The Incorporation of Inventory Expectations into Econometric Models

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Introduction

In economies dominated by private enterprise, changes in the levels of inventory play a key role in explaining short-run cyclical movements in the gross national product (GNP). While changes in inventories are small relative to many other components of GNP, the absolute errors in predicting these changes by econometric models for up to a year ahead in the United States have been several times larger than those for both housing and plant and equipment expenditures and of the same order of magnitude as for consumption. Since changes in the levels of inventory average about one percent of consumption, the percentage errors in estimating these changes are roughly 100 times those for consumption. Errors in forecasts of changes in the levels of inventory investment, which constitute the inventory investment component of GNP, are perhaps the single most important barrier to improving short-run predictions of GNP.

In an attempt to improve the accuracy of forecasts of inventory changes, the Bureau of Economic Analysis (BEA) of the U.S. Department of Commerce began in late 1957 to collect anticipation data on inventories from a large sample of manufacturing firms. Similar anticipation data on outlays for plant and equipment (P&E) resulted in substantial improvements in the accuracy of forecasts of such expenditures. While the initial tests of the predictive usefulness of these anticipations of inventories were promising, subsequent tests were so discouraging that the BEA stopped publishing aggregate data on inventory anticipations. The Census Bureau

is still collecting and publishing these data, but only on an experimental basis.

Unlike the P&E anticipations, the inventory anticipation data have only been analyzed comprehensively at the aggregate level. Since analyses at the micro or individual-firm level of P&E anticipations have led to substantial improvements in forecasts of aggregate P&E, it may well be that similar analyses of the inventory anticipation data will reveal how to use these data in forecasting. Before passing final judgment on the predictive value of the inventory anticipation data, these data should be evaluated at the firm level in much the same way as the P&E anticipation data have been analyzed. It is the purpose of this paper to undertake this task.

After a description of the inventory anticipation data and the steps taken to preserve the confidentiality of the responses, the paper will assess the usefulness of these data in predicting one and two quarter changes in the levels of inventories for individual firms. This analysis does indicate that these data are of value in improving the accuracy of forecasts of inventories at the micro level. In view of this finding, it should be possible to use these data on anticipations to improve inventory forecasts at the macro level. The original design of this paper envisioned the development of such macro forecasts, but unforeseen delays in obtaining the inventory anticipation data at the level of the firm forced us to curtail our plans.

II. The Anticipation Data

In late 1957, the BEA began collecting data on anticipated inventories every six months from a sample of about 1,250 to 1,400 manufacturing corporation. This sample included most firms with assets of \$10 million dollars or more and a limited number of firms under that size.

In early 1959, the sample was expanded moderately and the survey was changed to a quarterly one. Also in early 1959, companies were asked to report actual inventories and prior sales and expected inventories and sales for each of the following two quarters. They were also requested to characterize their current inventories as "high", "about right", or "low". Although total inventories were broken down into finished goods and other inventories, the analyses in the next sections will only examine total inventories.

Subsequently, the survey was expanded to include anticipated inventories three quarters ahead. If the current inventories were characterized as "high" or "low", the approximate imbalance was obtained. These imbalances were not available to us for a long enough period of time to warrant the incorporation of these data in the subsequent analyses.

The BEA made available to us machine readable copies of the survey responses of individual companies from the fourth quarter of 1964 (IV-1964) through I-1972. In order to preserve confidentiality, the authors were made special agents of the U.S. Government, making them legally responsible for any leakage. To be doubly certain, the BEA in addition deleted the identification codes of the individual companies from the survey responses.

The analyses in the next section treated the period from IV-1964 through IV-1969 as the base period in which the various inventory models were fitted. The models were then evaluated as to their forecasting ability over the subsequent quarters, I-1970 through I-1972. A firm to be in-

cluded in the analysis of one-quarter anticipations must have had: (1) at least fifteen quarters of data in the base period and at least four quarters in the evaluation period, (2) no substantial changes in accounting methods or in the nature of its business, and (3) complete information on all variables used in any of the analyses. Finally, any firm whose anticipated inventories were always the same as their past inventories was deleted. The same requirements were imposed upon the analyses of two-quarter anticipations. The analysis of one-quarter anticipations were thus based upon 545 firms; and that of two-quarter anticipations, upon 505 firms.

