conditions. One cannot say, _a priori_, what conditions are likely to prevail.

A. Gordon's Valuation Formula

Assuming Gordon's definition of regulation, we show that generally the distribution $\frac{\tilde{X}^t}{\bar{X}^t}$ does not have a constant distribution for all degrees of leverage. Using a notation similar to that employed by Elton-Gruber, we define:

M = Price of product set by regulatory agency for unlevered firm.

$\tilde{d}(M)$ = Quantity demanded at price M. It is assumed that $\tilde{d}(M)$ is a normally distributed random variable with mean, $\bar{d}(M)$, and standard deviation, $\sigma(\tilde{d}(M))$.[8]

C_{NL} = Average cost of production for an unlevered firm, when:[9]

1) Cost of fixed capital investment is excluded, i.e. only labor and other variable costs are included.

2) The firm is not allowed to vary fixed capital investment to meet random variation in demand, i.e. this is a short-run formulation of costs.

The term, C_{NL}, is assumed to be constant in the relevant range.

T_{NL} = Total fixed assets for the unlevered firm. For simplicity we assume that these assets do not depreciate, i.e. they can be sold for exactly T_{NL} in the far distant future. This greatly simplifies the tax treatment.

The post-tax earnings[10] of an unlevered firm are:

$$\tilde{X}^t_{NL} = (M - C_{NL})\,\tilde{d}(M)\,(1 - t). \tag{3}$$

The average post-tax earnings are:

$$\bar{X}^t_{NL} = (M - C_{NL})\,\bar{d}\,(M)\,(1 - t). \tag{4}$$

The probability distribution of $\frac{\tilde{X}^t_{NL}}{\bar{X}^t_{NL}}$ can then be expressed by the ratio:

$$\frac{\tilde{X}^t_{NL}}{\bar{X}^t_{NL}} = \frac{(M - C_{NL})\,\tilde{d}\,(M)\,(1 - t)}{(M - C_{NL})\,\bar{d}\,(M)\,(1 - t)} = \frac{\tilde{d}(M)}{\bar{d}(M)} \tag{5}$$

When an all equity firm levers, its post-tax profits will rise since interest payments are tax deductible. A regulatory agency will lower the firm's price in order to eliminate this profit increase. With a downward sloping demand curve, a decrease in price yields a rise in units sold. This will generally lead to an increase in all factors of production, including an addition to the total assets of the firm.[11] Adopting Gordon's view of regulation, the price for the levered firm, Y, will be set so that:[12]

$$\frac{\bar{X}^t_{NL}}{\bar{X}^t_L} = \frac{T_{NL}}{T_L} \tag{6}$$

where:

T_L = Total assets of the levered firm

$\bar{X}^t_L$ = Expected earnings after taxes but before interest for a levered firm.

For simplicity, we assume that the levered firm's short-run average costs of production, exclusive of capital costs are constant and equal to C_L in the range around $\bar{d}(Y)$.

Under these assumptions:

$$\bar{X}^t_L = (Y - C_L)\,\bar{d}\,(Y)\,(1-t) + trD. \tag{7}$$

The probability distribution of $\frac{\tilde{X}^t_L}{\bar{X}^t_L}$ can then be expressed by:

$$\frac{\tilde{X}^t_L}{\bar{X}^t_L} = \frac{(Y-C_L)\,\tilde{d}(Y)(1-t) + trD}{(Y-C_L)\,\bar{d}(Y)(1-t) + trD} \tag{8}$$

If we assume that the distributions of $\frac{\tilde{X}^t}{\bar{X}^t}$ are perfectly correlated throughout all degrees of leverage, we need only show that the means and standard deviations stay constant to imply that the distributions remain identical as leverage changes.[13] This will indicate that Gordon's valuation method is correct. Thus we must compare equation (5) with equation (8). The mean of each of the two equations equals one.

The variance of the distribution of equation (8) is:

$$(9)\qquad \sigma\left(\frac{\tilde{X}_L^t}{\bar{X}_L^t}\right) = \frac{(Y-C_L)\ (1-t)}{(Y-C_L)\ (1-t)\ \bar{d}(Y) + trD}\ \sigma(\tilde{d}(Y))$$

As long as the amount of debt, D, is greater than zero, the condition that

$$(10)\qquad \frac{\sigma(\tilde{d}(Y))}{\bar{d}(Y)} > \frac{\sigma(\tilde{d}(M))}{\bar{d}(M)}\ ,$$

is necessary but not sufficient for

$$(11)\qquad \sigma\left(\frac{\tilde{X}_L^t}{\bar{X}_L^t}\right) = \sigma\left(\frac{\tilde{X}_{NL}^t}{\bar{X}_{NL}^t}\right)\ .$$

However, this necessary condition is unusual.

