A Model for Corporate Debt Maturity Decisions

by

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Working Paper No. 13-74

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I. Introduction

Whenever the firm must borrow funds it must also decide maturity of the new debt. Yet, the decision models which have dealt with the debt maturity decision have done so almost incidentally, as an extension of the decision to exercise the call provision on outstanding bonds ([4], [8], [21]). There has been little direct examination of the corporate debt maturity decision. In an attempt to fill this gap, this paper is an exploration of the debt maturity decision for a firm which is concerned with minimizing the present value of the expected costs of borrowing. This paper develops a discrete dynamic programming model of the debt maturity decision, in a world where interest rates follow a finite Markov process, and where the yield curve is formed from expectations regarding the future course of interest rates. With this optimization model, the influence on the debt maturity strategy of variables such as flotation costs and liquidity premiums will be explored. There will be no consideration of the risks associated with alternative borrowing strategies.

One important issue to be analyzed is whether or not a least expected cost maturity strategy can be determined when the yield curve is determined according to the expectations model of term structure (see [13], [14], [20]). It will be noted that the existence of an optimal policy depends on the discounting scheme used, and thus, on the firm's opportunity cost for funds. This issue is discussed in further detail in Section II, along with the other factors which influence the costs of debt. Section III develops an explicit model of term structure which is then to be used in the dynamic programming model explained in Section IV. The results from using the model are discussed in Section V.

II. Interest Rates, Discount Rates, and Flotation Costs

The expectations model of the term structure of interest rates [3], [9], [12], [13], [14], [20]) contends that the current interest rate on a long term loan must be equal to the geometric mean of current and future expected short-term interest rates. One implication of the unbiased expectations model is that, in a world with no transaction costs, yields on bonds of different maturities will adjust so that an investor cannot expect to earn a higher average yield by investing in a particular maturity or sequence of maturities than would be expected from any alternative sequence of maturities over a given horizon. Similarly, for a borrower with no flotation costs, the expected average interest costs would be the same for all combinations of maturity sequences over a given borrowing horizon. Thus, with no flotation costs, if expected average interest costs are the sole consideration, the borrower would be indifferent between maturities.

In this paper it will be assumed that the borrower's objective is to minimize the present value of interest (and flotation) costs over the borrowing horizon, rather than minimizing expected average interest (and flotation) costs. The unbiased expectations model either assumes that the borrower is concerned only with expected average interest costs, or, if the borrower is concerned with the present value of the interest costs, then it is implicitly assumed that the firm's opportunity cost (discount rate) is equal to the yield to maturity (as of the issue date) on the borrowed funds. When this assumption holds and interest rates are determined according to the unbiased expectations model, then the expected average interest costs are the same for all borrowing sequences, and the present value of interest payments is the same for all borrowing sequences. It must be noted, however, that if the firm's opportunity cost for funds is different from the interest rate on the debt, then, even though the expected average interest cost will be the same for all borrowing sequences, the present value of interest payments may vary between borrowing sequences. Under these latter circumstances a maturity policy may exist which minimized present value of interest costs.

For example, assume the current rate on 1-year loans is $R(1) = .05$, the expected rate on 1-year loans commencing in 1 year is ${}_1r = .09$, and using the pure expectations model, let the current rate on 2-year loans be $R(2) = .06981$, where $R(2) = [(1+R(1))\ (1+{}_1r)]^{\frac{1}{2}} -1$. If the discount rate is assumed to be equal to the yield on the bond being issued, then the present value minimizing borrower will be indifferent between borrowing for two periods, where the present value of interest and principal is

$$PV = 69.81\ (1.06981^{-1}) + 69.81\ (1.06981^{-2}) + 1000\ (1.06981^{-2}) = 1000, \quad (1)$$

or using a sequence of two 1-period loans with present value

$$PV = 50\ (1.05^{-1}) + 90\ (1.05^{-1}\ 1.09^{-1}) + 1000\ (1.05^{-1}\ 1.09^{-1}) = 1000. \quad (2)$$

On the other hand, if the discount rate is assumed to be equal to the rate on the 2-year loan for both borrowing sequences, then a least cost borrowing strategy exists. If the 2-year loan is used, the present value is 1000 as shown in (1), but if the sequence of two 1-year loans is used, then the present value is

$$PV = 50\ (1.06981^{-1}) + 90\ (1.06981^{-2}) + 1000\ (1.06981^{-2}) = 999.13. \quad (3)$$

If the sequence of 1-period rates is reversed (R(1) = .09 and $_1r$ = .05), we have

$$PV = 90\ (1.06981^{-1}) + 50\ (1.06981^{-2}) + 1000\ (1.06981^{-2}) = 1001.558 \quad (4)$$

where we have used the same discount rate as before since the average rate over the two periods is unchanged. Using the rate on 2-period loans (equal to the average of rates on the 1-period loans) as the discount rate over both periods leads the present value minimizing borrower to prefer the 5%-9% sequence over the 9%-5% sequence of 1-period loans. Yet, according to the expectations model, borrowers should be indifferent between these two sequences. This indifference holds only if the borrower's discount rates are 5% and 9%, for periods 1 and 2 respectively, in the first sequence, and 9% and 5%, respectively, in the second sequence.

