On Corporate Debt Maturity Strategies

by

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I. Introduction

The literature of finance is replete with analyses of the corporate financing decision with regard to the optimal mix of debt and equity (e.g.: [4], [8], [13], [18]). Most of this literature does not distinguish between debt of different maturities;[1] yet, given the decision to use debt, each time the firm contemplates borrowing to meet its need for capital, it faces a decision regarding the term to maturity for its debt. The debt maturity decision involves a consideration of both cost and risk elements. This paper will explore one dimension of the risk associated with different maturity policies: the effects of bond maturity upon the variance of net income, and subsequently on the firm's cost of equity capital.

One element of the risk of borrowing is the risk that the firm's cash inflows will not be sufficient to cover the fixed outflows necessary to service the debt. One way in which firms attempt to deal with this risk is to follow a hedging policy whereby the maturity of the debt is chosen so as to approximately equal the life of the asset. By matching debt maturity to asset life the costs of financing the asset are known over the life of the asset, and it is expected that the cash flows generated by the asset will be sufficient to service and retire the debt by the end of the asset's life.[2] Debt of maturity shorter than asset life is considered more risky since there is some possibility the asset will not have generated sufficient cash flows by the maturity date to retire the debt. This possibility exists for longer maturities, but it is less likely and has the advantage of pushing the possible "crisis at maturity"[3] further into the future. Debt of maturity longer than the asset life is considered risky due to the uncertainty of the source and volume of the cash flows which are necessary to service the debt after the asset is retired.

Although these risks of departing from a hedging strategy are real, there may be countervailing reasons for not following the hedging strategy. For example, under certain circumstances, departing from the hedging maturity policy may actually reduce the risk to the equity stockholder which is inherent in the use of leverage to finance the firm. It is to this possibility that the analysis of this paper is directed. Without analyzing the expected costs associated with the different maturity policies, this paper develops a model which shows that the use of a short maturity policy may reduce the variance of net income, and thus, reduce the risk to the stockholder. If the covariance between the firm's net operating income and interest rates is sufficiently high, then use of short term debt may cause total interest costs to take on the character of variable, rather than fixed, costs so that the firm is not so subject to the tyranny of the "break even point" which is so often associated with fixed costs and the use of leverage.

II. The Effect of Debt Maturity on Variance of Net Income

First we consider the financing of a long term asset where we assume that the level of net operating income produced by the asset and short term interest rates are random variables which are normally distributed, independent and identically distributed between periods.

a. The Hedging Maturity Policy

An asset with an L-period life is expected to generate net operating income (earnings before interest and taxes), denoted by N, each period for the L periods. Assume N is a normally distributed random variable with mean $\overline{N}$, and variance σ_N^2, and N is independent from one period to the next and identically distributed in every period. With a pure hedging maturity policy the debt financing is obtained at the beginning of the asset's life and remains outstanding until the asset is retired. The interest cost in each period is a known constant equal to $I = r(L)B$, where $r(L)$ is the rate on the L period loan and B is the size of the loan. In each period we have expected net income

$$\overline{NI} \equiv E(NI) = (\overline{N}-I)\ (1-T) \tag{1}$$

and variance of net income

$$\sigma^2_{NI} = (1-T)^2\ \sigma^2_N \tag{2}$$

where NI denotes net income and T is the tax rate.

With the hedging policy the debt maturity decision affects expected net income by determining I, but since interest costs are fixed over the L-period horizon, variance of net income is independent of the interest payments and independent of the proportion of the total cost of the asset financed by debt. The mean and variance of net income with the hedging maturity policy serves as a standard of comparison for a shorter maturity policy. For simplicity we will consider a 1-period maturity policy.

b. Short Term Maturity Policy

Assume that the L-period asset is to be financed with a sequence of 1-period loans: each period a loan matures and another 1-period loan is obtained at the interest rate prevailing at that date. Interest rates vary over time so that, on an *a priori* basis, each period's interest rate is a random variable, although once the loan is obtained, it is at the currently prevailing riskless rate. Assume that the interest rate on a 1-period loan, $r(1)$, is a normally distributed random variable with a mean $\overline{r(1)}$ and variance $\sigma^2_{r(1)}$; the distributions are independent and identical in each period. With B dollars borrowed each period, the interest payments are independent, identically distributed random variables with mean $\overline{I} = B \cdot \overline{r(1)}$, and variance $\sigma_I^2 = B^2 \sigma_{r(1)}^2$.