Leland[14] provides the following general formulation of the variability of the demand curve:

$$(12)\qquad \tilde{d}(P) = a + f(P).\tilde{u}\ ,$$

where:

P is the price of the firm's product
$\tilde{u}$ is a random variable with $E(\tilde{u}) = 1$.

From (12) we derive $\bar{d}(P)$, the mean of $\tilde{d}(P)$:

$$(12')\qquad \bar{d}(P) = a + f(P)\ .$$

Subtituting equation (12') into equation (12) gives:

$$(12'')\qquad \tilde{d}(P) = a + [\bar{d}(P)-a].\tilde{u}\ .$$

This implies that:

$$(13)\qquad \frac{\sigma(\tilde{d}(P))}{\bar{d}(P)} = \frac{(\bar{d}(P)-a)}{\bar{d}(P)}\cdot\sigma(\tilde{u})$$

Taking derivatives, we have:

$$(14)\qquad \frac{d\left[\frac{\sigma(\tilde{d}(P))}{\bar{d}(P)}\right]}{d\,[\bar{d}(P)]} = \frac{+a}{[\bar{d}(P)]^2}\sigma(\tilde{u})$$

Equation (14) is greater than, less than, or equal to zero when $a > 0$, $a < 0$, $a = 0$, respectively. As mentioned previously, since the levered firm earns a tax subsidy on interest payments, its price, Y, is set lower than M in order for the after-tax profits of both firms to be equal. With a downward sloping demand curve, $\bar{d}(Y) > \bar{d}(M)$. Thus the relationship in (10) can obtain under Leland's model only when $a > 0$.[15] Though the value of a must ultimately be established empirically for each industry, it is interesting that the common assumption in the literature sets $a = 0$, yielding the so-called "multiplicative" model.[16]

Certain other models[17] postulate a demand curve with constant standard deviation of demand at all levels of price, i.e., that $\sigma(\tilde{d}(Y)) = \sigma(\tilde{d}(M))$. A constant standard deviation of demand implies that:

$$\text{(15)} \qquad \frac{\sigma(\tilde{d}(Y))}{\bar{d}(Y)} < \frac{\sigma(\tilde{d}(M))}{\bar{d}(M)},$$

which contradicts (10).

In summary, the traditional models mentioned in footnotes 14 and 16 have assumed that for a decline in price, the standard deviation of demand does not increase at a faster rate than the increase in demand. Gordon's valuation formula is inconsistent with this assumption. In a world where investors dislike variability of return, Gordon's valuation formula underestimates the value of a levered firm, since it implies a higher ρ. However, Gordon's valuation formula is consistent with equation (15) when $a > 0$. A discussion of the ideas of this section in the framework of the capital asset pricing model (CAPM) are presented in Appendix I. It is shown that our results are easily derived under that model too.

B. Elton-Gruber's Formula

Elton-Gruber argue that $\frac{\tilde{X}}{\bar{X}}$ is independent of leverage, given Gordon's view of regulation. The following shows that the truth of this statement cannot be judged without specific knowledge of the variability of the demand curves facing the firm. For example, it is correct in our model, when the variability of the demand curve follows the "multiplicative" formulation, while incorrect under other assumptions. From the terms defined in section IIA, the following relationships hold:

$$\tilde{X}_{NL} = (M - C_{NL})\ \tilde{d}(M)$$

$$\bar{X}_{NL} = (M - C_{NL})\ \bar{d}(M)$$

$$\tilde{X}_{L} = (Y - C_{L})\ \tilde{d}(Y)$$

$$\bar{X}_{L} = (Y - C_{L})\ \bar{d}(Y)$$

Thus:

$$\frac{\tilde{X}_{NL}}{\bar{X}_{NL}} = \frac{\tilde{d}(M)}{\bar{d}(M)}$$

and

$$\frac{\tilde{X}_{L}}{\bar{X}_{L}} = \frac{\tilde{d}(Y)}{\bar{d}(Y)} .$$

This implies that:

$$\frac{\tilde{X}_{NL}}{\bar{X}_{NL}} = \frac{\tilde{X}_{L}}{\bar{X}_{L}} ,$$

only if:

