Notes on the Theory of Optimal Public Investment in Pollution Control

by

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Environmental pollution created as an externality of either private market or public economic activities has become an issue of growing public concern. From this concern has emerged a wide range of policies and penalties that are designed to curb pollution by curbing polluters. Most of these have proved only partially effective. This paper is concerned with what happens when government decides to go directly into the business of "reprocessing" the polluted environment, restoring it to desirable levels, rather than solely relying on economic sanctions to protect its constituents. Our analysis, a theoretically oriented presentation, is concerned with the optimal public investment in a world of second best, and thereby attempts to integrate the recent economics of pollution literature[1] with that of public investment decisions.[2]

After a review of some of these failed policy instruments in Section I, our analysis in Section II will introduce a model for determining the socially optimal size for a pollution control plant under conditions of perfect foresight. In Section III, utilizing the time-state-preference framework, we will examine optimal policy when there is an uncertain future social demand for pollution control. Section IV will study the optimal control plant size over time, taking into account depreciation and reinvestment possibilities. Section V will explore the social welfare loss created by governmental investments in pollution control that are, by their very existence, "second best" solutions. Our general strategy for this presentation is to set-up a simple model, which we will subsequently modify by altering one assumption at a time.

I. CURRENT POLLUTION CONTROL POLICY

A. Regulating Polluters

The most common method for regulating pollution is to create an independent agency that determines the appropriate anti-pollution policy instruments, monitors pollution emissions, and enforces pollution control policy. The range of instruments for pollution control has been diverse.[3] For example, a frequently used anti-pollution policy instrument has been the setting of permissible pollution standards, such as for automobile emissions or discharges of effuents into rivers by factories.

The standards approach has many intrinsic difficulties. First, each violation may require enforcement activities by the agency that can involve significant administrative and legal resources. Second, legal recourse is often time consuming, offering the polluter an effective delaying tactic. Third, standards, once achieved, do not engender incentives for further pollution reductions or additional research and development for pollution control. Finally, unless the fines imposed for violations of standards are more than the costs of pollution reduction, it may be economic for the polluters to disregard the standards.

Another common set of pollution control policies tax the polluter according to the quantity of pollutants he creates. Such pollution taxation plans may not stop or necessarily reduce pollution, depending inevitably upon how the pollution tax schedule affects the polluters pecuniary incentives. Moreover, there are cases when it may be politically either unfeasible to utilize an appropriate tax scheme or unacceptable to increase an existing pollution tax rate sufficiently to achieve the socially optimal pollution level.

B. Other Dimensions of the Control Problem

Frequently, the pollution is caused by the public sector. In such a case, the decision to control pollution is already in the public domain. The control of publicly created pollution requires either a public adjustment in its direct production of goods and services or a reduction in the pollutant after it is generated as an externality to public production.

Finally, there may exist economies of scale in pollution reduction. For example, a single centralized pollution control plant may be more efficient than in-plant pollution control measures simultaneously taken in several spatially separated plants. Under current circumstances, polluters in the private sector may not have incentives to employ economies of scale for pollution control. This may be particularly true when (1) the total level of future pollution is uncertain; (2) the banding together of polluters may change the individual's propensity to pollute; and (3) the harnessing of the economies of scale requires a large level of current investment.

II. DIRECT GOVERNMENTAL POLLUTION CONTROL

From a societal purview, the standards and effluent tax approaches may have permitted sub-optimal pollution levels to persist. Hence, the public sector may deem it necessary to establish pollution control plants. Almost tautologically, the objective of the optimal pollution control plant will be to maximize the expected discounted net social benefits it will produce. If the precise extent of future pollution,[4] its social disutility in monetary terms, and the operating and capital costs for reducing these pollutants are known, the problem can be solved easily. The traditional economic shibboleth would suggest the expansion of the project such that the discounted net social marginal benefits would be zero.

A. A Simple Two-Period Model with Perfect Foresight

First, we shall formulate these notions in a simple two period model. We assume that the pollution control authorities construct the pollution control plant in the current period (0). This will be construed as the "public investment" in pollution control. The pollution itself is reduced by reprocessing the "polluted" environment during the subsequent period (1). The extent and the cost of the reprocessing in period 1 depends upon the size or capacity of the plant that has been built.