III. The Models

All of the inventory investment models analyzed were of the form

(1)
$$(l_{j,t+i} - l_{j,t}) = \mu_j + \gamma_j (l_{j,t+i} - l_{j,t}) + \epsilon_{j,t}$$

where $I_{j,t}$ is the actual level of inventory for firm j at time t and $I_{j,t+1}^*$ is the level of inventory planned for time t+i as of time t. The models varied in the formulation of $I_{j,t+1}^*$. The constant $I_{j,t+1}^*$ measures the extent to which actual inventories adjust to planned inventories for firm $I_{j,t+1}^*$. The constant $I_{j,t+1}^*$ allows for any consistent bias over time for firm $I_{j,t+1}^*$. The random variable $I_{j,t+1}^*$ is a mean-zero disturbance. To minimize potential inefficiencies due to heteroscedasticity, equation (2) was deflated by $I_{t,t}^*$ before estimation. The deflated model, which formed the basis of the models actually estimated, thus took the form:

(2)
$$\frac{1_{j,t+i}-1_{j,t}}{1_{j,t}} = \frac{\mu_{j}}{1_{j,t}} + \gamma_{j} \frac{(t_{j,t+i})}{1_{j,t}} - \gamma_{j} + \varepsilon_{j,t}$$

where $\varepsilon_{j,t}$ is $\varepsilon_{j,t}^{\prime}/l_{j,t}$.

The inventory investment model as embodied in (1) gains economic meaning from the formulation of $t^{\dagger}_{j,t+i}$. Perhaps, the most naive formulation is to set $t^{\dagger}_{j,t+i}$ equal to the current level of inventory $t^{\dagger}_{j,t}$. Substituting for planned inventory, one obtains the naive model

(3)
$$\frac{I_{j,t+i} - I_{j,t}}{I_{j,t}} = \frac{\mu_t}{I_{j,t}} + \varepsilon_{j,t}$$

In a no-growth situation, μ_t would be zero. Equation (3) thus leads to two naive models depending upon whether the value of $\mu_t/I_{j,t}$ is set to zero or de-

termined by the sample.

A more sophisticated formulation of planned inventory is

(4)
$$t_{j,t+i}^* = a_j + b_j S_{j,t}$$

where the symbol S indicates quarterly sales and the subscript t indicates the quarter ending at time t. Substituting (4) into (2) yields the estimatable regression

(5)
$$\frac{I_{j,t+i}}{I_{j,t}} = (\mu_j + a_j) \frac{1}{I_{j,t}} + \gamma_j b_j \frac{S_{i,t}}{I_{i,t}} - \gamma_j + \varepsilon_{j,t}$$

Model (5) can still be viewed as naive in that it does not include expectational data and except for possibly the functional form provides a valid theoretical construct. It also represents the form which on the basis of past research yields about as good a forecast of planned inventory as any which does not use expectational data. With (4) and two forms of (3), three naive models can be constructed to evaluate the predictive value of the anticipations data.

The models using anticipation data to assess planned inventory were of two basic types:

(6)
$$t_{j,t+i}^* = c_j + d_{j,t+i}^e$$

and

(7)
$$t_{j,t+i}^{*} = e_{j} + f_{j} s_{j,t+i}^{e}$$
,

where the superscript "e" connotes anticipated values. If the anticipated inventories are identical to planned inventories, c_j and d_j would be respectively 0.0 and 1.0. If anticipated inventories differed from planned inventories only by a multiplicative constant, c_j would be zero. Both of these special cases of (6) were examined by setting c_j , and in the first

instance, d_i, to the appropriate constants.

To gain some insight into the potential interaction of inventory models and a complete system, planned inventory was also given as a function of future sales assuming perfect foresight. In this case, planned inventory is given by

(8)
$$t^{j*}_{j,t+i} = g_{j} + h_{j} S_{j,t+i}$$

Similarly to the derivation of (5), substitutions of these several formulations of t^* into (2) yield estimatable regressions. In addition, regressions were estimated in which planned inventory was given as weighted averages of various combinations of (4), (6), (7) and (8). The weights were determined by the appropriate regressions. Table 1 indicates symbolically the several combinations examined by listing the variables used in the formulation of planned inventory.

