The Micro Foundations of Equilibrium in a Monetary Economy: A Transactions Cost Approach

by

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Working Paper No. 18-74

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The microfoundations of a neo-classical macroeconomic model have generally been derived from a general utility function with consumption each period and money balances as arguments. This may be illustrated as follows. Consider an individual wishing to maximize a three period utility function, subject to a budget constraint for each period and initial endowments of bonds and money of $\frac{Bo}{P}$ and $\frac{Mo}{P}$, in real terms. The approach used heretofore by most macro-monetary theorists, such as Patinkin [1965] and Metzler [1951], can therefore be viewed formally as maximizing

$$(1)\ U = U(C_1, \frac{M1}{P}, C_2, \frac{M2}{P}, C_3; \frac{M_o}{P}, \frac{B_o}{P}),$$

subject to

$$(2)\ C_1 + \frac{B_1}{P(1+r)} + \frac{M_1}{P} - \frac{B_o}{P} - \frac{M_o}{P} - \bar{Y}_1 = 0,$$

$$(3)\ C_2 + \frac{B_2}{P(1+r)} + \frac{M_2}{P} - \frac{B_1}{P} - \frac{M_1}{P} - \bar{Y}_2 = 0,$$

$$(4)\ C_3 - \frac{B_2}{P} - \frac{M_2}{P} - \bar{Y}_3 = 0.$$

where $\bar{Y}_i$ is the exogenous endowment each period and bonds are assumed to be one period bonds sold at a discount. Assuming that consumption in each period and money balances are normal goods and the utility function is well behaved and concave in consumption each period and money balances, one may obtain a unique maximum for the implied constrained maximization. Using these first order conditions as a system of simultaneous equations, demand functions for consumption goods, C, bonds in real terms, $(\frac{B}{P})$, and real money balances, $(\frac{M}{P})$, can be obtained. This results in demand functions of the general form:

(5) $C = c(Y, r, \frac{M_o}{P})$,

(6) $\frac{B}{P} = b(Y, r, \frac{M_o}{P})$,

(7) $\frac{M}{P} = m(Y, r, \frac{M}{P})$.

While this approach results in demand functions that satisfy both a priori conjecture as to the arguments that effect household demands and fits the empirical data on the determinants of demands, the derivation from a utility function with money balances as an element has been strenuously objected to by many. However, without such a construction no demand function can be obtained for both money balances and bonds.

It is well known by now that this dilemma results from the level of abstraction of the neo-classical model. Neither transaction costs nor uncertainty exist. Accordingly, the money asset "exists but serves no function," using the words of Savings [1970]. To remedy the situation research has been done both on the impact of uncertainty on demand for money as in Tsiang [1969], and Goldman [1973], and on the impact of transaction costs on money balances by Baumol [1952], Tobin [1956], Feige and Parkin [1971], Barro and Santomero [1972], and Santomero [1974A]. Yet, until very recently this work was always partial analysis of the monetary flow problem due to discrete transaction costs versus rate of return trade-offs. Underlying demands and supplies in the real sector were assumed exogenous.[1] However, to treat the problem in a general equilibrium context requires that the impact of cash management on the real sector be considered. This would involve a total reformulation of the basic formulation as contained in equations (1) through (4) above to allow for the interaction, at the individual maximization level, of real demands and the costs associated with the transactions resulting from these real flows. A move was made in this direction by Saving [1971] and [1972] where the cost of transactions is considered in a model of the household and firm respectively. However, the very construction of the transaction cost function, which was of the general form,[2]

(8) $T = T(PY,M,v)$,

where T = total transaction costs,

PY = nominal income,
v = inventory holdings,

and other variables are as previously defined, belies the very reason for adding the complexity of transaction costs into an aggregate model. Using such a general, non-specified, function leaves the model one step further in ambiguity. Now money balances exist to minimize transaction costs[3] but transaction costs, and particularly their specification, exist but serve no well-defined function.

To properly incorporate money into the analysis of a macroeconomic model that considers the minimization of transaction costs as the major cause of the demand for this asset, it is necessary that particularly careful attention be given to how such costs are incurred and their true nature. However, to do so requires the development of optimal micro-unit-behavior models with respect to frequency of transactions. The present analysis attempts such a construction. By so doing it will suggest an explicit alternative structure to the neo-classical paridigm that may be compared and contrasted relative to its comparative advantage in explaining macroeconomic phenomenon.

The present approach may be outlined as follows. The household is assumed to maximize a utility function such as

(9) $U = u(C_1, \ell_1, C_2, \ell_2)$

where C = consumption; ℓ = labor services.

It will be noted that this specification of U is more general than (1) above, as labor supply can also be derived. In addition money balances no longer enter the utility function, but rather will be held to minimize the cost of transaction. Budget constraints in this formulation would include (a) endogenously determined income, (b) return on wealth and (c) returns from cash balance behavior, all of which are net of taxes. Accordingly the model contains an intertemporal element, viz. wealth, an intersectoral element, viz. wage and profit income as well as taxes, and a money balance feedback, all directly into the utility maximization approach. The resultant demand functions and supply of labor function are well defined and contain generally accepted arguments as well as transaction costs and rates of returns.

Following a similar approach for the firm, a supply function for output can be derived as well as a demand for labor function that complements the household. Here profit maximization is assumed, rather than utility function maximization.

Government activity can be viewed as setting rates of return on monetary assets, demanding goods and levying taxes.

Once combined into a general equilibrium system, comparative static analysis can be conducted to determine the impact on equilibrium output, employment, and real balances of various disturbances. Here the analysis centers upon the appropriate view of the adjustment to a new comparative static equilibrium following some fairly traditional exogenous disturbances. Also disturbances previously left untreated in other general equilibrium models, variations in transactions costs on equilibrium, will be considered.

In all, the analysis derives what may be viewed as an alternative specification of the microfoundations of the now well accepted four market model that allows for similar reduced form specifications, in the limit, but very different interpretations.

I. Household Sector

The household is viewed as maximizing a utility function over time of the general form.

$$\text{(I.1)} \qquad V = \int_0^\infty U(C_t, \ell_t)e^{-\delta t} dt,$$

where C_t = consumption at period t,

ℓ_t = labor services at period t,

δ = discount rate.

In discrete time a two period maximization of (I.1) above is equivalent to a maximization of a time separable utility function in the form,

$$\text{(I.2)} \qquad U = U(C_t, \ell_t, C_{t+1}, \ell_{t+1}),$$

where t and t+1 refer to two successive discrete periods and $\frac{\partial U}{\partial C_{t+1}} = \frac{\partial V}{\partial C_{t+1}} e^{-\delta t} dt$ as well as the time separability conditions conditions, $\frac{\partial^2 U}{\partial C_t C_{t+1}} = \frac{\partial^2 U}{\partial C_{t+1} C_t} = \frac{\partial^2 U}{\partial \ell_t \ell_{t+1}} = \frac{\partial^2 U}{\partial \ell_{t+1} \ell_t} = 0.$

As has been shown elsewhere, Santomero [1974B], for a given initial wealth an intertemporal utility function as in (I.2) results in the following conditions at the point of maximization,

$$\text{(I.3A)} \qquad \frac{\partial U}{\partial C_t} = (1+\delta) \frac{\partial U}{\partial C_{t+1}}$$

$$\text{(I.3B)} \qquad \frac{\partial U}{\partial \ell_t} = (1+\delta) \frac{\partial U}{\partial \ell_{t+1}},$$

and the marginal rate of substitution at any time, t,

$$(I.3C) \qquad \frac{\frac{\partial U}{\partial \ell_t}}{\frac{\partial U}{\partial C_t}} = \frac{W}{P} .$$

Equating the time rate of discount with the borrowing rate, r_b, as in Hirshleifer [1958] and Fisher [1937], the quantities that achieve a maximum for equation (I.2) satisfy the two period budget constraints,

$$(I.4A) \qquad \frac{W}{P} \ell_t^* + \omega_o - C_t^* = \frac{\omega_t}{(1+ r_b)} ,$$

$$(I.4B) \qquad C_{t+1}^* - \frac{W}{P} \ell_{t+1}^* = \omega_t ,$$

where ω_o = initial wealth holdings of interest bearing assets,

ω_t = wealth holdings after income and consumption decisions for period t.

$*$ = optimal values from a maximization of (I.2).

and all bonds are bought at discount for the price $\left[\frac{1}{(1+r_b)}\right]$. From (I.4), therefore, it can be seen that selecting optimal values of the time dimensioned discrete period utility function (I.2) defines an optimal saving quantity during period one, equal to $(\omega_t - \omega_o)$, or equivalently for a given ω_t the optimization of the discrete time utility function (I.2) is equivalent to optimizing a one period objective function in C_t, ℓ_t and ω_t conditioned upon ω_o, viz.

$$(I.5) \qquad U = g(C_t, \ell_t, \omega_t; \omega_o),$$

where g_3 indicates the increased utility obtained from period t income in period t+1, i.e., consuming in excess of income in period t+1.[4] It should be stressed here that wealth balances in and of themselves are not assumed to yield any return to the individual utility maximizer but rather enter the objective function due to the utility yield obtainable from next period consumption of these balances. A utility or objective function of the form of (I.5), used elsewhere with no explicit derivation by Dutton and Gramm [1973], allows a multi-period maximization process to be contained in a less cumbersome one period functional form.

