Rates of Return on Bonds and Stocks, the Market Price of Risk and the Cost of Capital

by

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PRELIMINARY

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If *ex ante* and *ex post* (i.e., expected and realized) rates of return were always equal, it would be an easy exercise to obtain the average cost of capital for any corporation. In such a situation there would, of course, be no risk. Since the *ex ante* and *ex post* values are not equal--especially for corporate common stock, a comparison of *ex post* or realized rates of return on stock and on fixed-interest-bearing obligations is only the starting point in the determination of the risk differential between the expected returns on risky and risk-free assets required by the market or in the determination of the market price of risk.[1]

One new approach to this problem that we shall follow here is to determine whether at least over long periods of time the average relationship between returns on risky assets (common stocks in particular) and on risk-free assets can be reasonably explained by the dispersion of returns on risky assets and by a market utility function[2] which is consistent with the available data on the distribution of asset holdings

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[1]The market price of risk is the difference between the expected rate of return on the market for risky assets as a whole (R_M) and on a risk-free asset (R_F) per unit of risk on the market portfolio. For present purposes, it does not matter whether risk is measured by standard deviation of return, variance of return or variance of return multiplied by the market value of all risky assets.

[2]This is the relationship between the utility and size of wealth for investors as a whole.

among different wealth groups in the population. Since our primary interest is in explaining the risk differentials in return required on stocks and bonds, the next part of this paper (Section 2) will summarize the relevant implications of prior research in this area.

Section 3 will present realized rates of return and associated measures of risk on New York Stock Exchange stock and high-grade corporate bonds and the return on a relatively risk-free asset for various periods back to the 19th century. It will also develop extremely crude measures of *ex ante* rates of return on common stock for the same periods based on current dividend-yield plus the expected growth rate in stock prices (where the expected growth rate in stock prices is assumed to be equal to the actual or lagged growth rate in per share earnings.) This is equivalent to the assumption that investors do not anticipate any change in the price/earnings ratio for the market as a whole.[1]

Section 4 will present comprehensive new data on asset holdings bearing on the characteristics of the market utility function. An attempt will be made to determine whether any utility function consistent with these data can plausibly explain the long-run relationship between realized returns and dispersion of return, which is a pre-condition for using the *ex post* figures as *ex ante* values even in the long-run. Section 5 will contain some concluding comments on additional measures which will be required to make substantial further progress in estimating the required rates of return on risky assets and hence the market price of risk and the cost of capital.

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[1]The dividend payout ratio is also assumed to remain constant.

2. Earlier Work

The two best-known theoretical and empirical developments over the past fifteen years bearing on the relative costs of stock and bond financing have been those relating to the contributions by Professors Modigliani and Miller (MM) on the invariance of the cost of capital to the capital structure and the more recent evolution of the potentially more powerful market line theory. Neither of these theories provide a basis for estimating the market price of risk, but to the extent that they are applicable to real-world phenomena in spite of the perfect market assumptions they make, they could furnish insights into the relative costs of stock and bond financing.

Thus, it has been commonly asserted that MM theory indicates that the cost of capital is not affected by the mix of bonds and stocks in the capital structure and that the empirical evidence supports that theory.[3] Even if true, this would not provide a basis for estimating the cost of capital for any corporation, but it would imply that management cannot affect this cost by appropriately packaging its capital structure. The obvious fact, however, is that under the MM theory so long as corporate taxation exists the minimal cost of capital is achieved at a capital structure which consists entirely of debt. Since the average ratio of debt in the corporate capital structure seems to be of the rough

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[3]Franco Modigliani and M. H. Miller, "The Cost of Capital, Corporation Finance and the Theory of Investment," American Economic Review, June, 1958 and June, 1963, and Robert S. Hamada, "The Effect of the Firm's Capital Structure on the Systematic Risk of Common Stocks," Journal of Finance, May, 1972.

order of magnitude of 20%,[4] it is difficult to take seriously any empirical "verification" of the MM theory. Some of the reasons which have been advanced to explain the observed results appear to be incorrect (e.g., the proposition that the cost of retained earnings is substantially cheaper than the cost of debt), to make the empirical data irrelevant to testing the theory (e.g., the notion that management is attempting to maximize its own interests rather than those of the shareowners), or to undermine the usefulness of the theory (viz., that there are major institutional constraints or non-monetary costs associated with the issuance of debt which are not included in the framework of the theory).

In recent years, theoretical attention has properly been directed to the implications of the risks and costs of bankruptcy for the MM theory, leading to the not too surprising conclusion that the cost of capital is not invariant to the capital structure, at least when leverage is high, and that there is an optimal debt-equity ratio.[5] At least at the upper end of the capital structure range, "traditional" as distinct from MM theory has re-emerged. It can be questioned, however, whether bankruptcy risk alone is sufficient to explain the relatively low corporate debt ratio.

Similarly, the market line theory and its recent modifications have not been successful in explaining the relationship between the implied rate of return on low-risk assets and the observed return on low-risk bonds. The returns implied for low-risk assets by the observed

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[4]This 20% figure, which reflects the valuation of equity at market rather than book, was obtained from an estimate incorporated for the post-war period in the M.I.T.-Pennsylvania-S.S.R.C. econometric model. A crude independent estimate, based on <u>Statistics of Income, 1968: Corporation Income Tax Returns</u> and the revised SEC series on the market value of outstanding stocks, points to a somewhat higher figure for the end of 1968.

[5]E.g., Joseph E. Stiglitz, "Some Aspects of the Pure Theory of Corporate Finance: Bankruptcies and Take-Overs," <u>Bell Journal of Economics and Management Science</u>, Autumn, 1972.

relationships between returns and beta for all New York Stock Exchange stocks have tended to be higher on the average than the returns for comparably low-risk bonds or "risk-free" governments over a number of periods tested from the late 1920's to the late 1960's.[6]

These results, while not at all definitive, suggest that even without allowing for the tax advantages of debt financing, the risk-adjusted cost of bond financing may have been smaller than the risk-adjusted cost of stock financing or than the risk-adjusted cost of internal financing. Considering the sizeable tax advantage of bonds, the question arises why corporations did not place even more reliance on such financing. One answer may be that corporate management in its attempt to avoid the risk of bankruptcy and to preserve its own position has shied away from debt financing, in preference to the retention of earnings, whereas this risk is readily diversified by individual investors. However, even if the risk-adjusted cost of bond financing was lower on the average than that for stock financing or retained earnings over most of the period for which we have data, the relative costs of debt and equity financing may have been changed in recent years by such developments as the institutionalization of equity investment. Institutional investors have been committing a much higher proportion of their greatly expanded resources to investment in common stock, reflecting both an easing of legal restrictions and apparently also an increased willingness to assume equity risks.