IV. One-Period Inventory Models

Each of the various regressions was fitted to each firm separately in the base period, IV-1964 through 1-1972, resulting in 545 distinct regressions analyzing one-quarter anticipations and 505 regressions analyzing two-quarter anticipations. The one-quarter regressions will be examined in this section.

The model fitting the data most closely in the base period was the one in which planned inventory was an unconstrained function of the anticipated levels and the actual prior levels of sales and inventories as well as the actual sales in the anticipated period (Table 1). The average coefficient of determination adjusted for degrees of freedom (R - squared) was 0.40. The worst fitting model in the base period was the one in which planned inventories were set equal to anticipated inventories. This case corresponds to equation (6) with c; and d; set to zero and one respectively. The average R - squared was only 0.08.

If however c_j and d_j are not constrained, the average R - squared for the model using only anticipated inventories increases to 0.27. This average is slightly below those in which anticipated inventories are augmented by prior or anticipated sales. On the assumption that μ_j in (1) is zero, it is possible to derive estimates of γ_j , a_j and b_j for the model using only anticipated inventories. Summaries of the distributions of these estimates reveal the magnitude of these biases (Table 2). A critical question to be examined below is whether these biases are stationary over time, so that they can be used to improve forecasts.

It has been found that percentage errors in anticipated plant and equipment expenditures tend to be less for larger firms. 8 To determine whether the same phenomenon characterizes inventory anticipation data, the statistical fit of the different models was examined separately for small, medium, and

large firms. A firm was considered large if its maximum quarterly sales in the entire sample period was in excess of \$25,000,000 and small if less than \$10,000,000. The remaining firms were classified as medium. There are no substantial differences in the magnitude or patterns of the R - squareds according to size (Table 1). Thus, at least by this criterion, the accuracy of anticipated inventories does not vary with the size of the firm, unlike P & E anticipations data. In fact, this finding is observed in all of the subsequent tests and therefore only the results for the total sample will be discussed in the text.

The test which really counts in evaluating an inventory model is not how well it fits the data in a base period but how well it forecasts. On the basis of the coefficients derived in the base period up to nine forecasts of $(l_{j,t+i}-l_{j,t})/l_{j,t}$ were made for each firm for each model from I-1970 through I-1972. The ratio of the mean squared error of the forecasts from a specific model to the mean squared error of the forecasts from a naive model is a measure of the predictive value of that model. A value of this ratio less than 1.0 would imply an improvement in the forecasts over the naive model.

As mentioned above, there were three naive models: (1) future inventories are expected to be unchanged from past inventories, (2) future inventories are expected to grow at the same percentage as the average quarterly rate in the base period, and (3) future inventories are given by (2) and planned inventories are given by $a_j + b_j S_{j,t}$. Irrespective of the naive model, the inventory models which include only anticipated inventories performed better than those which include some variable for sales. The best inventory model is the one which assumes that anticipated inventories are unbiased estimates of planned inventories, i.e. $c_j = 0$ and $d_j = 1$. Depending upon the naive model, this last inventory model makes improvements in from 60.2 percent to 69.4 percent of the firms.

It should be recalled that this best inventory model in the forecasting period was the worst according the average R - squareds in the base period.

Moreover, the best model using anticipation data in the base period is the worst in the forecasting period. Furthermore, adding some measure of sales to the anticipation data appears to have improved the fit in the base period but to have worsened the accuracy of the forecasts -- a strange situation.

What all this suggests is that the non-stationarities in the coefficients in the inventory models with some sales variable in addition to anticipated inventory contribute more errors to the forecasts than the non-stationarities in the less complex models. The superior forecasting performance of the model in which anticipated inventories are taken to be unbiased estimates of planned inventories suggests that the nature of the bias in anticipated inventories changes over time.

The prior analyses have been based upon separate regressions for each firm. It might be argued that a more efficient estimation procedure would be to concatenate the data from all the firms in the base period into one regression. An example of such a pooled regression is

(9)
$$\frac{\int_{j,t+1}^{j} \int_{j,t}^{j} = 0.042 \frac{1}{1} + 0.919 \frac{\int_{j,t+1}^{z} - 0.876}{(3.41)} + 0.919 \frac{\int_{j,t}^{z} \int_{j}^{z} - 0.876}{(99.20)}, \quad \overline{R}^{2} = 0.50$$

where the numbers in parentheses are t-values. Based upon 10,715 observations, this regression is derived by assuming planned inventory is the same linear function of anticipated inventory for all firms and then substituting this function into (1), again assuming μ_i and γ_i are the same for all firms.

