Working Capital, Risk, Growth and Diversification

by

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In some areas of finance there are many popular, sometimes conflicting and controversial theories. However, there are relatively few theories of working capital and its effects on the firm.[1] Moreover, most of these theories do not consider the interactions between working capital, short term assets and the rest of the firm.[2] Instead, studies in the working capital area are often programming models or case studies which, while informative, may be primarily problem oriented. Alternatively, some analyses apply inventory theory, but consider sectors of the firm rather than the entire firm.

In this paper, a theory is developed depicting working capital as a liquid buffer stock protecting the permanent, risky activities of the firm and abosrbing the fluctuations of the firm associated with deviations of returns from expectations. As an integral part of the theory a firm's working capital position is related to the firm's long term asset position and to the risk-return relationship that the firm has accepted. In addition, it will be shown that the model of working capital developed here is consistent with some of the more popular theories of dividend policy and the use of leverage. Finally, the implications of growth and diversification due to investments on the margin are examined with respect to working capital policies.

II. Short-Term Assets and Liabilities and the Firm

Although each firm is generally accepted as a permanent entity if it appears, currently, to be a viable and going concern,

the assets within the firm are not permanent.[3] The separation between the assets of the firm and the firm itself is especially apparent when considering current assets and current liabilities as necessary, permanent portions of the firm. Discussions of these aspects of the firm indicate that liquidity is their prime feature. Although speed of turnover and liquidity are undoubtedly important, it is also necessary to recognize that the current components of the firm, as a group, are important to the firm in a permanent sense. In particular, the permanent presence of some minimum level of current assets and current liabilities facilitates the conversion of long term assets and investments of the firm into cash flows and income flows throughout the firm's existence. In addition, short term assets and liabilities are used to smooth out any differences in the timing of the inflows and outflows due to a project thereby enabling the cash flows to long term assets.[4] Additionally, there is some buffer stock of current assets or working capital that is maintained by the firm. This buffer stock is useful to the firm in that it reduces the chance that the firm's level of current assets will fall below some minimum viable level due to a depressed economy or a series of random, but bad events for the firm thereby impinging on the firm's operations and causing a decrease in the firm's ability to generate profits temporarily or permanently.

Work has been done determining the minimum acceptable or desired buffer stock levels of these assets and liabilities. The most typical analyses of cash or short term assets are the invetory

control analyses. In this case, there may be a regular flow of funds from a given account. When a given minimum buffer stock is reached, a replacement order is made. However, these analyses often do not handle risk fully and sometimes effectively assume certainty. Moreover, the typical inventory analyses assume that the firm has infinite capital available from which to replenish the account under examination.[5] Other studies, using ratio analysis, allude to the importance of the use of short term assets as a buffer to absorb the effects of adverse economic circumstances by covering the fixed cash obligations of the firm.[6] However, these studies are unrealistic in the sense that the ratios used suggest that the going concern cannot replenish its supply of current assets in some regular manner.[7] Instead, even if economic conditions are so severe that there are heavy outflows of working capital and current assets, there will be some inflows to the firm to slow this drain. Moreover, if it is necessary for the firm to replenish its supplies of these assets quickly in order to remain a going concern, then it can obtain these funds through external financing, dividend reductions or the sale of selected long term assets. These inflows would enable the firm to continue to convert its remaining long-term assets into cash flows and profits continuously.

The analysis of working capital to be developed here includes a framework through which working capital, long term assets and desired levels of risk interact with each other. In particular, associated with the expected return on its physical long term assets the firm accepts some risk. The risk perceived by the firm may contain two components; diversifiable or unique risk

and non-diversifiable or systematic risk. However, investors holding portions of several financial investments in their portfolios and operating in an environment with complete secondary markets, no transaction costs and divisible assets can diversify away any non systematic risks associated with any investment.[8] With full diversification, the relevant risk of an asset to an investor in equilibrium is the systematic risk of that asset, and the expected return on that asset, E(R), above the riskless rate, R_F, is a linear function of the expected return on the market portfolio, $E(R_m)$, in excess of the riskless rate or

$$(E(R)-R_F) = \beta(E(R_m)-R_F) \qquad (2.1)$$

where β is the measure of systematic risk.[9]

Let the total assets of the firm TA, be divided into two segments or working capital WC, and investment assets IA, where the investment assets include all long term assets and those current assets necessary to meet exactly the firm's current liabilities.[10,11] Additionally, if β_{TA} represents the systematic risk of the total assets of the firm as perceived by investors, then

$$\beta_{TA} = \frac{WC}{TA}\beta_{WC} + \frac{IA}{TA}\beta_{IA} \qquad (2.2)$$

where β_{IA} is the systematic risk associated with the investment assets of the firm and β_{WC} is the systematic risk associated with the firm's working capital. The coefficients of β_{WC} and β_{IA} weight each part of the firm appropriately.[12] In a typical situation β_{WC} is less than β_{IA}.