It is clear that a central issue in considering the bond maturity decision is the appropriate discount rate to use in discounting the interest and flotation costs. Elton and Gruber [4] and Weingartner [21] suggest using the yield on the bond being issued as the discount rate. It has been suggested that the yield on a bond with maturity equal to the asset being financed would be appropriate discount rate. The difficulty with this is that it is often difficult to associate the financing with any particular asset. Perhaps the term "opportunity cost" is the key to this discount rate problem. In this regard, the question is: in a given period, if interest costs are reduced by $1, what return (with the same level of risk) will be earned or imputed on this marginal investment of $1? Certainly not the firm's overall cost of capital, since this rate allows for risks unrelated to the debt maturity decision. We can not necessarily say that this extra $1 will be invested in a bond with maturity equal to that of the currently issued debt, although this may be the case. Obviously, the answer depends on the circumstances and the opportunities available to the firm. In lieu of more specific information, it would seem that a logical candidate for an opportunity cost would be the period-by-period rate on short term riskless securities. That is, assume that marginal funds are invested in short term riskless securities which mature in one period. In that case, discounting would proceed as shown in expression (2), and, in an "unbiased" expectations world, it would be possible to find a present value minimizing maturity policy.

Some authors ([3], [9], [13], [18], [20]) contend that the expectations model must be modified to take account of liquidity premiums which are added onto the rate determined according to pure expectations. The addition of liquidity premiums results in an upward bias in the yield curve so that the "normal" shape of the yield curve is upward sloping. With the addition of liquidity premiums which increase with time to maturity, the borrower may find, prior to considering flotation costs, that a sequence of short term loans would have lower expected average interest cost than a longer term debt maturity policy.

Once again, the firm's opportunity cost for funds must be brought into the analysis. Even though it is true, with the addition of liquidity premiums which increase with maturity, that the expected average interest cost of long term borrowing may be greater than from a sequence of short term loans, if the firm's opportunity cost for funds is equal to the interest rate on the borrowed funds, then the present value of expected interest payments will be the same for all borrowing sequences. But, if, for example, each future period's expected short term rate is used to discount costs for that period as in (2), then there could be a present value minimizing maturity policy. Extending the previous example, with $R(1) = .05$, ${}_1r = .09$, and letting $R(2) = .08$ which includes a liquidity premium, we have for the 2-period loan,

$$PV = 80\ (1.08^{-1}) + 80\ (1.08^{-2}) + 1000\ (1.08^{-2}) = 1000, \tag{5}$$

the same value as expression (2). But, if the appropriate discount rates are (a) the short term rates for each period, where

$$PV = 80\ (1.05^{-1}) + 80\ (1.05^{-1}\ 1.09^{-1}) + 1000\ (1.05^{-1}\ 1.09^{-1}) = 1019.83, \tag{6}$$

or (b) the average of the two rates (equal to the rate on a 2-year loan without a liquidity premium), where

$$PV = 80\ (1.06981^{-1}) + 80\ (1.06981^{-2}) + 1000\ (1.06981^{-2}) = 1018.42, \tag{7}$$

then, in either of these two latter cases, the borrower would prefer the two 1-period loan sequence represented by expression (2) or (3).

Borrowers normally incur flotation costs with each debt issue so that if long term debt is issued, flotation costs are incurred infrequently, and if a sequence of short term loans are used then flotation costs are incurred more frequently. If the discount rate is assumed to be equal to the rate on the debt being issued, then all maturity policies have the same expected present value of interest costs, and the flotation costs will be the only costs which enable us to distinguish between maturity policies. In this case, if flotation costs are the same for all maturities, then the longest maturity would always minimize the present value of interest and flotation costs; and, if flotation costs are an increasing function of the maturity, then the minimum cost policy must be determined by computation. If the discount rate is not equal to the yield on the debt, then interest costs and flotation costs will both influence the present value of costs, and the optimal policy must be determined by computation. The dynamic programming model which will solve these latter problems is presented in Section IV, and the term structure model to be used in the dynamic programming model is presented in Section III.