1. The Measure of Social Value

In this paradigm we shall subsume that the gross benefits to society are measured by the area under the social demand function for the "reprocessed environment." For example, if polluted water is reprocessed, the social demand function will evaluate the benefits to society in terms of tons of pollutants removed from the environment per year. If X is the quantity of the pollutant removed from the environment and V(X) is the social demand function of X, the total benefits to society are $\int_0^X V(y)dy$.[5] In other words, V(X) is a monetary measure of the decrease in the social disutility caused by the "removing" of the Xth ton of pollutant from the environment.

It should be noted that in this version of our paradigm V(X) is known with certainty. That is, in the future period 1, there is only one relevant state of the world. Therefore, it is appropriate to discount monetary flows over time by using the riskless social time preference interest rate. If r is this riskless interest rate, the discount factor for costs and benefits known with certainty occurring during period 1 will be

$$D(1) = \frac{1}{1 + r}$$

2. The Model

We will employ the following additional symbols:

K = the capacity of the pollution plant in terms of the volume of the environment it can reprocess per period.

C(X,K) = the operating and maintenance costs incurred by the society when reducing environmental pollutants by X units per year in a plant with a capacity of K, and $X \leq K$.

I(K) = the public investment for convenience, assumed to take place entirely in period 0; the initial cost of constructing the pollution project with a maximum pollutant reduction capacity of K units per year.

Mathematically, the constrained objective function of the pollution authorities is

$$\max_{X,K} \quad P = D(1)\left[\int_0^X V(y)dy - C(X,K)\right] - I(K) - \lambda(X - K) \qquad (1)$$

such that $X,K \geq 0$ and $X \leq K$

where λ is the Lagrangian multiplier. The constraint implies that one cannot reprocess more than the plant's capacity. The optimal solution can be found by simultaneously solving equations (2), (3) and (4):

$$D(1)\left[V(X) - \frac{\partial C(X,K)}{\partial X}\right] = \lambda \qquad (2)$$

$$D(1)\ \frac{\partial C(X,K)}{\partial K} + I'(K) = \lambda \qquad (3)$$

$$\lambda = 0 \text{ or } X - K = 0;\ K \geq 0;\ X \geq 0 \qquad (4)$$

3. Discussion about Marginal Conditions

Assuming that $K > 0$ (i.e., a plant will be constructed if $P > 0$ for some $K > 0$), it is clear that in this model one would never build excess capacity. Therefore, the constraint is binding and $\lambda \geq 0$. The optimal plant size will be $K^* = X^*$, and, by rearranging (2) and (3), can be found by solving

$$D(1)V(X^*) = D(1)\left[\frac{\partial C(X^*,K^*)}{\partial X} + \frac{\partial C(X^*,K^*)}{\partial K}\right] + I'(K^*) \qquad (5)$$

The left hand side of equation (5) is the discounted marginal social benefits of the pollution plant; the right hand side represents the components of discounted marginal social costs.

λ is clearly the shadow price for capacity creation. From equation (3), it is observed that the shadow price for capacity, for a given X, consists of two elements: the discounted savings in marginal costs created by capacity expansion and the marginal costs of constructing the new capacity (i.e., the change in the cost of the initial public investment). Combining equations (2) and (3) indicates that for the optimal $K = K^*$, in equilibrium, the net discounted benefits (i.e., the marginal "social" gain) will equal the discounted marginal "social" costs of expanding K. This is the conclusion drawn from equation (5) and is presented graphically in figures 1(a) and 1(b), where λ^*, X^*, and K^* (the asterisks denoting optimality values) are determined simultaneously.

Figure 1(a) and 1(b) illustrate the usual conditions that we would expect in terms of costs and benefits functions. However, second order conditions for maximization can be consistent with "pathological" cost and benefit functions. The second order conditions for a local maximization at λ^*, X^* and K^* require that

$$\begin{vmatrix} F_{XX} & F_{XK} & 1 \\ F_{KX} & F_{KK} & -1 \\ 1 & -1 & 0 \end{vmatrix} = |D| > 0$$

where we define at the point λ^*, X^*, K^*:

$$F_{XX} = D(1)\left(V'(X) - \frac{\partial^2 C}{\partial X^2}\right)$$

$$F_{XK} = F_{KX} = D(1)\,\frac{\partial^2 C}{\partial X \partial K}$$

$$F_{KK} = -D(1)\,\frac{\partial^2 C}{\partial K^2} - I''(K)$$

$|D| > 0$ will occur if

$$D(1)\;\; V'(X) < D\;(1)\left[\frac{\partial^2 C}{\partial x^2} + \frac{2\partial^2 C}{\partial X \partial K} + \frac{\partial^2 C}{\partial K^2}\right] + I''(K)$$