Some preliminary results, not reported in detail here, suggest that the pooled regressions fit much more closely when confined to non-durable manufacturers. The resulting R - squared for the same functional form as

(9) is 0.74.

An important test of a pooled regression is its predictive value. Since we have not yet developed macro-forecasts using the individual firm regressions and since any macro-forecast from a pooled regression would require adjustment for missing firms, it would at this point be premature to evaluate the pooled regressions on the basis of their forecasts.

V. Two-Period Inventory Models

All the same analyses which were carried out for the one-period models were also performed for two-period models. The comparative fits of the various models in the base period and the comparative accuracy of the predictions was roughly the same. The major differences were that most of the models fit less well in the base period and have larger errors in the forecast period relative to the naive models.

The only regression in which the average R - squared increased was the one in which planned inventory was given as a function of anticipated sales and inventory, past sales, and future sales. The average R - squared was 0.54, compared to 0.40 for the one-period model. All the other models gave lower or at best the same average R - squareds. If the no change model is used as the benchmark, the formulation that planned inventories in (2) were equal to anticipated inventories—again the best predictive model—improved the forecasts of 53 percent of the firms, compared to 60 percent for the corresponding one-period model.

Table 2 Distributions of Estimated Values of γ_j , c_j , and d_j where $t^{\dagger}_{j,t+1} = c_j + d_j t^{e}_{j,t+1}$ in $(l_{j,t+1} - l_{jt}) = \gamma_j (t^{\dagger}_{j,t+1} - l_{jt}) + \epsilon_j t^{e}_{j,t+1}$

	Percentiles						
Sample	10%	40%	50%	60%	90%		
. Distribution of γ							
All Firms	0.06	0.50	0.60	0.72	1.06		
Small Firms	0.12	0.54	0.62	0.73	1.08		
Medium Firms	0.06	0.48	0.60	0.71	1.04		
Large Firms	0.05	0.55	0.64	0.73	1.08		
. Distribution of c							
All Firms	-5.29	1.96	3.42	5.46	33.20		
Small Firms	-0.13	1.46	2.05	2.56	14.17		
Medium Firms	-3.44	1.87	3.29	5.15	21.84		
Large Firms	-26.81	8.52	16.90	23.89	137.16		
. Distribution of d							
All Firms	0.35	0.86	0.91	0.97	1.2		
Small Firms	0.02	0.74	0.81	0.91	1.13		
Medium Firms	0.35	0.88	0.92	0.98	1.34		
Large Firms	0.57	0.89	0.97	1.00	1.18		

Table 3 Distributions of Ratios of Mean Square Errors of Forecasts to Mean Square Errors of Naive Forecasts For All Firms

	t			Percentiles	•		Percentage of
Naive Model	Given As a Function of	10%	40%	\$08	\$09	306	Ratio < 1.0
A. No Change	S +	0.78	0.98	1.04	: ::	1.78	0.44
	t _ 0 ;	0.62	0.93	0.99	1.07	1.64	0.51
	t j, t+1 (c =0)	0.61	0.91	0.96	1-04	1.68	0.55
	t [, t+] (c =0, d =1)	0.63	0.92	0.97	1.00	1.25	0.60
	t j,t+1, S,t	0.61	0.94	1.02	1.11	1.88	0.48
	t j, t+1' t Se, t+1	0.62	0,94	1.03	1.13	1.87	0.47
	c j, t+1, c j, t+1, S, t	. 0,60	0.95	1.04	1.17	2.02	0.45
	t j, t+1' t j, t+1' Sj, t' Sj, t+1	0.60	0.97	1.08	1.21	2.09	0.42
B. Historical Mean							
	ي. د	0.69	0.92	0.98	1.04	1.54	0.55
	t j, t+1	0.56	0.86	0.92	0.98	1.47	0.63
	t e	0.60	0.90	0.95	0.99	1.22	0.65
	t e, t+1 (c = 0, d; * 1)	0.49	0.80	0.86	. 0.93	1.24	0.69
	t], t+1' S, t	0.55	0.87	0,93	1.03	1.61	0.57
	t]; t+1' t Se t+1	0.56	0.87	0.94	1.02	1.66	0.57
	t j, t+1' t se t+1' Sj, t	0.55	0.87	0,96	1.05	1.69	0.55
	t j, t+1, t S j, t+1, S j, t, S j, t+1	0.56	0.88	0.95	1.07	1.73	0.54
C. Function of Current Sales	· · · ·	0.52	0.88	0.94	1.00	1.34	0.61
	t e (c = 0)	0.46	0.87	0.92	1.00	1.45	0.60
	t j, t+1 (c, =0, d, =1)	0.42	0.80	0.88	0.94	1.25	0.67
	t j, t+1, S, t	0.63	0.92	0.98	1.00	1.28	0.59
	t j, t+1' t Se, t+1	0.56	0.89	0.94	1.02	1.47	0.57
	t j, t+1, t Se, t+1, S, t	0. 6 2	0.92	0.97	1.03	1.42	0.55
	[], t+1' t Se, t+1' Sj, t' Sj, t+1	0.59	0.91	0.98	1.06	1.59	0.52