Although the investor is not concerned about the unique features of any single firm if he has a fully diversified portfolio and there is no need by investors for firms to diversify by themselves, the incompletely diversified firm can be harmed by either the systematic or unique risks of the firm. Moreover, if the investment assets currently owned by a firm do not represent full diversification and if these assets are assumed to be unique and can be traded or sold only with great effort in a manner that is time-consuming with large transactions costs, then the investment assets are traded in a poor secondary market. Due to the large amount of friction involved with any sale of such an investment asset a firm is likely to decide that it can serve investors better by maintaining its current nondiversified physical asset structure while investors diversify via their selection of financial assets.[13] In this case, to ensure survival of the firm and the maintenance of the firm's current level of income, it is likely that management considers both the systematic and unique risks of the firm or the total risk of the firm.

If the firm accepts its unique risk as preferable to the expense of diversification, then the main purpose of working capital from the firm's viewpoint is to absorb the unique and systematic shocks that affect the firm thereby protecting the firm's investment asset structure and the firm's current levels of income and dividends generated by the investment assets. In this case, working capital must be sufficient to absorb a chosen proportion of

the unexpected fluctuations in assets suffered by the firm due to risks associated with the investments of the firm, and an important measure of risk to the firm is the probability that the working capital buffer may be breached during the comming period thereby decreasing income via forced sale of selected investment assets to maintain liquidity or

$$P(\Delta WC < -WC) \qquad (2.3)$$

where ΔWC represents the change in working capital during the period.

III. Working Capital and Risk

The two measures of risk described above can be used to study the effect of a firm's level of working capital on the risk associated with that firm's permanent level of income in a no growth situation where the firm cannot diversify away unique risks. It will be shown that, for a given level of systematic risk of the firm's investment assets and a given level of total assets, the firm can arbitrarily choose a given probability of maintaining some expected levels of income from its investment assets and dividends via its selection of a working capital buffer as the firm completes its total asset structure where the expected income level decreases with its increased safety.[14] This can best be shown through an example. In particular, it will be shown that a firm can increase its probability of maintaining some (decreased) level of income by increasing its working capital position at the expense of investment assets while not growing in total assets. The emphasis is on the differential

safety and risk generated by alternative allocations of funds to working capital and investment assets. Although the debt/equity ratio of the firm differs due to different uses of current liabilities, the values of the firms are not affected as they are functions of the firm's asset structure and not their financial structures.[15] These problems will be discussed more fully later.

Consider a firm with the interrelationships shown in Table 1. The firm's investment assets are a proportion, w, of the firm's total assets. Moreover, the firm has an expected investment asset turnover ratio $E(\gamma)$, a profit margin λ, and fixed costs F.

If expectations are met, if there is no external financing, and if dividends are 100% of expected returns, there will be no change in working capital. However, sales will not always be exactly equal to expectations as the actual market rate of return varies. From rows e and h of Table 1 the variance of returns to the firm, σ_R^2, can be stated as

$$\sigma_R^2 = \frac{\lambda^2}{TA^2}\sigma_S^2 = \frac{\lambda^2}{TA^2} IA^2\sigma_\gamma^2 . \qquad (3.1)$$

Moreover, from (2.1), the variance of the return to the firm is

$$\sigma_R^2 = \beta_{TA}^2 \sigma_m^2 + \sigma_u^2 \qquad (3.2)$$

where σ_m^2 represents the variance of the market return and σ_u^2 represents the unsystematic variance of the returns. Additionally, from (2.2) where β_{WC} is 0 due to the investment of working capital in a zero-beta, riskless portfolio

Table 1

The Structural Relationships of the Firm

a)	Working Capital	$WC =$	$= (1-w)TA$
b)	Investment Assets	$IA =$	$= wTA$
c)	Total Assets	$TA = WC + IA$	
d)	Expected Sales	$E(S) = IAE(\gamma)$	$= wTAE(\gamma)$
e)	Actual Sales	$S = IA\gamma$	$= w\gamma TA$
f)	Expected Operating Income	$\lambda E(S)-F$	$= \lambda wTAE(\gamma)-F$
g)	Net Operating Income	$\lambda S-F$	$= \lambda\gamma wTA-F$
h)	Return on Total Assets	$\frac{\lambda S-F+R_F WC}{TA}$	$= \lambda\gamma w+R_F(1-w)-\frac{F}{TA}$

$$\beta_{TA} = \frac{IA}{TA}\ \beta_{IA} = w\beta_{IA} \qquad (3.3)$$

Substituting for β^2_{TA} in (3.2) and combining with (3.1) yields the variance of sales in relation with variance of the market or

$$\sigma_S^2 = \frac{IA^2}{\lambda^2}\ \beta^2_{IA}\,\sigma^2_m + \frac{TA^2}{\lambda^2}\ \sigma^2_u = IA^2\sigma_\gamma{}^2 \ .^{16} \qquad (3.4)$$