III. The Markov Model of Term Structure

In order to develop a model for the debt maturity decision in Section IV, a model of the term structure of interest rates is presented, based on the assumption that the short term interest rates follow a finite Markov chain with known, stationary transition probabilities.[1] At each discrete point in time the one period rate, R, can take on a finite number (N) of possible values: R_1, R_2, ..., R_N and next period's rate depends only on the rate this period. Let p_{ij} be the probability that next period's rate will be R_j given that the current rate is R_i; $p_{ij}^{(t)}$ is the probability that the rate t-periods hence will be R_j given that the current rate is R_i. Let ${}_t r_i$ denote the one period rate expected to prevail t-periods hence, given that the current rate is R_i, where

$$ {}_t r_i = \sum_{j=1}^{N} p_{ij}^{(t)} R_j. \qquad (8) $$

Given the expectations mechanism as specified by the Markov transition probability matrix, $P = \{p_{ij}\}$, we can derive a unique yield curve associated with each of the N discrete one period interest rates R_i, $i = 1, \ldots, N$. The expectations model of the term structure of interest rates contends that the current long-term interest rate must be equal to the geometric mean of the current and future expected short-term interest rates. Letting $R_i(m)$ denote the current rate on an m-period loan when the current one period rate is R_i, according to the "unbiased" expectations model we have

$$ \left[1 + R_i(m)\right]^m = (1 + R_i)(1 + {}_1r_i)(1 + {}_2r_i) \ldots (1 + {}_{m-1}r_i), \qquad (9) $$

and thus

$$ R_i(m) = \left[(1 + R_i)(1 + {}_1r_i)(1 + {}_2r_i) \ldots (1 + {}_{m-1}r_i)\right]^{\frac{1}{m}} - 1. \qquad (10) $$

In order to allow for the case where the prevailing yield curve includes liquidity premiums, let $C_i(m)$ denote the borrowing rate for m-period loans when the 1-period riskless rate is R_i, and let $L_i(m)$ denote the the liquidity premium on the m-period loan when the rate is R_i, then we have

$$ C_i(m) = R_i(m) + L_i(m), \qquad (11) $$

where $R_i(m)$ is derived according to pure expectations as in (10).

Estimates of the transition probabilities for short-term interest rates over monthly transition periods were made for use in the dynamic programming model to be developed in Section III.

The sample used to estimate the transition probabilities consisted of monthly observations of yields to maturity on 90 day Treasury Bills over the 266 month period from January, 1952 to February, 1974.[2] Over the sample period the average rate was 3.55%, ranging from 0.56% to 8.34%, with standard deviation of 1.72%. With 265 observations, monthly first differences ($d_t = R_{t+1} - R_t$) ranged from -1.34% to +1.45% with mean of +0.023% and standard deviation of 0.3367%.[3] The transition probability estimates were approximated by the observed frequency distribution over monthly first differences.[4] It was assumed that there are N = 20 possible short-term rates, ranging from R_1 = 1.375% (state 1) to R_{20} = 8.50% (state 20), with the interval between rates equal to 0.375%.[5] Table 1 shows the transition probabilities.

The estimated transition probabilities were used to develop yield curves according to expression (10), where expected future short term rates were derived according to expression (8). For each state i there is a unique yield curve, (which excludes liquidity premiums), some of which are shown in Figure 1.

IV. The Debt Maturity Decision Under Uncertainty: An Infinite Horizon Model[6]

A general model of the bond maturity decision should provide us with a decision rule which would be valid under a wide variety of economic conditions. That is, it should prescribe the correct action whatever the level of interest rates or the shape of the yield curve. The infinite horizon dynamic programming model to be developed will take into account the level and shape of the yield curve at each decision date, along with expected fucure rates, in order

TABLE 1

STATE TRANSITION PROBABILITIES
FOR ONE MONTH TRANSITION PERIODS

		2		4		6		8		10		12		14		16		18		20
1	.814	.170	.006	.005	.005	0	0													
	.104	.710	.170	.006	.005	.005	0	0												
3	.016	.088	.710	.170	.006	.005	.005	0	0											
	.010	.006	.088	.710	.170	.006	.005	.0050	0	0										
5	.005	.005	.006	.088	.710	.170	.006	.0050	.005	0	0									
	0	.004	.010	.015	.095	.680	.175	.0100	.006	.005	0	0								
7	0	0	.003	.014	.020	.105	.650	.1840	.012	.007	.005	0	0							
		0	0	.002	.017	.022	.110	.6300	.192	.016	.007	.004	0	0						
9			0	0	.001	.018	.026	.1170	.599	.208	.019	.008	.004	0	0					
				0	0	.001	.019	.0260	.117	.598	.208	.019	.008	.004	0	0				
11					0	0	.001	.0190	.026	.117	.598	.208	.019	.008	.004	0	0			
						0	0	.0010	.021	.026	.125	.575	.218	.020	.010	.004	0	0		
13							0	0	.001	.024	.035	.130	.544	.228	.023	.012	.003	0	0	
								0	0	.001	.028	.040	.138	.510	.238	.028	.014	.003	0	0
15									0	0	.001	.032	.047	.146	.480	.245	.031	.016	.002	0
										0	0	.001	.035	.052	.155	.450	.255	.035	.016	.001
17											0	0	.001	.035	.052	.155	.450	.255	.035	.017
												0	0	.001	.035	.052	.155	.450	.255	.052
19													0	0	.001	.035	.052	.155	.450	.307
														0	0	.001	.035	.052	.155	.757