VI. Summary

Though the results in this paper are preliminary, they do suggest that inventory anticipation data may ultimately prove to be of value in improving forecasts of inventory investment. It is clear that there are substantial non-stationarities in the relationship of inventory investment to planned inventory investment within the confines of the simple models considered in this paper.

Perhaps in a more complete model, these apparent non-stationarities could be explained. In this regard, some tentative analyses suggest that there may be seasonal biases in the anticipation data. Finally, no effort has as yet been made to weight the anticipated inventories according to the track records of the individual firms. In view of the results in this paper, it would seem worth undertaking these more complicated analyses.

 $fable \ 1$ Distributions of Coefficients of Lorentziatlon Adjusted for Degrees of Freedom for the Grantziale Models Explaining $\frac{1}{j_1t+j_2-j_3}$

t ¹ j,t+i Siven As a Function of	No. of C	Percentiles					
	Number of Observations	10%	40%	50%	6 0%	90%	Average Value
A. All Finns							
⁵ J,t	545	-0.05	0.09	0.15	0.21	0.44	0.18
t ^{le} j,t+1	545	-0.05	0.14	0.23	0.32	0.69	0.27
$(c_{j,t+1}^{e}(c_{j}=0))$	545	-0.05	0.09	0.14	0.23	0.66	0.23
t ^{fc} j, (+1 (cj=0, dj=1)	545	-0.33	-0.03	0.02	0.11	0.60	0.08
t ^{le} j,t+1 ^{,S} j,t	545	-0.04	0.20	0.29	0.37	0.70	0.31
t	545	-0.04	0,21	0.27	0.35	0.72	0.31
t j,t+1' t se,t+1' sj,t	545	-0.03	0.23	0.31	0.39	0.73	0.32
t ^{ie} j,t+1' t ^{Se} j,t+1' ^{\$} j,t' ^{\$} j,t+1	545	0.01	0.31	0.39	0.48	0.80	0.40
3. Small Firms S _{j,t}	106	0.01	0,17	0.21	0.23	0.53	0.25
t j. t+1	106	0.00	0.16	0.26	0.35	0.76	0.32
$t_{j,t+1}^{e}(c_{j}=0)$	106	-0.05	0.11	0.19	0.25	0.76	0.28
te, (c; =0,d; =1)	106	-0.21	0.00	0,11	0.22	0.71	0.18
t i,t+!	106	0.03	0.24	0.33	0.39	0.76	0.36
t'j,t+1' t ^{se} j,t+1	106	0.05	0.23	0,30	8ز.ب	0.78	0.36
ι ^ρ , t+1' ι ^ς , t+1' ^ς , t	106	0.05	0.24	0.33	0.43	0.79	0.38
t j, t+1', (Sj, t+1', Sj, t', Sj, t+1	106	0.04	0.31	0.39	0.48	0.83	0.42
I. Medium Firms	336	-0.06	0.08	0,13	0.19	0.42	0.17
^S j,t _J e	336	-0.06	0.13	0.22	0.30	0.65	0.26
t	336	-0.05	0.07	0.12	0.23	0.65	0.20
('j,(+1(cj=0) [e [j,t+](cj=0,dj=1)	336	-0.35	-0.06	0,00	0.06	0.60	0.22
t',,t+,'-; ','; '' t 'j,t+;' S j,t	336	-0.05	0.19	0,28	0.36	0.69	0.30
= '	336	-0.04	0.20	0.26	0.34	0.70	0.30
t j, t+1	336	-0.06	0.22	0.31	u.37	0.74	0.32
t'j,t+1' t [*] j,t+1' [*] j,t t ^{le} j,t+1' t ^{Se} j,t+1' ^S j,t' ^S j,t+1	336	-0.02	0, 30	0.38	0.46	0.80	
Large Firms	7,7"	0,0%	W. J.	0. 70	0.40	0.00	0.39
S _{j.t}	103	-0.07	0.05	0.08	0.13	0.33	0.12
t ^{†e} j,t+}	103	.0.01	0.11	0.20	0.29	0.59	0.25
$\mathcal{E}_{j,t+1}^{\alpha}(c_{j}>0)$	103	-0.05	0.09	0.14	0.24	0.57	0.22
$t_{j,c+1}^{e}(c_{j=0},d_{j}=1)$	103	0.48	-0.02	0.00	0.09	0.56	0.05
t ^{l°} j,tH ^{, S} j,t	103	-0.66	0.17	0.28	0.32	0.61	0.28
(i, tet (; set)	103	-0.08	0.13	0,25	0.34	0.63	0.28
$t_{j,t+1}^{e}, t_{j,t+1}^{e}, s_{j,t}$	103	0.07	0.24	0.28	0.35	0.65	0.30
that shirt shirt	103	0.06	0.31	0.42	0.52	0.78	0.42