Accordingly it will be assumed that the representative household wishes to maximize an objective function as represented in (I.5) throughout this analysis. What remains is to define the constraints to the maximization of this equation. The view of the world to be captured in the model is one where transaction costs exist for all intersectoral transfers. These costs are modeled as purely time consuming, although fixed dollar transfer costs are possible in the present formulation. The household, then, in attempting to maximize its welfare function, takes cognizance not only of prices paid for consumption and labor services but also the implied time cost of securing them, and the positive or negative returns they yield while they are held. Let us now consider the prices, transaction costs and returns of each argument of the utility function.

Turning to labor, labor services are used to (a) produce output by engaging in market oriented activities at a wage rate of $\frac{W}{P}$, denominated in real dollars per time interval, and (b) to make transactions between money balances and commodities,

$$(I.6) \qquad \ell = \ell_p + \ell^h_{\dashv},$$

where Γ = transactions activity,

p = production activity,

ℓ = labor services over the period T, in pure number units.[5]

The cost of transacting between money balances and goods is assumed to be a fixed cost, β, denominated in units of time per goods purchase. Within each payment period, T,[6] the household may select, endogenously, the frequency of such purchase trips based upon β and the total value of consumption, C. Given C, the problem reduces to the simple Baumol-Tobin scenario of cash management, now between goods and money, rather than between money and a short term asset. In any case, the individual, within the maximization process, selects the number of such trips per income period, denoted by n, which in turn defines a flow of labor input to transactions activity defined as,

(I.7) $\ell^h_{\Gamma} = \beta n,$

for a given value of T, the payment period.

At the same time the individual is selecting its labor market supply, for which he receives a wage rate of $\frac{W}{P}$. Payment to wage earners are assumed to take place after they are accrued, and, as discussed below, at an interval set endogenously by the firm. Accordingly for the first payment period the household must either forego consumption at an imputed cost of δ per year or run down its real wealth balances at an opportunity cost of r_b per year. In a perfect capital market these two rates are equal; while in an imperfect market setting some divergence between these rates may occur (see Hirshleifer(1958)). In the present analysis it will be assumed that the representative household possesses positive wealth balances upon which to draw; therefore, the opportunity cost of non-payment is r_b. The gross cost[7] of deferred payment, defined as the forgone interest payments from these wealth balances, may be written as $\frac{W}{P}\ell_p r_b T$. Eventually, the household receives a lump sum monetary payment, at intervals of T days, equal to $\frac{W}{P}\ell_p$, but perceives this income as equal to $\frac{W}{P}\ell_p(1-r_b T)$ dollars due to the cost associated with its deferred receipt.

At the beginning of the first work period, the household is viewed as withdrawing $\frac{W}{P}\ell p$ dollars and distributing this quantity optimally between saving and consumption, according to the result of its intertemporal first order conditions. Funds allocated to consumption over the period would, in turn, be distributed into commodity inventories and money balances, just as if the funds had resulted from wage income receipts.

In addition to income received from wage earnings the individual also has two other sources of revenue, viz. equity yields and returns on bond holdings. Over the period the household is viewed as receiving a continuous flow of profit income at rate, π, where π is defined as the proportionate share of total firm profits accruing from its equity holdings of the existing capital stock, assumed constant in the short run.[8] In addition, income is received in terms of interest returns on liquid wealth assumed to be in the form of bond holdings, at a rate r_b. Finally taxes are purely per capita levies per unit of time to finance government activities.

Over a payment period of given size, therefore, total household income, denoted by X_h or in flow units Y_h, for the payment period T, may be written as,

$$(I.8) \quad X_h = Y_h T = \frac{W}{P}\ell_p - \frac{W}{P}\ell_p r_b T + \pi T + b r_b T - \tau T,$$

where b = real bond holdings entering the period.

This may be simplified and written in rate of flow terms as,

$$(I.8') \quad Y_h = \frac{W}{P}\frac{\ell p}{T}(1-r_b^T) + br_b + \pi - \tau.$$ [9]

From this perceived income and real wealth balances with which the individual enters the period, the household determines endogenously the quantity of funds to be used for consumption expenditure during the current period through maximization of the objective function contained in equation (I.5) and subject to (I.8). For simplicity assume that the decision period for the single period maximization implied above is coincident with the payment period, T.[10] For any consumption quantity, however, the individual must take cognizance of the implied cost of purchase and management of the goods to be consumed over the period. Due to the costs involved in the purchase of consumption goods and the (negative) return on these goods in the form of real depreciation, the household must simultaneously derive from behavior a pattern for transactions associated with the derived optimum consumption that results from this maximization. This problem can be viewed as an inventory maximization problem of optimal behavior of funds destined to be consumed. The control variable from the household's point of view is the frequency of transfer between money and commodities.[11] Frequent trips result in low inventories but high transaction costs, which as indicated above, are modeled as time costs of transfers. Less frequent transfers reduce the time transaction costs but leave relatively higher inventory balances at an assumed inferior rate of return relative to money. The optimal frequency of expenditure trip would so weigh these costs so that these two offsetting costs were equalized on the margin.

For any frequency of commodity shopping trips the procedure may be viewed formally as follows. Assuming a uniform consumption stream throughout the period, T, average working balance, $\overline{\Omega}$, defined as the average amount of money and commodity assets held by the household in anticipation of consumption, then becomes,

(I.9) $\overline{\Omega} = \overline{M} + \overline{G} = \frac{CT}{2}$,

where M = money balances,

G = commodity balances,

bars equal average balances.

Over the period the household engages in commodity purchase trips of total number n per T. Tobin [1956] has shown in a similar context that an optimal solution involves equally spaced trips, each of which concludes with commodity inventories of $G = \frac{C}{n}$. Therefore average inventories of commodities held by the household can be written as,

(I.10) $\overline{G} = \frac{C}{2n}$.

From (9) and (10) average money balances can be written as,

(I.11) $\overline{M} = (1 - \frac{1}{n}) \frac{C}{2}$.

The determination of the household's average money balances, therefore, is dependent upon the optimal value of n. The frequency of trips itself, however, is endogenously determined and dependent upon the rate of return yielded by these two short term intra-payments-period assets, G and M. Elsewhere, Santomero [1974A], it has been shown that the appropriate ex ante rate of return on commodity inventories is the sum of the expected rate of change in commodity prices, $(\frac{DP}{P})^e = \pi^e$ and the negative real depreciation rate, λ, so that $r_g = \pi^e - \lambda < 0$ iff $|\pi^e| < |\lambda|$.[12] The rate of return on money balances, denoted as r_m, is more difficult. It has been generally assumed that this rate is institutionally set at zero, e.g., see Tobin [1969], however, recent evidence, e.g., see Barro and Santomero [1972], O'Brien [1972], indicates that this rate is indeed positive, and may fluctuate with r_b. In any case it will be assumed here to be variable and generally greater than zero.

Combining the rate of flow of profits (losses) from the optimal management of a given consumption allotment, denoted as Φ, where,

$$\text{(I.12)} \quad \Phi = \overline{M}\, r_m + \overline{G}\, r_g,$$

to the total perceived household income obtained in equation (I.8') yields, in rate of flow units,

$$\text{(I.13)} \quad Y_H = \frac{W}{P}\frac{\ell p}{T}(1-r_bT) + br_b + \pi - \tau + \frac{C}{2}r_m - \frac{C}{2n}(r_m - r_g).$$

This equation sums the flow of income from all sources, viz. labor services offered in the production sector, $\frac{W}{P}\frac{\ell p}{T}(1-r_bT)$, returns to wealth, $br_b + \pi$, returns from cash management, $\frac{C}{2}\, r_m - \frac{C}{2n}(r_m - r_g)$, and taxes, $-\tau$. It will be noted that this income flow, Y_H, is jointly determined by ℓ, through ℓp, and C. Also, given entering wealth balances, all in the form of bond holdings and non-negoitable equity ownership,[13] the determination of ω is achieved. All interperiod wealth balances are in the form of bonds and equity ownership rather than money balances in this analysis because money serves only to minimize transaction costs rather than because of some utility yield.[14] At the end of each payments period the household's desired money balances are exhausted and net savings from one period to the next are carried in higher yielding assets.[15]

In sum, then, the household is viewed as, at the beginning of time t, selecting ℓ and C which in turn defines net saving for the payment period, T.

Given the initial values of wealth this determines the quantity of funds, denominated in real terms to be held to next period, the third argument in the objective function,

$$\text{(I.14)} \quad \omega_t = \omega_o + Y_h T - C,$$

where ω_o is the real value of wealth from the last period, which may be written as,

$$\text{(I.15)} \quad \omega_o = b_o + \frac{\pi}{r_b},$$

where $\frac{\pi}{r_b}$ = net present value of the society's fixed equity holdings,

b_o = maturity value of bonds sold at discount in the prior period.