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[6] F. Black, J.C. Jensen, and M. Scholes, "The Capital Asset Pricing Model: Some Empirical Tests," in Michael C. Jensen, ed., Studies in the Theory of Capital Markets (New York: Prager, 1972), and M. Blume and I. Friend, "A New Look at the Capital Asset Pricing Model," The Journal of Finance (March 1973).

3. Historical Measures of Return and Risk

Table 1 presents the average annual arithmetic rates of return (dividends plus price appreciation) and the standard deviations of annual returns for New York Stock Exchange common stocks over the past hundred years, 1872-1971, and over each of the decades in this period. These rates of return, which are shown both for all industry groups combined (the composite index) and for industrials alone, assume the quarterly reinvestment of dividends and are estimated from Cowles Commission and Standard and Poor's sources. They are probably less accurate prior to 1926 than in the subsequent period.

If the distribution of returns was stationary over this period, the arithmetic means presented represent an unbiased estimate of the universe mean return for a one-year holding period.[7] The mean annual return for all N.Y.S.E. stocks over the past 100 years was 9.75%, with a mean standard error of 1.75%. The mean returns would be raised somewhat if confined to industrial stocks or to the period after 1926 for which the data are presumed to be more reliable. There is no clear secular trend over these years.

Table 2 presents comparable data for high-grade corporate bonds and for risk-free instruments as measured by high-grade bonds with a one-year maturity, and also includes crude beta measures for bonds. The mean annual return on the bonds, with semi-annual reinvestment of interest

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[7] Since the arithmetic mean raised to the appropriate n^{th} power to obtain a n year return can represent a seriously upward biased estimate of expected return over an n year holding period (see Marshall Blume and Irwin Friend, "Risk, Investment Strategy and the Long-Run Rates of Return," Review of Economics and Statistics, forthcoming), an average of five year returns over the 1872-1971 period (yielding a return relative of 1.587 for the composite index) was compared with the fifth power of the average annual returns (1.592), with little difference in results.

Table 1

Average Annual Arithmetic Rates of Return and Standard Deviations of Annual Returns for New York Stock Exchange Common Stocks, 1872-1971[1]

	Composite Stocks		Industrial Stocks	
	Mean Return	Standard Deviation	Mean Return	Standard Deviation
1872 - 1881	.0953	.167	.0820	.122
1882 - 1891	.0429	.124	.0720	.111
1892 - 1901	.0879	.112	.0773	.154
1902 - 1911	.0794	.207	.1111	.302
1912 - 1921	.0532	.185	.1128	.281
1922 - 1931	.1020	.254	.1108	.275
1932 - 1941	.0866	.268	.1255	.330
1942 - 1951	.1725	.129	.1697	.100
1952 - 1961	.1804	.179	.1859	.193
1962 - 1971	.0742	.123	.0779	.129
1872 - 1961	.1000	.181[2]	.1163	.208[2]
1872 - 1971	.0975	.175[2]	.1125	.200[2]

[1]From 1926, the figures are estimated from the Standard & Poor's Stock Indexes. For earlier years, the data are from Common-Stock Indexes by the Cowles Commission for Research in Economics, 2nd edition (Bloomington, Indiana: Principia Press Inc., 1939). The Cowles Index was converted to the Standard & Poor's base by adjusting overlapping figures.

The return for the first quarter of the t^{th} year was taken to be

$$i_1 = \frac{P_1 + \frac{1}{4}\left(\frac{D_1}{P_1}\right)P_1}{P_0}$$

where P_1 and D_1/P_1 are stock price and annualized dividend yield at end of quarter and $i_t = i_1 \cdot i_2 \cdot i_3 \cdot i_4$.

[2]These standard deviations represent means of the decade averages for the period covered.

Table 2

Average Annual Arithmetic Rates of Return and Standard Deviations of Annual Returns for Fixed-Interest Assets, 1902-1971

	High Grade Corporate Bonds[1]		Bond β[3]	Risk-Free Rate[4]
	Mean Return	Standard Deviation		Mean Return
1902 - 1911	.0413	.0385	.176	.0409
1912 - 1921	.0363	.0553	.184	.0495
1922 - 1931	.0541	.0364	.025	.0447
1932 - 1941	.0690	.0342	.030	.0138
1942 - 1951	.0315	.0255	.000	.0127
1952 - 1961	.0200	.0408	-.036	.0315
1962 - 1971	.0178	.0611	.049	.0512
1902 - 1961	.0420	.0384[2]	.068	.0322
1902 - 1971	.0386	.0417[2]	.067	.0349

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[1] Rough estimates of mean annual rates of return where the semi-annual return for the first half of the t^{th} year is

$$i_1 = \frac{P_1 + \frac{1}{2}C}{P_0}$$, with P_1 = bond price per \$100 bond at end of half year, C = annual coupon payment

per \$100 bond, and $i_t = i_1 \cdot i_2$ is the annual rate of return. Bond prices and coupon interest payments per \$100 bond are from Standard & Poor's High Grade Corporate Bond Indexes. These S&P estimates, which assume a 4% coupon payment, are subject to a substantial margin of error but are the only ones available. It is planned to reestimate bond prices and interest payments by assuming that the percentage coupon is a simple average of the yields-to-maturity of the past twenty years.

[2] These standard deviations represent means of the decade averages for the period covered.

[3] The β's are derived from $[(R_B-R_F)_t - (R_B-R_F)_{t-1}] = \beta\,[(R_S-R_F)_t - (R_S-R_F)_{t-1}]$.

[4] Prime Corporate Bonds 1-Year Maturity February Average from The History of Interest Rates by Sidney Homer (New Brunswick, N.J.: Rutgers University Press, 1963), pp. 366-67, updated by Salomon Brothers.

payments, was 3.86% since the turn of the century, with a mean standard error of .50%. There was no clear trend in the level or dispersion of return, but some indication of a rise in dispersion from the early to the latter part of the period. For the risk-free rate, the mean annual return was 3.49%.

The beta measures for high-grade corporate bonds shown in the table are obtained by regressing $[(R_B - R_F)_t - (R_B - R_F)_{t-1}]$ on $[(R_S - R_F)_t - (R_S - R_F)_{t-1}]$, where R_B is the annual return on bonds, R_F is the return on high-grade debt instruments maturing in one year and R_S the return on the composite index for all N.Y.S.E. stocks. The beta coefficient for the period as a whole is very close to zero,[8] and does not deviate substantially from zero in any of the decades over the past century. Similar results, pointing to an extremely small beta coefficient for the period as a whole, were obtained when $(R_{Bt} - R_{B(t-1)})$ was regressed on $(R_{St} - R_{S(t-1)})$ without constraining the constant term to be zero, when $(R_B - R_F)$ was regressed on $(R_S - R_F)$ and R_B on R_S, and when a first order auto-regressive correction was made on the basis of the highest correlation found between lagged values of the dependent and independent variables.

