The Demand for Risky Assets

by

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Introduction

A striking characteristic of recent research on capital asset pricing is the paucity of empirical work on the determinants of the market price of risk[1] in contrast to the abundance of work on the interrelationships of the risk premiums among different risky assets. The market price of risk will in part depend on the utility functions of individual investors and, except for a special class of functions, on the distribution of wealth among investors.

The relationship between utility functions and wealth of all investors is obviously basic to constructing an aggregate demand function for risky assets. However, there are a multiplicity of other important potential uses, including the derivation of the form of household consumption or saving functions, measurement of the impact of different government, fiscal or insurance measures on economic welfare, and assessment of the problems in extending single-period financial and economic investment decisions to a multi-period world.

Until recently, most interested economists have seemed to believe that utility functions of individuals were characterized either by constant or decreasing absolute risk aversion and by increasing proportional risk aversion (Section I). The measure of proportional risk aversion used in this paper is the same as the elasticity of the marginal utility of wealth. In recent years, this traditional view of increasing proportional risk aversion has been vigorously questioned by advocates of the log utility function which, while still maintaining decreasing absolute risk aversion, implies that proportional risk aversion is constant rather than increasing.

This paper will, for the first time, systematically exploit cross-sectional data on household asset holdings to assess the nature of households' utility functions. The data used are detailed information on assets, income, and other socio-economic-demographic characteristics for a large sample of households.

Prior to analyzing the survey data, the paper will adapt and extend existing theory to obtain the relationships between the composition of household wealth (both human and non-human) and their utility functions. These relationships are suitable for statistical analyses at both the micro and macro levels (Section II). Part of the empirical work is based upon these survey data (Section III Other work analyzes *ex post* and *ex ante* market returns since the latter part of the 19th century to estimate the market price of risk (Section IV The last part of the paper synthesizes the analyses of the household and market data (Section VI).

Our main conclusions are: First, regardless of their wealth level, the coefficients of proportional risk aversion for households are on average well in excess of one and probably in excess of two. Thus, investors require a substantially larger premium to hold equities or other risky assets than they would if their attitudes toward risk were described by logarithmic utility functions.

Second, the paper concludes that the assumption of constant proportional risk aversion for households is as a first approximation a fairly accurate description of the market place. These first two findings imply that the utility function of a representative investor is quite different from those which have often been assumed in the literature. However, it should be pointed out that our conclusion of constant proportional risk aversion follows from our treatment of investment in housing. Other plausible treatments would imply either moderately increasing or moderately decreasing proportional risk aversion.

Third, under tenable assumptions, including that of constant proportional risk aversion, we develop a simple form of the aggregate equilibrium relationship between the relative demand for risky assets and the market price of risk. To determine the actual values of the required rate of return on risky assets, the market price of risk,and the relative value of risky assets would of course entail the specification of supply conditions. For a constant physical supply of both risky and non-risky assets, the relationship developed in this paper indicates how the required rate of return on risky assets as a whole is determined.

II. Earlier Studies

Economists have generally been convinced that the market utility function has risk aversion properties somewhere between a negative exponential utility function, with constant absolute risk aversion and increasing relative risk aversion, and a constant elasticity utility function, with decreasing absolute risk aversion but with constant proportional risk aversion. The authority generally cited for asserting these bounds for the utility function is Kenneth Arrow.[2]

While no one is likely to argue with the plausibility of decreasing absolute risk aversion, the widely-held assumption of increasing (or at most constant) relative risk aversion is open to question. At the theoretical level, the denial of the tenability of decreasing relative risk aversion is based on the assumed implausibility of a utility function which is unbounded either from above or from below.[3] However, arguments based upon such bounding conditions may be economically empty. The ultimate justification for such an assumption must rest on the empirical data. The only direct evidence we know of that has been cited to support the assumption of increasing or constant relative risk aversion are the studies which conclude that either the income elasticity or the wealth elasticity of demand for cash balances (usually money narrowly or broadly defined) is at least one.[4] Of this evidence, only the wealth elasticity is at all relevant. Moreover, in view of the existence of liabilities or negative assets in investors' balance sheets, it is more appropriate to relate the total of risky assets, rather than the customary cash balances, to net worth, but this does not appear to have been done in the relevant literature.

The available cross-section studies based on household surveys of assets and liabilities seem to point to a wealth elasticity of liquid assets of well below, or close to, one if all tangible assets including consumer durables are included in wealth and lower figures if tangible assets are excluded.[5] The aggregate time-series data reflecting changes in supply and demand conditions, including those arising from changes in wealth distribution among different groups in the population, appear to be much less pertinent than the cross-section information.[6]

In recent years, the major challenge to the widely held assumption of a market utility function characterized by increasing relative risk aversion has been posed by proponents of the log utility function which implies constant relative risk aversion (again associated with decreasing absolute risk aversion). The log utility function has been justified in this literature mainly on grounds of the reasonableness of the implicit growth maximization criterion as contrasted with the alleged unreasonableness of mean-variance efficient portfolios.[7]

More recently, a new set of empirical tests of the implications of the growth-optimum and mean-variance models found that the former performs well in comparison with the latter "but the test results are clouded by the close operational similarity of the two models."[8] Still another set of empirical tests obtained from experimental games suggested that a log utility function might characterize individuals with wealth in excess of $200,000,but below that level increasing relative (and decreasing absolute) risk aversion was exhibited.[9]

III. Theoretical Background for the Empirical Analysis

The literature has shown under various conditions that in equilibrium the market price of risk--the ratio of the expected risk premium on risky assets to the variance of return on those assets--equals the market value of all risky assets times a function of each individual's measure of absolute risk aversion. More precisely, this relationship can be developed for discrete planning periods by assuming either that investors' utility functions are quadratic or that their utility functions are exponential or logarithmic and end-of-period wealth is normally distributed.[10] This relationship takes the form

$$(1) \qquad \frac{E(r_m - r_f)}{\sigma^2(r_m)} = V_{mo}[\sum_k E(\frac{1}{R_k})]^{-1}$$

where r_m is the return on the market portfolio of all risky assets, r_f is the return on the risk-free asset, R_k is Pratt's measure of absolute risk aversion for the k^{th} investor, and V_{mo} is the market value of all risky assets at the beginning of the planning period. Stephen A. Ross has obtained a similar expression by assuming an infinitesimal planning horizon and no finite changes in value of any asset in an infinitesimal period.[11]

Although the various developments of (1) assumed that the net supply of the risk-free asset was zero, they can easily be generalized to the situation in which this asset has a positive net supply. For mathematical convenience, this generalization will be based upon an infinitesmal horizon model and will make the same assumptions as those made by Ross. Besides the assumption about the distribution of returns mentioned above, perhaps the most critical assumptions include homogeneity of expectations and a frictionless capital market. In a frictionless capital market, all financial assets--both risk-free and risky--are infinitely divisible, can be traded with no transaction costs, and can be sold short with the proceeds available to purchase another asset.

In parallel with Ross's development, the wealth conservation equation for the k^{th} investor, assuming no taxes, would be[12]

$$(2) \qquad W_{k,t+dt} = W_{kt}\ \{1 + [r_f + \alpha_k E(r_m - r_f)]dt + \alpha_k \sigma_m y(t)\sqrt{dt}\}$$

where α_k is the proportion of the net worth of investor k placed in the portfolio of risky assets, σ_m is the standard deviation of the returns on the portfolio of risky assets, and $y(t)$ is a standardized normal random variate.[13] By expanding $U(W_{k,t+dt})$ about W_{kt}, taking expected values and dropping terms involving dt to the power of 2 or more, one obtains

$$(3) \qquad E[U(W_{k,t+dt})] = U(W_{kt}) + U'(W_{kt})W_{kt}[r_f + \alpha_k\ E(r_m - r_f)]dt + \frac{1}{2} U''(W_{kt})W_{kt}^2\alpha_k^2\sigma_m^2 dt.$$

To determine the optimal value of α_k, the derivative of E[U] is set to zero:

(4) $$U'(W_{kt})E(r_m - r_f) + U''(W_{kt})W_{kt}\alpha_k\sigma_m^2 = 0.$$

Recalling that Pratt's measure of relative risk aversion, C_k, is defined as $W_{kt}[-U''(W_{kt})/U'(W_{kt})]$,[14] (4) can be rewritten as [15]

(5) $$\alpha_k = \frac{E(r_m - r_f)}{\sigma_m^2} \cdot \frac{1}{C_k}$$

In the absence of taxes, (5) could be used to estimate the coefficient of relative risk aversion for investor k given estimates of α_k and the market price of risk. However, if returns from both risky and risk-free assets are taxed at the same rates in any tax bracket[16] and if t_k is the average rate of tax for investor k, equation (2) could be modified to incorporate the tax effect. Repeating the logic underlying (2) through (5) would yield the tax adjusted relationship:

(6) $$\alpha_k = \frac{E(r_m - r_f)}{\sigma_m^2} \cdot \frac{1}{(1 - t_k)C_k}$$

Since r_m and r_f are measured before personal taxes, the assumption of homogeneous expectations assures that if $(1 - t_k)\alpha_k$ is constant over all investors, C_k will also be constant. This observation will be used in the next section in conjunction with cross-sectional survey data to assess how C_k varies with net worth.