In words, if the discounted marginal social gains are increasing less rapidly than the discounted marginal social costs, the local extrema will be

Figure 1(a)

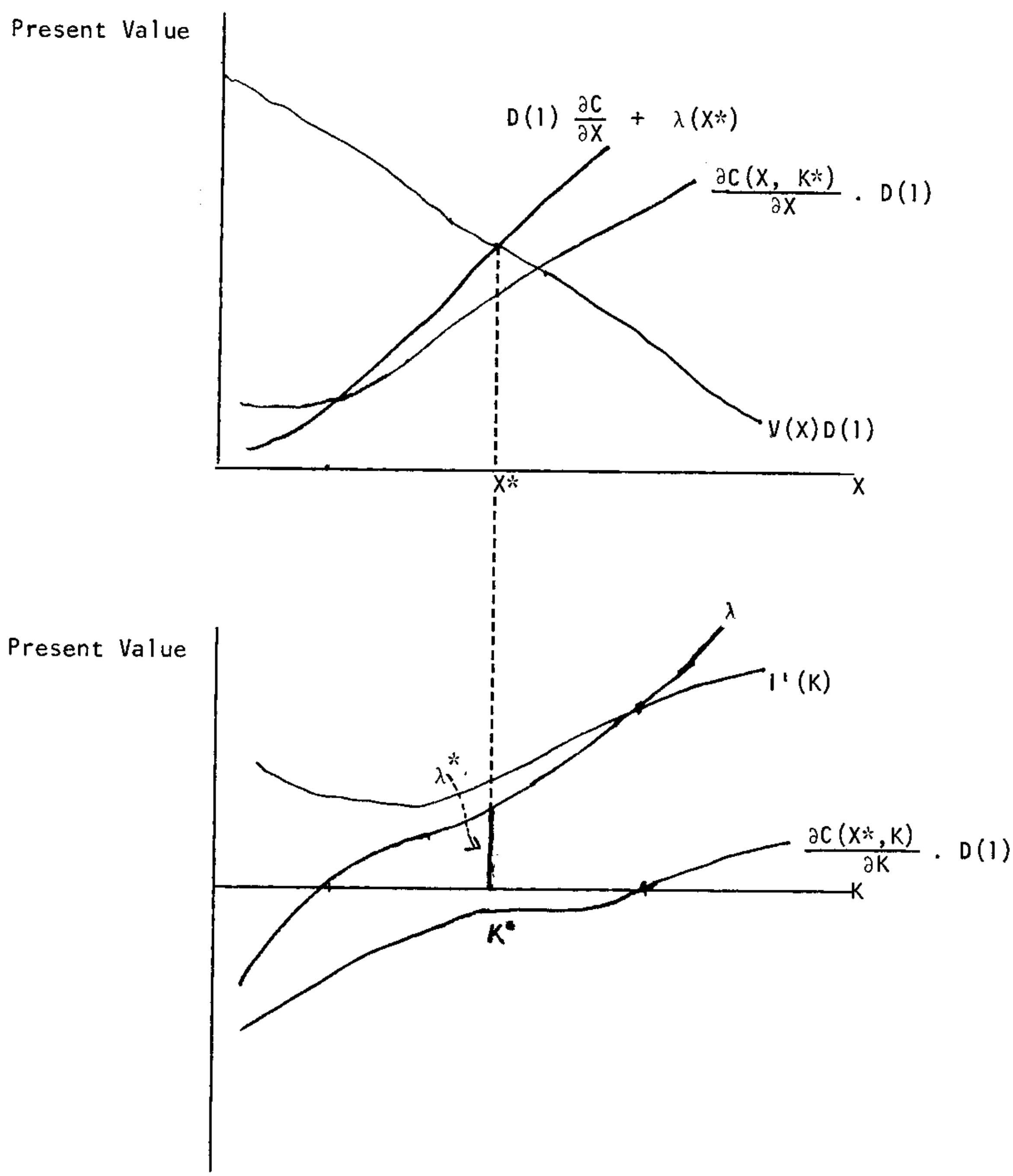


Figure 1(b)

a maximum. In microeconomic parlance, the maximization occurs if the discounted marginal cost curve intersects the discounted marginal benefit function from below. Therefore, this does not rule out the possibility, a relatively plausible one in cases of pollution analysis, that the social demand function may be in certain ranges upward sloping.