Thus, an analysis which uses the usual type of stock market index as a proxy for all risky assets indicates little co-variation in movement between the realized rates of return on stocks and on high-grade bonds.

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[8]This regression for the period as a whole is

$$[(R_B - R_F)_t - (R_B - R_F)_{t-1}] = \underset{(3.08)}{.067} [(R_S - R_F)_t - (R_S - R_F)_{t-1}]$$

$\bar{R}^2 = .121$ DW = 2.84

and the number in parentheses below the beta coefficient represents its t-value. The serial correlation in these long-run regressions of $R_B - R_F$ on $R_S - R_F$ or of R_B on R_F, which is somewhat surprising, may reflect inadequacies in the data used to estimate R_B, but these inadequacies are not likely to greatly affect the estimates of the beta coefficients.

However, if bonds and other risky assets were added to the market index, the beta coefficient for bonds would presumably be raised. Moreover, there is some evidence that in recent years the beta coefficient for (quarterly) returns on high-grade bonds as a function of (quarterly) stock returns may have been close to .3.[9]

A direct comparison of the annual rates of returns on stocks and those on bonds and risk-free assets is provided in Table 3. For the 1902-1971 period as a whole, the average annual returns on all N.Y.S.E. stocks and on industrial N.Y.S.E. stocks were, respectively, 6.8% and 8.9% in excess of the annual returns on high-grade corporate bonds, and 7.2% and 9.3% in excess of the returns on risk-free assets. However, a substantial part of these discrepancies between returns on stocks and on fixed interest-bearing obligations was attributable to the two decades from 1942 to 1961, and especially for that period the discrepancies may reflect in large part the difference between the realized and expected values of return. Presumably no significant differences exist between the realized and expected values of annual return on high-grade bonds maturing in one year, but the differences may be somewhat more important for high-grade bonds and can be very substantial for stocks.

Crude estimates of the expected rates of return on high-grade corporate bonds and on N.Y.S.E. stocks are presented in Table 4. In view of the crudeness of the approach, it is not clear that the approach followed provides an estimate of *ex ante* return for stocks which is superior to the realized rate of return as a measure of the expected rate except for short periods of time.

[9] This result for the period September 1966 through December 1972 is presented in Jane C. Tripp, *An Empirical Analysis of Risk Measurement in Corporate Bonds*, Appendix A, Wharton School, May 1, 1973. For the period 1960-1970, the beta coefficient for quarterly returns on high-grade bonds was estimated at .126 in Marshall Blume and Irwin Friend, "A New Look at the Capital Asset Pricing Model," *Journal of Finance*, March 1973. It was insignificantly different from zero in the early 1960's.

Table 3

Differences in Average Annual Arithmetic Rates of Return Between New York Stock Exchange Common Stocks and Fixed-Interest Assets, 1902-1971

	Composite Stocks less High Grade Corporate Bonds[1]	Composite Stocks less Risk-Free Rate[1]		Industrial Stocks less High Grade Corporate Bonds[1]	Industrial Stocks less Risk-Free Rate[1]
	Mean Return	Mean Return	Standard Deviation	Mean Return	Mean Return
1902 - 1911	.0380	.0385	.207	.0697	.0701
1912 - 1921	.0170	.0037	.184	.0765	.0632
1922 - 1931	.0479	.0574	.252	.0566	.0661
1932 - 1941	.0176	.0728	.267	.0565	.1117
1942 - 1951	.1410	.1598	.128	.1382	.1571
1952 - 1961	.1604	.1489	.183	.1659	.1545
1962 - 1971	.0564	.0230	.128	.0601	.0267
1902 - 1961	.0703	.0802	.203[2]	.0939	.1038
1902 - 1971	.0683	.0720	.193[2]	.0891	.0928

[1]See footnotes of Tables 1 and 2 for a complete explanation of the rates of returns used above.

[2]These standard deviations represent means of decade averages for the period covered.

Table 4

"Expected" Annual Rate of Return Estimates for Common Stock and Bonds, 1882-1971[1]

	New York Stock Exchange Composite Stocks: Estimate 1[2] Mean Return	Estimate 1[2] Standard Deviation	Estimate 2[3] Mean Return	Estimate 2[3] Standard Deviation	High Grade Corporate Bonds[4]
1882 - 1891	.0696	.0762	.0559	.0769	n.a.
1892 - 1901	.0972	.0994	.0626	.1038	n.a.
1902 - 1911	.1484	.0621	.1321	.0631	.0452
1912 - 1921	.1584	.1166	.1139	.0946	.0519
1922 - 1931	.1592	.1621	.0576	.1322	.0486
1932 - 1941	.0813	.1637	.0352	.1590	.0363
1942 - 1951	.1558	.1045	.1338	.1042	.0270
1952 - 1961	.1096	.0644	.0983	.0586	.0362
1962 - 1971	.0904	.0376	.0875	.0388	.0562
1882 - 1961	.1224	.1061[5]	.0862	.0991[5]	n.a.
1882 - 1971	.1189	.0985[5]	.0863	.0924[5]	n.a.
1902 - 1971	.1290	.1016[5]	.0941	.0929[5]	.0431

[1] Dividends yields were obtained and earnings growth rates calculated from Standard & Poor's Quarterly "Earnings Dividends and Price Earnings Ratios".

[2] This estimate is the decade arithmetic mean of annual expected returns where the expected return for the t^{th} year is taken to be

$$i_t = \frac{D_t}{P_t} + GE_t \quad \text{where } GE_t = \text{arithmetic mean of the previous 5 years of earnings growth.}$$

[3] This estimate is the decade arithmetic mean of annual expected returns where the expected return for the t^{th} year is taken to be

$$i_t = \frac{D_t}{P_t} + GE_t \quad \text{where } GE_t = \text{geometric mean of the previous 5 years of earnings growth.}$$

[4] This series is the mean annual yield to maturity obtained from Standard & Poor's Composite High Grade Corporate Bond Indexes.

[5] These standard deviations represent means of decade averages for the period covered.

For the bonds, the average annual yield to maturity, which has been taken as an estimate of their expected rate of return, is not much different from their average annual realized rate of return for the past seventy years, though for some of the ten year periods the differences between expected and realized returns are rather appreciable. For the stocks, as would be anticipated, the differences between expected and realized returns are considerably larger. Two series for the expected returns on stocks are shown in the table. The annual values for each series is the sum of the beginning of year dividend yield (D/P) plus an estimate of annual expected earnings growth (g). The first series estimates g for each year as the arithmetic mean of the annual growth rates over the preceding five years; the second estimates g for each year as the geometric mean over the preceding five years.[10] The first set of estimates unlike the second never implies a zero risk differential, but it is far from certain that the first set is any better and the substantial difference between the two series raises obvious problems. The standard deviations of both sets of estimates of "expected" return on stock seem unreasonably high, especially on the downside.