If sales are different from expectations and the anticipated dividends are paid, then the level of working capital for a given firm at the beginning of the next period is

$$WC_{t+1} = WC_t + \lambda S - F + R_F\, WC_t - (\lambda E(S) - F + R_F\, WC_t)\ ^{17}$$

or

$$\Delta WC = \lambda(S - E(S)) = \lambda wTA(\gamma - E(\gamma)). \qquad (3.5)$$

In this case, ΔWC is a random variable with a variance

$$\sigma^2_{\Delta WC} = \lambda^2\sigma_S^2 = IA^2\beta^2_{IA}\sigma^2_m + TA^2\sigma_u{}^2 = \lambda^2\sigma_\gamma{}^2\, w^2TA^2 \ . \qquad (3.6)$$

If market returns and unsystematic returns each follow a normal distribution, then ΔWC will be distributed normally with a mean 0 and a standard deviation $\sigma_{\Delta WC}$ where the variance of the working capital of the firm is completely dependent on the structure of the firm's portfolio of investment assets.

Since all fluctuations in the firm's total assets are subsumed into the variance of working capital, the safety of the firm's income from investment assets in a given period can be determined from consideration of the standardized distribution of changes in working capital. In particular, the probability of wiping out the complete working capital buffer during the period is $P(\Delta WC \leq -WC)$ or $P(t \leq t^*)$

where $t=\Delta WC/\sigma_{\Delta WC}$ and $t^* = -WC/\sigma_{\Delta WC}$ and is the standardized statistic associated with a change in working capital of -WC or erosion of the entire working capital buffer.

If, in this environment, a firm wants to increase working capital without growth and without a change in the systematic risk of the investment assets, $d\beta_{IA} = 0$, then the change in total assets, dTA, is 0 or

$$dTA = 0 = dWC + dIA \tag{3.7}$$

where dWC is the change in working capital and dIA is the change in investment assets. In this case dWC = -dIA. In addition, if the differentials of $t^* = WC/\sigma_{\Delta WC}$ and (3.6) are taken and are combined with (3.7), then the change in t*, dt*, is associated with the change in working capital by

$$dt^* = \frac{-\sigma^2_{\Delta WC} dWC + WC d\sigma^2_{\Delta WC}}{(\sigma^2_{\Delta WC})^2} \tag{3.8}$$

where

$$d\sigma^2_{\Delta WC} = 2\lambda^2\sigma^2_{\gamma}(wTA\ (TAdw + wdTA)) \tag{3.9}$$

The increase in working capital means a corresponding decrease in investment assets or $dw < 0$. In turn, from (3.9), $d\sigma^2_{\Delta WC} < 0$. Furthermore, since $dWC > 0$, $dt^* < 0$ from (3.8) or the critical t* value becomes more negative. This means that the deviation of inflows from expectations must now be more extreme than in the past to destroy the working capital buffer and that the change in total asset structure makes the firm's new level of income sager. Coincident with this change

is a change in the equilibrium rate of return required by investors and the systematic risk of the firm. The differentials of lines d and h of Table 1 and (2.1) give

$$dE(R_{TA}) = \frac{(R_F - \lambda E(\gamma))dWC}{TA} = d\beta_{TA}(E(R_m) - R_F)$$

Note that $dE(R_{TA})$ is less than 0 for an increase in working capital since $\lambda E(\gamma)$, the expected return on investment assets, is greater than R_F. Hence, $d\beta_{TA}$ is less than 0 and the systematic risk of the firm falls with the increase in working capital.[18]

Moreover, this analysis indicates that restructuring a firm's assets in order to gain increased investments and cash flows at the expense of decreased working capital, while not necessarily changing the risk of the typical investment asset held by the firm, may carry a more substantial threat to the firm than might be apparent from a review of asset levels alone.[19] These threats would appear in terms of increased sensitivities to unexpected changes in the economy, random shocks and the risk associated with the firm's higher chosen level of income implicit in its restructured portfolio.