Formally, the entire problem faced by the individual can be viewed as the maximization of (I.5) above, for a period of length T,

$$U = g(C_t, \ell_t, \omega_t; \omega_o),$$

subject to,

$$\ell_t = \ell_p + \beta n,$$

and,

$$\omega_t = b_o + \frac{\pi}{r_b} + \frac{W}{P}\ell_p(1-r_bT) + T[br_b + \pi - \tau + \frac{C}{2} r_m - \frac{C}{2n}(r_m - r_g)] - C .$$

The control variables of the system are ℓp, n, and C. Assuming the utility function is well behaved and can be treated as a continuous variate the first order conditions can be obtained. Neglecting time subscripts, these become,

$$\text{(I.16A)} \quad \frac{\partial g}{\partial C} - \frac{\partial g}{\partial \omega}\left[1 - \frac{r_m T}{2} + \frac{(r_m - r_g)\,T}{2n}\right] = 0$$

$$\text{(I.16B)} \quad \frac{\partial g}{\partial \ell} + \frac{\partial g}{\partial \omega}\left[\frac{W}{P}\,(1 - r_b T)\right] = 0$$

$$\text{(I.16C)} \quad \frac{\partial g}{\partial \ell}\beta + \frac{\partial g}{\partial \omega}\left[\frac{C}{2n^2}(r_m - r_g)\,T\right] = 0$$

The first of these indicates the optimum point of consumption. Notice that the condition implied by (I.16A) is that the increase in utility yielded by an additional unit of consumption within this period be equal to the net cost of the item, here pictured as the net reduction from wealth held for the next period. Under the assumptions used thus far, viz. $n > 1$ and $r_m > r_g$ this implies,

$$\left|\frac{\partial g}{\partial C}\right| < \left|\frac{\partial g}{\partial \omega}\right| ;$$

at the point of optimization, due to $\left[1 - \frac{r_m T}{2} + \frac{(r_m - r_g)T}{2n}\right] < 1.$ Accordingly, on the margin, the household is viewed as consuming more than the traditional neo-classical results would have indicated due to the positive net return from cash management.

On the other hand, the labor supply partial results indicate that the household works less than traditional theory would suggest, due to the discounting of future payments of labor income, viz. $(1 - r_b T) < 1$.

Solving the third partial equilibrium condition for the optimal frequency of transfers from money to commodities yields,

$$\text{(I.17)} \quad n = \left[\frac{\frac{\partial g}{\partial \omega}\, C\,(r_m - r_g)\,T}{2\beta \frac{\partial g}{\partial \ell}}\right]^{\frac{1}{2}} .$$

Reverting to rate of flow terms for the consumption variable, $\frac{C}{T} = c$, and substituting from (I.16B), this may be written as,

$$(I.17')\, n = T \left[\frac{\frac{C}{T}(r_m - r_g)}{2\beta \frac{W}{P}(1 - r_b T)} \right]^{\frac{1}{2}}$$

This result supports previous work by Barro and Santomero [1974] indicating the determinants of the shopping frequency. Here n is strictly proportional to T and depends directly upon both the volume of consumption and the relative interest rates, and inversely upon the time cost of transactions. This last result was also brought out by Karni [1974].

The first order conditions so derived may also be looked at simultaneously as a system of equations that jointly determine ℓp, n, and C. Given the three control variable's simultaneous solutions the general functional forms for ℓ, C, $\overline{M}$, and b can also be derived, and form the basis of the household's input into a complete general equilibrium model. Evaluation of such a system, however, involves a great deal of difficulty as second order effects offsetting direct effects often prohibit explicit sign determination.

For example consider the case of a rise in the rate of return on commodities, r_g.[16] Evaluation of the impact on consumption of such a disturbance results in ambiguity. Further analysis, however, suggests that what is involved are essentially second order effects. A rise in r_g results in a net increase in the flow of goods that can be obtained for a given consumption allotment. As shown elsewhere, Barro and Santomero [1974], this is viewed by the household unit as a reduction in the price of the good to the individual. Accordingly consumption of goods for the current period rises. However, at the same time, the rise in r_g decreases the frequency of commodity transactions, resulting in higher commodity inventories at the expense of money balances. The portfolio shift therefore somewhat offsets the direct income effect. To obtain a clear sign on the impact of this shift on consumption demand one must assume that the second order effect of smaller cash management profits due to higher commodity inventories is more than dominated by the income effect on current period consumption.

Assuming the second order effects are small, i.e., assuming transaction frequency effects do not swamp real effects,will allow the explicit, determination of signs in our demand functions. This assumption is equivalent to evaluating C and ℓ_p functions at dn = 0.

Using this approximation one can obtain,[17]

$$(I.18)\quad C^d = C(\overset{+}{\frac{W}{P}}, \overset{-}{\beta}, \overset{+}{r_m}, \overset{+}{r_g}, \overset{-}{T}, \overset{+}{\pi}, \overset{-}{\tau}, \overset{-}{r_b}, \overset{+}{b}),$$

$$(I.19)\quad \ell_p^s = \ell_p(\overset{+}{\frac{W}{P}}, \overset{-}{\beta}, \overset{?}{\overset{+}{r_m}}, \overset{?}{\overset{+}{r_g}}, \overset{-}{T}, \overset{-}{\pi}, \overset{+}{\tau}, \overset{?}{\overset{+}{r_b}}, \overset{-}{b}).$$

In equation (I.18) the arguments of household consumption are indicated. As in the traditional neo-classical model consumption responds positively to increases in real wages, $\frac{W}{P}$, and increases in real wealth, here indicated by b, and $\frac{\pi}{r_b}$. Further, consumption responds to transaction costs and rates of return in this formulation. An increase in the time cost of commodity purchaes, β, is perceived by the household as a net decrease in the real goods return from a given labor flow. Accordingly,it may be viewed as a reduction in real wages due to a perceived increase in the real cost of securing a given consumption flow during the period T. Consumption therefore declines. The rates of return on interim balances of money and commodity inventories may be viewed similarly. Here a given income allotment purchases more commodities, <u>as if</u> the price level of goods this period had fallen. The household, therefore, increases its consumption with a rise in these rates, r_m, and r_g. The profit and tax terms, π and τ respectively enter the demand function through the budget constraint on present consumption. Due to the normality of consumption this quantity increases with π and decreases with τ. The bond rate of of return enters the consumption function in three ways. An increase in r_b causes (a) the present value of the future stream of profit to fall, (b) the return during period T on existing bonds held by the household to rise (assuming variable rate bonds), and (c) a reduced present value of the future payment from the firm to the household for production labor. The total impact is assumed to be a reduction in consumption. Finally consider the impact of an alteration in T, the payment period. In considering this disturbance, however, care should be taken to isolate merely the changes in the flow behavior due to an alteration in T, and not the effect of increasing the time span of the utility maximization. That is, what is of interest here is what is the impact of an increase in T given all underlying flow transaction rates and profit, interest, and tax flows remain constant. So considered, the impact upon consumption is clearly negative as the household views the longer payment period as a reduction in the net present value of wage income, the latter being discounted over time at the rate r_b.

Turning to the labor supply function, equation (I.19) leads to an analogous view of the representative household. Labor supply for production increases with $\frac{W}{P}$, and decreases with wealth and endowment flow changes, π, τ, b. This latter effect is consistent with the recent and growing literature on the wealth impact on labor supply discussed in Phelps [1972] and Santomero [1974B]. Further, the cost of transacting enters here, as in Barro and Santomero [1974] as the increased cost of purchasing goods is viewed by the household as a reduction in the real wage paid for market labor. On the other hand increases in the rates of return earned by interim balances causes higher labor supply at a given real wage. Finally the impact of T on the system, following C^d above, decreases the present value of wages and reduces labor supply.

Given the supply of labor at any real wage rate, $\frac{W}{P}$, and the demand for consumption over the period, one may now turn to money demand. As indicated above, equation (I.11), money balances are a direction function of n, the frequency of transfers between money and commodities, and consumption over the period. Money demand therefore is modeled completely as a transactions demand with transfers only between money and commodities, rather than the traditional Baumol-Tobin transfers between money and bonds within the payments period. Accordingly substitution from (I.17') and (I.18) yields the general money demand function,

$$M^d = \frac{C^d(\cdot)}{T} - \left[\frac{\beta \frac{W}{P}(1-r_b T)}{2(r_m - r_g)} \quad \frac{C^d(\cdot)}{T}\right]^{\frac{1}{2}} \qquad \text{(I.20)}$$

or in general terms,

$$M^d = M^h(\overset{?}{\frac{W}{P}}, \overset{-}{\beta}, \overset{+}{r_m}, \overset{-}{r_g}, \overset{+}{T}, \overset{+}{\pi}, \overset{-}{\tau}, \overset{?}{\bar{r}_b}, \overset{+}{b}). \qquad \text{(I.21)}$$

The impact of real wages is left ambiguous because of two offsetting effects. Directly an increase in real wages increases the cost of transfer from money to commodities, resulting in a decrease in the frequency of transfer, and average money balances. On the other hand, increases in $\frac{W}{P}$ increase total consumption, and hence the transactions demand for money. The relative size of C may result in non-obvious results with respect to this partial derivative and it is accordingly left indeterminant. The sign of β, however, will be assumed determinant. Here an increase in the transfer costs has a direct positive effect on money balances due to the rise in the cost of transactions between money and commodities. This is offset by the household's perceived increase in the real price of commodities due to an increase in transfer costs. On net it would appear appropriate to assume that the portfolio shift would dominate the change in consumption brought about by a rise in transaction costs between two assets. The return on money balances has an unambiguous sign as increasing r_m leads to higher money holding for a given C^d, as well as the second order effect of the increase in C^d itself. The return on r_g is not unambiguous. However, using the same rationale as in the sign of β it will be taken to be negative. The impact of T, the frequency of payment, will be quite strongly positive in the model as, for a given consumption flow, more is initially placed into the medium of exchange. The signs of π, τ, b, however, while determinant in the system are solely the result of wealth on consumption. Finally the rise in r_b, the bond market interest rate, is ambiguous. Here wealth balances fall as r_b increases suggesting a decrease in C^d. At the same time, however, an increase in r_b causes the perceived real cost of transfers between money and goods to fall, resulting in higher money balances. For the present purposes the former will be assumed to dominate, with no explicit justification given.

