Measuring Capital Asset Returns and the Stable Probability Laws

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The contents of and opinions expressed in this paper are solely the responsibility of the authors.

*Associate Professors, Wharton School, University of Pennsylvania. We would like to acknowledge the work of Edwin Elton, Martin Gruber, and Paul Kleindorfer (see [4]). They have reached some similar conclusions with a different type of analysis. The comments of Marshall Blume have been helpful.

I. Introduction

The two parameter capital asset pricing models of Sharpe [26], Lintner [19] and Fama [7] make explicit assumptions about the probability distributions of one-period security returns. There is considerable disagreement among economists about which probability distributions best describe the empirical distributions of security returns. One aspect of this disagreement is a controversy over the best method of measuring security returns in developing theoretical and empirical probability distributions.[1] This issue is critical, since the measure of security return has important implications for the appropriateness of alternative probability assumptions.[2]

One procedure for measuring returns, most recently employed by Officer [21] and Blume [1] is in terms of discrete time percentage changes in investment value. For example, suppose one is interested in examining the change in investment value of a security over a particular time interval. The return is computed by taking the investment value at the beginning of the time period and dividing it into the investment value of the end of the time period -- resulting in a percentage investment return (i. e., return relative). Alternatively, the investment return of securities has been commonly defined in terms of continuous compounding. Here, the return of a security is measured by computing the percentage change in investment value over a time interval and taking natural logarithms. This procedure was used by Mandelbrot [20] when he introduced the "stable Paretian" probability hypothesis to cotton future contract returns, and later by Fama [8] for common stock returns and Roll [23] for Treasury bill returns. Moreover, much empirical work purports to suggest that security returns, whether measured in percentages or logarithms, conform to non-normal members of the symmetric, stable class of probability distributions with characteristic exponent α, $1 < \alpha < 2$.[3]

The primary purpose of this essay is to examine several implications of using the logarithm of returns and percentage returns in developing probability assumptions for asset pricing models. Our analysis leads to three main conclusions:

1. The logarithm of security returns is the only measure of returns for which returns can be literally stable;
2. If the logarithms of security returns can reasonably be expected to be stable, they will have characteristic exponent $0 < \alpha < 2$ and shape parameter $\beta = 1$ or be stable with characteristic exponent equal to 2, that is, normal. The logarithm of security returns will not obey a symmetric, non-normal stable law. Indeed it must be as asymmetric as it is possible to be!
3. The expectation of the logarithm of security return will not be the same as that for equilibrium instantaneous return for a security in a capital asset pricing model.

The formal presentations of section III are made in a technical appendix.

II. Security Returns and Stable Probability Laws

A. Individual Security Returns

Consider the price of an individual security P_t, $t = 1, 2, \ldots$ to be the result of a stochastic process such that

$$(1) \quad E_{t-1}\,(P_t | \theta_{t-1}) = P_{t-1} e^{\rho}, \; \rho \geq 0,$$

where E_{t-1} is the conditional expectation operator at time t-1 and θ_{t-1} represents all information available about the security's value at the time t-1. The variable ρ is an exogenously determined discount rate. Thus, the sequence $\{P_t\}$ constitutes a submartingale.[4] Then, for any T the return of an individual security can be defined as,

$$(2) \quad P_T/P_1 = \prod_{t=2}^{T} (P_t/P_{t-1})$$

and

$$(3) \quad \ln\,(P_T/P_1) = \ln P_T - \ln P_1$$

$$= \sum_{t=2}^{T} \ln\,(P_t/P_{t-1})$$

Notice that

$$(4) \quad \ln(P_T/P_1) = \ln\left(1 + \frac{P_T - P_1}{P_1}\right)$$

$$= \frac{P_T - P_1}{P_1} + 0\left(\frac{P_T - P_1}{P_1}\right)^2$$

as $\quad \dfrac{P_T - P_1}{P_1} \rightarrow 0.$

That is, $(P_T - P_1)/P_1$ is a good approximation to $\ln(P_T/P_1)$ if the former is small as it is likely to be for small T, say for a single transaction. However, if the number of transactions is large the approximation may be poor. Furthermore, $(P_T - P_1)/P_1$ is not additive, that is

$$(5) \quad \frac{P_T - P_1}{P_1} \neq \sum_{t=2}^{T} \frac{P_t - P_{t-1}}{P_{t-1}}$$

The interesting point is that if discrete time security returns are sums of many independent price changes from transaction to transaction, and if security returns conform to any probability distribution, the generalized central limit theorem states they will conform to stable distributions.[5] However, only the logarithms of individual security returns can be literally considered as sums of random variables. Thus, the central limit theorem cannot be used as an <u>a priori</u> argument for the percentage return of securities to conform to a stable law.