So far, it has been explicitly assumed that all assets are liquid in that they can be purchased or sold at no cost in any quantity. This assumption is clearly unrealistic for human wealth. David Mayers has shown in a mean-variance world of finite horizons how this assumption can be relaxed to allow for the non-marketability of human wealth. His insight can be readily applied to the continuous case. To do this, redefine r_m as the return on the portfolio of liquid risky assets and α_k as the proportion of investor k's liquid net worth (total net worth less human wealth) placed in the portfolio of liquid risky assets. Further, define the new terms: h_k as the ratio of the value of the human wealth of the k^{th} investor to his net worth and r_{hk} as the rate of return on the k^{th} investor's human wealth.

Under these definitions, the wealth conservation equation adjusted for taxes, t_k, again assumed the same for all types of income, would be

$$(7)\qquad W_{k,t+dt} = W_{kt}\Big[1 + (1 - t_k)\{(1 - h_k)[r_f + \alpha_k E(r_m - r_f)] + h_k E(r_{hk})\}dt + (1 - t_k)[(1 - h_k)^2\alpha_k^2\sigma_m^2 + h_k^2\sigma_{hk}^2 + 2\alpha_k h_k(1 - h_k)\mathrm{Cov}(r_m, r_{hk})]^{\frac{1}{2}} y(t)\sqrt{dt}\Big]$$

The covariance term allows for the presence of dependencies between the returns on liquid and human wealth. That both the variance of human wealth and the covariances are subscripted by k means that these statistics can vary from one individual to another. Repeating the logic underlying (2) through (5) would yield

$$(8)\qquad \alpha_k = \frac{E(r_m - r_f)}{\sigma_m^2} \cdot \frac{1}{(1 - t_k)(1 - h_k)C_k} - \frac{h_k}{1 - h_k}\beta_{hk,m} ,$$

where $\beta_{hk,m}$ is the ratio of $\mathrm{Cov}(r_m, r_{hk})$ to σ_m^2. Thus, $\beta_{hk,m}$ can be interpreted as the slope coefficient of the regression of r_{hk} on r_m. It is interesting to note that the ratio of α_k does not depend upon $E(r_{hk})$ but only upon the dependence between r_m and r_{hk}.[17]

Equations (6) and (8) are micro-functions for the individual investor and are not the same kinds of relationships as (1), which is a equilibrium condition for the market as a whole--a macro-economic concept. Such macro-relationships can be developed from these micro-functions by forming a weighted average of the α_k's over all investors, the weights defined as the ratio of the investor's liquid wealth to the total liquid wealth in the market. If γ_k equals this proportion, if α is defined as the ratio of liquid risky assets to all liquid assets, and if all assets are liquid, the micro-relationship (6) would imply

$$(9) \qquad \Sigma \gamma_k \alpha_k = \alpha = \frac{E(r_m - r_f)}{\sigma_m^2} \Sigma \frac{\gamma_k}{(1 - t_k)C_k} .$$

Relation (9), which is easily solved for the market price of risk, differs from (1) mainly in that it takes into account taxes, allows the net supply of the risk-free asset to be positive, and is expressed in terms of relative rather than absolute risk aversion.

If one includes human wealth and further assumes that this wealth is illiquid, a similar macro-relationship to (9) can be derived by averaging the α_k's given by (8). The resulting average is

$$(10) \qquad \Sigma \gamma_k \alpha_k = \alpha = \frac{E(r_m - r_f)}{\sigma_m^2} [\Sigma \frac{\gamma_k}{(1 - t_k)(1 - h_k)C_k}] - \Sigma \gamma_k \frac{h_k}{1 - h_k} \beta_{hk,m} .$$

The macro-relationship (10), again easily solved for the market price of risk, is quite different from (1) and (9) in that it takes explicit account of the non-marketability of human wealth.

It should be noted that equations (9) and (10), and the simplifications of these equations presented subsequently, represent aggregate equilibrium relationships between the relative demand for risky assets and the market price of risk. The market price of risk at the macro level is not given, as it may be at the micro level, but is determined jointly with the aggregate ratio (α) of liquid risky assets to all liquid assets. For example, assuming a constant physical supply of both risky and non-risky assets, α is determined by $E(r_m)$ and r_f. Equations (9) and (10) indicate how $E(r_m)$

and α are related to C_k, the distribution of wealth (γ_k), tax rates (t_k), σ_m^2 and r_f.[18]

If an investor's coefficient of relative risk aversion is assumed independent of γ_k and what almost follows immediately that this coefficient is also independent of his average tax rate, (9) can be vastly simplified. This assumption of constant proportional risk aversion on the average is considerably more general than the assumption that all investors have the same value for C_k in that C_k can be a function of characteristics of investors which are independent of γ_k and t_k.

Specifically, let it be assumed that C_k is given by

$$(11) \qquad \frac{1}{C_k} = \frac{1}{C} + \varepsilon_{ck}$$

where ε_{ck} is a mean-zero disturbance independent of γ_k and t_k. The symbol C defined by (11) can be interpreted as the harmonic mean of C_k. Let us first simplify (9) by substituting (11) into it to obtain

$$(12) \qquad \alpha = \frac{E(r_m - r_f)}{\sigma_m^2} \left[\frac{1}{C} \Sigma \frac{\gamma_k}{1 - t_k} + \Sigma \frac{\gamma_k \varepsilon_{ck}}{1 - t_k}\right]$$

The application of expected value operators to (12) causes the second summation in the brackets to go to zero. Since the sum of the γ_k's is one, the reciprocal of the first summation might be termed the weighted harmonic mean of $(1 - t_k)$. Designating this mean as $1 - t$ and solving for the market price of risk, (12) takes the simplified form

$$(13) \qquad \frac{E(r_m - r_f)}{\sigma_m^2} = \alpha C(1 - t) \ .$$

If in addition to the assumption of constant proportional risk aversion on the average, it can further be assumed that $\beta_{hk,m}$ and ε_{ck} are independent of the level of human wealth of the k^{th} individual, (10) can likewise be vastly simplified. To do this, note first that the terms α, $(\gamma_k h_k)/(1 - h_k)$ and $\gamma_k/(1 - h_k)$ can be rewritten as R/L, H_k/L and W_k/L respectively where H_k is the value of the k^{th} investor's human wealth, W_k his net worth, R the total value of all risky liquid assets, and L the total value of all liquid assets. With these identities, the simplification of (10) proceeds by substituting (11) into (10), taking expected values, substituting these identities, and solving for the market price of risk. The resulting expression is

$$(14) \qquad \frac{E(r_m - r_f)}{\sigma_m^2} = [\frac{R}{W^*} + \beta_{hm} \frac{H}{W^*}]C$$

where W^* is $\Sigma W_k/(1 - t_k)$ or a tax adjusted sum of all wealth and β_{hm} is defined as $E(\beta_{hk,m}|H_k)$, assumed the same for all investors.[19]

These simple equilibrium expressions are valid if investor's utility functions are characterized by constant proportional risk aversion on the average as specified in (11). If valid, (13) and (14) would seem to offer tractable ways to estimate C from time-series data.[20] They also provide extremely simple forms of the aggregate equilibrium relationship between the relative demand for risky assets and the market price of risk.

IV. The Analysis of the Survey Data

The micro-relationships just developed lend themselves to the use of cross-sectional data in ascertaining whether or not C_k, the coefficient of proportional risk aversion, is invariant on the average to investors' net worth. After describing the cross-sectional data, the section will first examine the behavior of C_k employing (6) which implicitly assumes that all wealth is liquid and then will assess it employing (8) which explicitly accounts for the non-marketability of human capital.

The cross-sectional data to be analyzed here come from the 1962 and 1963 Federal Reserve Board Surveys of the Financial Characteristics of Consumers and Changes in Family Finances. These surveys, which oversampled the upper income groups, collected for more than 2,100 households detailed information on the value of their assets and liabilities at the end of both 1962 and 1963 and on the sources and amounts of income in both of these years.

From these data, the study constructed three different types of balance sheets at the end of 1962. The first type included all assets and associated liabilities with the exception of human wealth and homes.[21] The second excluded only human wealth. The third included not only homes, but also an estimate of human wealth. These balance sheets, expressed as ratios to the corresponding measures of net worth, are summarized in Tables 1-3 by net worth categories. Changes in the definition of wealth will, of course, move households from one wealth class to another.