From C^d and ℓ^s one can also analyse the behavior of the bond market demand function of the model. From the definition of wealth and the household budget constraint one may write this in general form as,

$$\text{(I.22)} \quad b^d = b(\overset{+}{\frac{W}{P}}, \overset{-}{\beta}, \overset{+}{r_m}, \overset{+}{r_g}, \overset{-}{T}, \overset{+}{\pi}, \overset{-}{\tau}, \overset{+}{r_b}, \overset{+}{b}),$$

where the partial derivative signs follow directly from the assumption of normality and separability in our utility function.

This, therefore, defines the household's demand functions in three markets, and its labor supply. Schematically the procedure outlined may be characterized as in Figure 1. The household simultaneously determines ℓ^s, C^d, and n, which accordingly determines Y_h. At the same time by the household's budget constraint b^d is defined. Finally the breakdown between m^d and G follows directly from the household's choice of C^d and n.

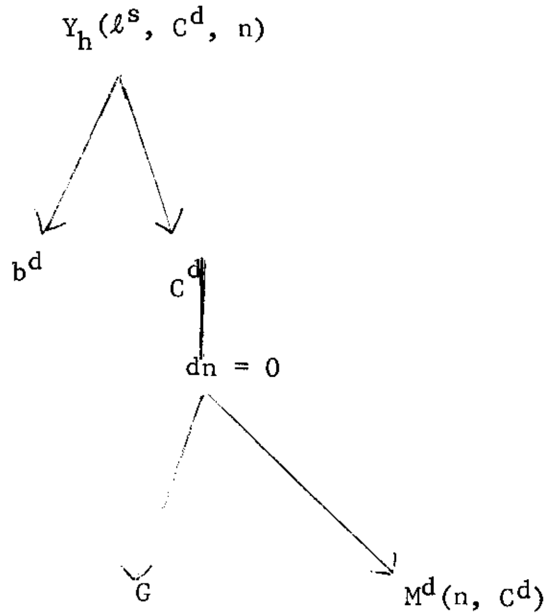

Figure 1

II. The Firm Sector

The firm sector is the production side of the economy, viewed as producing a homogeneous real commodity, y^s, using both capital, assumed fixed and labor inputs, ℓ_o. The objective of management is to maximize the value of the firm, which is consistent with the maximization of profit, given that households value firms as equal to the present value of their discounted profit flows. It has two control variables to accomplish this objective, viz. (a) the total quantity of labor employed and (b) the frequency of payment to labor for services rendered.

In production for a fixed real capital the aggregate production function may be written as

(II.1) $$y^s = f(\ell_o)$$

with ℓ_o used to produce output, and

$$f'(\ell_o) > 0,$$

$$f''(\ell_o) < 0,$$

all evaluated for a given T.[18] For all labor services employed, the homogeneous labor is paid a single real wage $\frac{W}{P}$, determined by equilibrium in the labor market. These wages are paid periodically at a frequency determined by the firm after consideration of the implicit cost associated with deferment in terms of higher wage rates as discussed above. In any case, wages are not paid continuously due to non-zero transaction costs associated with payment. Here the cost associated with payment to labor will be modelled as a time cost necessitating the employment of transactions workers, ℓ_Γ^f equal to the man hours necessary to process payments. Transaction costs accrue for both output workers, ℓ_o, and transaction workers themselves, ℓ_Γ^f. Assuming that the payment technology can be viewed as a linear function of total employment, one may write this transaction cost function as

(II.2) $$\ell_T^f = \gamma_o + \gamma_1(\ell_o + \ell_T^f),$$

or

(II.2') $$\ell_T^f = \frac{\gamma_o + \gamma_1\ell_o}{(1-\gamma_1)} \quad ,$$

where γ_o = fixed cost per payment period,

γ_1 = the marginal cost of an additional worker ($\gamma_1 \ll 1$),

all denominated in man hours per transaction period for wage payments.

Total labor employed by the production sector through the labor market, therefore may be written as the sum of output and transaction labor,

(II.3) $$\ell_p = \ell_o + \ell_T^f.$$

By substitution from (II.2) we obtain,

(II.4) $$\ell_p = \ell_o \left[1 + \frac{\gamma_1}{1-\gamma_1}\right] + \frac{\gamma_o}{1-\gamma_1},$$

where the second term in the brackets of (II.4) represents the additional (or marginal) labor employment due to the (non-zero) cost of paying a production worker staff, ℓ_o, and the last term indicates the minimum fixed cost of the funds transfer.[19] This may also be written in rate of flow terms, for any value of T, as

(II.4') $$\frac{\ell_p}{T} = \frac{\ell_o}{T} \left[1 + \frac{\gamma_1}{1-\gamma_1}\right] + \frac{\gamma_o}{(1-\gamma_1)T} \quad .$$

The firm is assumed to act as a perfect competitor in the goods and labor market. Initially funds flow into the firm as a result of sales of production using labor services. No inventories are held.

At some future endogenous date payments are made to labor. In the interim, however, the firm keeps these funds in money balances, receiving the short term rate of interest, r_m, on these balances.[20] At the same time the households, however, are undergoing positive costs due to this delay in the form of foregone consumption or foregone interest, at a rate of r_b. The explicit wage rate, therefore, must consider the delay involved in payment given a perfectly competitive factor market. Essentially the firm must increase the wage rate paid employees as a result of the lag between services and payment so that it maintains the competitive real wage as viewed by the labor suppliers. The household would be indifferent between sets of $\frac{W}{P}$ and T if and only if its utility levels as seen in the maximization contained in section I above were unaffected. This, however, could only be the case if the firm were compensated at the rate r_b for its deferred payment. This rate reflects the cost to the household of deferred payment, as seen in equation (I.8), due to the loss of interest return on net savings. This rate would completely exhaust the cost of deferred payment, as all intraperiod cash and inventory management were shown to be independent of T (see equation I.17). The problem of setting the payment period, therefore, involves balancing the rate at which the nominal wage rate varies with T, i.e., r_b, with both the return earned from interim balances held by the firm, i.e., r_m, and the fixed cost of wage payment, i.e., ℓ^f_Γ. An optimum would be achieved where the differential in return just balanced the costs of making payments to labor.

Summing the options of the firm, one may view the producing sector as determining labor for any T so as to maximize profit from output production and setting the payment frequency, T, so as to maximize profit from cash management. The former involves a marginal product, marginal wage tradeoff while the latter involves fixed costs versus rates of return. Formally, the problem of the firm becomes the maximization of its rate of flow of profits by selecting both total employment and the frequency of payment simultaneously. This is equivalent to the maximization of the profit function in rate of flow terms, which may be written as,

$$\text{(II.5)} \qquad \pi^{f}\left(\frac{\ell_p}{T}, T\right) = \frac{y_s}{T} - \left(\frac{W}{P}\right)_T \frac{\ell_p}{T} e^{-r_m T},$$

where the real wage varies with T according to the functional form,

$$\text{(II.6)} \qquad \left(\frac{W}{P}\right)_T = \left(\frac{W}{P}\Big|_{T=0}\right) e^{r_b T},$$

and production in flow terms follows the form,

$$\text{(II.7)} \qquad \frac{y_s}{T} \equiv z^s = g\left(\frac{\ell_o}{T}\right),$$

with $g' > 0$,

$g'' < 0$.