B. Portfolio Returns

The "log stable" model does not carry over into portfolio theory. The log return R and the percentage return Q are related:

$$R_t = \ln (1 + Q),$$

$$Q = \frac{P_T - P_1}{P_1}$$

$$= e^R - 1$$

Consider a portfolio consisting of two securities with equal value, say 1/2 each.

Then

$$Q = 1/2 \ (Q_1 + Q_2) = 1/2 \ (e^{R_1} + e^{R_2} - 2)$$

and

$$R = \ell n\ (1 + Q) = \ell n\ (1/2(e^{R_1} + e^{R_2})).$$

If R_1 and R_2 have stable laws clearly R will not! Thus there is no a priori reason for log portfolio returns to follow a stable law.

Next, consider the percentage return for a portfolio in a single time period. Assume that the portfolio consists of K securities in amounts of value $a_1, a_2, \ldots a_k$. Assume without loss of generality that

$$\sum_{k=1}^{K} a_k = 1,$$

that is, that the units of measurement are so chosen that the opening value of the portfolio is 1.

Let Q + 1 be the closing value of the portfolio. That is, Q is the percentage return on the portfolio. Let Q_k be the percentage return on the k^{th} security. Then

$$Q = \sum_{k=1}^{K} a_k Q_k.$$

The assumption which we make is that $Q_1, Q_2, \ldots Q_K$ are mutually independent. The random variables Q and Q_k must be between -1 and ∞ with probability 1, and the expected value of the closing value is the same as the opening value for each of the securities as well as for the portfolio itself. Thus

$$EQ = EQ_k = 0,$$

$k = 1, 2, \ldots K$.

We can argue that the risk approaches 0 and that Q approaches a stable law as the portfolio becomes more diversified, that is, as

$$\max_{1 \leq k \leq K} |a_k| \to 0.$$

It would be very easy to prove this if we could assume that each of the Q_k and therefore Q had a finite variance. This may not be justified and so we will not assume it.

Let

$$Q_k^- \equiv -\min\ (0,\ Q_k)$$

and

$$Q_k^+ \equiv \max\ (0,\ Q_k).$$

Then, by definition

$$Q_k = Q_k^+ - Q_k^-$$

where one of the terms Q_k^+ and Q_k^- must be 0 according as $Q \geq 0$, or ≤ 0. Both are nonnegative. Furthermore, Q_k^- is bounded between 0 and 1. It follows that

$$EQ_k^- < \infty.$$

Since

$$EQ_k = 0,$$

$$0 < EQ_k^- = EQ_k^+ < \infty$$

and therefore

$$E|Q_k| = EQ_k^- + EQ_k^+ < \infty$$

where $|Q_k|$ is the absolute value of Q_k. It follows from this and independently by similar reasoning that

$$E|Q| < \infty.$$

It follows by the Law of Large Numbers that

$$EQ \to 0$$

as

$$\max_{1 \leq k \leq K} |a_k| \to 0$$

that is the portfolio becomes less risky in the limit. (See Feller [11] page 231.) On the other hand under these assumptions the central limit theorem obtains and the distribution of Q, suitably normalized, approaches that of a stable law if a limiting distribution exists.

Under the preceding analysis we would find virtually no variability in the returns of well diversified portfolios of common stocks and since these is, the assumption of "many small independent effects" does not seem to be valid. These must be at least one indivisible effect which is market wide and substantial. However, there is no reason why this market wide effect would follow a stable law, thus there is no *a priori* reason to expect portfolio percentage returns to follow a stable law.[6]

III. Implications of Natural Logarithms of Return and Stable Probability Laws

Since the logarithm of individual security returns measured over any finite interval in calendar time can be thought of as sums of random variables, it is not unreasonable to expect that they would conform to a stable law. Specifically, it can be shown that the logarithms of returns should conform to a non-normal asymetric stable law with characteristic exponent $0 < \alpha < 2$ and shape parameter $\beta = 1$, or they will conform to a normal stable law.