One set of ex ante or expected rates of return on stocks implied by this type of analysis was higher than the realized rates of return, while the other was lower. Depending on which of the two sets of estimates is used, the average annual expected rate of return on all N.Y.S.E. stocks was 11.9% or 8.6% for the past ninety years as a whole compared with a realized rate of return of somewhat over 9.75%. In other words, according to these two rough estimates of the expected return on stocks, the long-run realized rate of return over the past century does not provide

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[10] The decade and longer period averages of the annual expected rates of return are arithmetic means. Geometric means are slightly lower.

a clearly implausible estimate of the expected rate. Moreover, the application of somewhat different lag structures to past and current earnings growth to obtain revised estimates of expected growth rates did not yield an appreciably different ex ante rate of return on stock for the period as a whole,[11] nor more plausible decade results.

Similarly, correlations between the realized rates of return on N.Y.S.E. stock or any of four ex ante series tested (Table 4 and Appendix Table 1) and both the risk-free rate and the difference between the bond rate and the risk-free rate provide little basis for choosing among the different estimates of the required rate of return on stock. Using annual data for the entire period, the correlations with these two interest rate variables were quite small and the regressions equally unsatisfactory for all of the series both ex post and ex ante. Using decade averages, the correlation was very much more marked for the realized rates of return ($\overline{R}^2$ = .51) than for the ex ante series, with both the risk-free rate and the difference between the bond rate and the risk-free rate negatively related to the realized rate of return over the past century.

While the method used above to derive ex ante rates of return on stock does roughly correct for unanticipated changes in the price-earnings ratio and makes the plausible assumption that investors ordinarily tend to anticipate that some average of the current and past rates of growth

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[11]The major difference in the estimates of ex ante returns reflected the type of mean (i.e., arithmetic or geometric) derived from the lagged values of growth rates for prior years. Thus, if the expected growth rate g for each year is estimated as the arithmetic mean of the annual growth rates over the preceding 10 years, the ex ante return for the period as a whole becomes .116; if g is estimated from the 10 year geometric mean, the ex ante return is .080 (Appendix Table 1). However, while these numbers are reasonably close to those based on 5 year growth rates, the standard deviations are substantially reduced to .062 and .056, respectively. In spite of the lower standard deviations, the decade results are no more plausible, especially when g is estimated from the 10 year geometric mean.

in per share earnings will continue in the future, it is obvious -- especially for a relatively short period of time -- that the resulting estimates of the expected growth rate and the associated required rate of return on stocks may deviate substantially from the expectations and requirements actually held by investors. Thus, a survey[12] of the anticipated long-run growth rates as of June 30, 1972 used by seven of the largest New York City banks to determine the desirability of investing in 17 high-grade N.Y.S.E. industrial and 24 electric and gas utility stocks indicated that the estimated growth rates per share averaged 7.9% and 5.0% respectively, implying required rates of return of 11.3% and 11.5%. These figures were well above the 9% or somewhat lower estimate of the required rate of return over the past decade presented in Table 4 and probably reflect the expectations of a higher rate of long-run price inflation than the average rate of inflation reflected in historical earnings. The expectation of a higher rate of inflation was also reflected in the much higher level of interest rates in 1972 than the average level over the preceding decade.

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[12]This survey was conducted by the New York Telephone Co. at my suggestion.

4. Market Utility Function

Since the available data do not generally permit us to obtain very satisfactory measures of the required rates of return on common stocks as compared with those on risk-free assets, we shall investigate whether any additional insights can be obtained by the determination of the characteristics of the market utility function, i.e., the relationship between the utility and size of wealth for investors as a whole.[13] The risk differential between the return on the market portfolio of all risky assets and the return on risk-free assets depends of course on the dispersion of returns on the market portfolio as well as on the attitudes of investors towards such dispersion as indicated by the form of the market utility function.

It is interesting to note that there has been very little attempt to determine the form of the utility function from the available data on asset holdings by different types of individuals. Economists in recent years have generally been convinced that the market utility function has risk aversion properties somewhere between a negative exponential utility function, with constant absolute risk aversion and increasing relative risk aversion, and a constant elasticity utility function, with decreasing absolute risk aversion but with constant proportional risk aversion. The authority generally cited for asserting these bounds for the utility function is Professor Arrow.[14]

While no one is likely to argue with the plausibility of decreasing absolute risk aversion, the widely-held assumption of increasing

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[13]The development of a market utility function would of course be highly useful for other purposes as well, including the analysis of saving and capital budgeting.

[14]E.G., see S.C. Tsiang, "The Rationale of the Mean-Standard Deviation Analysis, Skewness Preference, and the Demand for Money," American Economic Review, June, 1972.

(or at most constant) relative risk aversion seems questionable. At the theoretical level, the denial of the tenability of decreasing relative risk aversion is based on the assumed implausibility of a utility function which is unbounded either from above or from below.[15] However, it is not clear that a utility function should be bounded in this manner. The ultimate justification for such an assumption must rest on the empirical data. The only behavioral evidence I know of that has been cited to support the assumption of increasing or constant relative risk aversion are the studies which conclude that either the income elasticity or the wealth elasticity of demand for cash balances (usually money either narrowly or broadly defined) is at least one.[16] Of this evidence, only the wealth elasticity is at all relevant since the income elasticity of total wealth may be greater than the income elasticity of cash balances even if the latter elasticity is in excess of one.

To the best of my knowledge, the available cross-section information seems to point to a wealth elasticity of liquid assets of well below, to close to, one if all tangible assets including consumer durables are included in wealth and lower figures if tangible assets are excluded.[17] The aggregate time-series data reflecting changes in supply and demand conditions, including those arising from changes in wealth distribution

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[15] Kenneth Arrow, Essays in the Theory of Risk-Bearing, Markham, 1971.

[16] Ibid.