Substituting from equations (II.4), (II.6) and (II.7) into (II.5) and approximating the continuous flows, this may be written as, [21]

$$\text{(II.8)} \qquad \pi\left(\frac{\ell_p}{T}, T\right) = g\left(\frac{\frac{\ell_p}{T} - \frac{\gamma_o}{(1-\gamma_1)T}}{\frac{1}{1-\gamma_1}}\right) - \left(\frac{W}{P}\Big|_{T=0}\right)\frac{\ell_p}{T}\left[1 + \frac{1}{2}(r_b - r_m)T\right].$$

Assuming that this can be treated as a continuous variate and a unique maximization obtains, the first order conditions become,

(II.9A) $$g'(1-\gamma_1) - \left(\frac{W}{P}\Big|_{T=0}\right)\left[1 + \frac{1}{2}(r_b-r_m)T\right] = 0$$

(II.9B) $$T = \left[\frac{g'\gamma_0}{\frac{1}{2}\left(\frac{W}{P}\Big|_{T=0}\right)\frac{\ell p}{T}(r_b-r_m)}\right]^{\frac{1}{2}}$$

The first of these conditions indicate the optimal quantity of labor employment as seen by the firm. Here it is demonstrated, as in Barro and Santomero [1973], that the usual point of maximization is somewhat altered by the existence of a non-zero payments cost. Employment of one unit of labor results in only $(1-\gamma_1)$ units of output producing labor. Its product must equal not only the wage rate at T = 0 but also the difference in interim returns over the period T. Accordingly $g' > (\frac{W}{P}\Big|_{T=0})$ at the point of maximization, due to the fact that $(r_b-r_m) > 0$.

Equation (II.9B) on the other hand indicates the length of the discounting period. Here it is demonstrated that the payments period will vary directly with the marginal product lost from production $(g'\gamma_0)$, and inversely with the volume of payments and the net cost of delaying, $(\frac{W}{P}\Big|_{T=0})\frac{\ell p}{T}$, and (r_b-r_m), respectively.

The last of these proves crucial, for if $(r_b-r_m) \leq 0$ the payment to wage earners could be delayed indefinitely; from (II.9B) T would equal infinity. This is as should be expected. If the firm could more than compensate the household for delay in payment it would be optimal to indefinitely suspend payments, thereby profitably investing the funds withheld. However, this problem does not arise in this formulation as the condition necessary for the existence of a bond market, i.e., $r_b > r_m$, preclude such a situation.

The first order conditions contained in equations (II.9) may also be looked upon as a system of simultaneous equations determining the firms demand for labor services over any period. In this connection one may examine the impact of variations in the exogenous variables of the system on the firms output supply and labor demand for both output production and transaction services. Assuming that variations in transactions frequencies does not offset direct effects of such exogenous shifts the flow demand function for labor for output production becomes

(II.10) $$\frac{\ell_o}{T} = \frac{\ell_o}{T}\ (\overset{-}{\gamma_o},\ \overset{-}{\gamma_1},\ \overset{-}{\frac{W}{P}},\ \overset{-}{r_b},\ \overset{+}{r_m}).$$

The assumption necessary to obtain equation (II.10) is weaker than the evaluation around a transactions frequency, i.e. $dT = 0$, used above, but is qualitatively equivalent. Essentially, in the model, alterations in cash management costs, either rates of return or transaction time costs, will effect both the underlying labor cost associated with any output level (labor quantity) and effect the optimum frequency of transactions. What is necessary to obtain (II.10) is that the latter not completely offset the former. For example consider the effect of an increase in transaction costs, either γ_o or γ_1. Such an increase will raise the cost of the existing labor quantity as payments to labor enter the total cost of labor services as perceived by the firm. This increased cost will lead the firm to reduce the quantity of labor demanded. At the same time when an increase in transaction costs occurs the firm also reduces the payment frequency itself, in an effort to maintain optimal profit levels resulting from cash management. This alteration in the payment frequency will somewhat offset the cost increase associated with a shift in γ_o or γ_1. It is assumed, however, that such a transaction shift does not totally offset such an increase so that the real cost of labor services rises to the firm, causing it to reduce its desired quantity as evidenced by the negative partial derivatives in equation (II.10). A similar argument could be presented for the rates of return, r_b and r_m. For the first of these, increases in the cost of delayed payment will increase labor cost for any T and alter T itself. Following the analysis above this will lead to a decrease in desired labor for the firm. Concerning r_m, higher returns on interim balances lead to lower perceived cost, at any $\frac{W}{P}$, and less frequent payments to labor. The former dominates the latter in the partial derivative given in equation (II.10).

Using the results obtained for the impact of the exogenous variables upon T one may also obtain a labor demand function for transaction labor. This follows from the form of ℓ_T^f given by equation (II.2'),

$$\text{(II.12)} \qquad \frac{\ell_T^f}{T} = \frac{\gamma_o + \gamma_1 \ell_o}{(1-\gamma_1)T} ,$$

written here in flow terms.[22] From equation (II.9B), however, T may be written as

$$\text{(II.13)} \qquad T\Big|_{d\ell_o=0} = T\Big|_{d\ell_o=0} (\overset{+}{\gamma_o}, \overset{\approx 0}{\gamma_1}, \overset{-}{r_b}, \overset{+}{r_m}, \overset{\approx 0}{\tfrac{W}{P}}).^{23}$$

By substitution, equation (II.12) can be written as,

$$\text{(II.14)} \qquad \frac{\ell_\Gamma^f}{T}\Big|_{d\ell_o=0} = \frac{\ell_\Gamma^f}{T}\Big|_{d\ell_o=0} (\overset{+}{\gamma_o}, \overset{+}{\gamma_1}, \overset{-}{r_b}, \overset{+}{r_m}, \overset{\approx 0}{\tfrac{W}{P}}).$$

It should be noted that the signs indicating partial derivative of this equation differ significantly from those contained in equation (II.10). This results from the fact that the firm is faced, here, directly with the increased costs, and will, in general, not totally offset variations in payment costs by variations in payment frequency. Therefore increases in γ_o while increasing the cost of labor to the firm and resulting in less frequent transfers to labor, still increases the firms demand for transaction labor. Also increases in γ_1, while leaving the payment frequency uneffected so alters the labor input cost of payment as to require additional labor services for transactions purposes.

Relaxation of the constraint on equation (II.14) relative to the evaluation of a transaction labor function around fixed labor employment would result in

$$\text{(II.15)} \qquad \frac{\ell_\Gamma^f}{T} = \frac{\ell_\Gamma^f}{T} (\overset{+}{\gamma_o}, \overset{+}{\gamma_1}, \overset{-}{r_b}, \overset{+}{r_m}, \overset{-}{\tfrac{W}{P}}).$$

Contained in the partial derivatives of equation (II.15) is the assumption that variations in real flows, contained in equation (II.10) do not alter total output sufficiently to offset the direct substitution associated with payment frequency changes and their impact upon ℓ_{Γ}^{f}.[24] Therefore the first four partials follow the assumed dominant effect. The one exception to this dominance assumption is the impact of $\frac{W}{P}$ on $\frac{\ell_{\Gamma}^{f}}{T}$. An increase in $\frac{W}{P}$ will increase the cost of production workers. This would be expected to reduce total labor employment, decreasing total transaction labor required.

Summing the two functional relationships for labor demand and assuming $\ell_{\Gamma}^{f} << \ell_{o}$ for any T a total demand for labor function is obtained, in flow terms,

$$\text{(II.16)} \qquad \frac{\ell_p^d}{T} = \frac{\ell_o}{T} + \frac{\ell_{\Gamma}^{f}}{T} = \frac{\ell_p^d}{T}(\overset{?}{\gamma_o}, \overset{?}{\gamma_1}, \overset{-}{\tfrac{W}{P}}, \overset{?}{r_b}, \overset{+?}{r_m}).$$

On net the firm is viewed as perceiving γ_o, γ_1 and r_b as net increases in the real wage, $\frac{W}{P}$, and therefore it reduces total demand. The return on money balances, however, reduces total cost and increases demand at a given $\frac{W}{P}$.[25]

Further, given the single factor short run production function in equation (II.1) and the functional form for ℓ_o in equation (II.10), total output supply can be written as, in flow terms,

$$\text{(II.17)} \qquad \frac{Y^s}{T} = z^s = z^s(\overset{-}{\gamma_o}, \overset{-}{\gamma_1}, \overset{-}{\tfrac{W}{P}}, \overset{-}{r_b}, \overset{+}{r_m}).$$

Given the firm's total demand for labor, determined by production labor demand and transaction labor demand, and the firm's real supply of output, the behavior of the firm's money balances can be determined. As indicated above, the firm is viewed as selling output at a continual rate through time, given by z^s above. For a period of time, endogenously determined, the total revenue, net of continuously distributed profits, accumulates in the form of money balances. After a period of length T these funds are distributed to the labor sector. Assuming a continuous flow of sales and production,[26] then, mean money balances will average one-half the labor distribution over period T,

$$\overline{M} = \frac{y^s}{2} = \frac{z^s T}{2} \tag{II.18}$$

This may be written in functional form as,

$$\overline{M} = M^f \left(\overset{+}{\gamma_o}, \overset{-}{\gamma_1}, \overset{-}{r_b}, \overset{+}{r_m}, \overset{-}{\frac{W}{P}} \right) \tag{II.19}$$

The signs of the partial derivatives of equation (II.19) follows directly from equations (II.13), and (II.17). An increase in the fixed cost of payment results in less frequent distributions to the labor sector and higher money balances at the firm. This is somewhat mitigated by the "income effect" as discussed above where higher transaction costs reduce the real demand for labor for production. The next variable considered, γ_1, has a non-obvious result. Here an increase in γ_1, the per unit labor cost of making payment, reduces money balances, whereas the fixed cost of transfer,

γ_o, increases them. This results from the fact that γ_1, does not directly enter the determination of T, but rather enters indirectly through the marginal product of labor at T^*. Accordingly, an increase in γ_1, reduces the optimum labor demand at any $\frac{W}{P}$, resulting in a lower level of total disbursements and a reduction in average money balances. With regard to rates of return, movements in the interest differential, (r_b-r_m), effect payment frequency and firm output, through labor demand. Finally movements in $\frac{W}{P}$ effect M^f primarily through the volume of gross transactions so that increases in the real wage decreases average money balances.