Since future interest rates are unknown, one might be inclined to assume that the variance of net income is greater with the short (1-period) maturity policy than with the longer (hedging) policy. Although this can be true, it is not necessarily true, and in fact it may be the case that a short maturity policy will result in lower variance of net income. Given our normality assumptions, and calculating expected net income and variance of net income for the 1-period maturity policy we have:

$$\overline{NI} = \overline{N}(1-T) - \overline{I}(1-T) \tag{3}$$

and

$$\sigma_{NI}^2 = E\left\{\left[(N-I)\ (1-T) - E\Big((N-I)\ (1-T)\Big)\right]^2\right\}$$

$$= (1-T)^2\ \sigma_N^2 + (1-T)^2\ \sigma_I^2 - 2(1-T)^2\ \mathrm{Cov}(N,I). \tag{4}$$

If interest rates are independent of, or negatively correlated with N ($\mathrm{Cov}(I,N) \leq 0$), then we can say unambiguously that the variance of net income is greater with the 1-period maturity policy than with the L-period (hedging) maturity policy. That is, with $\mathrm{Cov}(I,N) \leq 0$, the sum of the terms $(1-T)^2\sigma_I^2 - 2(1-T)^2\ \mathrm{Cov}(I,N)$ will be positive so that $\sigma_{NI}^2 > (1-T)^2\ \sigma_N^2$, where the latter term is the variance of net income associated with the hedging policy as noted in expression (2). Under these circumstances, a 1-year maturity policy would increase the financial risk for the equity stockholders, and thus may increase their required rate of return. The company might be willing to undertake this extra risk if the rate on one year loans was expected to be considerably lower than the rate on the long term loans used for the hedging policy.

It is likely that the covariance between N and interest rates would be positive ($Cov(N,I) > 0$).[4] That is, we observe interest rates reaching their peak during periods of economic prosperity when corporate revenues also are at their peak, and during recessions interest rates and corporate revenues tend to go down together. With positive Cov(N,I), and annual refinancing of debt, the variations in net income would tend to be dampened. When N is high, interest charges would tend to be high since interest rates are high, thus the increase in NI would be mitigated. During a recession when N is low, interest rates and thus interest charges would be low, limiting the decrease in Net Income. With interest costs behaving more like variable than fixed costs, there is less risk of dropping below the break even point which is usually associated with fixed costs.

When an asset with a life of 1-period, which must be replaced every period over the planning horizon, is financed with debt, the hedging strategy involves a sequence of 1-period loans, and the same type of covariance effects are present as are associated with the short maturity policy for financing longer lived assets. Conversely, if the short term asset is financed with longer term borrowing, then interest costs are stabilized over several lives of the asset, and the covariance effects of short term borrowing are eliminated.

For ease of exposition the preceding analysis made the simplifying assumption that <u>levels</u> of net operating income and interest rates were normally distributed. This analysis is easily expanded to a more general case where first differences of net operating income ($\Delta N_t = N_t - N_{t-1}$) and interest rates ($\Delta r_t = r_t - r_{t-1}$) are normally distributed, independent and identically distributed between periods. The foregoing analysis is a special case of

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While this paper is not intended as a statement of positive financial theory to be tested with empirical analysis, one can ask whether there is any evidence that companies with high values of Cov(N,r) (or $\rho_{N,r}$) attempt to reduce variability of net income by following a short maturity policy. If such a strategy is used then we would expect that companies with high, positive values of $\rho_{N,c}$ (or $\rho_{\Delta N,\Delta c}$) would have a larger proportion of their assets financed with short term debt. This proposition was investigated with a simple linear regression, and the results indicate that the correlations have a negligible influence on the corporate debt maturity policy.[6]

III. Debt Maturity and the Cost of Capital

We can relate the maturity decision to the firm's cost of equity capital and the market value of the equity in the context of the Sharpe [17] -Lintner [9] capital asset pricing model. Let the required rate of return on the firm's stock (the cost of equity capital) be denoted by R, where, according to the capital asset pricing model

$$E(R) = r_f + \lambda \, \mathrm{Cov}(R, R_m), \tag{5}$$

where r_f is the single period riskless rate, R_m is the return on the "Market Portfolio," and $\lambda = (E(R_m) - r_f)/\sigma^2_{R_m}$ is the "Market price of risk."