In checking the validity and reasonableness of the data, it was observed that a few households received substantial salaries from closely-held businessess in which they had active interests. Since part of some of these salaries might more properly be classified as dividends or return on capital, any salaries from such closely-held businesses in excess of $25,000 were

TABLE 1-AVERAGE RATIOS OF ASSETS AND SELECTED ITEMS TO HOUSEHOLD NET WORTH EXCLUSIVE OF HOMES (AND ASSOCIATED MORTGAGES) FOR HOUSEHOLDS CLASSIFIED BY NET WORTH[a]

December 31, 1962

Type of Item	Net Worth ($000's) 1-10		10-100		100-200		200-500		500-1000		Over 1000	
RISK FREE ASSETS TO NET WORTH												
CHECKING ACCOUNTS	0.079		0.026		0.020		0.014		0.018		[illegible]	
OTHER CASH BALANCES[b]	0.301		0.192		0.062		0.022		0.027		0.021	
SAVINGS BONDS	0.069		0.040		0.034		0.012		0.012		[illegible]	
LIFE INSURANCE (CASH VALUE)	0.238		0.071		0.042		0.025		0.031		0.013	
OTHER RISK-FREE ASSETS[c]	0.127	0.814	0.053	0.382	0.004	0.163	0.010	0.069	0.017	0.103	0.012	0.057
MIXED-RISK ASSETS TO NET WORTH												
STATE AND LOCAL BONDS	0.0		0.001		0.001		0.013		0.014		0.032	
OTHER MIXED-RISK ASSETS[d]	0.042	0.042	0.077	0.078	0.131	0.132	0.084	0.098	0.064	0.078	0.099	0.131
RISKY ASSETS TO NET WORTH												
COMMON AND PREFERRED STOCK	0.050		0.111		0.272		0.363		0.283		0.316	
EQUITY IN UNINC. BUSINESS	0.081		0.308		0.286		0.370		0.315		0.328	
OTHER RISKY ASSETS[e]	0.140	0.271	0.167	0.586	0.186	0.743	0.104	0.858	0.265	0.863	0.225	0.869
TOTAL ASSETS TO NET WORTH		1.128		1.046		1.039		1.025		1.044		1.057
LIABILITIES TO NET WORTH		0.128		0.046		0.039		0.025		0.044		0.057
ANCILLARY STATISTICS												
$(1-t_k)\alpha_k$ [f]	0.285		0.601		0.749		0.763		0.677		0.650	
NUMBER OF HOUSEHOLDS	523.		477.		68.		100.		69.		105.	

[a] These averages are weighted by the inverse of the sampling probability for each household. These sampling probabilities have been adjusted for non-response and other factors. The net worth used in classifying households is defined in the same terms as the balance sheet.

[b] Includes checking and other commercial bank accounts, savings and loan savings accounts, credit union savings accounts, and mutual savings accounts.

[c] Includes U. S. Treasury bills, notes and certificates, the withdrawal value of profit sharing and retirement plans, credit balances in brokerage accounts, and risk-free assets held in trust accounts. Risky assets held in trust accounts are included in the appropriate category of direct holdings, except that when they could not be classified in this manner, they were included in miscellaneous risky assets.

[d] Includes long-term corporate, state and local and U. S. Government bonds (other than savings bonds).

[e] Includes investment real estate assets and miscellaneous assets, such as patents, etc.

[f] The symbol α_k is defined as the ratio of the sum of the mixed-risk and risky assets to net worth for Investor k. The symbol t_k is the average federal tax rate for Investor k as estimated by the procedure devised by the Survey of Financial Characteristics of Consumers. The only difference was that realized capital gain income or losses were included in adjusted gross income in this paper unlike the original procedure. The original survey ignored this type of income in calculating the tax rate.

TABLE 2-AVERAGE RATIOS OF ASSETS AND SELECTED ITEMS TO HOUSEHOLD NET WORTH INCLUSIVE OF HOMES FOR HOUSEHOLDS CLASSIFIED BY NET WORTH[a]

December 31, 1962

e of Item	Net Worth ($000's) 1-10		10-100		100-200		200-500		500-1000		Over 1000	
K-FREE ASSETS TO NET WORTH												
CHECKING ACCOUNTS	0.040		0.019		0.017		0.012		0.023		0.009	
OTHER CASH BALANCES	0.138		0.109		0.084		0.032		0.023		0.019	
SAVINGS BONDS	0.029		0.025		0.020		0.020		0.0[illegible]		0.003	
LIFE INSURANCE (CASH VALUE)	0.097		0.056		0.033		0.025		0.033		0.013	
OTHER RISK-FREE ASSETS	0.055	0.359	0.033	0.243	0.013	0.167	0.008	0.096	0.010	0.097	0.012	0.056
D-RISK ASSETS TO NET WORTH												
STATE AND LOCAL BONDS	0.0		0.000		0.001		0.005		0.037		0.029	
OTHER MIXED-RISK ASSETS	0.016	0.016	0.038	0.039	0.084	0.085	0.077	0.082	0.043	0.081	0.090	0.119
Y ASSETS TO NET WORTH												
MARKET VALUE OF HOMES	1.539		0.657		0.216		0.098		0.112		0.046	
COMMON AND PREFERRED STOCK	0.012		0.052		0.208		0.289		0.306		0.2[illegible]2	
EQUITY IN UNINC. BUSINESS	0.045		0.147		0.271		0.395		0.235		0.339	
OTHER RISKY ASSETS	0.035	1.631	0.101	0.957	0.108	0.803	0.084	0.866	0.213	0.866	0.215	0.892
ASSETS TO NET WORTH		2.006		1.238		1.055		1.044		1.043		1.007
ILITIES TO NET WORTH												
MORTGAGE LIABILITIES	0.924		0.204		0.041		0.016		0.008		0.003	
OTHER LIABILITIES	0.083	1.006	0.034	0.238	0.014	0.055	0.028	0.044	0.035	0.043	0.064	0.067
LARY STATISTICS												
$(1-t_k)\alpha_k$ (EQUITY IN HOMES)[b]	0.669		0.718		0.724		0.780		0.642		0.665	
$(1-t_k)\alpha_k$ (INVESTMENT IN HOMES)[b]	1.495		0.897		0.758		0.793		0.646		0.667	
NUMBER OF HOUSEHOLDS	459.		731.		94.		103.		76.		10[illegible].	

. footnotes for Table 1.

cludes automobiles.

TABLE 3-AVERAGE RATIOS OF ASSETS AND SELECTED ITEMS TO HOUSEHOLD NET WORTH INCLUSIVE OF HUMAN WEALTH AND HOMES CLASSIFIED BY NET WORTH[a]

December 31, 1962

Type of Item	Net Worth ($000's) 1-10		10-100		100-200		200-500		500-1000		Over 1000	
RISK-FREE ASSETS TO NET WORTH												
CHECKING ACCOUNTS	0.027		0.007		0.003		0.005		0.014		[illegible]	
OTHER CASH BALANCES	0.116		0.042		0.014		0.021		0.019		0.016	
SAVINGS BONDS	0.011		0.011		0.003		0.006		0.005		0.003	
LIFE INSURANCE (CASH VALUE)	0.027		0.013		0.010		0.014		0.021		0.013	
OTHER RISK-FREE ASSETS	0.000	0.181	0.004	0.077	0.006	0.036	0.013	0.061	0.009	0.068	0.012	0.052
MIXED-RISK ASSETS TO NET WORTH												
STATE AND LOCAL BONDS	0.0		0.000		0.000		0.001		0.019		0.023	
OTHER MIXED-RISK ASSETS	0.013	0.013	0.014	0.014	0.008	0.008	0.032	0.033	0.023	0.041	0.070	0.093
RISKY ASSETS TO NET WORTH												
MARKET VALUE OF HOMES	0.470		0.183		0.101		0.104		0.080		0.063	
COMMON AND PREFERRED STOCK	0.002		0.017		0.018		0.085		0.171		0.253	
EQUITY IN UNINC. BUSINESS	0.055		0.070		0.022		0.085		0.220		0.277	
HUMAN CAPITAL	0.334		0.656		0.850		0.649		0.349		0.146	
OTHER RISKY ASSETS	0.018	0.878	0.038	0.964	0.014	1.006	0.035	0.959	0.1[illegible]	0.920	0.169	0.919
TOTAL ASSETS TO NET WORTH		1.072		1.056		1.049		1.052		1.135		1.055
LIABILITIES TO NET WORTH												
MORTGAGE LIABILITIES	0.052		0.044		0.043		0.039		0.013		0.004	
OTHER LIABILITIES	0.021	0.072	0.012	0.056	0.006	0.049	0.013	0.052	0.022	0.035	0.051	0.055
ANCILLARY STATISTICS												
NUMBER OF HOUSEHOLDS	114.		819.		455.		230.		9[illegible].		135.	

[a]Cf. footnotes for Table 2.

valued as perpetuities at ten percent and added to equity. To determine how sensitive the results are to this adjustment, the subsequent analyses were replicated on a subsample which excludes any households which own such businesses. The conclusions were virtually unchanged.[22]

Also only households with a net worth in excess of \$1,000 were included. Since such households have a predominant impact upon the market for assets, this restriction would have little effect upon the value of the variables in the macro-relationships (13) or (14). Yet, it appears to improve the quality of the survey data by eliminating those households who may have under-reported their assets or whose assets were temporarily lower than normal.

A crude estimate of human wealth was given by the discounted value of the average labor income in 1962 and 1963[23] which was then assumed to grow at four percent per year. The discount rate was taken to be ten percent. If a person was less than 65, he was assumed to retire at 65. If he was working and over 65, he was assumed to continue to work for four years; if over 69, three years; if over 74, two years; and if over 79, one year. The sensitivity of the subsequent results to this arbitrary method of calculating human wealth will be examined, and it will turn out that within broad ranges of discount rates and growth rates the results are substantially unaffected.