Consider a boundedness condition taken from the familiar sub-martingale model of equation (1)

$$(6)\quad E_{t-1}(P_t/P_{t-1}|\theta_{t-1}) < 1 + \delta_t < \infty$$

and under a continuous compounding measure of returns, its analogue

$$(7)\quad E_{t-1}(e^{R_t}|\theta_{t-1}) \leq 1 + \delta_t < \infty$$

where

$$R_t \equiv \ln(P_t/P_{t-1}).$$

This last condition requires that (dropping the θ_{t-1})

$$(8)\quad E_{t-1}(\max(0, R_t))^2 \leq CE_{t-1}\exp(\max(0, R_t)) < \infty$$

since $R_t^2 \leq Ce^{R_t}$ for some $C < \infty$, when $R_t \geq 0$.

Inequality (8) requires that $E_{t-1}(R_t)^2 < \infty$ if the distribution is symmetric, hence it will be satisfied for a symmetric stable law only when $\alpha = 2$ and the moment of order $\alpha = 2$ exists. It follows that (8) will not be literally true under a symmetric, stable law where $\alpha < 2$. It is reasonable to ask under what conditions (8) can be satisfied when $\alpha < 2$.[7] We show that under a stable law this can only be true if the shape parameter (or asymmetry parameter) β is equal to 1, and $0 < \alpha < 2$. (A formal proof is provided in the appendix).

We conclude that if the distribution of R_t can have a non-normal stable distribution, it can only have a completely asymmetric one with $\beta = 1$. However, it should be pointed out that under a stable law completely asymmetric distributions need not be greatly skewed in terms of the common measures of skewness. For example, if α is close to 2, say 1.95, and if $\beta = 1$, the distribution of R_t will be approximately symmetric. Some preliminary analyses by Marshall Blume of the University of Pennsylvania indicates that for stable laws with $1.5 < \alpha < 1.7$ and $\beta = 1$ about 40% of the empirical distribution will be below the mean and 60% will be above the mean.

IV. Measuring the Expected Return:

One important practical implication of using logarithms to measure returns is for assessing expected returns. A common error has been to assume the expectation of return for a security is the same as the continuous time equilibrium return for the security.

Recall that

$$R_t = \ell n \, (P_t / P_{t-1})$$

and

$$E_{t-1} \, (e^{R_t} | \theta_{t-1}) = e^{\rho}$$

with probability 1, in practical terms. This expression is the familiar martingale model of Samuelson [24] and Roll [23] (recall equation (1)).

Thus, for any θ_{t-1} it follows that

$$E_{t-1} \, (e^{(R_t - \rho)} | \theta_{t-1}) = 1$$

and that

$$E_{t-1}(e^{(R_t-\rho)} - 1 \mid \theta_{t-1}) = 0.$$

Notice that

(9) $$e^x - 1 \geq x$$

for all x, $-\infty < x < \infty$, with equality only when $x = 0$. This can be seen by observing that

$$d(e^x-1)/dx = e^x < \text{or} > dx/dx = 1$$

as $x < \text{or} > 0$. Let us add the trivial assumptions that $R_t - \rho|\theta_{t-1}$ is not concentrated at 0 with probability 1. Then by (9) with $x = R_t - \rho$,

$$E_{t-1}\ (R_t-\rho|\theta_{t-1}) < E\left[(e^{(R_t-\rho)} - 1)\ |\theta_{t-1}\right] = 0.$$

That is

$$E_{t-1}\ (R_t|\theta_{t-1}) < \rho\ .$$

This is true no matter what the distribution of R_t may be and has implications in testing on the value of ρ by use of sample averages.