Using assumptions similar to Hamada [4], let $E(R) = E(NI)/V$, where V is the total market value of equity at the beginning of the period and NI is net income, then we can rewrite (5) as this more general approach.[5] Using first differences we get the same basic conclusions regarding the effects of the covariance between ΔN and Δr upon the variance of $\Delta NI = NI_t - NI_{t-1}$ as were obtained previously.

While this paper is not intended as a statement of positive financial theory to be tested with empirical analysis, one can ask whether there is any evidence that companies with high values of $Cov(N,r)$ (or $\rho_{N,r}$) attempt to reduce variability of net income by following a short maturity policy. If such a strategy is used then we would expect that companies with high, positive values of $\rho_{N,c}$ (or $\rho_{\Delta N,\Delta c}$) would have a larger proportion of their assets financed with short term debt. This proposition was investigated with a simple linear regression, and the results indicate that the correlations have a negligible influence on the corporate debt maturity policy.[6]

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$$E(R) = r_f + \lambda \text{ Cov}(\frac{NI}{V}, R_m)$$

$$= r_f + \frac{\lambda}{V} \text{Cov}(NI, R_m)$$

$$= r_f + \frac{\lambda}{V} \text{Cov}(N(1-T)-I(1-T), R_m)$$

$$E(R) = r_f + (1-T) \frac{\lambda}{V} \left[\text{Cov}(N, R_m) - \text{Cov}(I, R_m)\right] \quad (6)$$

Since $I = B \cdot r$, we have

$$E(R) = r_f + (1-T) \frac{\lambda}{V} \left[\text{Cov}(N,R_m) - B \cdot \text{Cov}(r,R_m)\right] \quad (7)$$

where r denotes the interest rate on the firm's debt, B.

With long term borrowing the interest cost is fixed and $\text{Cov}(I,R_m) = 0$ in every period over the life of the debt, and

$$E(R) = r_f + (1-T) \frac{\lambda}{V} \text{Cov}(N,R_m). \quad (8)$$

Even though this is a single period model, and after the firm obtains its loans, r is not a random variable; on an a priori basis, with short term borrowing, the interest rate in the debt can be considered a random variable. Thus, with short term borrowing the systematic risk of the firm's stock is measured by

$$\text{Cov}(R, R_m) = (1-T) \frac{1}{V} \left[\text{Cov}(N,R_m) - B \cdot \text{Cov}(r, R_m)\right].$$

If $\text{Cov}(r,R_m) > 0$, then the systematic risk of the equity is reduced with short term borrowing, and the cost of equity capital, E(R), will be reduced below the level consistent with long term borrowing as indicated by (8).

At first glance it appears to be unsatisfactory to apply this single period capital asset pricing model to what is inherently a multi-period

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$$= r_f + \frac{\lambda}{V} Cov(N(1-T)-I(1-T), R_m)$$

$$E(R) = r_f + (1-T) \frac{\lambda}{V} \left[Cov(N, R_m) - Cov(I, R_m) \right] \quad (6)$$

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$$Cov(R, R_m) = (1-T) \frac{1}{V} \left[Cov(N,R_m) - B \cdot Cov(r, R_m) \right].$$

If $Cov(r,R_m) > 0$, then the systematic risk of the equity is reduced with short term borrowing, and the cost of equity capital, E(R), will be reduced below the level consistent with long term borrowing as indicated by (8).