There are several corollaries to this result. In particular, $P(t \geq t^*)$ represents the degree of certainty associated with a firm's level of income or the probability that, regardless of the risks associated with the firm's risky assets, the firm can use its buffer stocks to smooth out results for the coming period and still expect to have at least the same income from investment assets for the next period. Moreover, probabilities can be provided to summarize the chance of suffering any chosen decrease in the level of the firm's

income or the probability of one firm suffering a decrease in permanent income while the other firm does not suffer any damage in income.[20] Also, because changes in working capital are cumulative, compound probabilities can be used to determine the chance of using up all the available working capital over several periods and conditional probabilities can be used to determine the risks of the firm in the future given specific results in coming periods.

IV. The Appropriate Level of Working Capital

In general, increasing the size of the working capital buffer on the margin will always reduce the chance of suffering a decreased income in the future for a given level of total assets. However, continued increases have a decreasing marginal effect on reducing the risk of such a decrease. The additional cost in terms of income foregone or a reduced level of income to achieve this safety is the difference between the expected rate of return on the firm's investment assets and the rate of return on working capital. This cost increases linearly despite the decreasing marginal benefits associated with incurring the cost. In this case, a firm can decide on the appropriate level of working capital via an examination of the trade-offs presented in Table 2.

If the selection of an optimal level of working capital for the firm is viewed as creation of negative leverage by the firm for its own safety, then the investor can use homemade leverage to create his own desired level of risk separate from that of the firm and is indifferent to the firm's choice of a working capital

Table 2

Probabilities of a Decrease in Income Due to Inferior Operating Results in One Period and the Income Foregone to Achieve This Safety for a Given Level of Total Assets*

Initial Working Capital	t	Probability	Expected Income Foregone
0	0	50.00%	0
$\sigma_{\Delta WC}$	1	15.866	$\{\lambda E(\lambda)-R_F\}\sigma_{\Delta WC}$
$2\sigma_{\Delta WC}$	2	2.275	$2\{\lambda E(\lambda)-R_F\}\sigma_{\Delta WC}$
$3\sigma_{\Delta WC}$	3	0.135	$3\{\lambda E(\lambda)-R_F\}\sigma_{\Delta WC}$
$4\sigma_{\Delta WC}$	4	0.003	$4\{\lambda E(\lambda)-R_F\}\sigma_{\Delta WC}$

*This table assumes that the distirbution of changes in working capital is $N(0,\sigma_{\Delta WC})$.

buffer.[21] In this case, selection of the optimal level of working capital becomes a decision of the corporate management independent of investors or management is effectively selecting the risk reward relationship it is willing to accept for its own undiversified portfolio.

V. Working Capital and Growth

Thus far the analysis has been confined to a situation where total assets are not expected to change over time. However, if a firm's target dividend rate is less than 100 percent of total earnings and perhaps less than 100 percent of earnings on investment assets, then the firm expects to accumulate additional working capital through operations increasing its total assets and is a growing firm. In this section the effect of growth due to regular additions to retained earnings will be examined. If the firm does not convert these inflows to investment assets, then the firm's asset structure, systematic risk, and equilibrium rate of return will change over time. However, it is possible that the firms wants to maintain a constant safety level and/or a constant systematic risk and invests funds continuously as they are accumulated by the firm and that the length of the firm's planning period remains constant through time at one period into the future.[22]

In the growth case, it is convenient to restate the safety level of the firm's income, from (3.6), as

$$t^* = \frac{-WC}{\sigma_{\Delta WC}} = \frac{-WC}{\lambda\sigma_S} = \frac{(1-w)}{(w^2\beta_{IA}^2\sigma_m^2 + \sigma_u^2)^{\frac{1}{2}}} \qquad (4.1)$$

Since the firm now expects its total assets to increase (dTA>0), and since investment is continuous, additions to retained earnings are divided into new investment assets and the new working capital to protect the income to be generated by the new investment assets.[23] If the firm wants to retain its current safety and systematic risk of the total assets at the end of the planning period and if the effects of diversification are nil, then

$$dt^* = \frac{-dw}{(w^2\beta_{IA}^2\sigma_m^2 + \sigma_u^2)^{\frac{1}{2}}} + \frac{(w-1)(w\beta_{IA}^2\sigma_m^2 dw + \beta_{IA}w^2\sigma_m^2 d\beta_{IA})}{(w^2\beta_{IA}^2\sigma_m^2 + \sigma_u^2)^{3/2}}$$

$$= 0 \qquad (4.2a)$$

and, from (3.3),

$$d\beta_{TA} = \beta_{IA}dw + wd\beta_{IA} = 0 \qquad (4.2b)$$

must be satisfied. In this case dw=0 and $d\beta_{IA}$=0. This implies that accumulations of new funds must be invested so that the firm retains its current proportions of working capital to total assets at the end of the period. Moreover, because the length of the firm's planning period is constant through time, the firm always invests this way.