APPENDIX

Asymmetric Non-Normal Stable Laws and Continuous Time Security Returns

A random variable X is said to have a stable law with characteristic exponent α, $0 < \alpha < 2$, if

$$(1) \quad \ell n\, E(e^{ixt}) = i\gamma t - c|t|^{\alpha} \{1 + i\beta \frac{t}{|t|} \omega(t, \alpha)\}$$

where $-\infty < \gamma < \infty$, $c > 0$ and

$$\omega(t, \alpha) = \tan \frac{\pi}{2} \alpha, \quad \alpha \neq 1,$$

$$= \frac{2}{\pi} \ell n|t|, \quad \alpha = 1.$$

The parameters γ, $c^{1/\alpha}$, and β are respectively location, scale and shape parameters (see Gnedenko and Kolmogorov [13] p. 164).

The stable laws are infinitely divisible and so they admit a unique Lévy representation. We assume without loss of generality that the location parameter $\gamma = 0$. Then

$$(2) \quad \ell n\, E(e^{ixt}) = c_1 \int_{-\infty}^{0} (e^{itu} - 1 - \frac{itu}{i+u^2}) |u|^{-(1+\alpha)} du$$

$$+ c_2 \int_{0}^{\infty} (e^{itu} - 1 - \frac{itu}{i+u^2}) |u|^{-(1+\alpha)} du$$

where $c_1, c_2 \geq 0$, $c_1 + c_2 > 0$.

The shape parameter $\beta = (c_1 - c_2)/(c_1 + c_2)$ which equals 1 (-1) if and only if

$$c_1(c_2) > 0, \quad c_2(c_1) = 0$$

(see [13], page 168). Any stable variate Z can be represented

$$Z = X + Y$$

where X and Y are also stable with parameters $\beta = -1, +1$ respectively. It is clear from the characteristic function that if X has the stable law with parameters $(\gamma = 0, c, \alpha, \beta)$ then -X has the stable law with parameters $(\gamma = 0, c, \alpha, -\beta)$. In our representation $Z = X + Y$ the possible distributions for Y are the same ones as the possible distributions for -X.

Now consider a completely asymmetric non-normal stable law, that is one with $0 < \alpha < 2$, $\beta = -1$. Since the random variable is stable, hence infinitely divisible, it is a limit of convolutions of Poisson random variables. So, using the Lévy representation, the logarithm of the moment generating function

$$(3) \quad \ell n\, E(e^{x\theta}) = c_2 \int_0^\infty \left(e^{\theta u} - 1 - \frac{\theta u}{1+u^2}\right) u^{-(1+\alpha)} du,$$

substituting θ for it. Thus

$$E(e^{x\theta}) < \infty$$

for those and only those values of θ for which the integral (3) converges.

Convergence: Consider convergence at 0.

Now:

$$e^{\theta u} = 1 + \theta u + \frac{\theta^2 u^2}{2} + 0\,(u^3)$$

and

$$\theta u/(1+u^2) = \theta u + 0(u^3)$$

as $u \to 0$ and so the integrand

$$u^{-(1+\alpha)}\left(e^{\theta u} - 1 - \frac{\theta u}{1+u^2}\right) = u^{-(1+\alpha)}\left(1 + \theta u + \frac{\theta^2 u^2}{2} - 1 - \theta u + 0\,(u^3)\right)$$

$$= u^{-(1+\alpha)}\left(\frac{\theta^2 u^2}{2} + 0\,(u^3)\right)$$

$$= \frac{\theta^2}{2} u^{(1-\alpha)} + 0(u^{(2-\alpha)}).$$

But

$$\int_0^1 u^{(1-\alpha)} du = (2-\alpha)^{-1} u^{(2-\alpha)} \Big|_0^1 = (2-\alpha)^{-1} < \infty$$

for all α, $0 < \alpha < 2$, and so the integral converges at 0 for all θ. Consider convergence at ∞. If $\theta > 0$ the dominant term is $e^{\theta u}$ and

$$\lim_{u \to \infty} u^{-(1+\alpha)} \left(e^{\theta u} - 1 - \frac{\theta u}{1+u^2} \right) = \infty .$$