B. A Multi-Period Model with Perfect Foresight

Our model can be expanded to yield conclusions for the optimal pollution control plant in a multi-period context. We shall continue to assume a world with certainty for both costs and benefits in all future periods. That is, there is only one state possible for each future period. Our notations will be modified for the multi-period model as follows:

$D(t)$ = the riskless social time preference discount factor for period t; that is, the social present value of a dollar for certain t periods hence; it will be the appropriate discount factor for both social costs and social gains.

$X(t)$ = the quantity of the pollutant reduction in period t

$V(X(t),t)$ = the social demand function for processing $X(t)$ in period t

$C(X(t),K,t)$ = the operating and maintenance costs for reprocessing $X(t)$ in a plant with capacity K.

$I(K)$ = as before, the initial public investment needed to construct a plant with a capacity for processing $X = K$ per period; implicitly, the cost of capital for the project is assumed to be the social time preference interest rate.

A further word about the social demand function $V(X(t),t)$. It is alleged to be the measure of gross social benefit that is derived in each period for the last unit of pollutant treated if X(t) is the quantity of reduced pollutants. It is clear that this formulation is intended to include intra-temporal externalities. For example, there are obvious social benefits in period t for the consumers of reprocessed or purified water for drinking. Also, the purified water may facilitate the existence of beautiful flora and fauna, thereby creating a positive externality in period t to fisherman, hunters, and so forth. The principal and externality benefits are included in the value assigned to the social demand function for each X(t). Also, the "cleaning up" of the environment in period 1 may provide future generations with a socially desired clean environment, and thereby generates an inter-temporal externality benefit. For the moment, however, we will assume away inter-temporal externalities. Clearly, these latter types of externalities exist and will be discussed in a later section.

The objective function will be transformed in the multiperiod model to

$$\max_{X(t),} \quad P = \sum_{t=1}^{n} D(t) \left[\int_{0}^{X(t)} V(Y(t), t)\,dY - C(X(t), K, t) \right] - I(K) - \sum_{t=1}^{n} \lambda(t)(X(t) - K) \qquad (6)$$

where n is the relevant life of the project under consideration. The marginal conditions for optimization are

$$D(t) \left[V(X(t),t) - \frac{\partial C(X(t), K, t)}{\partial X(t)} \right] = \lambda(t);\ t = 1, \ldots, n \qquad (7)$$

$$\sum_{t=1}^{n} D(t) \frac{\partial C(X(t), K, t)}{\partial K} + I'(K) = \sum_{t=1}^{n} \lambda(t) \qquad (8)$$

$$\lambda(t) = 0 \text{ or } X(t) - K = 0;\ X(t) \geq 0;\ K \geq 0;\ t = 1, \ldots, n \qquad (9)$$

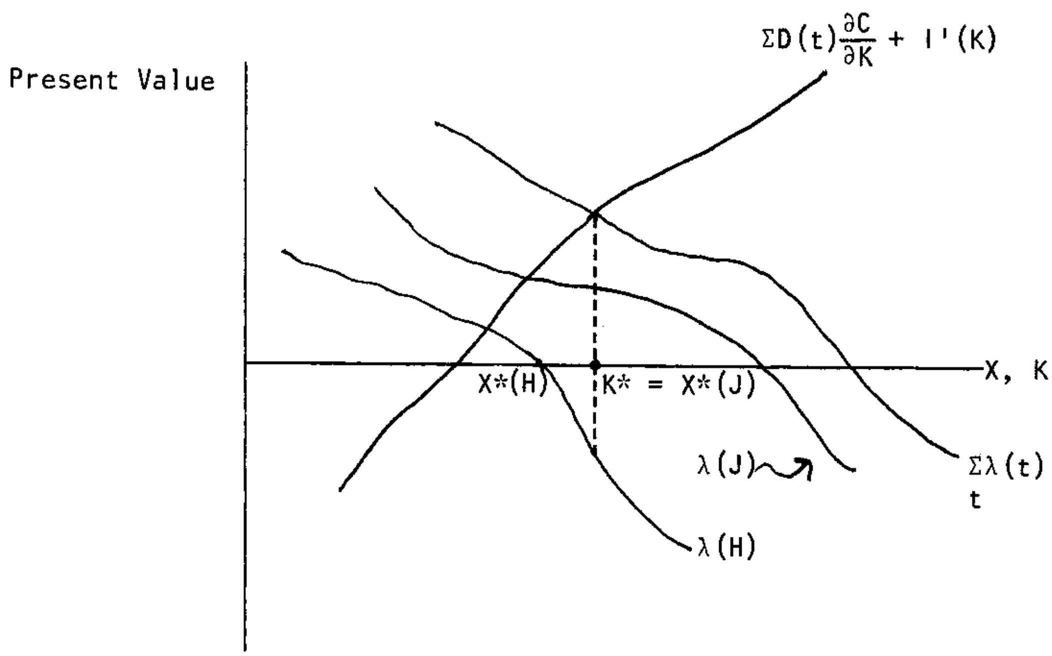
1. Discussion about Marginal Conditions for Multi-Period Model