In the growth case, the firm's inflows accumulate at some expected rate. However, if the firm's net inflows are greater than expected and if the firm still follows this investment plan, then income and growth are greater than expected at the beginning of the period, but the risk of the firm does not change. If the

same policy is followed and if there is a net accumulation that is less than is expected, then growth is less than expected, but the firm's safety is intact. There is a problem only if the firm experiences a net outflow of working capital. In this case, there are two possible results. If the firm wants to retain its level of income from invested assets in the future, then the net outflows represent a deterioration of the firm's safety level and an increase in the systematic risk of the firm's total assets as the firm's investment assets are now a larger portion of the firm. However, there is no decrease in the firm's income from investment assets. If future periods develop according to expectations, then the firm will accumulate new working capital to replace the working capital lost in this period to achieve the desired safety level before it can plan to expand again. Alternatively, if possible, the firm may convert some investment assets to working capital, thereby retaining its safety and systematic risk but suffering reduced earnings from investment assets. However, due to the poor secondary market in physical assets this result may not be easily obtainable.

This solution does not account for the effects of diversification. In particular, the variance of changes in working capital at any point in time is a function of the firm's current structure and size and can be stated as

$$\sigma^2_{\Delta WC} = TA^2\beta^2_{TA}\sigma^2_m + TA^2\sigma^2_u \quad (IA)$$

from (3.3) and (3.6) where the unsystematic variance affecting

changes in working capital is now a function of the investment assets of the firm. Due to the poor secondary market in physical assets the firm can diversify most easily through growth. With growth and diversification the last term of this equation is likely to decrease with marginal increments in the number of investment assets of the firm as $\sigma_u^2(IA)=0$ for a portfolio of the entire economy while TA is still finite. In this case, it is possible to diversify some of the unsystematic variance away with each new investment. If $\sigma_u^2(IA)$ decreases toward zero or $d\sigma_u(IA)<0$ with increasing investment assets as a firm grows, then the total risk that must be subsumed into the variance of the changes in working capital for a given level of systematic risk grows at a slower rate. In this case, the total differential of equation (4.1) is

$$dt^* = \frac{-dw}{\{w^2\beta_{IA}^2\sigma_m^2 + \sigma_u^2(IA)\}^{\frac{1}{2}}} + \frac{(w-1)\{w\beta_{IA}^2\sigma_m^2 dw + \beta_{IA}w^2\sigma_m^2 d\beta_{IA} + \sigma_u(IA)d\sigma_u(IA)\}}{\{w^2\beta_{IA}^2\sigma_m^2 + \sigma_u^2(IA)\}^{3/2}} \quad (4.3)$$

If increased diversification accompanies growth, then (4.3) describes the interaction among a firm's safety, the systematic risk of its investment assets and the proportion of the firm in working capital. For example, if the firm wants to retain the same level of safety, $dt^*=0$, any change in the firms' asset structure must satisfy

$$dw = \frac{(w-1)\{\beta_{IA}w^2\sigma_m^2 d\beta_{IA} + \sigma_u(IA)d\sigma_u(IA)\}}{w\beta_{IA}^2\sigma_m^2 + \sigma_u^2(IA)} \quad (4.4)$$

Typically w is between 0 and 1 and w-1 and the coefficients of $d\beta_{IA}$

and $d\sigma_u(IA)$ are negative.[24] Additionally, if $d\beta_{IA}=0$, then $dw>0$ and the firm can retain the same level of safety as it grows while investing larger amounts of new funds in investment assets than the firm had, on average, invested in such assets in the past. In turn, this means that working capital grows at a slower rate than total assets while providing the desired safety and that investment assets grow at a faster rate than total assets. But, from (4.2b), this means that $d\beta_{TA}$ is positive. In this case, associated with the constant safety level and benefits of diversification, there is an increase in the systematic risk of the firm. If the firm wants to maintain its current safety level and its current level of systematic risk for its total assets, then, in the presence of the diversification effect, it must decrease the risk of its investment assets and $d\beta_{IA}$ must be negative from (4.2b). However, if the firm wants to maintain constant systematic risks for both its investment assets and its total assets, then dw must be 0 from (4.2b). In this case, the diversification effect will increase the safety of the firm's income from (4.3). In each case described here, the firm has been able to set some goals but has not been able to set goals for everything as the diversification effect encourages a change in the firm's structure with growth.