(16) $$\frac{\tilde{d}(M)}{\bar{d}(M)} = \frac{\tilde{d}(Y)}{\bar{d}(Y)}$$

Assuming that $\tilde{d}(M)$ and $\tilde{d}(Y)$ are perfectly correlated, we need only show that the means and standard deviations of the two sides of equation (16) are equal to imply that the two distributions are identical. The mean of each side is one. Inspection of equation (16) indicates that the standard deviations of the two ratios are equal only when the standard deviation of quantity demanded increases in proportion to an increase in quantity demanded. As previously mentioned, this occurs in the Leland model only when $a = 0$, i.e., the multiplicative model holds. When $a > 0$ ($a < 0$) the standard deviation of quantity demanded increases more (less) than in proportion to an increase in quantity demanded. In addition, the additive model implies (15), which is inconsistent with the E-G formula.

The above reasoning indicates the consistency or inconsistency between E-G's formulation and received models of demand variability. However, the existence in the real world of the necessary assumptions for E-G's model to hold must ultimately be determined empirically. For the valuation formula of Elton-Gruber to be correct, the equality of standard deviations of the two sides of (16) must be satisfied for each degree of leverage. As it is unlikely that this equation will hold in the real world for each degree of leverage, one might not feel confident in employing Elton-Gruber's formula in an empirical problem. Rather, one should estimate the variability of the demand curve before selecting either Gordon's or Elton-Gruber's formulae.

III. GENERAL FORMULAE FOR THE DISCOUNT RATE AND THE VALUATION OF A LEVERED FIRM IN A REGULATED INDUSTRY

Assuming that $\frac{\tilde{X}}{\bar{X}}$ remains constant for all ranges of leverage, Elton-Gruber argue that the discount rate in a regulated industry for the distribution of pre-tax earnings at all ranges of leverage is ρ. However, as we have shown that $\frac{\tilde{X}}{\bar{X}}$ is generally not constant as leverage changes, we must express ρ as a function of leverage. To illustrate this, we define B such that:

$$\sigma\left(\frac{\tilde{X}_L}{\bar{X}_L}\right) = B \cdot \sigma\left(\frac{\tilde{X}_{NL}}{\bar{X}_{NL}}\right) ,$$

where B is a function of the degree of leverage.

If $\tilde{X}_L$ and $\tilde{X}_{NL}$ are perfectly correlated, it follows that:

$$\frac{\tilde{X}_L}{B\bar{X}_L} + \frac{B - 1}{B} = \frac{\tilde{X}_{NL}}{\bar{X}_{NL}} . \quad ^{18}$$

Rearranging we have:

$$\frac{\tilde{X}_L}{\bar{X}_L} = B \cdot \frac{\tilde{X}_{NL}}{\bar{X}_{NL}} + (1 - B) . \tag{17}$$

Thus, the distribution of $\frac{\tilde{X}_L}{\bar{X}_L}$ can be viewed as the weighted sum of two distributions. The first of the two distributions is $\frac{\tilde{X}_{NL}}{\bar{X}_{NL}}$, while the second is the certain return, $1 - B$.

We define:

ρ_L = discount rate for a pre-tax earnings stream with a distribution of $\frac{\tilde{X}_L}{\bar{X}_L}$

ρ_{NL} = discount rate for an earnings stream with a distribution of $\frac{\tilde{X}_{NL}}{\bar{X}_{NL}}$

r = interest rate.

As the distributions of the two sides of equation (17) are equal, the market values of the two sides are equal. From the definitions above, it follows that the value of an earnings stream with a distribution of $\frac{\tilde{X}_{NL}}{\bar{X}_{NL}}$ is $\frac{1}{\rho_{NL}}$ and the value of an earnings stream with a distribution of $\frac{\tilde{X}_L}{\bar{X}_L}$ is $\frac{1}{\rho_L}$. The value of the certain stream of income, $1 - B$, is $\frac{1 - B}{r}$.