This, therefore, defines the firm sector's behavior. It offers output, y^s in the commodity market, demands labor in the labor market for output, ℓ_o and transactions, ℓ_T^f, and holds money balances on average, M^f. If these functional forms are representative of the firm sectors behavior then the above analysis for the individual firm may be viewed as representative of the behavior of all firms.

III. The Government Sector

The government will be treated in the model following the lines of Christ [1967, 1973].[27] It will be assumed throughout that the government operations are subject to the same type of budget constraints facing the firm and household. Specifically the government expenditure plus interest payments must equal its tax revenue and new debt issue. For simplicity it will be assumed that the government has complete control over banking facilities and accordingly issues money balances, and pays interest on these balances at the rate r_m. Further, government debt, b, is issued with variable coupons and their wealth effects is assumed fully discounted as in Patinkin [1965] and Grossman [1967]. The above conditions are fully satisfied by a budget constraint of the form, in flow terms,

$$\text{(III.1)} \qquad g^d + br_b + M^s r_m = \tau + \Delta m + \Delta b$$

for any discrete period, T.

The behavioral assumptions covering this sector will be quite minimal. It will be assumed that the government sets government expenditure and income exogenously and changes in money stock and bonds outstanding are, as well, exogenous to the system. It should be noted, however, that monetary and fiscal policy flows directly from the budget constraint, and must at all times satisfy it. This point has been made well by Christ [1967] and Branson [1974].

IV. The Properties of the General Equilibrium Model

Having derived the demand and supply functions of the representative agents of each sector under the regime of positive transaction costs one may combine these functional forms, through aggregation over economic agents of each sector, to obtain market equilibrium conditions. The result will be a complete general equilibrium model of the economy based upon assumed household and firm behavior quite different than neo-classical theory alleges. The reduced form equilibrium loci, however, will look quite similar, as will be discussed below.

To derive aggregate demand and supply functions it is necessary to sum over all individual units of the household and firm sectors to obtain aggregate functional forms. These are, then, added to government demand and the exogenous supply quantities to obtain market equilibrium conditions. This aggregation procedure is subject to one additional constraint , however. In the aggregate all existing stocks must be held, so that wealth summations must equal existing stocks. This proves to be of considerable importance. In addition, care must be taken to assure consistent units. Accordingly, the labor and commodity markets are expressed in flow units, while the market for financial assets remains a stock equilibrium.[28]

Viewing the financial market as stock-equilibrium determined needs further elaboration. The bond market lends no difficulties as bonds are held only for inter-temporal transfers and the stock of wealth to be held in bonds is an explicit control variable of the model. Money balances, however, are analytically more difficult. To obtain the individuals' and firms' money demand function average money holding of the representative unit were obtained. Each individual units' actual money balances, however move in a predetermined "saw-tooth" fashion from an endogenously determined positive value associated with consumption to zero. Average money holdings, then, are not equivalent to the stock demand for bonds above. Rather it is necessary to assume some distribution of households, paid at different points in time, but not necessarily at different intervals. Each micro units' cash balances vary through its payment period but in the aggregate the demand for money function is implied by the representative units' average demand function.

The resultant equilibrium conditions from the model, then, become,

A) The Labor Market,

$$\text{(IV.1)} \quad \sum_{j=1}^{n} \frac{\ell^{d}_{pj}}{T}(\cdot) = \sum_{i=1}^{m} \frac{\ell^{s}_{pi}}{T}(\cdot),$$

B) The Commodity Market,

$$\text{(IV.2)} \quad \sum_{j=1}^{m} \frac{C^{d}_{i}}{T}(\cdot) + g^{d} = \sum_{j=1}^{n} Z^{s}_{j}(\cdot),$$

C) The Money Market,

$$\text{(IV.3)} \quad \sum_{i=1}^{m} M^{h}(\cdot) + \sum_{j=1}^{n} M^{f}(\cdot) = \frac{M}{P},$$

D) The Bond Market,

$$\text{(IV.4)} \quad \sum_{i=1}^{m} b^{d}(\cdot) = b^{s},$$

where firms are numbered from j = 1...n and households are numbered from i = 1...m.

The summation occurs over units that are individually either at the beginning of their payment period or presently engaging in the transactions previously derived from their optimization process. For the household sector the individuals in the first group hold only maturing bonds and their present period receipt of income from labor services, while those in the second group hold a stock of both bonds, money balances and commodities. For the firm sector money balances are held by all producing sector units that have payment commitments at some finite time in the future to labor suppliers. Accordingly, while no one individual unit begins its period of optimizing behavior with money balances at <u>any one point in time</u> the existing stock of money balances is being held. This implies that aggregation across economic units at any point in time must result in the sum of total wealth, defined as $\frac{M}{P} + b$, being held by the society as a whole.[29] Accordingly one may rewrite the market clearing equations as:

A) The Labor Market

$$\text{(IV. 5)} \quad \frac{\ell^d}{T^p}\left(\overset{-}{\gamma_o}, \overset{-}{\gamma_1}, \overset{-}{\frac{W}{P}}, \overset{-}{r_b}, \overset{+}{r_m}\right) = \frac{\ell^s}{T^p}\left(\overset{+}{\frac{W}{P}}, \overset{-}{\beta}, \overset{+}{r_m}, \overset{+}{r_g}, \overset{-}{\pi}, \overset{+}{\tau}, \overset{+}{r_b}, \overset{-}{b}, \overset{-}{\frac{M}{P}}\right).$$

B) The Commodity Market

$$\text{(IV. 6)} \quad \frac{C^d}{T}\left(\overset{+}{\frac{W}{P}}, \overset{-}{\beta}, \overset{+}{r_m}, \overset{+}{r_g}, \overset{+}{\pi}, \overset{-}{\tau}, \overset{-}{r_b}, \overset{+}{b}, \overset{+}{\frac{M}{P}}\right) + g^d = z^s\left(\overset{-}{\gamma_o}, \overset{-}{\gamma_1}, \overset{-}{r_b}, \overset{+}{r_m}, \overset{-}{\frac{W}{P}}\right).$$

C) The Money Market

$$\text{(IV. 7)} \quad M^d\left(\overset{?}{\frac{W}{P}}, \overset{-}{\beta}, \overset{+}{r_m}, \overset{-}{r_g}, \overset{+}{\pi}, \overset{-}{\tau}, \overset{-}{r_b}, \overset{+}{b}, \overset{+}{\frac{M}{P}}\right) + M^f\left(\overset{+}{\gamma_o}, \overset{-}{\gamma_1}, \overset{-}{r_b}, \overset{+}{r_m}, \overset{-}{\frac{W}{P}}\right) = \frac{M}{P}$$

D. The Bond Market

$$\text{(IV. 8)} \quad b^d\left(\overset{+}{\frac{W}{P}}, \overset{-}{\beta}, \overset{+}{r_m}, \overset{+}{r_g}, \overset{+}{\pi}, \overset{-}{\tau}, \overset{+}{r_b}, \overset{+}{b}, \overset{+}{\frac{M}{P}}\right) = b^s$$

The revised model has four equilibrium conditions, one for each market, however, by Walras' Law only three are independent. Selecting equation (IV. 7) to be neglected the system of equilibrium conditions can be viewed as determining the nominal wage rate in the labor market, the commodity price level in the goods market, and the market clearing interest rate, r_b, in the bond market.

Following neo-classical tradition, the equilibrium locus for each market can be derived in interest rate, r_b, price level, P, space by totally differentiating the equilibrium condition and solving for $\frac{dr_b}{dP}$.[30] This results in

(IV.9) $$\frac{dr_b}{dp} = - \frac{\frac{\partial \frac{\ell_P^s}{T}}{\partial \frac{M}{P}} \frac{M}{P^2}}{\frac{\partial \frac{\ell_P^d}{T}}{\partial r_b} - \frac{\partial \frac{\ell_P^s}{T}}{\partial r_b}} < 0,$$

(IV.10) $$\frac{dr_b}{dp} = \frac{\frac{\partial \frac{C^d}{T}}{\partial \frac{M}{P}} \frac{M}{P^2}}{\frac{\partial \frac{C^d}{T}}{\partial r_b} - \frac{\partial z^s}{\partial r_b}} < 0,$$ [31]

(IV.11) $$\frac{dr_b}{dP} = \frac{\frac{\partial b^d}{\partial \frac{M}{P}} \frac{M}{P^2}}{\frac{\partial b^d}{\partial r_b}} > 0,$$

for the labor market, commodity market and bond market respectively, all evaluated at $d\gamma_o = d\gamma_1 = dr_m = dr_g = d\beta = d\pi = d\tau = db = d\frac{W}{P} = 0$. The last of these is of particular note given the sign of $\frac{dr_b}{dP}$ for the labor market equilibrium. The traditional neo-classical models of the labor market, usually derived in a rather <u>ad</u> <u>hoc</u> fashion, have $\frac{dr_b}{dP} = 0$. This does not obtain here as labor supply is derived directly from the utility maximization of the household and therefore the model allows for variations in its quantity due to variations in any of the exogenous and endogenous variables in the system. In the present context it is of particular note that as real money balances vary due to price level change the labor supply shifts as well. The relevance of the impact of variations in wealth upon the labor market has recently been examined in the neo-classical tradition by Santomero [1974B] and Phelps [1972] with similar results as those presented here.