The growth plans and patterns for the company depends on the priorities it places on the factors discussed here. In general, any change in total assets can be summarized by (4.5a), 4.5b) and (4.5c).

$$dWC = (1-w)dTA - TAdw \tag{4.5a}$$

$$dIA = wdTA + TAdw \tag{4.5b}$$

$$dTA = dWC + dIA \tag{4.5c}$$

In this case, if the growth of a firm is such that dw=0, then additions to working capital are in the same proportion as the firm has historically averaged. If dw is greater than 0, then marginal additions to working capital are at a lower than average rate. Also, if dw is less than 0, then marginal additions to working capital are at a higher than average rate. Finally, note that in some cases, $d\beta_{IA}$ and/or $d\sigma_u(IA)$ may be negative enough in (4.4) so that TAdw is greater in an absolute sense than is (1-w)dTA and, from (4.5a), dWC is negative. In this case, the firm can reduce the amount of its working capital at the same time that it develops new investment assets and can yet retain the same level of safety for its new, higher level of income. In general, note that the key features of a firm's growth plans and growth patterns include the firm's desires to change the systematic risks associated with parts of the firms, the safety of any given level of permanent income and the expected strength of the diversification effect.

VI. Inclusion of the Debt/Equity Ratio and a Dividend Policy

The target debt/equity ratio has been ignored.[25] However, as new funds are converted into investment assets, there will be an increase in current liabilities. Adjustment for this problem and an allowance for external financing can be made easily by describing dTA more fully. In particular, let

$$dTA = D' + CL' + EE' + \lambda E(S) + R_F WC - Div$$

where D' represents new debt, CL' represents new current liabilities, EE' represents new external equity and $\lambda E(S) + R_F WC - Div$ represents the expected new funds to be generated internally after payment of dividends, Div, according to some dividend policy and the refurbishing the firm's present set of investment assets. In this case, expansion plans are based on consideration of these additional flows instead of internal flows alone. Since the firm may maintain a target debt/equity ratio over time but may not try to maintain it exactly through time due to issue cost patterns and risk associated with planning for any given period, the firm can set D' and EE' to 0 whenever it is useful thereby avoiding the new issues market. Moreover, since $\lambda E(S) + R_F WC - Div$ does not represent a fully certain source of funds, this and CL' will vary with corporate expectations and realized results.

The dividend policy used in this paper is included primarily to allow the firm to discard excess funds. However, if external funding is available costlessly and continuously, then the dividend policy becomes irrelevant in this model and dividends can follow any arbitrary pattern.[26] In cases where there are transactions costs for external funds or where the firm cannot go to the market regularly, then the dividend pattern becomes important. For example, if no external funds except current liabilities are obtainable and one firm has a lower target dividend rate than does another, otherwise identical firm, it is likely to be able to grow at a faster rate than can the second firm. In this case, the first

firm will feel a stronger diversification effect and may commit a lower proportion of new funds to working capital than will the second firm. In addition, if experience is bad and working capital erodes and if external sources of funds are limited, then commitment to the lower dividend rate will allow firm 1 to rebuild its working capital position more quickly than can firm 2.

VII. Conclusion

This paper develops a theory about the importance and usefulness of working capital as an integral part of the firm. In particular, it is shown that if working capital can function as a protective buffer for the entire firm, then the firm can maintain a commitment to an income level with some safety, a permanent debt/equity ratio and a given level of systematic risk in the no growth and growth situations based, at least in part, upon the size of its working capital position which has been determined independently of investors. Moreover, note that the model allows for fluctuation in the dividend payout ratio and the debt equity ratio about the target ratios as the firm's experiences deviate from normal and as the firm grows.

In addition, the growth case suggests that due to the diversification effect, a firm might accidentally accept goals that are not compatible. In particular, it was shown that attempts to maintain a safety level for permanent income, a constant ratio or working capital to total assets, and a constant level systematic risks for both the firm's total and investment assets leads to a

situation where the firm's policies are over-determined. In this case, the firm cannot attain all its goals at once and must ultimately set a pattern of priorities for these goals.

Moreover, since the diversification effect is likely to be stronger on the margin for small firms with small numbers of investments than for larger firms, this model can be used to describe the problem of business failures among small firms. Even if a small, undiversified firm has the same systematic risk as a larger firm, the small firm has not yet diversified away any of its total risk. Hence, if the small firm has the same proportion of working capital to total assets as the larger firm, it can be shown via an application of the growth case that the small firm has more total risk for its size than does the large firm and its working capital buffer therefore provides less protection than does the buffer of the large firm.[27] In this case, the small firm is more likely to suffer a loss of income than is a large firm. Also, since it has less income to start with, it may go bankrupt under market conditions that might not even affect the income from the large firm's investment assets adversely. This suggests that small firms, which are chronically cash short, should actually have more working capital as a proportion of the firm's total assets than a large, but otherwise identical firm just to have the same chance to maintain their levels of income. Alternatively, the smaller firm with the same proportion of working capital to investment assets as a larger firm must have less systematic risk than the large firm to compensate for its inability to

diversify.