Equating the market values of the two sides, we have:

$$\frac{1}{\rho_L} = \frac{B}{\rho_{NL}} + \frac{1 - B}{r} .$$

Rearranging we have:

$$\rho_L = \frac{1}{\frac{B}{\rho_{NL}} + \frac{1 - B}{r}} . \tag{18}$$

As B is assumed to change as leverage changes, ρ_L will vary with leverage. Note that

$$\frac{d\rho_L}{dB} = \left[\frac{B}{\rho_{NL}} + \frac{1 - B}{r}\right]^{-2} \left[- \frac{1}{\rho_{NL}} + \frac{1}{r}\right] .$$

The first term on the right-hand side is always positive as the expressions inside the brackets are squared in the equation.

Assuming $\rho_{NL} > r$, the second term must also be positive. Thus $\frac{d\rho_L}{dB} > 0$, as one would expect if investors dislike variability of earnings.

Once ρ_L, the discount rate for pre-tax earnings of the levered firm, is derived as in equation (18), the value of the levered firm can be calculated as follows:

$$V = \frac{\bar{X}_L(1 - t)}{\rho_L} + tD \; .$$

V. CONCLUSIONS

This paper examines the problem of valuation of a firm in a regulated industry under Gordon's definition of regulation and concludes that:

1) The valuation formula for a regulated firm cannot be derived without specifying its demand and supply curves. This occurs because the regulated price changes with leverage.

2) Gordon's and Elton-Gruber's formulae can be correct only for special cases of demand and supply conditions.

3) Under traditional assumptions of demand such as either a constant variability of demand or a multiplicative model of demand and with an increasing slope of the average cost curve, both Gordon and Elton-Gruber underestimate the value of levered firms under regulation.

In addition we have formulated generalized formulae for both the discount rate of a levered firm's earnings, ρ_L, and the valuation of a levered firm.

APPENDIX I [19]

Our Results in the Framework of the Capital Asset Pricing Model.

In the text of our paper, as well as in MM, Gordon, and EG, the variability of the distributions of $\frac{\tilde{X}^t}{\bar{X}^t}$ and $\frac{\tilde{X}}{\bar{X}}$ is analyzed. For example, EG state that if the variability of $\frac{\tilde{X}_L^t}{\bar{X}_L^t}$ is equal to the variability of $\frac{\tilde{X}_{NL}^t}{\bar{X}_{NL}^t}$, the post-tax earnings (inclusive of interest) of both the levered and the unlevered firm should be capitalized at a constant rate, ρ. To analyze our problem in terms of the CAPM, we will talk of the covariability of these distributions with $\tilde{R}_m$, the percentage return on the market.[20] Let us look at the distribution, $\frac{\tilde{X}^t}{\bar{X}^t}$, in terms of the capital asset pricing model. We will show below that if

$$\text{(A1)} \qquad \text{cov}\left(\frac{\tilde{X}_L^t}{\bar{X}_L^t}, \tilde{R}_M\right) = \text{cov}\left(\frac{\tilde{X}_{NL}^t}{\bar{X}_{NL}^t}, \tilde{R}_M\right),$$

the post-tax earnings of the two firms should be capitalized at the same rate.

We employ the definition of expected return on the non-levered firm:

$$\text{(A2)} \qquad E(\tilde{R}_{NL}) = \frac{\bar{X}_{NL}^t}{S_{NL}},$$ where S_{NL} is the market value of the unlevered firm. From the CAPM,

$$\text{(A3)} \qquad E(\tilde{R}_{NL}) = R_F + \lambda \text{ cov }(\tilde{R}_{NL}, \tilde{R}_M),$$

where

R_F = return on a riskless asset

$\tilde{R}_M$ = return on the market portfolio

$E(\tilde{R}_{NL})$ = expected return of the non-levered firm

$$\lambda = \frac{E(\tilde{R}_M) - R_F}{\sigma^2(\tilde{R}_M)} \quad .$$

From (A2) and (A3) it follows that:

(A4) $$S_{NL} = \frac{\bar{X}^t_{NL}}{R_F + \dfrac{\lambda \text{cov}\ (\tilde{X}^t_{NL},\ \tilde{R}_M)}{S_{NL}}}$$

Similarly, for the levered firm, we have:

(A5) $$S_L = \frac{\bar{X}^t_L}{R_F + \dfrac{\lambda \text{cov}\ (\tilde{X}^t_L,\ R_M)}{S_L}}$$