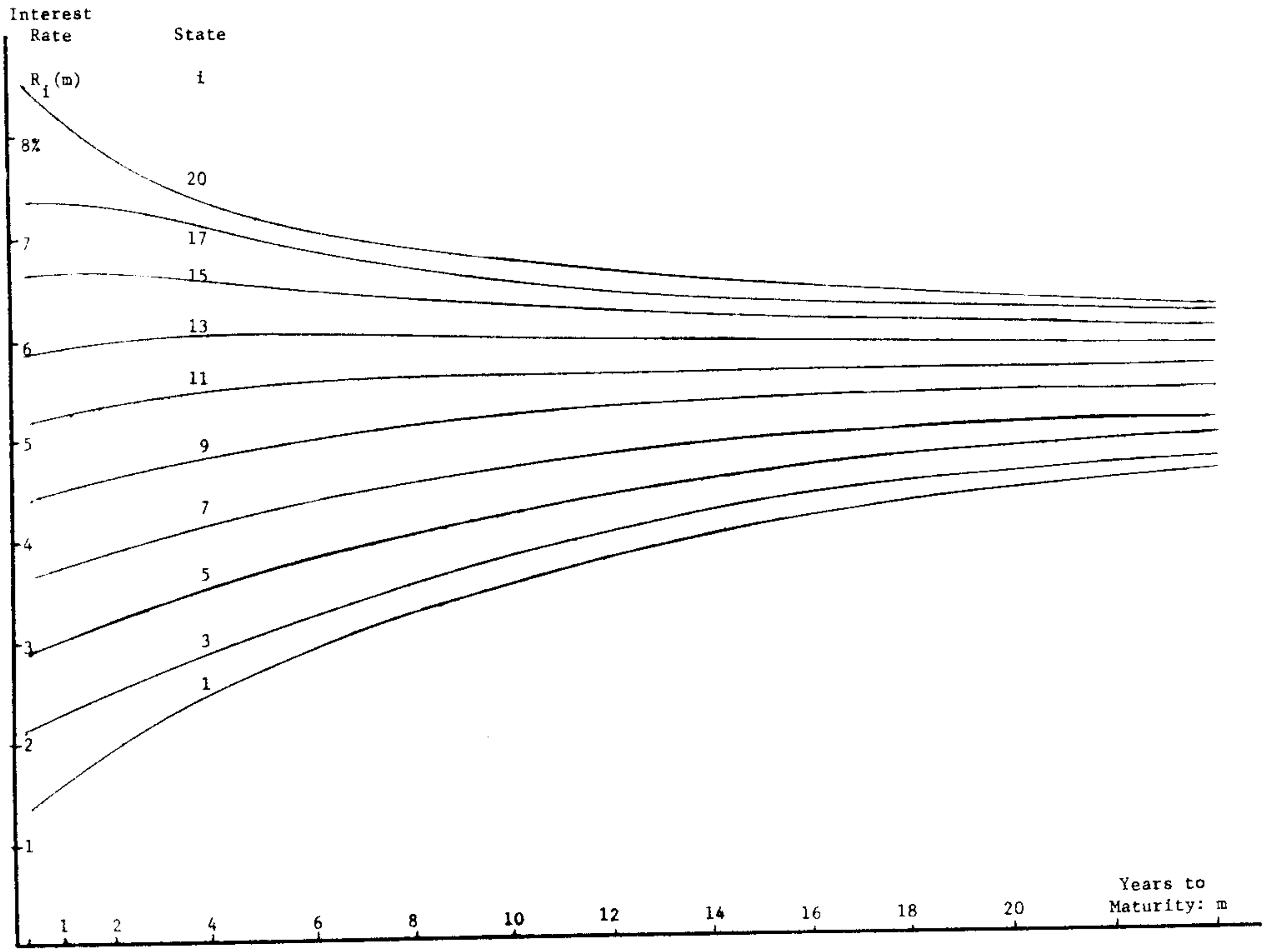

FIGURE 1

Yield Curves

Note: Not every yield curve is shown.

to calculate an optimal maturity policy. Each time outstanding bonds mature, the current state of the economy, as characterized by the current level and shape of the yield curve, is observed, and based on this observed state and expectations regarding future transions of the economy, a new maturity for the refunding bonds is prescribed. Note that the maturity may vary so that a maturing short-term bond may be refinanced with a long-term bond, depending on prevailing economic conditions. That is, that this is an infinite horizon model and should not be taken to imply that the same maturity is used at each refunding date.