The equilibrium loci of the system, together with its point of general equilibrium in all markets is presented graphically in Figure 2.

It should be immediately noticed that the equilibrium loci derived from the model, under the assumption of no change in transaction costs or other rates of return is identical in sign to the neo-classical version. This suggests that,while the economic process being described by the loci differs significantly, comparative static analysis centering on exogenous expenditure or liquidity preference shifts results in shifts in r_b and P of the same sign from both models.

Consider, for example an increase in government expenditure financed completely by monetary expansion. Both models result in an increase in the price of commodities and a rise in the bond interest rate. However, the method of arriving at this conclusion differs. In the neo-classical model[32] an increased demand for commodities puts pressure upon prices due to excess demand. At the same time, the increased money supply initially goes into the household's portfolio as it is paid the higher money wage. The household, in turn, perceives the marginal utility of the excess money balances falling and spreads this new addition to wealth into both the commodity and bond market. The former puts continued pressure on prices while the latter causes bond interest rates to fall as their prices are bid up. Eventually prices rise so as to restore equilibrium at a higher bond rate and higher price level.

The transactions approach, on the other hand presents a different picture. As above, the increased demand for commodities puts pressure upon prices. However, this is where the parallel with the neo-classical model ends. As the increased demand and money supply enter the economy and are paid to households, the latter is viewed as revising its real consumption decision upward by a fraction of the higher income. Money balances just sufficient to facilitate the

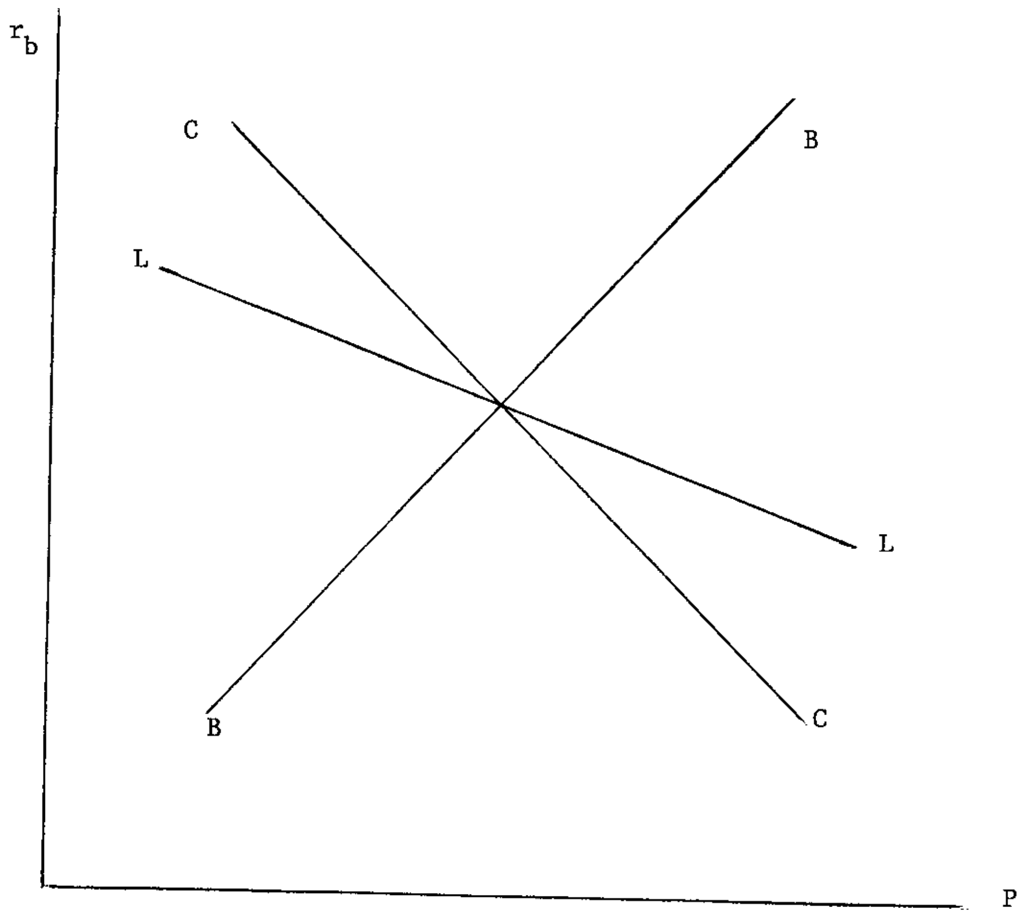

Figure 2

expected additional consumption levels are added to transaction balances. The remaining increase in the money supply goes directly to the bond market for intertemporal transfer, resulting in interest rate adjustments downward. The increases in demand, due to government expenditure, derivative consumer expenditure, and interest induced expenditure, cause price movements and reduced real demands to the new equilibrium condition.[33]

The major difference in the two approaches outlined is one of causality and the ordering of effects. In the traditional model desired money balances rise initially as well as desired bond holdings. In the transactions model desired money balances can only be altered by a change in desired consumption. Accordingly the household is viewed as initially wishing to rid itself of these excess balances by portfolio investment. Therefore, interest rate adjustments are viewed as occurring much earlier and more swiftly in the transaction costs than in the traditional model where some of the increased money balances are held for their utility return.

V. Variations in Transactions Costs and Rates of Return

In addition to the traditional disturbances analyzed elsewhere using the neo-classical version of the general equilibrium model the present approach may be used also to examine the macroeconomic effects of other alterations in the economic environment. Specifically, where our traditional model was unable to explicitly consider the impact of transaction costs variation on the aggregate price level and interest rates the present model is particularly suited to that end. As such it may be viewed as a generalization of the partial equilibrium analysis of the micro effects of transfer cost variation analyzed elsewhere by Saving [1972], Karni [1974] and Barro and Santomero [1974]. Here the analysis centers on the equilibrium impact of transaction cost changes, or rates of return on aggregate price level and bond interest rate.

Consider first an exogenous increase in γ_0 or γ_1, the cost of payments to labor absorbed by the firm in terms of additional labor hours. This will effect the equilibrium conditions in all markets, as the firm now views the effective wage paid to workers, including both the explicit wage and costs of payment, as having risen. The firm will, therefore, reduce the quantity of labor demanded and cut back production in the real goods market. Neglecting the nominal wage market and centering our interest upon the equilibrium r_b and P, this will result in an excess demand for commodities and, therefore, upward price pressure. Equilibrium will be restored when aggregate demand falls to equilibriate with the reduced supply. The comparative static result is a reduction in real wealth and a higher interest rate equilibrium. Formally this result may be seen by taking the total differential of the system evaluated at the equilibrium wage rate in the labor market.

Table 1

Effects of Exogenous Disturbances

	P	r_b	z^s
β	–	?	–
r_m	?	–	+
r_g	+	?	+
γ_o	+	+	–
γ_1	+	+	–

The table reports the results of the exogenous disturbances, listed vertically, upon the endogenously determined variables of the system. In all cases the sign is evaluated at labor market equilibrium value of $\frac{W}{P}$.

In a similar fashion the impact of variations in the time cost of transfers between money balances and commodities can be considered. Here increases in β reduce labor income, and consumption demand. The last of these results in downward price pressure which forces real wealth up and equilibrium in the commodity market at a lower price level. In the bond market, however, things are more ambiguous. Decreases in the flow of income reduces bond demand. However, as prices fall due to excess supply in the commodity market, wealth rises forcing bond demand higher. The resultant effect upon r_b is dependent upon the relative elasticities of the system.

In a similar fashion other economic disturbances may be examined. The results follow directly from the above analysis and are included in Table 1. In all cases the equilibrium conditions are evaluated for small changes in the exogenous disturbance neglecting second order effects on the labor market. They indicate that shifts in transaction cost time or rates of return on assets other than bonds will have an effect upon the equilibrium in the economy. Unfortunately the net impact of many of the disturbances is ambiguous and depend upon cross elasticities in the system. However this should not be surprising. The general equilibrium approach offered here includes interactions neglected in other partial equilibrium models, many of which serve to offset the initial effect of variations in the exogenous variables. Accordingly explicit treatment or estimation of the interrelationships is required.