At first glance it appears to be unsatisfactory to apply this single period capital asset pricing model to what is inherently a multi-period problem. But, Bogue and Roll [1] provide the multi-period framework for analyzing the debt maturity problem in the capital asset pricing context.[7] Assume that expected dividend is equal to expected net income, and following the Bogue and Roll development, we have for two periods

$$V_1 = \frac{E(NI_2) - \lambda_1 \, Cov(NI_2, V_{m2})}{1 + r_{f1}} + \frac{E(V_2) - \lambda_1 \, Cov(V_2, V_{m2})}{1 + r_{f1}} \qquad (9)$$

and

$$V_o = \frac{E(NI_1) - \lambda_o \, Cov(V_1, V_{m1})}{1 + r_{fo}} + \frac{E(V_1) - \lambda_o \, Cov(V_1, V_{m1})}{1 + r_{fo}} \qquad (10)$$

where

V_{mt} = market value of the "Market Portfolio" as of the end of period t,

V_t = market value of the equity of the firm as of the end of period t,

$\lambda_t = \left[E(V_{m,t+1}) - (1 + r_{ft})\, V_{mt}\right] / \sigma^2_{Vmt}$ = the market price per unit of risk evaluated at time t.

r_{ft} = the risk free interest rate for the one period from t to t+1.

As of t = 0, V_1, NI_2, λ_1, $Cov(NI_2, V_{m2})$, and r_{f1} are all random variables. Rearranging (9) and taking expectations with respect to probability beliefs held at t = 0 (denoted by E_o), we have

$$E_o(V_1) = \frac{E_o(NI_2) - E_o\left[\lambda_1 \, Cov(NI_2, V_{m2})\right]}{E_o(1 + r_{f1})} - \frac{Cov_o(V_1, r_{f1})}{E_o(1+r_{f1})}$$

$$+ \frac{E_o(V_2) - E_o\left[\lambda_1 \, Cov(V_2, V_{m2})\right]}{E_o(1+r_{f1})} \quad , \qquad (11)$$

where the subscript on $Cov_o(V_1, r_{f1})$ denotes that the covariance is evaluated according to probability beliefs held at t = 0.

Substituting (11) into (10) we have

$$V_o = \frac{E_o(NI_1) - \lambda_o \, Cov(NI_1, V_{m1}) - \lambda_o \, Cov(V_1, V_{m1})}{1 + r_{fo}} +$$

$$+ \frac{E_o(NI_2) - E_o\left[\lambda_1 \, Cov(NI_2, V_{m2})\right] + E_o(V_2) - E_o\left[\lambda_1 \, Cov(V_2, V_{m2})\right]}{(1+r_{fo}) \, E_o(1+r_{f1})}$$

$$- \frac{E_o\left[Cov(V_1, r_{f1})\right]}{(1+r_{fo})E_o(1+r_{f1})} \quad . \tag{12}$$

Now, let NI = N-I, where N is net operating income and I is the interest cost, and where we will ignore taxes to simplify the notation. Noting that $Cov(N\text{-}I, V_m) = Cov(N,V_m) - Cov(I,V_m)$, we have

$$V_o = \frac{E_o(N_1) - \lambda_o \, Cov(N_1, V_{m1})}{1 + r_{fo}} - \frac{E_o(I_1) - \lambda_o \, Cov(I_1, V_{m1})}{1 + r_{f1}} - \frac{\lambda_o \, Cov(V_1, V_{m1})}{1 + r_{fo}}$$

$$+ \frac{E_o(N_2) - E_o\left[\lambda_1 Cov(N_2, V_{m2})\right]}{(1+r_{fo}) \, E_o(1+r_{f1})} + \frac{E_o(V_2) - E_o\left[\lambda_1 \, Cov(V_2, V_{m2})\right]}{(1+r_{fo}) \, E_o(1+r_{f1})}$$

$$- \frac{E_o(I_2) - E_o\left[\lambda_1 \, Cov(I_2, V_{m2})\right]}{(1+r_{fo}) \, E_o(1+r_{f1})} - \frac{E_o\left[Cov(V_1, r_{f1})\right]}{(1+r_{fo})E_o(1+r_{f1})} \quad . \tag{13}$$