If the micro-relationship (6) were to hold, an estimate of the market price of risk times the reciprocal of the coefficient of relative risk aversion, $[E(r_m - r_f)/\sigma_m^2]C_k^{-1}$, is given by $\alpha_k(1 - t_k)$. The assumption made in the last section that all investors agree on the value of the market price of risk means that $\alpha_k(1 - t_k)$ provides an extimate of C_k^{-1} up to a multiplicative positive constant, so that $\alpha_k(1 - t_k)$ can be used to assess how C_k^{-1} and, thereby, how C_k varie with net worth.[24] Using either of the first two balance sheets, which do not include human capital, the cross-sectional data provide estimates of $\alpha_k(1 - t_k)$. Since (6) does not take account of the non-marketability of human wealth, the analysis based upon (6) will not include this kind of wealth. It might be mentioned that virtually all empirical applications of portfolio theory have ignored human wealth in spite of its obvious importance to the demand for risky assets.

In interpreting the meaning of these numbers, it should be noted that the derivation of (6) assumed that all assets are acquired for investment purposes, can be unambiguously dichotomized as risky or risk-free, are subject to the same schedule of income taxes, and are infinitely divisible. For analyses of the composition of wealth inclusive of homes, these assumptions may be so unrealistic as to place serious restrictions on the validity of (6).

Contrary to the implicit treatment of assets in our model and in capital asset pricing theory generally, homes are typically acquired by households for consumption as well as investment purposes. Moreover, the lack of tax on the imputed income from owning a home as well as other subsidies make the tax rate assumption highly questionable. Furthermore, a house involves a substantial investment and for a person with limited liquid assets, the incurrence of more debt than he might normally want to have. In the absence of an adequate rental market, the purchase of a home may be the only way of obtaining the desired type of housing. Holding utility functions constant, these factors would tend to increase the values of α_k for the lower wealth groups relative to those for the upper wealth groups if the investment in a home is assumed to be risky.[25]

Since investments in homes are not strictly compatible with the assumptions underlying (6), homes have been treated in three different ways in the empirical work--not as an investment asset, as a risky asset where the relevant investment is measured by the household's equity in the home, and as a risky asset where the investment is measured by the gross market value of the home. Another possibility would be to assume that households consider homes to be riskless assets but this seems a less tenable assumption than the ones we have utilized.

Within the context of the model embodied in (6), perhaps the most appropriate measure of a household's investment in housing is its equity value. First, if an investor views a mortgage and a house as a package, it may well be that he regards only his equity at risk. Second, the use of equity is more in the spirit of infinitely divisible assets than the use of total assets. Nonetheless, the results of alternative assumptions are presented.

The paper presents in Tables 1-2 averages of the estimated tax adjusted ratio $(1 - t_k)\alpha_k$ by net worth classes for varying treatments of housing. Human wealth is not included. In addition, Table 4 presents the slope coefficients of the regressions of $(1 - t_k)\alpha_k$ on the logarithm of net worth.[26] The α_k's in these tables are ratios of risky and mixed-risk assets to net worth. Both simple regressions and regressions which control with dummy variables for the possible interaction between net worth and three socio-economic factors (age, education and occupation) are presented. Whether or not these dummy variables are included, the slope coefficients on the logarithm of net worth are not much different.[27]

The relationship of $(1 - t_k)\alpha_k$ to net worth varies according to whether housing is included, and if included, how measured. For the narrowest definition of wealth exclusive of human wealth and homes, there is overall a

TABLE 4-REGRESSIONS OF $(1 - t_k)\alpha_k$ ON THE LOGARITHM OF NET WORTH WITH AND WITHOUT HOLDING CONSTANT AGE, EDUCATION AND OCCUPATION

December 31, 1962

Net Worth Including Homes	Housing[a] Measured by	Simple Regression			Regression with Dummy Variables for Age, Education and Occupation[b]		
		Coefficient on ln(NW)		$\bar{R}^2$	Coefficient on ln(NW)		$\bar{R}^2$
		estimate	t-value		estimate	t-value	
No		0.063	11.27	0.09	0.055	7.76	0.15
Yes	Equity	-0.006	-1.52	0.00	-0.001	-0.23	0.05
Yes	Full Value	-0.185	-15.30	0.13	-0.155	-9.77	0.18

[a] Includes automobiles.

[b] The dummy variables, assuming the value of 1.0 if the characteristic is applicable to the head of the household and 0.0 otherwise, are for age (less than or equal to 25 and 5-year intervals from 25 to 65), for education (grade school or less, some high school, high school graduate, some college, and college graduate), and for occupation (self-employed, employed by others, retired, farm operator, not gainfully employed, and employer unknown).

significant positive relationship to net worth (Table 4). A closer analysis, however, indicates that most of this positive relationship stems from households with net worth of less than $500,000 and that for households of greater means, the ratio shows a slight tendency to decrease (Table 1). This behavior would be consistent with decreasing proportional risk aversion at least through net worth classes up to $500,000. This result is not affected appreciably if mixed-assets were combined with risk-free rather than with risky assets. For the broader definition of wealth including housing as a risky asset but excluding human wealth, the ratio of $(1 - t_k)\alpha_k$ is practically invariant to net worth if housing is measured by its equity value, and negatively related to net worth if housing is measured by its full value (Table 4). These findings are consistent with constant proportional and increasing proportional risk aversion respectively.[28] If housing is included as a risk-free asset, the tendency toward decreasing proportional risk aversion would be even greater than indicated by Table 1.

There are several potentially important biases in these findings. First, contrary to the assumptions of the model, individuals do not all hold the same portfolio of risky assets. Consider an individual whose portfolio of risky assets is perfectly correlated with the market portfolio of risky assets but is less volatile--that is, has a portfolio beta coefficient of less than 1.0. Since the risky portfolio of reference is the market portfolio, the calculated value of α_k would overstate the relative degree of risk in his portfolio. Likewise, if he were to hold a portfolio of risky assets with a beta greater than one, the calculated value of α_k would understate the degree of risk. The evidence in M. E. Blume and I. Friend indicates that households with lower incomes and presumably less wealth tend to hold portfolios of stocks with smaller portfolio betas than their richer counterparts although their portfolios ~e less well diversified.

Second, life nsurance is not handled satisfactorily in this analysis since only the immediat y realizable portion is treated as a risk-free asset. Life insurance does nc conveniently fit into the dichotomy of risk-free and risky assets, and some ot mode of analysis is required to handle appropriately this type of asset. H ever, the net surrender value of life insurance probably represents a signif ant understatement of the value of insurance as a device for reducing risk. S ce life insurance is relatively much more important in the lower than in the upper et worth classes, correction for this deficiency of the model would again tend change the results in the direction of decreasing proportional risk aversion.

Third, the ca culated α_k might differ from its permanent value because of transitory elemer If a transitory reduction in net worth left unchanged the value of risky as s, the calculated α_k would be biased upwards. This situation might apply to ar dividual who was faced with a temporary reduction in net worth and because of tra action costs chose to reduce his risk-free assets. However, if he reduced only risky assets, the calculated α_k would be biased downwards. Since the former case is probably more prevalent for families of lower net worth and this type of transitory phenomena is more likely to apply to these households, correcting for this possible bias would be expected to change the results in the same way as the previous bias.

Fourth, the model made the assumption that the tax rate on risky assets was the same as that on risk-free assets. In fact, the tax rate on risky assets tends to be less than that on risk-free assets. While the magnitude of this bias seems to be small, it results in an understatement of C_k.[29] However, it is not clear how this understatement varies with net worth.

To this point, the empirical analyses have ignored human wealth. The micro-relationship (8) incorporates such wealth and can be used to examine the behavior C_k. Rearranging (8) yields

$$\text{(15)} \qquad \frac{E(r_m - r_f)}{\sigma_m^2} \, \frac{1}{C_k} = (1 - h_k)(1 - t_k)\alpha_k + h_k(1 - t_k)\beta_{hk,m}.$$

If $\beta_{hk,m}$ were known, the right-hand side of (15) could be used to analyze the relationship of C_k^{-1} to net worth. The variable $\beta_{hk,m}$, however, is not known, so that some other tack must be taken.

This other tack will be to assume the validity of two stochastic relationships and use these in conjunction with (15) to derive an empirically estimatable regression. For net worth class i, the first relationship takes the form

$$\text{(16)} \qquad (C_k)^{-1} = (C^i)^{-1} + \eta_{ck}$$

where η_{ck} is a mean-zero disturbance independent of h_k and t_k, and $(C^i)^{-1}$ is the harmonic mean of the C_k's in the i^{th} net worth class. This assumption is like the assumption of constant proportional risk aversion on the average, as embodied in (11), except that it only pertains to a single net worth class. Again for net worth class i, the second relationship takes the form

$$\text{(17)} \qquad \beta_{hk,m} = \beta^i_{hm} + \eta_{hk}$$

where η_{hk} is a mean-zero disturbance independent of h_k and t_k, and β^i_{hm} is $(C^i)^{-1}$ the mean of the $\beta_{hk,m}$'s in the i^{th} net worth class.