The objective of the bond maturity decision is to minimize the expected present value of all future debt costs, which include interest and costs of bond flotation. Within the context of the infinite horizon model it is assumed that debt will remain outstanding forever; that is, every time bonds mature, new bonds are issued to refinance the maturing debt. In reality there is some trade-off between risk and cost as they relate to the bond maturity decision,[7] but these considerations will be ignored here.

In order to develop our model, the following notation is defined:

F = the fixed cost of bond issue which is incurred each time a bond is issued, but is independent of the size of the bond issue or the maturity.

f(m) = the variable cost of issuing a bond of maturity m, expressed as a proportion.

B = the size of the bond issue.

$C_i(m)$ = the coupon interest rate on the firm's m-period bond when the short term riskless rate at time of issue is R_i. The system (economy) is said to be in state i when the current one period rate is R_i.

$\alpha_i(t,m)$ = the present value of \$1 to be received t periods hence when the current state of the system is i and a bond of maturity m is issued. In the application of the model two discounting schemes were used, first $\alpha_i(t,m) = [1 + C_i(m)]^{-t}$, where $C_i(m)$ is the coupon rate on the bond which, as in (11), includes a liquidity premium; and second, $\alpha_i(t,m) = [1 + R_i(m)]^{-t}$, which excludes the liquidity premium.

$\alpha_i(m)$ = $[1 + C_i(m)]^{-m}$ (or $= [1 + R_i(m)]^{-m}$) is the present value of \$1 to be received m periods hence, when the current m-period bond matures.

If a bond issue of size B is issued to mature in m periods, the total flotation costs are $F + f(m)\cdot B$. If the state is i at the issue date, the present value of all interest costs until maturity is $\sum_{t=1}^{m} \alpha_i(t,m)\, C_i(m)\cdot B$, where we assume annual discounting and annual coupon payments. The present value of flotation and interest rates incurred from time of issue until maturity is

$$q_i(m) = (F + f(m)B) + \sum_{t=1}^{m} \alpha_i(t,m)\, C_i(m)B. \qquad (12)$$

When the m-period bond matures m periods hence, another bond will be issued to refinance the maturing debt. The new bond will have a maturity as prescribed by the maturity policy then in force, and it should depend on the then prevailing level of interest rates, the shape of the yield curve and expectations regarding the future course of the economy, that is, the state of the system at that time.

A maturity policy π is a decision rule which prescribes a bond maturity for each possible state of the economy i=1, ..., N. That is, π is a function: $\pi(i) = m$. In the infinite horizon context, a policy will be carried on indefinitely so that whenever a bond matures, the currently prevailing state is observed and the new refinancing bond is issued with the maturity corresponding with the current state as prescribed by the policy. For example, suppose there are two possible states, i = 1, 2, and four possible maturities, m = 1, 3, 5, 7. Policy π might be as follows:

Policy π

State i	Maturity m
i = 1: Prosperity $R_1 = .08$	$m = \pi(1)$ = 3 year maturity
i = 2: Recession $R_2 = .03$	$m = \pi(2)$ = 7 year maturity

If policy π is used, then each time a bond matures and therefore must be refinanced, the state of the economy is observed, if prosperity (state 1) prevails, then a 3 year bond is issued, if recession (state 2) prevails, then a 7 year bond is issued. This policy is followed forever. An alternative policy would prescribe a different maturity for at least one state.

In order to determine the optimal maturity policy, the expected cost of using the policy must be calculated. The cost, or "value" of a policy will be the present value of all expected costs associated with the policy, over an infinite horizon. Define $v_i(\pi)$ as the value of policy π when the current state is i, where $v_i(\pi)$ is the expected present value of all flotation and interest costs incurred over an infinite horizon, when the current state is i and the current and all future maturity decisions are made according to policy π. Suppose policy π prescribes maturity m when the current state of the economy is i (i.e. $\pi(i) = m$), then $v_i(\pi)$ is the sum of:

(a) the present value of flotation and interest costs until the m-year bond matures, $q_i(m)$, and,

(b) the expected present value of all future flotation and interest costs associated with following policy π after the current m-year bond matures, over an infinite horizon.