Interest rates effects are reflected (for the second period) in two countervailing ways. The last term of (13) takes account of the systematic relation between the firm's intermediate (period 1) market value and the capitalization rate, r_{f1}. If $Cov(V_1, r_{f1}) > 0$, then the value of equity is less than it would be otherwise. This risk is not directly affected by the firm's debt maturity policy. The next to the last term in (13) is influenced by the firm's maturity policy, and has an effect opposite to that of $Cov(V_1, r_{f1})$. This term

$$E_o\left[\lambda_1 Cov(I_2, V_{m2})\right] = E_o\left[B\lambda_1 Cov(r_{f1}, V_{m2})\right]$$

reflects the systematic relation between interest costs, $I = rB$, and the value of other assets, V_{m2}. With long term borrowing, interest costs in each period will be fixed so that

$$Cov(I_1, V_{m1}) = Cov(I_2, V_{m2}) = 0.$$

But, if short term borrowing is used, then interest payments (at t=2) become random variables, since the borrowing rate, r_{f1}, is unknown at t=0, so that $Cov(I_2, V_{m2})$ may be nonzero. If $Cov(I_2, V_{m2}) > 0$, then the term

$$E_o(I_2) - E_o\left[\lambda_1 Cov(I_2, V_{m2})\right]$$

is reduced and V_o increases relative to its value with longer term borrowing. To the extent that $Cov(I_2, V_{m2}) < 0$, the use of short term borrowing will cause interest costs to vary inversely with the value of the market portfolio, and net income will vary directly with V_m, increasing the systematic risk of the firm's equity.[8] It is important to note, also, that $Cov(I_2, V_{m2})$ is affected by both $Cov(r_{f1}, V_{m2})$ and the amount borrowed, B.

IV. Conclusion

This paper has explored two aspects of the risks associated with alternative debt maturity policies: the effects of debt maturity upon variance of net income, and upon the systematic risk of the firms equity. It was shown that if interest rates are highly correlated with the firm's net operating income, then with short term borrowing interest costs take on the aspect of variable, rather than fixed costs. Thus, even though a policy of financing with a sequence of short term loans increases the uncertainty of the interest costs in future periods, it can, under certain conditions, decrease the uncertainty of net income by decreasing the variance of net income. For example, with a high covariance between interest rates and net operating income, in recessionary periods interest rates and net operating income may tend to decrease simultaneously, so that with a short maturity policy interest costs will decrease with net operating income, mitigating the decline in net income. In periods of prosperity NOI and interest rates may tend to increase together so that the increase in net income is limited. The result is that, with the short term borrowing policy, the variation of net income tends to be smaller for those firms where the covariance of NOI and interest costs is large. These same conclusions can be stated in terms of the first differences in interest rates, net operating income and net income.

Relating debt maturity to asset life, a hedging policy where the debt maturity is approximately equal to the asset life is not necessarily the least risky maturity policy. For long term assets, the hedging policy decreases the uncertainty of interest costs over the life of the asset, but a shorter debt maturity policy may decrease the uncertainty of net income derived from the asset. For an asset with a short life, a hedging policy would involve short term borrowing, and would offer the potential benefits of less variable net income, available when there is a high covariance between Net Operating Income and interest costs. If short term assets, to be purchased in sequence over the long term are financed with long term debt, the level of interest costs are fixed and certain over the life of the debt, and if interest rates are negatively correlated with the income produced by the asset, this long term borrowing policy could reduce the variance of net income.

In the context of the capital asset pricing model it was noted that the covariance between interest rates and the return on the market portfolio was positive, then the systematic risk of the stock could be reduced with short term borrowing. In the multi-period context of the Bogue and Roll model it was shown that if the covariance between short term interest rates and the value of the market portfolio was positive, then short term borrowing would increase the market value of equity.

FOOTNOTES

*Assistant Professor of Finance, The Wharton School of the University of Pennsylvania. Financial support from the Rodney L. White Center for Financial Research is gratefully acknowledged.