[17] E.g., see Jean Crockett and Irwin Friend, "Determinants of Investment Behavior," Determinants of Investment Behavior, National Bureau of Economic Research, 1967, pp. 37 and 55-57.

among different groups in the population, appear to be much less pertinent than the cross-section data.[18]

In conjunction with Professors Marshall Blume and Jean Crockett, I am currently working on a study which is using new detailed information from the 1963 Federal Reserve Board surveys of the Financial Characteristics of Consumers and Changes in Family Finances and from a large sample of 1971 Federal personal income tax returns to analyze the risk characteristics of investment by different wealth (and other socio-economic and demographic) groups in the population. The FRB surveys, which oversampled upper income groups, collected 1963 information for more than 2,100 households in all income classes not only for income (1962 as well as 1963), saving and the value of all major categories of assets held at the beginning and end of the year but also for the amounts of individual stocks held. The relevant results from the analysis of the FRB surveys will be presented in this paper.[19]

Data from the FRB surveys indicate that as of the end of 1962 the ratio of cash balances (including time and all types of savings deposits) to net worth decreased as net worth increased from under

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[18] Even the time-series analyses are not consistent in indicating a wealth elasticity of cash balances broadly defined equal to or greater than one in the period following World War II. See Allan H. Meltzer, "The Demand for Money: The Evidence for the Time-Series," Journal of Political Economy, June, 1963, p. 236.

[19] A few preliminary results from these surveys were presented in Irwin Friend, Mythodology in Finance, The Journal of Finance, May 1973. See also Dorothy S. Projector and Gertrude S. Weiss, Survey of Financial Characteristics of Consumers, Federal Reserve Board, August 1966.

$10,000 to over $1 million.[20] It should be noted that net worth in this comparison as in all published studies of the wealth elasticity of cash or other risk-free assets does not include the value of human wealth.

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[20]The data on currency holdings were unavailable from these surveys. This ommission however, should not affect substantially any of the following results since from aggregate data it is estimated that, as of the end of 1962, currency holdings amounted to well under 10% of household cash balances and a very much smaller fraction of net worth (see SEC Statistical Bulletin, May 1963). Moreover, there is no reason to believe that the ratio of currency holdings to household wealth deviates markedly in its relationship to wealth from the ratio of other cash balances.

If equity in housing and consumer durables (i.e., their market value less related debt) is eliminated from net worth on the grounds that they are acquired for their provision of current consumption services at least as much as for investment purposes and are not generally considered as risky or non-risky assets by their owners, the ratio of cash balances to net worth at the end of 1962 decreased dramatically from 43% for the under $10,000 net worth class to 3% for households with net worth over $1 million (Table 5). The net surrender value of life insurance[21] and equity in savings bonds and Treasury bills[22]--other relatively "non-risky" assets--also declined markedly as a ratio of net worth less housing and consumer durables as net worth rose. In contrast to these tendencies for non-risky assets, the ratio to net worth of the value of such "mixed-risk" assets as long-term bonds increased moderately, while the ratio to net worth of "risky" assets such as common stock increased markedly, as net worth rose. The qualitative nature of these results is not affected if family characteristics other than net worth (such as age, occupation, employment status, region, education, size of family, etc.) are held constant. Nor does it seem plausible that the qualitative nature of these results is likely to be substantially influenced by well-known limitations of cross-section data--notably family tastes effects, possible transitory elements in the total and composition of net worth, and measurement errors.

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[21] Information on pension and retirement funds was not available.

[22] Holdings of Treasury bills were negligible.

Table 5

Average Ratios of Value of Different Assets to Household Net Worth Exclusive of Homes and Automobiles, for Households Classified by Net Worth[1]

December 31, 1962

Type of Asset	Net Worth: Less than $10t[6] (Ratios of Value of Specific Type of Asset to Net Worth)	$10t - $100t	$100t - $200t	$200t - $500t	$500t - $1m	Over $1m
Cash balances[2]	.425[7]	.218	.085	.042	.052	.029
Checking accounts	.103	.026	.020	.015	.020	.009
U.S. sav. bonds & Treas. bills[3]	.077[7]	.040	.034	.012	.013	.003
Life insurance net surr. val.	.240[7]	.072	.044	.024	.034	.011
Mixed-risk assets[4]	.045[7]	.078	.130	.099	.088	.141
State & local bonds	.000	.001	.001	.014	.015	.034
Risky assets, excl. homes & autos[5]	.271[7]	.586	.741	.842	.838	.864
Common & preferred stock	.034	.111	.277	.372	.297	.344
Equity in unincorp. businesses	.140	.308	.285	.348	.270	.269

[1] Exclusive of equity in homes and autos.

[2] Includes checking and other commercial bank accounts, savings and loan savings accounts, credit union savings accounts, and mutual savings accounts.

[3] Treasury bills are negligible compared to U.S. savings bonds.

[4] Includes long-term corporate, state and local and U.S. Government bonds (other than savings bonds).

[5] Includes common and preferred stock and equity in non-farm and farm unincorporated businesses.

[6] These are ratios of the mean value of the indicated asset in this net worth class to the mean value of net worth, unlike most of the other figures in Tables 5-7 which are averages of the ratios for individual households. The average ratios of assets to net worth were inflated by a number of households owning homes but reporting virtually no net worth. The average ratios in this net worth class, which had extremely large standard errors, were the only ones substantially affected by this problem.

[7] The sum of these percentages exceed 100% reflecting the existence of household debt unrelated to homes and autos.

The impression one obtains from this tabulation is much more consistent with decreasing than with increasing relative risk aversion. I suspect that further analysis both of the end of 1962 and the 1971 data will confirm our earlier finding that the riskiness of the stock portfolio held by individuals generally increases with rising wealth.[22] This would appear to support further the thesis of decreasing relative risk aversion. However, the analysis so far has not considered the impact of the tax laws on the portfolio preferences of different income and wealth classes, and only non-human wealth other than equity in housing and consumer durables has been covered. The remaining deficiencies in our analysis--measurement errors in the data used and differences in tax-adjusted investment opportunities open to households in various wealth classes--seem much less important. Survey data are subject to substantial random error and biases in measurement, but the known relevant deficiencies of measurement (viz., understatement of financial assets, especially cash balances, with some evidence of relatively greater understatement of non-equity assets in the lower income brackets) would not help explain the observed results.

To estimate the likely impact of differences in tax rates (t) on the ratio (α) of household wealth invested in risky and mixed assets vs. the ratio invested in riskless assets requires knowledge of the utility function. If we assume as a first approximation that the utility function is characterized by constant proportional risk aversion, since with such a utility function differences in tax rates would lead to larger relative holdings of risky assets by the wealthier households, we would expect $\alpha(1-t)$ to be roughly

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[22] Jean Crockett and Irwin Friend, "Characteristics of Stock Ownership," Proceedings of Business and Economics Statistics Section, American Statistical Association, 1963.

equal for the different net worth classes.[23] Actually, $\alpha(1-t)$ varies from 30% [$\alpha(1-t)$ = .32 X .94] for households with net worth under \$10,000, to 60% (.66 X .92) for households with net worth from \$10,000 to \$100,000 and 63% (1.01 X .62) for households with net worth of over \$1 million.[24]

Thus, on a tax-adjusted basis, there is still some evidence of decreasing proportional risk aversion if only non-human wealth other than equity in housing and consumer durables is considered relevant to investment decisions. However, for households with net worth of \$10,000 or over, the observed α's are consistent with the assumption of constant proportional risk aversion.