If state j prevails m periods hence, when the current bond matures, then a new bond will be issued with the maturity appropriate for that state as prescribed by the policy π, and the expected present value (evaluated at that date) of the policy from that date forward will be $v_j(\pi)$. Since we do not know which state will occur at that time, we calculate the expected value

$$\sum_{j=1}^{N} p_{ij}^{(m)} v_j(\pi). \tag{13}$$

Since $v_j(\pi)$ is the present value evaluated at a time m periods in the future, it must be discounted back to the present. This (current) expected present value is

$$\alpha_i(m) \sum_{p=1}^{N} p_{ij}^{(m)} \quad v_j(\pi), \tag{14}$$

where $\alpha_i(m)$ is the expected present value of $1 to be received m periods in the future, when the current state is i, as previously defined. Combining terms, if the policy prescribes an m-year bond when the state is i, then the value of the policy is

$$v_i[\pi(i)=m] = q_i(m) + \alpha_i(m) \sum_{j=1}^{N} p_{ij}^{(m)} \quad v_j(\pi). \tag{15}$$

In order to be more general, let $q_i(\pi)$ be the present value of flotation and interest costs until maturity for a bond of the maturity prescribed by policy π when the state is i, let $\alpha_i(\pi)$ be the present value of $1 received at the maturity date of the bond prescribed by policy when the current state is i, and let $p_{ij}(\pi)$ be the transition probability over the number of periods to the maturity of the bond. We rewrite (15) in more general form as

$$v_i(\pi) = q_i(\pi) + \alpha_i(\pi) \sum_{j=1}^{N} p_{ij}(\pi) \quad v_j(\pi), \tag{16}$$

for i=1,..., N.

Expressions (16) represent a system of N linear equations in N unknowns, where the unknowns are the values of the policy $v_i(\pi)$, i=1,...,N; the terms $q_i(\pi)$, $\alpha_i(\pi)$ and $P_{ij}(\pi)$ are all known. We solve for values $v_i(\pi)$, and by iterating over maturity policies according to the policy improvement technique of Howard [7], the least cost maturity policy is determined.

For example, with two states, i=1,2, and a policy π where

$\pi(i=1)$ = 3 year maturity and $\pi(i=2)$ = 7 year maturity, we have

$$q_1(\pi) = q_1(m=3) = [F = f(3)B] + \sum_{t=1}^{3} \alpha_i(t,3)\ C_1(3)B,$$

and

$$q_2(\pi) = q_2(m=7) = [F + f(7)B] + \sum_{t=1}^{7} \alpha_2(t,7)C_2(7)B.$$

Thus, to determine the value of policy π we solve for $v_1(\pi)$ and $v_2(\pi)$:

$$v_1(\pi) = q_1(\pi) + \alpha_1(3)\ p_{11}^{(3)}\ v_1(\pi) + \alpha_1(3)\ p_{12}^{(3)}\ v_2(\pi)$$

$$v_2(\pi) = q_2(\pi) + \alpha_2(7)\ p_{11}^{(7)}\ v_1(\pi) + \alpha_2(7)\ p_{12}^{(7)}\ v_2(\pi).$$

After solving for $v_1(\pi)$ and $v_2(\pi)$, a new policy π' defined where the maturity prescribed for at least one state i is varied, and values $v_1(\pi')$ and $v_2(\pi')$ are determined. By iterating over alternative policies, the optimal policy, which yields the minimum value for every state i, is determined.

V. Application of the Model

The dynamic programming model presented in Section IV was implemented in order to explore the effects of the discount rate, liquidity premiums, and flotation costs on the debt maturity decision. The transition probabilities shown in Table 1 and the yield curves shown in Figure 1 were used in the model. The general method and the results follow.

Two different discounting schemes were used. First, it was assumed that the discount rate was equal to the coupon rate on the bond being issued (at par). In this case, if the yield curve included a liquidity premium, then this liquidity premium is also included in the discount rate. Second, the discount rate was assumed to be equal the average of the future expected short term rates over the term to maturity of the bond. In this case, any liquidity premium is excluded from the discount rate although it is included in the coupon paid on the bond. Thus, the first discounting scheme proceeds as shown in expressions (2) and (5) of Section II, and under the second scheme discounting proceeds as shown in (3) and (7).

In applying the model five different flotation cost structures and various liquidity premium structures were considered.[8]

Using the first discounting scheme where the discount rate is equal to the coupon rate on the bond being issued (as derived according to expression (11) so as to include the liquidity premium), the structure of liquidity premiums does not influence the maturity decision. That is, the level of interest rates and the shape of the yield curve should not influence the firm's debt maturity decision, providing: (a) the firm's opportunity cost for funds is equal to the yield (including liquidity premium) on the debt being issued; (b) flotation costs are zero; and (c) the yield curve is derived from expectations and the firm shares these expectations.

When the discount rate is equal to the coupon rate on the bond being issued, the structure of flotation costs is the primary determinant of the optimal maturity policy. If flotation costs are zero, then all maturity policies have the same present value of interest costs. If flotation costs are the same for all maturities, or are a decreasing function of maturity, then the longest possible maturity is dominant. If the flotation cost is an increasing function of maturity, then the optimal maturity policy depends on the particular structure of flotation costs. Increasing the slope of the flotation cost function, f(m), has the effect of decreasing the optimal maturity. An increase in the fixed costs of flotation, F, has the effect of increasing the optimal maturity.