[1]Analysis of the debt maturity decision has consisted of work in two separate areas; first, the term structure of interest rates, and second, optimization models of the bond refunding decision. The "unbiased expectations" theory of the term structure of interest rates [12] contends that the current long term interest rate is equal to the average of the current and expected furture short term rates. In the absence of flotation costs, all maturities should have the same expected costs. The "expectations plus liquidity premium" theory of the term structure of interest rates [7], [11], [16], [19], contends that long term rates are systematically higher, on the average, than short term rates, so that a short maturity policy should have a lower expected average cost, prior to considering flotation costs.

Dynamic programming models of the bond refunding decision ([2], [6], [21]) analyze the bond maturity problem as an expected cost minimization problem without considering possible risks associated with different maturity policies.

[2]Grove [3] analyzes the maturity problem in terms of its effect on mean and variance of net worth, and shows that a hedging policy minimizes the variance of net worth. In this context, given an expected change in interest rates, expected net worth can be increased by departing from the hedged position, at the cost of accepting increased variance of net worth.

[3]"Crisis-at-maturity" is discussed by Johnson [5], and Van Horne [19].

[4]Correlation coefficients between r(1) and N, $\rho_{N,r}$ were estimated for each company in a sample of 535 companies, each with total assets exceeding $100 million in 1970. The average of the estimated correlation coefficients was $\bar{\rho}_{N,r}$ = + ,58 with standard error of .39. The 4-6 month prime commercial paper rate prevailing at the beginning of each year was used as a measure of the short term borrowing rate, r(1). Net Operating Income data was obtained from the COMPUSTAT Annual Industrial Tapes, for the period 1952-1970.

Estimates for correlation coefficients were used rather than covariance since covariance ($Cov(N,r) = \rho_{N,r}\sigma_N\sigma_r$) takes into account the standard deviation of Net Operating Income, which is in dollar terms, and is thus a function of the size of the company or industry, making inter-industry comparisons difficult.

[5]The assumption that first differences are normally distributed can be expressed with the models $N_t = N_{t-1} + u$ and $r_t = r_{t-1} + \epsilon$, where u and ϵ are normally distributed random variables. The case where N and r are normal, independent and identically distributed between periods is a special case where $E(u) = E(\epsilon) = 0$.

The assumption that Δr is normally distributed is consistent with the Martingale model used by Kalymon [6] and Roll [15] where $r_t = r_{t-1} + \epsilon$, where ϵ is a normally distributed random variable and $E(\epsilon) = E[\Delta r]$. This is also consistent with Pye's [14] model where interest rates are assumed to follow a Markov process.

[6]The ratio Short Term Debt/Total Assets, STD/TA, (for 1970) was regressed first, against $\rho_{N,r}$ and second, on $\rho_{\Delta N,\Delta r}$ for a cross sectional sample of 535 companies, where the correlations were estimated from data for the period 1952-1970. The results were:

$$STD/TA = .07 - .003\,\rho_{N,r} \qquad R^2 = .0004$$
$$(t=-.46)$$
$$STD/TA = .07 + .02\,\rho_{\Delta N,\Delta r} \qquad R^2 = .01$$
$$(t=2.42)$$

The coefficient in the latter equation is significant at the 1% level. In order to determine if a stronger relationship was evident for firms with particularly high or low values for $\rho_{N,r}$ or $\rho_{\Delta N,\Delta r}$, the same regressions were estimated for a sample of 40 companies on the extremes of distributions of $\rho_{N,r}$ and $\rho_{\Delta N,\Delta r}$. The resulting coefficients were not significantly different from zero at the 10% significance level.

[7]I am indebted to Ahmed Foda of the University of Pennsylvania for pointing out the relevance of the Bogue and Roll model for analyzing the debt maturity decision.

[8]Over a sample period of 58 quarters from January 1960 to June 1974, $Cov(r_f,V_m)$ was negative as shown by the regression of first differences (to correct for auto correlation):

$$(r_t - r_{t-1}) = .00025 - .0135\,(V_{mt} - V_{m,t-1}) \qquad R^2 = .07$$
$$(t = -2.05)$$

where r_t = 1 + the rate of return on 90 day Treasury Bills in quarter t, and V is the aggregate market value of the Standard and Poor's 500 stocks (expressed in \$ trillions) for quarter t.

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