Rearranging (A4) and (A5) yields:

(A4') $$S_{NL}\ R_F + \lambda\ \text{cov}\ (\tilde{X}^t_{NL},\ \tilde{R}_M\ ) = \bar{X}^t_{NL}$$

(A5') $$S_L\ R_F + \lambda\ \text{cov}\ (\tilde{X}^t_L,\ \tilde{R}_M) = \bar{X}^t_L \ .$$

Substituting equation (A1) into equation (A5'), we get:

(A5") $$S_L\ R_F\ \frac{\bar{X}^t_{NL}}{\bar{X}^t_L} + \lambda\ \text{cov}\ (\tilde{X}^t_{NL},\ \tilde{R}_M) = \bar{X}^t_{NL} \ .$$

Comparing (A4') with (A5"), we see that:

(A6) $$S_{NL} = S_L \cdot \frac{\bar{X}^t_{NL}}{\bar{X}^t_L} \quad \text{or}$$

(A7) $$\frac{\bar{X}^t_L}{S_L} = \frac{\bar{X}^t_{NL}}{S_{NL}} \quad .$$

Now $E(\tilde{R}_L) = \dfrac{\bar{X}^t_L}{S_L}$ and $E(\tilde{R}_{NL}) = \dfrac{\bar{X}^t_{NL}}{S_{NL}}$ so that:

(A8) $$E(\tilde{R}_L) = E(\tilde{R}_{NL}) \ .$$

Hence (A8) shows that the post-tax earnings of the two firms should be capitalized at the same rate, if (A1) holds.

Thus, to use the CAPM on Gordon's valuation formula (section IIA), the development of the first 8 equations would be identical to that given in the text. However, for equations (9), (10) and (11), we would note that

$$(9') \qquad \operatorname{cov}\left(\frac{\tilde{X}_L^t}{\bar{X}_L^t}, \tilde{R}_M \right) = \frac{(Y-C_L)\ (1-t)}{(Y-C_L)\bar{d}(Y)(1-t) + trD} \operatorname{cov}\ (\tilde{d}(Y), \tilde{R}_M).$$

As long as the amount of debt, D, is greater than zero, the condition that:

$$(10') \quad \operatorname{cov}\left(\frac{\tilde{d}(y)}{\bar{d}(y)}, \tilde{R}_m\right) > \quad \operatorname{cov}\left(\frac{\tilde{d}(M)}{\bar{d}(M)}, \tilde{R}_M\right),$$

is necessary but not sufficient for

$$(11') \qquad \operatorname{cov}\left(\frac{\tilde{X}_L^t}{\bar{X}_L^t}, \tilde{R}_M \right) = \operatorname{cov}\left(\frac{\tilde{X}_{NL}^t}{\bar{X}_{NL}^t}, \tilde{R}_M \right)$$

Therefore, our results under the CAPM formulation are similar to the results in the text, as we show that there can exist conditions of demand such that Gordon's valuation formula holds. However, as in the text, it is easy to show that also under the CAPM formulation, Gordon's formula holds only under unusual conditions. Thus the results of section IIA would be little changed in the CAPM framework. Similar adjustments could be made later in the paper if one wishes to employ the CAPM.

FOOTNOTES

1. Modigliani-Miller [19].

2. Modigliani-Miller [20].

3. See Barges [2], Sarma-Rao [25], Miller-Modigliani [16], Hamada [12], Weston [29], Brown [6], Brigham-Gordon [5], Wippern [30], Robichek, Higgins and Kinsman (RHK) [24]. Because of problems such as the Gordon and Elton-Gruber controversy raises, RHK state that their measurement of the effect of leverage on the cost of equity capital is not a test of the MM propositions.

4. See Miller-Modigliani [16], Weston [29], Brown [6], Brigham-Gordon [5], Barges [2], Robichek, Higgins and Kinsman [24].

5. Gordon [11]. In footnote 5, Gordon states:

   > "... in all the testimony on this and other cases I have read, the rate of return in question is earnings with taxes excluded and interest included divided by total assets."