The substitution of (16) and (17) into (15) with some simplification yields the regression

$$(18) \qquad (1 - h_k)(1 - t_k)\alpha_k = \frac{E(r_m - r_f)}{\sigma_m^2} \frac{1}{C^i} - \beta_{hm}^i h_k(1 - t_k) + \mu_k$$

where μ_k is defined to be $[E(r_m - r_f)/\sigma_m^2]\eta_{ck} + h_k(1 - t_k)\eta_{hk}$. The independent variable, $h_k(1 - t_k)$, and the dependent variable, $(1 - h_k)(1 - t_k)\alpha_k$, can be estimated from the cross-sectional data. The constant term allows an analysis of the relationship of the coefficient of proportional risk aversion to net worth class, and the slope coefficient gives an estimate of β_{hm}^i. It is easily verified that under the assumed stochastic processes, μ_k has an expected value of zero and is uncorrelated with the independent variable, so that the estimated coefficients will be unbiased.[30]

Using the total sample of households, regression (18) was first estimated for each of the six net worth classes inclusive of human wealth and with homes measured by their equity value (Table 5).[31] If unbiased, the constant terms indicate that the coefficient of proportional risk aversion on the average increases with net worth. The magnitude of the increase is not large. The estimate of $(C^i)^{-1}$ for those households with net worth in excess of one million dollars is 33 percent less than the estimate for those households with net worth between one and ten thousand dollars. The estimates of β_{hm}^i tend to decrease as net worth increases starting at 0.775 for the lowest net worth class and ending at 0.603 for the highest net worth class.

There are several potentially important biases in these results. The first type pertain to violations of the assumptions used in developing (18). The second type pertain to limitations of the survey data. Turning to the first type, the assumptions implied that μ_k, which is a weighted sum of the

TABLE 5-ESTIMATED COEFFICIENTS AND OTHER STATISTICS FOR REGRESSIONS OF THE FORM[a]

$$(1 - h_k)(1 - t_k)\alpha_k = \frac{E(r_m - r_f)}{\sigma_m^2}\frac{1}{c^i} - \beta^i_{hm} h_k(1 - t_k) + \eta_k$$

Net Worth Class ($000)	Estimates of $\frac{E(r_m - r_f)}{\sigma_m^2}\frac{1}{c^i}$	Estimates of β^i_{hm}	$\bar{R}^2$	Unweighted Means $(1 - t_k)\alpha_k$	Unweighted Means $(1 - h_k)(1 - t_k)\alpha_k$	Unweighted Means $h_k(1 - t_k)$	Unweighted Means $(1 - t_k)$	Number of Households
A. Total Sample[b]								
1-10	0.742 (0.035)	0.775 (0.069)	0.52	0.582	0.496	0.319	0.982	114
10-100	0.707 (0.009)	0.775 (0.014)	0.80	0.595	0.255	0.583	0.912	819
100-200	0.611 (0.010)	0.697 (0.013)	0.85	0.675	0.145	0.668	0.867	485
200-500	0.612 (0.016)	0.743 (0.029)	0.76	0.602	0.292	0.431	0.814	200
500-1000	0.529 (0.020)	0.613 (0.072)	0.44	0.584	0.412	0.192	0.705	90
over 1000	0.499 (0.021)	0.603 (0.146)	0.11	0.525	0.440	0.097	0.616	135
B. Households with Income from Salaries or Wages								
1-10	0.759 (0.048)	0.796 (0.071)	0.66	0.466	0.314	0.559	0.976	65
10-100	0.650 (0.012)	0.704 (0.015)	0.76	0.564	0.153	0.706	0.908	676
100-200	0.595 (0.010)	0.676 (0.014)	0.83	0.678	0.113	0.712	0.869	455
200-500	0.565 (0.018)	0.671 (0.031)	0.73	0.590	0.227	0.504	0.815	171
500-1000	0.567 (0.022)	0.712 (0.070)	0.60	0.615	0.387	0.254	0.714	68
over 1000	0.515 (0.024)	0.676 (0.154)	0.13	0.536	0.441	0.110	0.522	119

[a] Cf. footnotes for Table 1.

[b] Numbers in parenthesis are standard errors.

deviations of $(C_k)^{-1}$ and $\beta_{hk,m}$ from their group means, is independent of $h_k(1 - t_k)$ within a net worth class. This implication probably is not strictly true. For instance, households in the same net worth class with a lower proportion of their wealth in human capital might be expected to have lower tax rates which would tend to induce a correlation between n_k and $h_k(1 - t_k)$. The possibility that the coefficient of proportional risk aversion might vary with socio-economic factors would only cause a misspecification of (19) if these factors were correlated to h_k and t_k within a net worth class. Perhaps the most obvious socio-economic factor which might be correlated with h_k and probably t_k within a class is the employment status of the members of the household. To determine the importance of this obvious misspecification, regression (18) was rerun on the sample of households reporting some income from salaries or wages. The results are not much different from the regressions for the total sample(Table 5). For three out of six classes, the constant term was larger than before, while for the other three it was less, and the changes are not systematically related to net worth.

Let us now turn to the second type of biases which stem from limitations of the sample data. There are at least five principal problems. First, the balance sheets collected do not contain any estimate of the value of social security or pension fund benefits. Second, the value of relief payments is omitted. The importance of this omission is likely to be significant only in the lowest net worth class. Third, the value of human wealth may be misstated due to incorrectly assumed discount rates and growth rates.[32] Fourth, there may be transitory elements in the value of human capital. Fifth, the data do not allow an unambiguous determination of the return from closely-held businesses attributable to wages. Taking the last problem first, (18) was re-estimated for households with no closely-held businesses. The results were unchanged.[33]

The first four problems all reduce logically to misstatements of the values of h_k and $\beta_{hk,m}$. In the model, social security and pension funds are assets over which a household has no control, so that formally these assets should enter into a more broadly construed h_k and would increase the value of this term. If it is assumed that the return on social security is uncorrelated with the return on the market portfolio of liquid risky assets, the inclusion of social security would reduce the value of a more broadly defined $\beta_{hk,m}$. Since pension funds are generally tied more directly to wage income, as a first approximation it is probably not too far from the mark to assume that their inclusion would not change $\beta_{hk,m}$ by much. The use of an incorrect discount rate or the inclusion of transitory elements would probably cause a misstatement in h_k but not necessarily in $\beta_{hk,m}$.

The total differential of (15) provides a way to measure the impact of these potential problems. Assuming dt_k and $d\alpha_k$ are zero in that t_k and α_k, derived from the household data, are known, the total differential takes the form

$$(19) \qquad d\,\frac{E(r_m - r_f)}{\sigma_m^2}\,\frac{1}{C_k} = [(1 - t_k)\beta_{hk,m} - (1 - t_k)\alpha_k]dh_k + h_k(1 - t_k)d\beta_{hk,m}.$$

The coefficients on $d\beta_{hk,m}$ can be estimated by net worth class as the means of $h_k(1 - t_k)$ given in Table 5. Likewise from Table 5, the coefficients on dh_k by net worth class starting with the lowest can be estimated as: 0.179, 0.112, -0.071, 0.003, -0.152, and -0.154.

Replacing the total differentials in (19) by finite differences and recalling that h_k is measured as a proportion, it is seen that $|\Delta h_k|$ would have to be extremely large to have much effect upon the relationship of the market price of risk times $(C^i)^{-1}$ to net worth. Thus, the conclusions would appear robust to large errors in the assumed discount and growth rates.

Correcting for the omission of social security benefits may have an important impact and together with the biases previously discussed might easily change the conclusion to one of the constant proportional risk aversion on the average. To determine the magnitude of such a correction, let us assume that the average value of social security benefits within a net worth class in 1962 was \$15,000,[34] that the value of social security does not change in any predictable way with changes in the value of the market portfolio, and that the net worth of the typical household in a net worth class is the midpoint except for two million dollars for the class with the largest net worth. On the basis of these assumptions, the estimates of $[E(r_m - r_f)/\sigma_m^2](C^i)^{-1}$ would change to 0.609 for the 1-10 thousand dollar class,[35] 0.583 for the 10-100 class, 0.561 for the 100-200 class, 0.592 for the 200-500 class, 0.521 for the 500-1000 class, and 0.495 for the class of greatest wealth. These adjusted figures show considerably less evidence of increasing proportional risk aversion on the average. Assigning a larger value to social security benefits and adjusting for the other biases discussed above might even change the conclusion to decreasing proportional risk aversion.

To summarize, for what we regard as the most appropriate treatment of investment in homes and human wealth within the context of our model, the unadjusted results give some evidence of increasing proportional risk aversion on the average. Yet, the magnitude of this increase is not great. Corrections for the various deficiences of the theoretical model, the econometric analyses, and the sample data would on balance tend to make the results closer to constant proportional risk aversion and might even produce evidence of moderately decreasing proportional risk aversion. Perhaps the most accurate single statement is: if there is any tendency for increasing or decreasing proportional risk aversion, the tendency is so slight that for many purposes the assumption of constant proportional risk aversion is not a bad first approximation.

V. The Market Price of Risk

To assess numerical values for the coefficients of proportional risk aversion, the results in the previous section must be augmented with an assessment of the market price of risk, namely $E(r_m - r_f)/\sigma^2(r_m)$. This section will employ two different techniques to formulate such an assessment. The first will utilize realized rates of return on stocks and fixed interest obligations, while the second will attempt to develop direct *ex ante* measures. Common stocks on the NYSE (New York Stock Exchange) will be used as the principal proxy for the market portfolio of risky assets partially because of the importance of these stocks in the actual market portfolio, but more pragmatically because of the lack of data on returns for most other risky assets. Nonetheless, the market price of risk will also be estimated using similar techniques with data from the bond market to confirm the estimates from the NYSE data.