Working capital has been discussed as if it is the only protection a firm has for its level of income. In practice, some investment assets are recognized as expendable and will be sold first in order to protect and maintain the profitability of the remaining, more important assets of the firm. This represents more of an accounting problem than an economic one. Within the context of this model the definition of working capital can be extended to include other expendable and readily saleable current assets, investment assets and emergency lines of credit thereby increasing the firm's safety and reducing the level of income it feels committed to maintaining. In an accounting sense, this implies that the buffer stock for the firm may include some long term assets while some of the investment assets are not long term assets in their fullest sense and suggests that the division between current assets and long term assets may not be an appropriate division for analyzing the future of the firm and its policies.[28]

Moreover, it would be useful to develop ratio analyses to probe the firm as a truly going concern and to consider working capital as an important feature of the firm in studies of the firm. Note that this analysis also suggests that accounting ratios, being management statistics, cannot, in their present form, be expected to estimate the systematic risks of the firm, of interest to owners, on a firm by firm basis. However, it does suggest that as a firm diversifies the difference between the systematic risk of the firm, as portrayed by beta, and the accounting vari-

ables must narrow.

Finally, the discussion of changes in working capital within the context of systematic and unsystematic risks suggests that a corporate liquidity crisis may be individual or that it may be a symptom of a systematic economic crisis. In the latter case several corporations will suffer the problem concurrently as part of a national liquidity crisis.

Footnotes

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1. See Van Horne (9) and Walker (10) for two analyses of working capital that are typically cited.

2. One example is Miller and Orr(6) where the interaction among cash, short-term securities and long-term securities is analyzed.

3. Land is the major exception to this rule. Other assets must be replaced, rebuilt or refurnished after some time has elapsed in order to maintain the assets earning power. This is recognized through depreciation whereby the firm's future service potential is adjusted regularly.

4. Typically expenses (and current liabilities) accumulate before sales (and current assets) can accumulate. These comments are equally valid, whether the firm is converting physical or human capital into cash flows and profits.

5. Miller and Orr (6) use a stochastic set of flows in their analysis and link the appropriate account management policies to the variance of the value of the sales of the firm and include both short-term and long-term assets and an opportunity cost in their analysis. However, they effectively assume that the firm has an infinite supply of these assets and can always replenish the account in question without affecting the basic risk structure of the firm.

6. See Beaver (2) and Altman (1) for two empirical studies of bankruptcy (the reduction of the permanent level of income to a negative level thereby absorbing any equity in the firm). Both studies suggest that the current asset or working capital measures may contain some information about the fate of the firm.

7. One possible exception in this area is the defensive interval where projected flows are considered.

8. The presence of complete secondary market, the lack of transaction costs and the divisibility of assets enables any corporation or individual to develop costlessly a portfolio of any size based on the riskfree asset and the market portfolio thereby eliminating unique risks. See Sharpe (8).

9. See Fama (3) and Sharpe (8) for materials about this model.

10. Here and throughout the paper it is assumed that accounting measures of value are equivalent and equal to economic measures of value. This is a relatively minor problem for the current portion of the balance sheet, but could be a larger problem for the long-term portion of the balance

sheet. Alternatively, it could be assumed that there is some constant average adjustment factor which turns accountant's long-term asset valuations into economic valuations. This adjustment which might be appropriate if inflation rates are constant, if the aging process of machinery is recognized and constant, and if the firm maintains the same levels of risk and permanent income, would be subsumed in an appropriate measure.

11. The assumption that a firm's current assets exceeds its current liabilities may be too restrictive in reality as some firms, e.g. utilities, often have current assets less than current liabilities. However, in this monopoly situation, the turnover of current assets is high enough so that the pattern of maturities of current assets may be the same as that for the current liabilities. Moreover, this assumption is, at least partially, for convenience. The model can be reworked with the same results, but additional complexity.

12. Insofar as book values of assets reflect market values, book values can be used in the analysis. At the time of purchase book value equals market value. Moreover, if book value of an asset reflects the present value of the service potential of that asset in an equilibrium situation, then book values would reflect market values.

13. The typical firm has a large amount of unique risk. As soon as a firm selects an initial project (and is not a mutual fund), it must consider unique risk. Moreover, typical diversification appears to take place among products and projects closely related with the firm's current projects except in the case of conlomerates (where firms are doing the unnecessary task of diversifying for the investor). One possible reason for this can be seen in the theory of comparative advantage and specialization where investors and corporations are separate entities each exporting their most favorable goods. In this case, the firm and management class export their particular expertise to investors in return for the export of investment funds.

14. Since it is assumed that working capital has less risk than the firm's investment assets their level of systematic risk is generally less than that for the firm's investment assets. However, theoretically the firm could maintain a short position in working capital thereby increasing the risk of the firm.