Therefore, the integral diverges and

$$E(e^{x\theta}) = \infty$$

for all $\theta > 0$. If $\theta < 0$, then

$$u^{-(1+\alpha)} \left(e^{\theta u} - 1 - \frac{\theta u}{1+u^2} \right) = -0 \left(u^{-(1+\alpha)} \right)$$

as $u \to \infty$ and so the integral converges and

$$E(e^{x\theta}) = E(e^{(-\theta)(-x)}) < \infty .$$

When $\theta = 0$,

$$\ln E(e^{x\theta}) = 0$$

and as always

$$E(e^{x\theta}) = E(e^{x0}) = 1.$$

<u>Conclusion</u>: To summarize, if $\beta = -1$,

$$E(e^{x\theta}) < \infty$$

if and only if $\theta \leq 0$. By symmetry, if $\beta = +1$,

$$E(e^{x\theta}) < \infty$$

if and only if $\theta \geq 0$. But, for any value of a finite random variable and for all θ,

$$0 < e^{x\theta} < \infty .$$

Consider a stable law with $0 < \alpha < 2$, $-1 < \beta < 1$. Then if Z has such a distribution we can write

$$Z = X + Y$$

where X and Y are non-constant, independent, and stable with common characteristic exponent α and β, respectively -1 and +1. Then

$$E(e^{Z\theta}) = E(e^{X\theta})\ E(e^{Y\theta}) = \infty$$

for all $\theta \neq 0$.

Thus, if X has a stable law with characteristic exponent α, $0 < \alpha < 2$,

$$E(e^{X}) < \infty$$

if and only if $\beta = +1$.

If $\alpha < 1$, there exists a finite lower limit a such that

$$P\ \{X \geq a\} = 1$$

if and only if $\beta = -1$. There exists a finite upper limit b such

$$P\ \{X \leq b\} = 1$$

if and only if $\beta = +1$. The lower (upper) limit is $-(+)\ \infty$ if $\alpha \geq 1$ or if $\alpha < 1$ and $-1 < \beta < 1$.

Footnotes

1. One aspect of this controversy is the subject of an article by S. C. Tsiang [27] with a reply by Fama [7].
2. For example, Jensen [15] has argued that under one set of assumptions the market-trading horizon might be infinitely small. He concludes that security returns should be measured in terms of natural logarithms.
3. There is no published work on asymmetric stable laws; hence no procedure for estimating the parameters of an asymmetric stable law has been provided. For recent empirical evidence purporting to show that security returns are not non-normal stable see Clark [2] .
4. One necessary condition for a stochastic process to be a submartingale is that expectations exist. Samuelson [24] and Roll [23] provide a discussion of martingale models for security returns.
5. For example - Fama states:
   "Since the daily, weekly or monthly price changes of a security are just sums of price changes from transaction to transaction, in empirical models the central limit theorem has often been used to justify a normality assumption of returns . . . . It is clear that such limiting arguments also suggest the more general presumption of a stable distributions of returns." [6, p.33].

6. In this case the percentage return of a portfolio might follow the "market model" which asserts that percentage returns, R_{kt} for asset k at time t can be expressed as a linear function of some measure of a market wide effect, M_t,

$$R_{kt} = \alpha_k + \beta_k M_t + Q_{kt} \; ; \quad E_{t-1}(Q_{kt} \quad M_t) = 0$$

where α_k and β_k are constants appropriate to asset k. The percentage returns of a portfolio is given by

$$W_t = \sum_k^K a_k \quad (\alpha_k + \beta_k M_t + Q_{kt})$$

The distribution of W_t becomes more like a linear function of M_t as the portfolio becomes more diversified and Q becomes more concentrated.

7. Consider the case

$$E_{t-1}(P_t | \theta_{t-1}) = P_{t-1}$$

where $\{P_t\}$ constitutes a margingale.

Thus, the expected return is

$$E_{t-1}(e^{R_t}) = 1$$

From previous arguments, the following is obtained:

$$E_{t-1}(e^{R_t}) = \int_{-\infty}^{\infty} e^x dF(x) \geq \int_0^{\infty} e^x dF(x)$$

where R_t are independent identically distributed random variables.

Now we choose an arbitrary ρ and note that for some C > 0,

$$\int_{\rho}^{\infty} x^2 dF(x) \leq C \int_{\rho}^{\infty} e^x dF(x) \quad \leq 1.$$

However, if $F(x)$ is stable with characteristic exponent α, $1 < \alpha < 2$ and shape parameter $\beta \neq 1$ (i.e. it does not have a thin upper tail) then

$$\int_{0}^{\infty} x^2 dF(x) = \infty$$

Thus, $F(x)$ cannot be symmetric stable with α, $1 < \alpha < 2$.

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