For both industrial and all or composite NYSE common stocks, Table 6 shows for the past hundred years, 1872-1971, and selected subperiods, arithmetic averages of annual realized rates of return[36] as well as the standard deviations of these rates. The annual returns, which assume the quarterly reinvestment of dividends, are estimated from the work of the Cowles Commission prior to 1926 and, thereafter, from Standard and Poor's sources. The returns prior to 1926 may be less accurate. Over the past hundred years, the average annual return for the composite stocks was 9.7 percent and for the industrial stocks, 11.3 percent. The standard deviations of these annual returns were 0.19 and 0.22 respectively.

Also included in Table 6 are corresponding mean annual returns and standard deviations for high grade corporate bonds. The only differences are that coupons are reinvested semiannually rather than quarterly and only the

TABLE 6-AVERAGE ANNUAL ARITHMETIC RATES OF RETURN AND STANDARD DEVIATIONS OF ANNUAL RETURNS

1872-1971

Period	New York Stock Exchange Stocks[a]				High Grade Corporate Bonds[b]			Risk-free[d] Rate	Calculated Market Price of Risk		
	Composite		Industrial						Stocks		Bonds
	Mean Annual Return	Standard Deviation	Mean Annual Return	Standard Deviation	Mean Annual Return	Standard Deviation	Bond[c] Beta		Composite	Industrial	
1872-1881	0.0953	0.177	0.0820	0.129							
1882-1891	0.0429	0.130	0.0720	0.117							
1892-1901	0.0879	0.118	0.0773	0.163							
1902-1911	0.0794	0.218	0.1111	0.318	0.0436	0.0391	0.136	0.0409	0.806	0.6[illegible]2	1.712
1912-1921	0.0532	0.195	0.1128	0.296	0.0415	0.0545	0.179	0.0495	0.097	0.7[illegible]	-2.71[illegible]
1922-1931	0.1020	0.268	0.1108	0.289	0.0572	0.0369	0.061	0.0447	0.798	0.789	5.226
1932-1941	0.0866	0.282	0.1255	0.348	0.0572	0.0376	0.031	0.0138	0.925	0.915	37.70[illegible]
1942-1951	0.1725	0.136	0.1697	0.105	0.0244	0.0281	0.044	0.0127	8.673	[illegible]	14.60[illegible]
1952-1961	0.1304	0.189	0.1859	0.203	0.0177	0.0456	-0.055	0.0315	14.165	3.74[illegible]	[illegible]
1962-1971	0.0742	0.130	0.0779	0.136	0.0259	0.0596	0.131	0.0512	1.372	1.447	-7.1[illegible]5
1902-1961	0.1124	0.216	0.1360	0.263	0.0419	0.0431	0.072	0.0322	1.716	1.50[illegible]	5.24[illegible]
1902-1971	0.1069	0.206	0.1277	0.249	0.0396	0.0457	0.078	0.0349	1.702	1.50[illegible]	2.276
1926-1971	0.1155	0.212	0.1256	0.234	0.0381	0.0420		0.0232	1.963	1.703	5.207
1872-1971	0.0974	0.186	0.1125	0.221							

[a] From 1926, the figures are estimated from the Standard & Poor's Stock Indexes. For earlier years, the data are from Common-Stock Indexes by the Cowles Commission for Research in Economics, 2nd edition (Bloomington, Indiana: Principia Press Inc., 1939). The Cowles Index was converted to the Standard & Poor's base by adjusting overlapping figures.

The relative for the first quarter of the t^{th} year was taken to be

$$i_1 = \frac{P_1 + \frac{1}{4}\left(\frac{D_1}{P_1}\right)P_1}{P_0}$$

where P_1 and D_1/P_1 are stock price and annualized dividend yield at end of quarter and the return for the year derived from the product of the four quarterly returns.

[b] Semiannual relatives were estimated as $(P + \frac{1}{2}C)/100$, where C is the coupon rate on a new 20 year bond sold at par and P is the price at the end of the six months. The coupon rate C was estimated as the yield to maturity given by Standard & Poor's for High Grade Corporate Bonds. The price P was calculated using the coupon rate C and the yield to maturity given by Standard & Poor's at the end of the six months on the assumption that such a yield would be appropriate for a bond with nineteen years and six months to maturity. Multiplying successive pairs of these relatives yields estimates of the annual returns.

[c] The β's are derived from semi-annual regressions of the form $r_b = a + \beta r_m$ where r_b and r_m are annual rates of return on corporate grade high grade corporate stocks, respectively.

[d] Prime Corporate Bonds 1-Year Maturity February Average from The History of Interest Rates by Sidney Homer (New Brunswick, N.J.: Rutgers University Press, 1963), pp. 366-67, updated by Salomon Brothers.

last seventy years are covered. The mean annual return on these bonds over the years 1902-71 was 4.0 percent which compares to a range of 10.7 to 12.8 percent for common stocks over the same period. In fact, in every one of the seven decades covered, stocks beat bonds and sometimes by substantial margins. However, the standard deviations of annual returns on bonds were always less than on stocks.

As a further check on the relationship of the bond and the stock markets, the semiannual returns for the bonds were regressed upon the corresponding returns for the NYSE composite stocks.[37] The slope coefficients or beta coefficients are displayed in Table 6 and indicate that the returns on bonds usually tend to fluctuate with those on stocks except in the 1952-61 decade when the coefficient is negative. Over the entire seventy-year period, the beta coefficient is 0.078 which would imply, if the capital asset pricing model held, that the expected risk premium on bonds is 7.8 percent of that on stocks.[38]

Using the mean annual returns and the standard deviations calculated in the corresponding period, estimates of the market price of risk were derived. These estimates--as well as the risk-free rates--are contained in Table 6. The availability of a reliable series for the risk-free rate confined these estimates to the period after 1902. The estimates of the market price of risk over the longest period 1902 to 1971 were 1.7 for the composite stocks and 1.5 for the industrials, with not much different values for the 1902-61 period immediately preceding the date of the FRB survey. The estimate for the bond market was 2.3. It might be argued that these estimates are biased downwards because the variance of returns is estimated from a segment of the market of risky assets which would not reflect the full benefits from diversification. Additionally, assets which are both more and less risky then NYSE common stocks have been excluded in deriving the average risk premiums. It is difficult to assess the bias associated with these exclusion.[39]

The sampling errors in these estimates are undoubtedly large. To obtain some idea of the magnitude, the estimates of the market price of risk might be conceived of as drawings from some underlying probability distribution. Using this approach, the average of the seven decade estimates for the composite stocks is 2.4 which is somewhat larger than the estimate derived from the total period reflecting the large sampling error and possibly some non-stationarities in the distribution over time. The standard deviation of this average is 1.2. If the market price of risk were normally distributed, this would mean that there is an 0.88 probability that the true value of this parameter is greater than 1.0. The corresponding average and standard deviation for the industrials are 3.2 and 1.9, respectively.

An alternative way to assess the market price of risk is to develop ex ante measures of the expected returns on risky assets and the variance of those returns. To this end, ex ante measures of expected return were developed by adding the current yield to an estimate of the future growth rate of earnings per share.[40] The future growth rate was estimated by the arithmetic and by the geometric mean of the annual growth rates over the prior ten years.[41] For the seventy years ending in 1971, the mean estimate of the expected annual return for the composite stocks ranged from 13.0 to 9.0 percent, depending upon the way in which the future growth rate of earnings was estimated.[42] It might be recalled that the corresponding mean of annual realized returns was 10.7 percent. For comparison, the yield to maturity on high grade corporate bonds is also given in Table 7.

Using these estimates of expected rates of return for composite stocks, the paper derived an estimate of the market price of risk for each of the

TABLE 7—"EXPECTED" ANNUAL RATES OF RETURN FOR COMMON STOCKS AND BONDS

1902-1971

Period	High Grade Corporate Bonds Yields to Maturity	New York Stock Exchange Composite Stocks		Calculated Market Price of Risk for Stocks Using	
		Arithmetic Estimate[a]	Geometric Estimate[b]	Arithmetic Estimate	Geometric Estimate
1902-1911	0.0452	0.157	0.132	6.090	4.674
1912-1921	0.0519	0.132	0.097	1.989	1.120
1922-1931	0.0486	0.177	0.059	3.639	0.319
1932-1941	0.0363	0.064	0.011	0.518	-0.042
1942-1951	0.0270	0.160	0.126	3.829	3.075
1952-1961	0.0362	0.137	0.122	4.472	3.843
1962-1971	0.0562	0.086	0.080	1.228	1.047
1902-1961		0.138	0.091	4.423	2.165
1902-1971	0.0431	0.130	0.090	3.109	2.005

[a]The "Arithmetic Estimate" is the arithmetic mean of annual expected returns where the expected return for the t^{th} year is taken to be

$$i_t = \frac{D_t}{P_t} + GE_t,$$ where GE_t = arithmetic mean of previous 10 years of earnings growth.

Dividends yields were obtained and earnings growth rates calculated from Standard and Poor's Quarterly "Earnings, Dividends and Price Earnings Ratios."

[b]The "Geometric Estimate" is the same except that GE_t is the geometric mean of the previous 10 years of earnings growth.