When flotation costs are an increasing function of maturity, the optimal maturity is shorter when interest rates are high and longer when interest rates are low. This is due to the fact that when a higher discount rate is used, the present value of flotation costs decreases, and the "relative disadvantage" of incurring flotation costs more frequently with a shorter maturity policy decreases or disappears. For example, assume that the flotation cost on a 10 year bond is $5 and for a 20 year bond it is $8, then if we discount total flotation costs over a 20 year horizon at 3%, the present value for the sequence of two 10 year bonds is $8.72 and for the 20 year bond it is $8.00. If the discount rate is 8%, then the 10 year policy has present value of flotation costs equal to $7.315, versus $8.00 for the 20 year bond. This conclusion is valid when there is uncertainty regarding future interest rates, although the greater the probability that rates will move upward when they are currently low, or downward when they are high, the weaker the effect of the level of interest rates on the choice of maturities. Note that the fact that the shorter maturity is used when interest rates are high is due to the effects of flotation costs, and <u>not</u> from an effort on the part of management to "outguess the market" by issuing short maturities at high rates, to later be refuned with longer maturities when rates have decreased.

To the extent that the firm's opportunity costs are different from the coupon rate on the bond being issued, then interest rate considerations are relevant to the debt maturity decision, and an optimal maturity policy can exist, independent of flotation costs. The dynamic programming model was used with the discount rate equal to the rate derived from current expectations, over the life of the bond being issued (according to expression (10), so that the discount rate is equal to the average of expected future short term rates, excluding liquidity premiums. The coupon rate on the bonds is assumed to include a liquidity premium (as in expression (11)). In cases where the liquidity premium is increasing with maturity and is independent of the level of interest rates, if the firm's opportunity cost for funds excludes the liquidity premium and is, thus, less than the yield on the debt, and there are no flotation costs, then the least cost maturity policy is to issue short term debt regardless of the level of rates or the shape of the yield curve.

When we ignore flotation costs and consider the cases where liquidity premiums are dependent on the level of rates, then we find the relative advantage of short term maturities decreases as the liquidity premiums decrease as a function of the level of rates. That is, when liquidity premiums are high, then the short maturities are dominant, when liquidity premiums are zero, then all policies have the same cost, and no policy is dominant. For example, Cagan [3], and Kessel [9] have suggested that the liquidity premium varies directly with the level of rates: when rates are high, the liquidity premium is high. Under this scheme, when the discount rate excludes the liquidity premium, if the level of rates is high, the short maturity is dominant. If rates are low (and $L_i(m) > 0$) the advantage of short maturities is small relative to longer maturities. The reverse would be true if liquidity premiums are greatest when rates are low as suggested by Malkiel [13] and Van Horne [20]. These conclusions require that the liquidity premium for longer maturities is always greater than for shorter maturities at every level of rates.

It is occasionally suggested that when rates are very high then liquidity premiums could be negative. In this case, (when the discount rate excludes liquidity premiums) the long maturities are dominant when interest rates are high, and when rates are low the short maturities are dominant. In the middle range of rates, where the liquidity premium is zero, the relative advantage of any particular maturity policy disappears.

VI. Conclusion

The purpose of this paper has been to explore the debt maturity decision and, hopefully, to elicit further discussion of this interesting and important problem. The intent has been to present the problem so as to focus on the basic issues relating to the debt maturity decision. These issues involve the effects and interaction of interest costs, debt flotation costs, and the firm's opportunity cost for funds (which is used as the rate for discounting the costs). A basic problem in this context is whether or not an optimal maturity policy exists in a world where the yield curve is formed from investors' expectations regarding future interest rates.

The objective of the debt maturity decision was assumed to be to minimize the present value of expected interest and flotation costs over an infinite horizon. The dynamic programming model utilized to explore this decision assumed that interest rates follow a finite Markov process which is stationary so that the probabilities do not change over time. It was assumed that the yield curve at each decision point is formed according to the expectations model of term structure, and that these expectations are specified by the same transition probabilities as are used in the dynamic programming model. Thus, the firm shares the markets' expectations regarding the future course of interest rates, and the debt maturity decision does not depend on an attempt to "outguess the market" regarding future interest rates. Finally, taxes were ignored in the analysis, and there was no consideration of the risks involved in different debt maturity decisions.

The primary focus of the dynamic programming example was upon the effects and interaction of interest costs, debt flotation costs and the discount rate applied to these costs. Flotation costs are an important factor in the debt maturity decision regardless of what discount rate is used. If the flotation cost is a constant, independent of the maturity chosen, then the longest maturity will minimize the present value of flotation costs. If flotation costs are an increasing function of the maturity, then the optimal maturity must be determined by computation. Generally, increasing the fixed costs of flotation will have the effect of increasing the optimal maturity, and increasing the slope of the variable flotation cost function, $f(m)$, will have the effect of decreasing the optimal maturity.