   Similarly, Bryant and Hermann [7, p. 189] state

   > "The Supreme Court decided, in the Galveston Electric Case [Galveston E. Co. vs. Galveston, 258 U.S. 388, 66 L. ed. 678, 42 Sup. Ct. 351, P.U.R. 1922 D, 159] that the Federal income tax was like any other tax, being a part of the operating expense of the utility, but that the payment of this expense by the utility instead of the holders of the securities should be considered in the allowance for the return that the utility is permitted to earn. All decisions of the Supreme Court since the Galveston Case have concurred in this opinion; those of the commission and lower courts have tried to explain and verify these decisions."

   The reader is referred to Bonbright [4, p. 242-43 and 403], Phillips [23, p. 206-213] and Davis-Sparrow [8, p. 552] for a further discussion in support of Gordon's observation. Depending on jurisdiction, total assets are measured on either a (1) depreciated value of initial cost basis, (2) replacement cost basis, or (3) some combination of the two.

6. Elton -Gruber [10]. See also Miller-Modigliani [17].

7. Modigliani-Miller [20] have shown that an appropriate way to define a risk class is in terms of the distribution of the ratio of the random variable $\tilde{X}^t$ (or $\tilde{X}$) to its expected value $\bar{X}^t$ (or $\bar{X}$). See also Modigliani-Miller [17, p. 1295].

8. If the preferences of individuals are quadratic, any two parameter distribution may suffice. See Tobin [27].

9. Both the short-run and long-run average cost curves in economics commonly include fixed capital investment. A downward sloping long-run cost curve (including capital costs) is a common assumption for regulated industries. It can be shown that when a long-run average cost curve, including fixed capital costs, is downward sloping over a certain range of production, the short-run average cost curve, excluding fixed capital costs, can be horizontal over the same range.

10. Just as microeconomic theory explains how an unregulated firm selects its price and its number of units of output, capital, and labor, the standard and much-explored Averch-Johnson [1] model explains how a regulated firm selects these same variables. Thus, the relevant variables in (3) can be determined by an optimization process, though we do not reproduce the procedure. Actually the regulated and the unregulated firm maximize related functions. The basic difference is that the regulated firm satisfies a required rate of return constraint, implying that for a given output the regulated firm will have a capital-labor ratio in excess of that which minimizes expenses. Further discussion and extensions of the AJ model can be found in Baumol-Klevorick [3], Edelson [9], Takayama [26] and Zajac [32].

11. These statements follow from the Averch-Johnson [1] model of regulation.

12. The Averch-Johnson model shows how both price and total assets are simultaneously determined. For our purposes, we need only accept their results that both of these variables can be determined. As can be seen, exact values of Y and T_L are not needed for the results of this paper.

13. Previous work has assumed perfect correlation of distributions. (See Elton-Gruber [10]). However, if distributions are not perfectly correlated throughout all degrees of leverage, we can reach the same conclusions by assuming that investors' valuation of the firm is based on the standard deviation and the mean of the distribution. Under the assumptions of imperfect correlation and valuation based on the above parameters, we need only show that the mean and standard deviation are constant to imply that the value of the firm to investors remains the same as leverage changes. Similar conclusions can be reached by assuming that investors are interested in the mean of the distribution and the covariance of the distribution with some general market portfolio (See the Appendix).

14. See Leland [14, p. 286].

15. In particular, it can be shown that Gordon's model holds only when

$$1-\frac{a}{\bar{d}(M)} = \frac{(Y-C_L)\ (1-t)\ [\bar{d}(Y)\ -a]}{(Y-C_L)\ \bar{d}(Y)\ (1-t)\ +\ trD}$$

16. See Nevins [22], Zabel [31] and Meyer [33] for treatments of the multiplicative model.

17. Models which utilize this assumption are presented in Mills [18] and Nelson [21]. Though it is possible to combine Leland's model and the additive model into one formulation, this will not be done as there are theoretical

problems with the additive model. In particular, in his interesting article, Leland has discussed the intuitive and theoretical appeal of a demand curve where, as one moves down the curve, the change in total revenue has the same sign as the change in variability of this total revenue. He states that this demand curve satisfies the principle of increasing uncertainty (PIU). Models with a constant standard deviation for all prices may be criticized for not satisfying PIU in general. In fact, it can be shown that, for a large class of these models, PIU is not satisfied in the region where the demand curve is elastic.

18. See MM [20, p. 436] equation (3).

19. Much of the discussion in this appendix follows Hamada [12].

20. See Hamada (12, p. 16].

21. See Hamada [12], equation (3).

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