70 years from 1901 through 1971. Subtracting the risk-free rate appropriate to the year from these expected returns yielded 70 risk premiums, which were then divided by estimates of the variance of the return for these stocks. These variances were estimated from the ten realized annual returns immediately preceding the year for which the expected return was assessed. Table 7 presents the average of these 70 estimates of the market price of risk as well as averages for selected subperiods. The average covering the 70 years ending in 1971 was 3.1 using the arithmetic estimate of ex ante expected return and 2.0 using the geometric, which are again greater than 1.0 though somewhat higher than the figures derived from realized returns.

To obtain some feeling for the magnitude of the sampling errors associated with these averages for the overall period, the standard deviation of the mean was calculated using every tenth estimate of the market price of risk from 1901 through 1971. Using every estimate instead of every tenth would have resulted in a downward biased estimate of the standard deviation unless explicit account was taken of the fact that successive estimates are based in part upon the same data. The standard deviation of the mean was 0.99 for the market price of risk based upon the arithmetic rate of growth in earnings and 0.87 for the geometric rate of growth. Assuming these standard deviations of the mean can be applied to the means of the 70 annual estimates (though there is reason to believe the true standard deviations would be less), the probability that the market price of risk using arithmetic rates of growth is greater than 1.0 would be 98 percent. The corresponding probability for the market price of risk based upon geometric rates of growth would be 87 percent.

This section has used several different approaches to estimating the market price of risk. All of these methods indicate that the market price of risk is greater than 1.0. The probabilities attached to this conclusion were assessed and, while not always at the frequently cited 5 percent level of significance, were consistent with a high level of significance. To the extent that there is some independence among the various estimates, the probability that the market price of risk is greater than 1.0 would be considerably larger than those reported for each of the individual estimates.

VI. The Synthesis

The empirical results in Section IV indicate that the assumption of constant proportional risk aversion for households is a fairly accurate description of the market place. More specifically, the study found that with a plausible treatment of investment in homes the net worth of a household had little value in explaining the level of its coefficient of relative risk aversion. Before taking account of potential biases, there was a slight tendency of this coefficient to increase with net worth. After adjusting for the biases, little such tendency remained.[43]

The main exception to this finding of constant proportional risk aversion occurred in the analysis based upon an extremely narrow definition of wealth which excluded homes, the associated mortgages and human wealth. In this case, there was evidence of decreasing proportional risk aversion. This tendency would be more pronounced if homes were considered a risk-free asset. Since an investor probably considers the value of his home and at least to some extent his human wealth in formulating his investment decisions, the analyses based upon broader definitions of wealth are more pertinent in assessing how the coefficient of proportional risk aversion varies with net worth.

In addition, Section IV derived various estimates of the product of the market price of risk and the reciprocal of the coefficient of proportional risk aversion. Regardless of how broadly wealth was defined, the estimate of this product was, with one unimportant exception, always less than 1.0. Though the value obtained depends on the definition of wealth, the product of the market price of risk times the reciprocal of the coefficient of proportional risk aversion appears to be between 0.5 and 0.8.

Coupling this conclusion with an estimate of the market price of risk allows an assessment of the coefficient of relative risk aversion. The evidence in Section 5 showed with a high level of significance that the market price of risk is in excess of 1.0. The implication is that the coefficient of relative risk aversion for the typical household is in excess of 1.0--contrary to the properties of the log utility function. Since the market price of risk is probably around 2.0 or more, the coefficient of proportional risk aversion is more likely to be in excess of two.[44]

Households, of course, only represent part of the demand for risky marketable assets. The aggregate demand function would have to take into account the impact of other investors such as eleemosynary institutions and financial intermediaries. Regardless of how their utility or more accurately decision functions are derived, the fact that the ratios of risky assets to net worth for these institutions are usually less than 1.0 and not uncommonly considerably less than 1.0 would seem to imply that their coefficients of relative risk aversion would tend to be at least 2.0. If, as is probably correct, the riskiness of institutional portfolios as a whole has remained less than that of households, their coefficients of relative risk aversion would tend to be greater than those of households.

In concluding, it is worthwhile to review some of the more important limitations of the models under which the empirical data were interpreted. Such limitations point the way to further work. First, the model made no adjustment for inflation. The presence of unanticipated inflation would mean that no asset denominated in nominal dollars could be considered risk-free. Second, the model, in assuming that the changes in the value of an asset were continuous, could not properly incorporate insurance. Third, the model did not incorporate adequately the unique characteristics of housing. Fourth, the model assumed that an investor made his portfolio choice in a one-period context. Thus, the model, in contrast to a multi-period model, is not rich enough to capture the possibilities of hedging against future changes in the investment opportunity set.[45] For households, perhaps the most widely available hedges are homes which protect against future changes in the cost of housing.

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Footnotes

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[1] The market price of risk is the difference between the expected rates of return on the market portfolio of risky assets as a whole and on a risk-free asset per unit of risk of the market portfolio. For purposes of this paper, it turns out that the relevant measure of risk is the variance of returns.

[2] For example, see S. C. Tsiang.

[3] See K. J. Arrow.

[4] Ibid.

[5] For example, see J. Crockett and I. Friend, pp. 37 and 55-57.

[6] Even the time-series analyses are not consistent in indicating a wealth elasticity of cash balances, broadly defined, equal to or greater than one in the period following World War II. See A. H. Meltzer, p. 236.

[7] E. g., H. A. Latane; N. H. Hakansson; and H. Markowitz. For a different theoretical position, see P. A. Samuelson and R. Merton.

[8] See R. Roll.

[9] See M. J. Gordon, G. E. Paradis and C. H. Rorke.

[10] See J. Mossin; J. Lintner; and A. P. Budd and R. H. Litzenberger.

[11] This assumption is non-trivial and would exclude such distributions of returns of individual assets as non-normal stable.

[12] An intuitive rationale of why the standard deviation enters with $\sqrt{dt}$ rather than with dt is that if a time period is subdivided into n subperiods each of duration $\tau = 1/n$, and if the returns over each of these subperiods are independently distributed, then the variance of returns over τ, σ_τ^2, is equal to 1/n times the variance of returns σ_t^2 over the initial period. As n approaches ∞, 1/n approaches dt, so that σ_τ approaches $\sigma_t\sqrt{dt}$.

[13] Equation (2) implies that the k^{th} individual invests α_k of his assets in risky assets and $1 - \alpha_k$ in the risk-free asset. In the type of continuous time model used in this paper, the separation theorem holds.

[14] This measure of relative risk aversion can also be interpreted as the wealth elasticity of the marginal utility of wealth as indicated in Section I.

[15] Samuelson has obtained a similar expression for the individual investor.

[16] This assumption is tantamount to assuming that capital gains and ordinary income are taxed identically and abstracts from the special tax treatment of homes and tax-exempt securities. However, see footnote 44.

[17] Since the dependence between r_m and r_{hk} is a factor in determining the optimum ratio of risky liquid assets to all liquid assets, it would be anticipated that an investor might be able to improve his expected utility by altering the proportions of the risky liquid assets he holds from those implicit in the market portfolio to take into account differences in the values of the covariances of human wealth and individual liquid assets in the market portfolio. The work of Y. Landskroner indeed shows this to be the case. Whether (8) results in an expected utility close to the optimum depends upon

the correlation of the optimal portfolio of risky liquid assets and the market portfolio. If this correlation were very close to 1.0, for instance, it would be possible to construct a portfolio of the market and the risk-free asset which would have a probability distribution very similar to the optimal portfolio of liquid assets. Numerous empirical studies have demonstrated that the returns on portfolios of a large number of securities are highly correlated with the market portfolio.

[18]This can easily be seen by writing in a one-period model

$$\alpha = \frac{E(R_m)}{1 + E(r_m)} \div \left[\frac{E(R_m)}{1 + E(r_m)} + \frac{F}{1 + r_f} \right]$$

where $E(R_m)$ represents the expected dollar return over the period on the available supply of liquid risky assets at the beginning of the period, and F represents the corresponding known dollar return on riskless assets. The constant physical supply of liquid risky assets and risk-free assets are measured by $E(R_m)$ and F, respectively.

[19]Though (14) was developed under the restrictive assumption that the sole decision of the individual was how much of his liquid assets to place in the market portfolio of liquid risky assets, Landskroner has shown that if r_o, the return on an asset uncorrelated with the return on the market portfolio of risky assets, is substituted for r_f and is also uncorrelated with the return on human capital, (14) holds in the more general case where an investor's decision variables include not only how much to put into risky liquid assets, but also how much to place into each risky liquid asset.

[20]To utilize (13) in a time-series, the weighted harmonic mean of $(1 - t_k)$ might be approximated by the ratio of total disposable income to total adjusted gross income. For the survey of households to be examined in the next section, the weighted harmonic mean would be 0.787 while the ratio of total disposable income to total adjusted gross income would be 0.859 which is close

if this harmonic mean is representative of the population, the ratio of total disposable income to total adjusted gross income would be about 10 percent greater than the harmonic mean. This figure of 10 percent might be useful in adjusting the time-series available from the Statistics of Income to that required by (13).

[21]With homes are included automobiles which except possibly for the lowest wealth groups represent only a minor proportion of assets.

[22]Cf. footnote 28.