This solution is easy to interpret, and is similar to the two-period model above. Note that $\lambda(t)$ will be zero for periods when $X(t) < K$. In such a case, equation (7) suggests that if the pollution control plant is operating at less than capacity, it is socially desirable to equate discounted marginal benefits with discounted marginal operating costs. This is analogous to the theory of the firm where in the short run it does not consider its fixed costs, when equating discounted marginal costs and revenue in order to maximize the present value of profits.

Of course, there must be at least one time period for which capacity is binding, implying at least one $\lambda(t)$ will be non-negative. The equilibrium condition, represented by equation (8), states that the difference between savings on discounted operating costs and the marginal costs of the public investment induced by an expansion in capacity is equal to the sum of the shadow prices. Hence, using equation (7), the sum of the λ's equals the sum of the net discounted marginal benefits for all periods when the capacity constraint is binding. In other words, the sum of the shadow prices represents the discounted net social value for a marginal expansion of capacity, and, therefore, suggests that capacity will be expanded until the discounted marginal social gains are equal to the social marginal costs.

Graphically the multi-period model is illustrated in figure 2. The $\lambda(t)$ terms represent the values consistent with equation (7) above. Therefore, $\sum_t \lambda(t)$ is the net discounted marginal benefits which is consistent at the margin with varying the capacity level K. The upward sloping curve is similar to that presented in figure 1(a) above; and is derived

Figure 2



Note: $\lambda(t) = D(t)\left[V(X(t),\ t) - \frac{\partial C(X(t),\ t,\ K)}{\partial X(t)}\right]$

from the left hand side of equation (8). It is the net discounted marginal cost associated with each capacity level K. The marginal conditions for equilibrium occur where the two functions intersect, implying the net discounted marginal benefits and marginal costs are equal, and no further capacity expansion is socially desirable.

The implications for the optimal level of pollution reduction for two particular time periods are illustrated in figure 2. In time period J, capacity is binding, and $K^* = X^*(J)$, with the $\lambda^*(J) > 0$. In period H, capacity is "optimally" underused for $X^*(H) < K^*$ and $\lambda^*(H) = 0$.

C. An Illustration of the Multiperiod Model with Perfect Foresight

As an example for the multiperiod model, let us consider the optimal level of pollution control under the following restrictive assumptions:

1) $D(t) = D^t = \frac{1}{(1+r)^t}$ for all t

2) $V(X(t), t) = V(X(t))$ for all t

3) $C(X(t), K, t) = C(X(t), K)$ for all t

Assumption 1 indicates that the social time preference discount rate is unchanging over time. Assumption 2 describes a world for which the value of pollution control (in current dollars) is independent between periods, and evaluated similarly in all periods. Assumption 3 signifies that, for each capacity level for the pollution plant, the variable cost structure of the project is independent between time periods and unchanging over time.

Using these assumptions, the marginal conditions represented by equations (7) and (8) can be rewritten as equations (10) and (11).

$$D^t \left[V(X(t)) - \frac{\partial C(X(t),\ K)}{\partial X(t)} \right] = \lambda(t) \text{ for } t = 1, \ldots, n \qquad (10)$$

$$\sum_{t=1}^{n} \left(D^t \cdot \frac{\partial C(X(t),\ K)}{\partial K} + I'(K) \right) = \sum_{t=1}^{n} \lambda(t) \qquad (11)$$

It is clear from equation (10) that for all time periods, if $\overline{X} = X(t)$, then $|\lambda(t) - \lambda(t+\varepsilon)| \geq 0$ for ε, a positive integer. This is true because $V(\overline{X}) - \frac{\partial C(\overline{X},K)}{\partial X(t)}$ is a constant, while D^t must decrease with time. Therefore, $\lambda(t) > 0$, for some T such that $1 \leq t \leq T \leq n$; and $\lambda(t) = 0$ thereafter (i.e., $t > T$). In words, if the project is built, the capacity constraint will be binding for the initial T periods (and at least the first period), and not binding for $n \geq t > T$. The value of $\lambda(t)$ will change such that