If equity in housing and automobiles is included in net worth,[25] the ratio of cash balances to net worth is again negatively related to net worth. Table 6 shows that the ratio of cash balances to net worth

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[23]For such utility functions, it is easy to show that in the absence of personal taxes α is approximated by

$\frac{E(r_m - r_f)}{C \text{ var } r_m}$, where r_m is the rate of return on risky assets, r_f the rate of return on risk-free assets, E is expected value, and var the indicated variance. If r_m and r_f are assumed subject to the same average rate of personal taxation, t, for a given level of wealth,

then $\alpha = \frac{E(r_m - r_f)}{(1-t)C \text{ var } r_m}$. C is the harmonic mean of the Pratt measure of proportional risk aversion.

[24]The overall Federal income tax ratio for all households combined estimated from survey data (10%) was somewhat lower than the ratios of taxes to adjusted gross income reported in Statistics of Income, 1962 (12.8%).

[25]The market value of other consumer durables is not available from the 1963 survey data.

Table 6

Average Ratios of Value of Different Assets to Household Net Worth Including Homes and Automobiles, for Households Classified by Net Worth[1]

December 31, 1962

Net Worth

Type of Asset	Less Than $10t[9]	$10t - $100t	$100t - $200t	$200t - $500t	$500t - $1m	Over $1m
	(Ratios of Value of Specific Type of Asset or Liability to Net Worth)					
Cash balances[2]	0.162	0.128	0.102	0.045	0.050	0.029
Checking accounts	0.039	0.019	0.017	0.012	0.025	0.009
U.S. sav. bonds & Treas. bills[3]	0.030	0.024	0.020	0.020	0.010	0.003
Life insurance net surr. val.	0.092	0.056	0.033	0.025	0.038	0.012
Mixed-risk assets[4]	0.017	0.039	0.087	0.081	0.090	0.131
State & local bonds	0.000	0.000	0.001	0.005	0.041	0.033
Risky assets, excl. homes & autos[5]	0.104	0.299	0.587	0.767	0.721	0.842
Investment in homes & autos[6]	1.434	0.658	0.204	0.103	0.113	0.044
Risky assets, incl. inv. in homes & autos	1.538	0.957	0.791	0.869	0.834	0.886
Risky assets, incl. equity in homes & autos[7]	0.721	0.754	0.758	0.852	0.828	0.882
Common & preferred stock	0.013	0.052	0.219	0.284	0.341	0.327
Equity in unincorp. businesses	0.053	0.147	0.261	0.382	0.200	0.269
Mortgage liability	0.817	0.203	0.032	0.017	0.005	0.004
Other liability[8]	0.074	0.034	0.014	0.032	0.026	0.071

[1]Inclusive of equity in homes and autos.

[2]Includes checking and other commercial bank accounts, savings and loan savings accounts, credit union savings accounts, and mutual savings accounts.

[3]Treasury bills are negligible compared to U.S. savings bonds.

[4]Includes long-term corporate, state and local and U.S. Government bonds (other than savings bonds).

[5]Includes common and preferred stock and equity in non-farm and farm incorporated businesses.

[6]Market value of principal and other residences and autos.

[7]Equity in homes and autos equals investment in homes and autos less related debt.

[8]Mainly short- and intermediate-term.

[9]These are ratios of the mean value of the indicated asset in this net worth class to the mean value of net worth, unlike most of the other figures in Tables 5 - 7 which are averages of the ratios for individual households. The average ratios of assets to net worth were inflated by a number of households owning homes but reporting virtually no net worth. The average ratios in this net worth class, which had extremely large standard errors, were the only ones substantially affected by this problem.

inclusive of equity in homes and automobiles is once again negatively related to household wealth or net worth, with the ratio of cash balances to total wealth at the end of 1962 now showing a less dramatic but still impressive decrease from 16% for the under $10,000 net worth class to 3% for households with net worth over $1 million. Decreases characterized both time and saving accounts and checking accounts. However, once household debt, of which home mortgages is the most important form, is introduced into the analysis, it is theoretically preferable to relate risky assets, rather than the customary cash balances, to net worth. If investment in homes and autos were governed by the same considerations as other household investment, it would be appropriate to include the market value of homes and autos with risky assets, and the ratio of risky and mixed assets to net worth would decline with increases in household wealth--markedly so on a tax-adjusted basis. Since investment in homes and autos is required for satisfaction of current consumption services, and particularly for homes necessitates a large scale investment normally associated with the incurrence of debt, it may be more appropriate to consider net equity (market value of investment less debt) rather than investment as part of the relevant total of risky assets. If this is done, the unadjusted ratios of risky and mixed assets to net worth would rise substantially with increases in wealth and decline modestly on a tax-adjusted basis.

On an unadjusted basis, the ratio of risky and mixed assets to net worth, with both assets and net worth inclusive of equity in homes and autos, increased from 74% in the under $10,000 to 100% in the over $1 million net worth classes (Table 6). Abstracting from investment in housing and automobiles, the riskier the asset the more pronounced in general was its tendency to increase as a ratio of total net worth with increases in household wealth. Thus the ratio of the market value of stock to net worth inclusive of equity in homes and autos rose from 1% in the under $10,000 net worth class to 33% for households with net worth over $1 million. In contrast, the corresponding ratios were 10% and 1% for the net surrender value of life insurance.[26] Of the mixed assets, consisting largely of long-term bonds, the tax-exempt state and local governments not surprisingly had a special appeal for the wealthier households. On a tax-adjusted basis, the ratios of risky and mixed assets to net worth multiplied by the complement of the tax rate decreased erratically from 70% (.74 X .94) in the under $10,000 wealth class to 63% (1.02 X .62) in the over $1 million class.[27] For stock alone, the corresponding percentages were 1% and 20%.

Before we attempt to draw the implications of these results for the characteristics of the investors' utility function, it is still necessary to expand the coverage of wealth to include human wealth, i.e., the current discounted value of expected labor income. For this purpose, human wealth or that part of life-time labor income which should be capitalized and added to non-human wealth for analysis of investment decisions has initially been estimated by multiplying one year's wage and salary income by a factor of ten. This is equivalent to using a

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[26]The strong negative relationship between the insurance ratio and household wealth might be reduced by the inclusion of pension and retirement funds.