[23]If $E\left[C_k^{-1}\right]$ were to decrease with increases in net worth, W_k, it is not necessary that $E(C_k)$ would increase. Yet, if the distributions of C_k given W_k, $P(C_k|W_k)$, remain unchanged as W_k increases except for location, it can be shown that $E(C_k)$ would increase with increases in W_k. This proposition follows from noting that if for fixed ε_k, $\left[E(C_k) + \varepsilon_k\right]^{-1} > \left[E(C_k)' + \varepsilon_k\right]^{-1} > 0$, $E(C_k) + \varepsilon_k < E(C_k)' + \varepsilon_k$. Since $P(C_k|W_k)$ is unchanged except for location, the distributions of ε_k will be the same at each level of W_k. Taking the definite integral of these two inequalities with respect to the distribution of ε_k yields the desired result.

[24]Housing presents other difficulties in applying the micro-relationships developed in the last section. First, since an investor values a house not only for its monetary rewards but also for its consumption value, it may be difficult to aggregate returns from housing with those from other assets. Second, the purchase of a home with a large mortgage permits an investor to hedge against changes in housing costs and in the costs of borrowing funds.

[25]In the first attempt at running these regressions, the sampling probability associated with each household was used to adjust for heterosce-

dasticity on the assumption that the reciprocal of the sampling probabilities in each stratum was picked to be negatively related to the variation within the stratum. The correlations of the absolute value of the residuals with these reciprocals revealed that there was substantial heteroscedasticity present in the regressions. All regressions reported in the paper are unweighted and exhibited little evidence of heteroscedasticity.

[26]Though these comparative results indicate that the socio-economic factors are roughly orthogonal to the logarithm of net worth, this does not mean that C_k^{-1} could not vary with these factors.

[27]The qualitative results are unchanged if households with closely-held businesses are excluded. For instance, following the same order as in Table 4, the estimated coefficients on the logarithm of net worth would be: 0.076, -0.004 and -.250 with respective t-values of 8.4, -.61 and -11.48. These coefficients are roughly the same as those reported for the full sample. The regressions were also rerun with dummy variables included for age, education, and occupation. There is little change in the estimated coefficients on the logarithm of net worth.

[28]Cf. footnote 43.

[29]The formula for μ_k would on the surface suggest that (19) suffers from some form of heteroscedasticity. Though this is true because of the way in which (17) and (18) were defined, it would be easy to redefine the variance of η_{hk} as a function of $h_k(1 - t_k)$ so as to make (19) homoscedastic. Thus, the apparent heteroscedasticity of (19) is artificial. Further, the discussion of (19) at this point abstracts from measurement error in h_k.

[30]If housing is measured instead by market value, the results are not much different. From lowest to highest net worth class, the constant terms would have been 0.810, 0.793, 0.634, 0.623, 0.530, and 0.500.

[31]The calculation of the value of human wealth made no adjustment for the probability of death before normal retirement. Formally, this is equivalent to assuming an incorrect discount rate and will be subsumed under this problem.

[32]From lowest to highest net worth class, the constant terms were estimated respectively as 0.711, 0.635, 0.541, 0.580, 0.522, and 0.449.

[33]This estimate of value is based upon the following assumptions: The average household is headed by a person 45 years old and consists of a spouse and two children with average age of 10 years. Using the 1941 CSO table as in CRC Standard Mathematical Tables with a 2.5 percent interest rate and assuming maximum benefits with no increases in the future, the retirement portion was estimated roughly to be $13,000 which represents an overstatement in view of the assumption of maximum benefits. The ancillary survivorship and disability benefits are more difficult to estimate but seem well under the 50% of retirement benefits as shown in government compilations. As a result, we have used an overall, probably somewhat conservative estimate of $15,000 for the combined value of retirement and ancillary benefits. The use of 2.5 percent interest rate is consistent with the maintenance of the real value of the benefits over time.

[34]This number was calculated by noting that the inclusion of social security benefits would cause an increase of 0.487 in the value of h_k and a decrease of 0.690 in the value of β_{hm}. Applying these changes to (19) results in a decrease of 0.133, which when subtracted from 0.742 gives the adjusted estimate in the text.

[35]These average rates of return would represent unbiased estimates of one-period holding returns if stationarity is assumed. To assess the sensitivity of these numbers to different horizons and since the arithmetic mean raised to the appropriate n^{th} power to obtain an n-year return can yield a seriously upward biased estimate of expected return over an n-year period (see M. E. Blume, 1974), an average of five year returns over the 1872-1971 period (yielding a return relative of 1.587 for the composite index) was compared with the fifth power of the average annual returns (1.592), with little difference in results.

[36]For a discussion of the specification of this type of regression, the reader is referred to Blume (1970).

[37]Over the 1902-71 period, the average annual risk premium on NYSE Composite Stocks was 0.0720. Multiplying this figure by 0.078 would yield an expected risk premium for bonds of 0.0056, which compares to the actual average annual risk premium on bonds of 0.0047.

[38]The market price of risk was also estimated using, instead of variance of returns, the variance of the difference of the annual return and the risk-free rate. If the risk-free rate were constant over time, the two variances would, of course, be the same. These alternative estimates were marginally lower. For the period 1902-71, this alternative estimate was, for instance, 0.04 smaller for the composite stocks than that presented in Table 6.

[39]Implicit in this formulation are the assumptions: (1) Earnings grow at a constant rate in perpetuity. (2) The payout ratio and discount rates are constant.

[40]Estimates using only five prior years of data gave unreasonably low rates of return in the early twenties and middle thirties which with the appropriate dividend yield would have implied expected returns as low as -22 percent. As long as cash is available, no investor if he really held this belief would hold stock. Using ten years of prior data yielded negative es-

timates of the expected market return in only a few cases and then only with the geometric.

[41]This range is consistent with that found in a survey of stockholders by W. G. Lewellen, R. C. Lease and G. C. Schlarbaum.

[42]These conclusions do not preclude the possibility that the coefficient of proportional risk aversion varies with socio-economic factors which are uncorrelated with net worth.

[43]This conclusion is based upon the assumption that the tax rates on all sources of income are the same when in fact tax rates differ. The effect is that the estimates of the coefficients of proportional risk aversion reported in the text are downward biased, although the bias is not large and does not vary much among net worth classes.

With differential taxes and not including human wealth, an estimate of $\alpha_k C_k$ would be

$$\frac{(1 - t_m)E(r_m) - (1 - t_f)r_f}{(1 - t_m)^2\sigma_m^2}$$

where t_m is the tax rate on risky liquid assets and t_f the tax rate on risk-free assets. Assuming the same tax rate on both kinds of assets, the estimate of $\alpha_k C_k$ would be given by

$$\frac{E(r_m) - r_f}{(1 - t_c)\sigma_m^2}$$

where t_c is a weighted average of t_f and t_m. Noting that $(1 - t_m) > (1 - t_f)$ and $E(r_m) > r_f$, the following inequality holds

$$\frac{(1 - t_c)(1 - t_m)}{(1 - t_m)^2} E(r_m) - \frac{(1 - t_c)(1 - t_f)}{(1 - t_m)^2} r_f > E(r_m) - r_f.$$

Upon dividing by $(1 - t_c)\sigma_m^2$, the inequality shows that the estimate of $\alpha_k C_k$ derived under the assumption of the same tax for both kinds of assets is biased downwards.

If human wealth is included and assumed taxed at the same rate (t_{hw}) as risk-free assets, $\alpha_k C_k$ would be given by the same formula as the first one in the footnote times the quantity

$$[(1 - h_k) + \frac{h_k}{\alpha_k} \cdot \frac{1 - t_f}{1 - t_m} \cdot \beta_{hk,m}]^{-1}.$$

If the same rate is applied to all assets, $\alpha_k C_k$ is given by the second formula times the quantity

$$[(1 - h_k) + \frac{h_k}{\alpha_k} \cdot \beta_{hk,m}]^{-1}.$$

Since $(1 - t_f) < (1 - t_m)$, not taking account of differential taxes will result in a downward biased estimate of the second term in the product. Since the first term is subject to the same biases as before, the net effect will be that estimates of C_k which do not adjust for the differential taxes associated with human and risk-free wealth will be downward biased.

Under reasonable estimates of the relevant variables, it can be shown that the assumption that tax rates on all sources of income are the same biases downward the coefficient of proportional risk aversion for the broadest definition of wealth by about eight percent for the \$10,000-\$100,000 net worth class.and by nine percent for the \$500,000-\$1,000,000 class. These results assume: $E(r_m) = 0.10$, $r_f = .05$, and $\sigma_m^2 = .025$ for all households; for the low net worth households, $t_f = t_{hw} = .10$, $t_m = .02$ in view of the heavy weight of housing in the marketable risky assets of this group, and $t_c = .08$; and for the high net worth households, $t_f = t_{hw} = .35$, $t_m = .25$ in view of the heavy weight of unincorporated business and the small weight of housing and tax-exempts.

[44] Merton has developed a theoretical multi-period model which does incorporate continuous changes in the investment opportunity set. E. F. Fama and J. D. MacBeth have developed some preliminary tests of the economic content of Merton's model. It might be noted that a utility function characterized by constant proportional risk aversion is consistent with myopic behavior so long as investment yields in different periods are serially independent.