VI. Summary and Conclusions

The present paper has developed a general equilibrium model of the economy from microeconomic profit and utility maximization functions in a world with non-zero transaction costs. It resulted in an alternative micro foundation for the generally accepted reduced form market equilibrium conditions that appears just as acceptable as the neo-classical micro assumptions. Further it subsumes within its general formulation a special case, where a large number of exogenous variables are assumed constant, a reduced form that is indistinguishable from the neo-classical version. However, its underlying behavior differs substantially. The result should be somewhat alarming, for it suggests that our traditional model, while itself a heterogeneous collection of different variants, may in fact be a reduced from of a substantially different underlying "true model." If this be the case the previously considered stochastic disturbances studied elsewhere[34] may in fact be variations in variables completely neglected in the currently accepted neo-classical tradition (for example perceived returns on alternative assets, or transaction cost variability). It was demonstrated in Section V that variations in these previously ignored variables alter output, prices, and the bond rate of return. At that time, a new approach to the impact of such variations was presented where the total <u>macroeconomic</u> impact of alterations in transaction costs and rates of return was considered in a general equilibrium context. It was demonstrated that while many of these shifts have impacts that are difficult to evaluate at this point their effect is clearly not zero.

Presumably two things remain to be done. First one may wish to construct viable empirical tests of the two approaches that would discern differences in the two models to investigate the relative merits of each. In addition, one may wish to expand the present approach to explicitly consider the costs, as well as benefits, of variations in the transactions costs or rates of return. The analysis of section V centered upon the impact of alterations in these costs and returns. It remains to be determined if such a variation is socially optimal in a welfare sense.

Footnotes

*Assistant Professor of Finance, University of Pennsylvania, Financial support from the Rodney L. White Center for Financial Research is gratefully acknowledged.

[1]1. In Feige and Parkin [1971] cash management returns were incorporated into the income constraint. Yet no attempt was made to obtain a supply-demand equilibrium as is suggested here. Accordingly while theirs was a first step in this direction the question of the behavior of the market equilibrium and general equilibrium with transaction costs must still be examined.

[2]The exact specification given by Saving is

$$T_t = \gamma\ (P_t X_t,\ \sum_i W_{it},\ V_{it},\ \overline{X}_t,\ \overline{V}_t,\ \overline{M}_{it},\ M_{dt})$$

where X_t output at time t,

P_t = price,

M_{ct}, M_{dt} = money holdings in the form of currency. and deposits respectively,

V_t = inventory holdings,

W_{it} = wealth of individual i at time t,

bars refer to initial values. The specification is simplified to equation (8) above for expositional simplicity.

[3]In the model, $\frac{\partial T}{\partial M} < 0$.

[4]The notation adopted throughout the remainder of the paper, since subscripts have been dropped and the analysis converted into a one period procedure, is that numbered subscripts refer to partial derivatives of the function with respect to the numbered argument.

[5]The dimensions of the variables should be considered carefully. Here ℓ is in a pure number over the arbitrarily defined period of unit length. Accordingly it takes on values of (e.g.) 5 units over the unit time length. The entire analysis may be converted to flows over an arbitrary time interval with no loss in generality.

[6]The dimension of T is in units of time within the arbitrary time interval above, e.g. 1/10 of year or 10 days.

[7] From this opportunity cost one must subtract the positive return (if any) obtained from balances in working capital during the period in order to obtain net costs. It is in this sense that $\frac{W}{P}\ell p r_b T$ may be defined as a gross cost. However, it is gross cost that is relevant to the household in its determination of the perceived real wage as these funds will always be outstanding from wealth balances.

[8] π is included in the analysis to close the model. It will be assumed throughout, however, that the household views π as exogenously set and not a function of its microeconomic profit maximizing behavior. This point becomes relevant when the suppliers of labor change offer prices. This action, for any output level, will of necessity reduce profits. It is assumed, however, that no recognition is given to this fact.

[9] The units of Y_h are real dollars per time period. The units of $\frac{\ell_p}{T}$, following the discussion in footnote number 5, is work units per unit of time, a rate of flow.

[10] The coincidence of the decision unit period with T is of no substantive importance here as all decisions made within T are independent of the number of such units with the household's decision time span. Only the $(\frac{C}{T})$ quantity is of relevance to behavior within T.

[11] Transfers between bonds and money balances for intra-period interest are assumed to be too costly given the transfer cost relative to potential interest return differentials.

[12] The model presented below will assume $\pi^e=0$ throughout the analyses. However, r_g may be defined as including $\pi^e \gtrless 0$ for generality.

[13] The non-negotiable nature of equity shares is a simplifying assumption to free the present analysis of transaction cost foundations from the consideration of relative rates of returns of bonds and equities. For analysis of the latter see Tobin (1969) and Branson (1973). One may skirt this issue completely by assuming that all investment made by the firm is financed by bond issue so that only economic profit accrues to equity holders and on the margin the rates of return are equalized. This assumption is consistent with the remaining analysis.

[14] This statement, of course, assumes that $r_b > r_m$, a condition for the preference of bonds over money for inter-period wealth holdings.

[15] Throughout the payments period, while money balances held by the household are diminishing the firm's balances are rising due to the receipt of revenues from commodity sales. Accordingly aggregating over all firms and households results in a steady desired money balance over the period. This heuristic discussion becomes clearer below and is pointed out here to illustrate a steady state demand for money can result from this behavior even though the micro-unit always exhausts these balances within T.

[16] The disturbance considered is a movement of r_g closer to zero as $r_g<0$ in the present analysis.

[17] It should be noticed that $\frac{M}{P}$ does not enter as an argument in the functions reported in (I.18) and (I.19). This is a result of the fact that no money balances are used for interperiod transfers of wealth. This does not, however, imply no wealth effects exist in the present framework vis-a-vis money balance. This point will be addressed in section IV below.

[18] The units of y^s and ℓ_o are real output and man hours, respectively. The production function is defined over some fixed period of time T. In terms of section I above the analysis may be viewed as determining both the quantity of output over any payment period, T, and the appropriate length of the period, T, itself.

[19] ℓ_p is equation (II.3) is equal to ℓ_p of section I as this is the labor market quantity. Some assymmetry exists here as the firm hires transaction workers, ℓ_Γ^h through the labor market while the household transaction labor, ℓ_Γ^h, is not marketed.

[20] Transfers between money balances and bonds for intraperiod returns are assumed to result in higher transaction costs than potential return differentials.

[21] The approximation given by equation (II.8) neglects the cross term $(r_m r_b)$ and the intraperiod discounting of γ_o and γ_1. The analysis and implications are in no way effected by this approximation.

[22] The analysis has shifted to flow units to make the resultant functions more amenable to aggregation over firms with different values of T.

[23] The sign on $\frac{W}{P}$ results from a competitive factor market assumption so that variations in $\frac{W}{P}$ effect both $g'\gamma o$, the cost of payment and $\frac{W}{P}\frac{\ell}{T}P$, the volume of payment. The impact of $\frac{W}{P}$ in the system depends upon the relative impact, which in the limit goes to zero.

[24] This assumption is consistent with the behavior of the household in Section I, where variations in the cost of goods purchased, while altering the present period consumption bundle, did not result in a fall in consumption sufficient to completely offset the transactions pattern alteration and its effect upon ℓ^s. This assumption exists throughout the transaction cost literature and may be viewed as its version of the dominance of substitution over income effects.

[25] The specification of the signs of equation (II.16) can be objected to as completely arbitrary. A priori the dominance of real flows over monetary effects on total labor demand is not required in the present analysis for most results. Where these signs become crucial, Section V below, this fact will be noted.

[26] With Households making commodity purchases at discrete intervals through time it must be assumed that the firm serves a large number of households, each of whom shop at discrete intervals. Given this veiw then the firm receives a continuous flow of demands consisting of discrete units.

[27] Entering the government sector, while behaviorally superfluous, is required to close the system. Both taxes and interest payments have been explicitly treated in the sections above, requiring explicit mention of their implied interrelationship as contained in (III.1) below.

[28] By analyzing the model's financial sector in purely stock terms, as in Tobin [1969], the assumed flow increases in capital stock, net wealth, and money stock demand, if any, are assumed negligible.

[29] Summing over each sector results in money balances being held by both the firm sector and the household sector. However, given that the equity of all firms is held by the household sector, that portion of money balances held by the firm sector can just be added to the owners' wealth position.

[30] The equilibrium loci are defined for a given $\frac{W}{P}$ and π, evaluated at equilibrium.

[31] It will be assumed that the impact of interest rate change upon the demand for output dominates the output variation desired by the firm due to the marginal increase in the cost of labor. While this result is not required, it appears plausible.

[32] To contrast the present transaction cost model from frictionless models where demand functions are derived by more traditional macroeconomic theory I will lable the existing models neo-classical, for lack of a better phrase. It may well be protested that labeling all expenditure, stock, and flow models under one title results in a fairly heterogeneous grouping.

[33] It is assumed that the real wage moves so that the labor market is always in equilibrium. This is done for primarily heristic simplicity in both models.

[34] For example see Cooper and Fischer [1973].

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