[27]The corresponding figures are 69% and 62% if the ratios of the mean values of risky and mixed assets to the mean values of net worth in each net worth class are substituted for the averages of the ratios for individual households.

capitalization factor of about 11.8% if we assume that the average household in each non-human wealth class has an expected work life of 20 years and a 4% expected annual growth ratio in labor income. The results showing the implied ratios of non-risky and risky assets to total assets, with risky and total assets both now inclusive of human wealth, are presented in Table 7.

These results, which are unadjusted for differences in tax-rates, still point to a decline in the ratio of cash balances to total assets (inclusive of human wealth) for the richer households, and the decline persists even if transaction balances in the form of checking accounts are omitted from the cash balances. The sum of the two remaining household assets which are typically regarded as comparatively riskless--U. S. savings bonds and Treasury bills and insurance--also evidence an inverse relationship between their relative importance in household assets and the total amount of assets held.[28] Similarly, the ratio to net worth of risky and mixed assets, including equity in homes and autos, again increases with wealth, though the variations across wealth groups are no longer very marked. However, on a tax-adjusted basis, this ratio of risky and mixed assets to net worth decreased from 83% (.89 X .94) in the under $10,000 net worth class to 63% (1.01 X .62) in the over $1 million class. For stocks alone, the corresponding percentages were 0% and 19%.

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[28]Eventually, we also plan to compute crude beta coefficients and other risk measures for the totality of assets held by each wealth group.

Table 7

Average Ratios of Value of Different Assets to Household Net Worth Including Homes and Automobiles and Human Wealth[1], for Households Classified by Net Worth[2]

December 31, 1962

Type of Asset	Net Worth: Less Than $10t	$10t - $100t	$100t - $200t	$200t - $500t	$500t - $1m	Over $1m
	(Ratios of Value of Specific Type of Asset or Liability to Net Worth)					
Cash balances[3]	0.080	0.065	0.081	0.037	0.045	0.026
Checking accounts	0.021	0.009	0.013	0.010	0.023	0.008
U.S. sav. bonds & Treas. bills[4]	0.006	0.014	0.015	0.018	0.009	0.003
Life insurance net surr. val.	0.038	0.019	0.025	0.020	0.032	0.010
Mixed-risk assets[5]	0.004	0.023	0.073	0.076	0.081	0.120
State & local bonds	0.000	0.000	0.000	0.005	0.040	0.031
Risky assets, excl. homes & autos & human wealth[6,8]	0.008	0.128	0.382	0.611	0.588	0.758
Investment in homes & autos[7]	0.202	0.248	0.147	0.083	0.099	0.038
Investment in human wealth[8]	0.925	0.661	0.337	0.199	0.191	0.114
Risky assets, incl. inv. in homes & autos & human wealth[9]	0.976	0.937	0.834	0.880	0.851	0.896
Risky assets, incl. equity in homes & autos & human wealth[10]	0.882	0.885	0.810	0.867	0.846	0.893
Common & preferred stock	0.001	0.029	0.158	0.261	0.307	0.297
Equity in unincorp. businesses	0.022	0.109	0.221	0.334	0.145	0.225
Mortgage liability	0.094	0.052	0.024	0.013	0.004	0.003
Other liability[11]	0.014	0.014	0.008	0.023	0.021	0.062

[1] Estimated at 10 times wages and salaries.

[2] Inclusive of equity in homes and autos but not inclusive of human wealth.

[3] Includes checking and other commercial bank accounts, savings and loan savings accounts, credit union savings accounts, and mutual savings accounts.

[4] Treasury bills are negligible compared to U.S. savings bonds.

[5] Includes long-term corporate, state and local and U.S. Government bonds (other than savings bonds.

[6] Includes common and preferred stock and equity in non-farm and farm unincorporated businesses.

[7] Market value of principal and other residences and autos.

[8] These are ratios of the mean value of the indicated asset to the mean value of net worth. The corresponding averages of the ratios for individual households were not available.

[9] For the reason indicated in footnote 8, this row will not be equal to the sum of the preceding three rows. The difference is most marked in the first column.

[10] Equity in homes and autos equals investment in homes and autos less related debt.

[11] Mainly short- and intermediate-term.

Thus, the analysis thus far suggests that the market utility function is characterized (1) by decreasing to constant relative risk aversion if the relevant measure of household wealth is net worth exclusive of equity in homes and autos and human wealth; (2) by constant or increasing relative risk aversion if wealth is inclusive of equity in homes and autos but exclusive of human wealth; and (3) by increasing relative risk aversion if wealth is all-inclusive. Preliminary results available from two attempts to further refine this analysis reinforce the first two findings and modify the third suggesting that if the relevant measure of household wealth is net worth inclusive of equity in homes and autos and human wealth, the market utility function is characterized by either increasing or constant relative risk aversion.

One of these two attempts to refine the analysis consisted of the derivation of the utility function for different age groups. If age of head is held constant as wealth increases, the decline in the estimated tax-adjusted ratios to net worth of risky and mixed assets, including equity in homes and autos and human wealth, is substantially reduced. Thus, for households with a head over 50 years old their tax-adjusted ratios declined only from 69% to 63%.

The other attempt to refine the analysis consisted of reestimating the value of human wealth by obtaining the present value of future after-tax wage and salary income for each household based on the age of its head and other family characteristics and on aggregate trends in income, and by more carefully distinguishing between labor and non-labor income as reported in the FRB surveys to minimize double-counting of human and non-human wealth for entrepreneurs and related groups in the population. The assumed (in this case after-tax) capitalization factor was 10%, and the expected annual growth rate in wage and salary income was predicated to be 4%. The expected work life was made a function of age (but not yet of occupation). It is interesting to note that this more careful estimate of human wealth eliminates about half of the remaining difference between the tax-adjusted ratios of risky and mixed assets to net worth for the lowest and highest wealth classes.

To sum up this discussion of the market utility function, the assumption that investors are characterized by constant proportional risk aversion seems tenable. For such a function, it is easy to show that if end-of-period wealth has a normal distribution, (1) $\frac{E(r_m - r_f)}{(1-t)\text{var } r_m}$ for a short period is roughly approximated by αC where C is an average (harmonic mean) of the Pratt measure of proportional risk aversion for individual investors or (with a change in sign) of the wealth elasticity of the marginal utility of wealth.[1] For a longer planning period, the approximation becomes (2)

$$\frac{E(r_m - r_f)}{(1-t)\text{var } r_m} = \frac{\alpha C}{\alpha[1 + (1-t)E(r_m)] + (1-\alpha)[1 + (1-t)r_f]}.$$

To estimate the value of C, we need to know the values of $E(r_m)$, r_f, var r_m,