It was apparent from the application of the dynamic programming model that the level of interest rates has little effect on the optimal maturity, except through its effect on the present value of flotation costs. In the cases where the flotation cost was an increasing function of maturity, at higher interest rate levels, shorter maturities were chosen. This is not attributable to using short maturities until rates go down in order to minimize interest costs, rather at higher discount rates the present value of flotation costs is reduced, and those policies which incur flotation costs more frequently become less expensive relative to the longer maturity policies which incur flotation costs less frequently.

The expected interest costs may be an important determinant of the optimal maturity policy, depending on the firm's opportunity cost for funds which is used as a rate for discounting the interest costs. If the discount rate is equal to the yield on the debt, then all maturity policies will have the same present value of expected interest costs; this is true regardless of whether or not the yield curve includes liquidity premiums, and regardless of the shape or level of the yield curve, so long as the yield curve is formed according to the expectations model. On the other hand, if the discount rate is different from the yield on the bonds, then the present value will vary between maturity policies, and an optimal maturity policy can exist in a world where the yield curves are formed according to investor's expectations. That is, if the discount rate is not equal to the yield on the bonds, then the present value minimizing borrower may not be indifferent between two borrowing sequences with the same expected average interest cost.

Footnotes

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[1]The assumption that interest rates follow a Markov process is also used by Kalymon [8], Kraus [10], Pye [15], and to some extent by Elton and Gruber [4]. This assumption is consistent with the sub-martingale model of interest rates used by Roll [18].

[2]Yields were computed assuming the Bill was purchased at the asked price on the first of the month and held 90 days upon maturity. If, on the observation date, no Bill existed with 90 days to maturity, the yield was computed as the average of the yields on two Bills with maturity days adjacent to 90 days. The data was provided by John Bildersee of The Wharton School of the University of Pennsylvania.

[3]According to Roll [18] Treasury Bill rates, over weekly differencing periods, follow a sub-martingale process, and the first differences appear to be generated from a symmetric, stable (but non-normal) distribution. Roll's findings are consistent with the assumption used in the present model that rates follow a Markov process. Roll (p. 68) notes that when the distribution is symmetric stable (but non-normal), then the sampling distribution of the .5 Truncated Mean has a low sampling variance. This parameter was estimated from the present sample of monthly first differences as 0.0312%, which is somewhat greater than the sample mean. Roll's scale (dispersion) parameter, s, is estimated as

$$\hat{s} = (\hat{d}_{.72} - \hat{d}_{.28}) / 1.654,$$

where $\hat{d}_{.72}$ and $\hat{d}_{.28}$ denote the sample estimates of the .72 and .28 fractiles of the ordered sample of first differences. With $\hat{d}_{.28} = -0.08\%$ and $\hat{d}_{.72} = +0.16\%$ for the present sample of monthly first differences, we have $\hat{s} = 0.144$, where $2s^2$ is the variance in the case of the normal distribution, and s is the semi-interquartile range for the Cauchy distribution.

[4]Anderson and Goodman [1], Hoel [6], and Lee, Judge, and Zellner [11] show that maximum likelihood estimates of Markov transition probabilities are obtained directly from the observed frequency distribution of state transitions. This requires that (a) $p_{ij} > 0$ for all i and j, and (b) there be at least 5 observed transitions from each state i to every other state j. These requirements make it difficult to use the direct estimation method on interest rate data over an extended sample period due to the wide variation of rates over the period. Consequently, the frequency distribution of monthly first differences was used as an approximation of the transition probabilities.

[5]The frequency distributions of monthly first differences were estimated (a) for the whole 265 month period, (b) for the months when rates were below the 3.55% average for the period, and (c) for the months when rates were above 3.55%. The transition probabilities for the middle of the range of rates were set equal to the estimates from (a), and for the lower and upper range of rates the transition probabilities were approximated by the estimates from (b) and (C), respectively.

[6]Weingartner [21], Elton and Gruber [4], and Kalymon [8] have presented dynamic programming models focusing on the bond refunding decision (that is, when to call an outstanding bond). Their models include the maturity decision, but do not explicitly develop the issues relating to bond maturity.

[7]Some aspects of trade off between risks and costs of different maturity policies have been explored in James Morris, "A Note on Hedging as a Debt Maturity Strategy," Working Paper No. 4-74, Rodney L. White Center for Financial Research, University of Pennsylvania, February, 1974.

[8] Both the liquidity premium functions and the flotation cost functions were somewhat arbitrarily specified, and are not based on empirical data. However, the total flotation costs are of the same approximate magnitude as underwriting commissions. The liquidity premiums are of the same order of magnitude as those suggested by Malkiel [13] and Van Horne [20], or Cagan [3] and Kessel [9], although ours are somewhat greater in most cases than those suggested by the above authors.

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