15. See Hamada (4) and Modigliani and Miller (7).

16. Strictly speaking, there may be some risks and returns in excess of the riskless rate for working capital. However, this is a relatively minor concern for the corporation and inclusion of this would add unnecessary complications to the model. Also, the return to working capital can be in several forms. For example, whereas accounts receivable generate income via finance charges, cash may generate returns via guaranteeing an emergency line of credit or via credits against the service charges of a checking account.

17. This effectively assumes that the riskfree rate is earned on all working capital available at the beginning of the priod only. If a steady inflow or outflow of working capital is assumed, (3.5) is adjusted by a constant.

18. The differential $d\sigma_\gamma$, does not have to equal zero in this case. This particular example is structured so that there is no change in the systematic risk of the firm's investment assets or $d\beta_{IA} = 0$ and the expected return and price of these assets remains constant. In this case any change in the investment asset turnover ratio is unsystematic or total risk could change. In this situation increases in the investment asset turnover ratio could counteract any increase in safety due to the increased working capital.

19. There are many industries where the effort to take full advantage of a strong economy has led to an attempt to increase profits. In turn, this leads to increasingly efficient usage of total assets (through increased investments) and then to severe over-capacity problems when the economy enters a recession or when the industry falls from favor.

20. For example, it could be assumed that the forced sale of committed current assets or of long-term assets to cover a shortage of working capital can be made quickly at 50 cents on the dollar. In this case $2 of assets are required to cover a $1 deficiency in working capital. The resultant decrease in investment assets will have a detectable effect on future income.

21. See Modigliani and Miller (7) for development of the homemade leverage argument.

22. It is effectively assumed that dividends are paid continuously and that transferals of working capital into dividends and investment assets is costless.

23. Because of the assumption of continuous investment of new funds there were never any excess funds in the firm which might have, at least temporarily, served as a buffer to protect the firm's permanent income. However, if a firm intends to let funds accumulate before it makes periodic investments, then these expected inflows could immediately serve to protect the firm's income.

24. If w is between 0 and 1 then (w-1) is negative. It is assumed that β_{IA} is positive. Then $(w-1)\ \beta_{IA}\ w^2\sigma_m^2$ and $(w-1)\ \sigma_u\ (IA)$ are positive as is the denominator. However, if w is greater than 1 as is often the cause for utilities with negative working capital, then coefficients of $d\beta_{IA}$ and $d\sigma_u(IA)$ are positive. Even in this case the interactions described in the text are suggestive of what would occur although the sign of dw changes.

25. The analysis of the interaction of working capital and the firm has been limited to an analysis of the risk associated with miscellaneous asset structures of the firm. However, investors do not purchase these assets

directly when they invest in firms. In particular, Hamada (4) shows that the return required by owners of the firm in an equilibrium situation is conditioned by the firm's financial and asset structure as R_c, the equilibrium return on equity, is

$$R_c = R_{TA} + (R_{TA} - R_F)\ (D/E)$$

where R_{TA} is the equilibrium return on total assets in the non leverage case and D/E is the debt to equity ratio. From (2.1) this equation can be restated as $\beta_c = \beta_{TA}(1 + D/E)$. If the current liabilities of the firm, CL, represent the total debt of the firm, then the systematic risk associated with the common stock of the i^{th} firm, ${}_i\beta_c$, is

$${}_i\beta_c = {}_i\beta_{TA}(1 + CL_i/E_i)$$

where $CL_i + E_i = TA_i$ and, just as working capital generates a return R_F, committed current assets have a return R_F and current liabilities have a cost, R_F.

In the first analysis ${}_1\beta_{TA}$ and TA_2 are greater than ${}_2\beta_{TA}$ and TA_1 respectively. Since current liabilities are the same for each firm, E_1 is less than E_2 and the systematic risk associated with the common stock of firm 1 is more than that for firm 2. In the second example, ${}_1\beta_{TA}$ and CL_1 are greater than ${}_2\beta_{TA}$ and CL_2 due to the increased presence of investment assets in firm 1. Moreover, E_1 is less than E_2 as the committed current assets included in the extra investment assets of firm 1 are financed by the extra current liabilities and the total assets of the firms are the same. In this case the common stock of firm 1 is riskier than that for firm 2. In each case, the ordering of the relationship observed here can be traced directly to the firm's working capital-investment asset decision the same results can be obtained for any level of debt financing common to both firms.

26. See Miller and Modigliani (5) for a discussion of the irrelevance of dividends.

27. This difficulty does not bother the stockholders as they can diversify away any unsystematic risk not diversified away by the company.

28. If the transferal of assets is as described, then the current breakdown may, by coincidence, be approximately correct in a given situation.

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