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[1] It has been shown in the literature that for discrete planning periods

$$\frac{E(r_m - r_f)}{\text{var } r_m} = V_{om}\left[\sum_K E\left(\frac{1}{R_K}\right)\right]^{-1}$$

for the quadratic, and (if end-of-period wealth is normally distributed) for the exponential and log utility functions, where R_K is the Pratt measure of absolute risk aversion for the K^{th} individual and V_{om} is the market value of all risky assets at the beginning of the planning period and equal therefore to αW_o where W_o is total initial wealth. Stephen Ross, using a continuous time solution, has obtained an equivalent expression

$$\frac{E(r_m - r_f)}{\text{var } r_m} = \left[\sum_K \frac{\gamma_K}{W_K R_K}\right]^{-1}$$

for all utility functions (again assuming a normal distribution of wealth), where W_K is the wealth of the K'th individual and γ_K his ratio of total wealth, and it is assumed all wealth is invested in risky assets. If not all wealth is invested in risky assets, then the right-hand expression in the above equation should be multiplied by α, the overall ratio of risky assets to wealth. For constant proportional risk aversion C, it is easy to show in the continuous case that

$$\frac{E(r_m - r_f)}{\text{var } r_m} = \alpha C,$$ if it is assumed, as seems plausible for investors characterized by such a utility function, that γ_K and $W_K R_K$ are independent. Introducing taxes, at the micro-level for the K^{th} household, $\frac{E(r_m - r_f)}{(1-t_K)\text{var } r_m} = \alpha_K C_K$. At the macro-level, $\frac{E(r_m - r_f)}{\text{var } r_m} = \alpha C\left[\sum \frac{\gamma_K}{(1-t_K)}\right]^{-1}$ which can be roughly approximated by the micro-relationship omitting the K subscripts (though this approximation somewhat understates the value of C).

t and α. A more precise expression for (1) is $\frac{E(r_m - r_f)}{\text{var } r_m} = \alpha C \left[\Sigma \frac{\gamma_K}{(1-t_K)}\right]^{-1} = \alpha C(1-t')$.

The value of t is estimated at 10% and t' about 15%[1] by the application of the appropriate Federal income tax rates to the survey data. The overall ratio of risky and mixed assets to net worth, α, is 79%, according to the survey data, if risky assets and net worth do not include equity in homes and automobiles and human wealth; 85% if equity in homes and automobiles but not human wealth is included; and 95% for the most comprehensive measures of risky assets and net worth, i.e., inclusive of human wealth.[2] We shall use figures of .85 for (1-t') and .90 for α in estimating C, but possible errors in these figures will not markedly affect our estimate of C.

For the past 70 years, $E(r_m - r_f)$ for all stocks on the NYSE has averaged about .072 annually if estimated from realized returns and var r_m has averaged .037 so that the ratio $\frac{E(r_m - r_f)}{\text{var } r_m} = 1.95$ on this basis (Table 3). The value of this ratio is nearly as high for the 60 year period preceding 1962, the year covered by our survey data. The ratio would be very much higher (over 5) if estimated from realized returns for the two decades preceding 1962, 1942-61, and considerably lower (about one) if estimated from a period two decades earlier, 1922-41. The annual ratio $\frac{E(r_m - r_f)}{\text{var } r_m}$ would not be substantially changed if $E(r_m)$ is estimated over the period since the turn of the century from the "expected" rates of return based on current dividend yield and past growth rates (Table 4).

If monthly realized rates of return in the period after World War II are used to estimate $\frac{E(r_m - r_f)}{\text{var } r_m}$[3], where r_f is now the Treasury bill rate,

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[1] It is 12% on the most inclusive definition of net worth and 20% on the least inclusive.

[2] Time series data point to somewhat smaller figures.

[3] Monthly returns permit a somewhat more accurate estimate of C than annual figures, but they are not available as far back historically.

the ratio is over 5 when estimated from the 1947-62 period and over 3 when estimated from the 1947-71 period.

Obviously, the data do not permit a satisfactory estimate of $\frac{E(r_m - r_f)}{\text{var } r_m}$ as of the end of 1962 or the end of 1971 but the value of this ratio seems to have been well in excess of one if the ratio is derived from NYSE common stocks. An estimate of 2 seems to be as good as the data permit, but the possible range of error in this figure can substantially affect our estimate of C. With an estimate of 2 for $\frac{E(r_m - r_f)}{\text{var } r_m}$, C would be estimated to be somewhat over 2.5 in 1962 and 1971. Given the margin of error in our estimate of $\frac{E(r_m - r_f)}{\text{var } r_m}$, the error in estimating C is quite large. However, our evidence strongly suggests that the elasticity of the marginal utility of wealth is well in excess of one, i.e. that investors are considerably more risk averse than would be indicated if the market utility function were logarithmic.

Two qualifications of the above conclusion should be pointed out. First, the assumption is made that the ratio of $E(r_m - r_f)$ to var r_m for NYSE stocks is the same as for all risky (i.e. all non-risk-free) assets combined. Human wealth and equity in non-corporate enterprises (because of market imperfections if for no other reason) probably have a higher capitalization rate than NYSE stocks, and equity in housing may and long-term fixed interest bearing obligations do have a lower rate.[1] For the aggregate of risky assets, the expected rate of return is not likely to be appreciably below the expected rate for NYSE stocks, and may well be higher, though of course it could be appreciably below the observed

[1] This is likely to be true on a tax-adjusted as well as before tax basis.

of these equities. While a continuous series of such information would be most desirable, even a small number of carefully designed spot surveys of this nature would be extremely helpful. The necessary data are known to exist for institutional investors, and there is reason to believe that they also do for at least the large individual investors who together with institutions dominate stock holdings and transactions. It might also prove possible to use this approach to obtain useful new insights into the elements entering into the risk evaluation of equities by investors. More satisfactory *ex ante* measures of return and risk would, of course, further illuminate the nature of the market utility function.

Appendix Table 1

Additional "Expected" Annual Rate of Return Estimates for NYSE Composite Common Stocks, 1882-1971

	Estimate 3		Estimate 4	
	Mean Return	Standard Deviation	Mean Return	Standard Deviation
1882 - 1891	.066	.025	.051	.025
1892 - 1901	.067	.043	.037	.037
1902 - 1911	.157	.031	.132	.034
1912 - 1921	.132	.042	.097	.032
1922 - 1931	.177	.058	.059	.044
1932 - 1941	.064	.070	.011	.057
1942 - 1951	.160	.041	.126	.045
1952 - 1961	.137	.051	.122	.047
1962 - 1971	.086	.010	.080	.011
1882 - 1961	.120	.064	.080	.060
1882 - 1971	.116	.062	.080	.056

Note: The sources in this table are the same as those in Table 4 except that in Estimate 3 the expected growth rate for any year (GE_t) is taken to be the arithmetic mean of the previous 10 years of earnings growth and in Estimate 4 GE_t is the geometric mean of the previous 10 years of earnings growth.