FOOTNOTES

- *The authors are Professor of Finance and Richard K. Mellon Professor of Finance and Economics, respectively, The Wharton School, University of Pennsylvania.
- 1Cf. Irwin Friend and Robert Jones, "Short-Run Forecasting Models Incorporating Anticipatory Data," Models of Income Determination, Studies in Income and Wealth, Vol. 28, Princeton, 1964; and Irwin Friend and Paul Taubman, "A Short-Term Forecasting Model," The Review of Economics and Statistics, Vol. XLVI, August 1964, No. 3.
- ²Irwin Friend and William Thomas, "A Reevaluation of the Predictive Ability of Plant and Equipment Anticipations," <u>Journal of the American Statistical Association</u>, Vol. 65, June 1970, No. 330.
- ³Murray F. Foss, "Manufacturers' Inventory and Sales Expectations, a Progress Report on a New Survey," <u>Survey of Current Business</u>, U.S. Department of Commerce, August 1961 and Friend and Taubman, <u>op. cit.</u>
- ⁴The U.S. Department of Commerce News Report as of June 11, 1974 (CB 74-140) stated "Effective with this report, we are planning to discontinue this survey. The survey results have not proven to be satisfactory either as a predictor of sales and inventory levels or as an aid in forecasting changes. If information of this type is deemed necessary for economic forecasting, the Bureau of the Census would consider future experimentation in survey methods to help produce such data."
- ⁵Albert A. Hirsch and Michael C. Lovell, <u>Sales Anticipations and Inventory Behavior</u>, John Wiley & Sons, Inc., 1969, measure the accuracy of inventory anticipations for a small sample (83) of firms in five manufacturing industries, but do not compare their predictive record with that of an expost model.
 - ⁶For a more complete description, see Foss, <u>op</u>. <u>cit</u>.
- ⁷Friend and Jones, <u>op. cit.</u> show that the addition of unfilled orders in aggregate inventory models leads to a modest improvement in forecasts of inventory investments. These data however were not available to us on an individual firm basis.
- ⁸Friend and Bronfenbrenner, "Plant and Equipment Programs and their Realization," <u>Short-Term Economic Forecasting</u>, Studies in Income and Wealth, Volume 17, Princeton University Press, Princeton, 1955.
- 9 We are currently in the process of obtaining results from an even simpler model, $l_{j,t+1} = \frac{l^e}{t^j,t+1}$, which gives the best forecasts though not the best base-period fits for P&E investment (Friend and Thomas, op. cit.).