$\lambda(t) = \lambda(1)D^{t-1}$ for $1 \leq t \leq T \leq n$

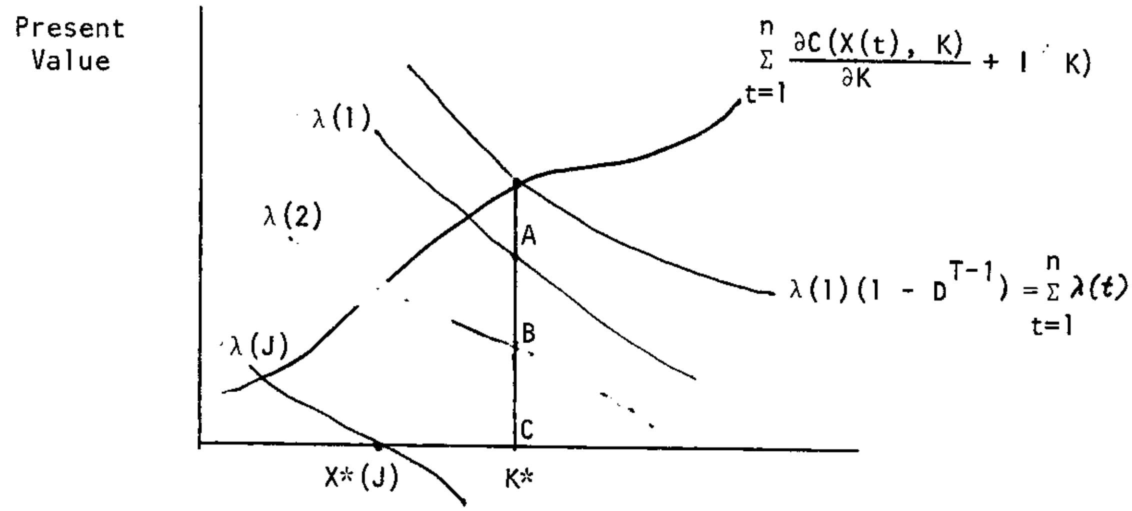
$\lambda(t) \equiv 0$ for $n \geq t > T \geq 1$

The contribution from each time period to the total shadow price for plant expansion for time periods when capacity is binding decreases in each period by a factor of the social discount rate D (i.e., geometrically decreasing over time). The total shadow price for capacity expansion is the sum of the λ's:

$$\sum_{t=1}^{n} \lambda(t) = \sum_{t=1}^{T} \lambda(t) = \lambda(1) \sum_{t=1}^{T} D^{t-1} = \frac{\lambda(1)}{r} (1-D^{T-1}) \qquad (12)$$

where $r \geq 0$ is the social time preference interest rate. This situation is illustrated in figure 3 which is similar to figure 2 above. In periods

Figure 3



$K^* = X^*(1) = X^*(2)$, $*(1) = AC > \lambda^*(2) = BC$

$X^*(3) < K^* \Rightarrow \lambda^*(3) = 0$

Note: $D \cdot \lambda(1) = \lambda(2)$

1 and 2, with $D \cdot \lambda(1) = \lambda(2) > 0$, the optimal pollution control requires that environmental pollutants be reduced such that $X^*(1) = X^*(2) = K^*$. Period J is an example for which it is optimal to process less than K^* per period because $\lambda^*(J) \equiv 0$ for $X^*(J) < K^*$.

Finally, there is an interesting sub-case in terms of intergenerational equity. If, as any catechumen of intertemporal equalitarianism would argue, the "intergenerational" social time preference interest rate should be zero for evaluating the protection of resources through pollutant reduction, In terms of our model this means $D^t \equiv 1$ for all t, and therefore $\lambda(1) = \lambda(t)$ for all t. Under these assumptions, for a project with a finite life time, the quantity of pollutants treated will be equal in each period. In this type of project, $\lambda(1) < \sum_{t=1}^{n} \lambda(t) = n \cdot \lambda(1)$ indicates that the relative amount of pollution reduction in period 1 will be smaller than under projects where $D < 1$. Also, it is clear that $X^*(1)$ will be greater in absolute value when $D \equiv 1$ as compared to $D < 1$, thereby implying that the absolute quantity of pollution reduction will be increased for everybody in all generations when the social discount rate does not have an intergenerational "bias".

D. An Extension: Temporally Interdependent Social Demand for Pollution Reduction

Costs and benefits have been assumed to be temporally independent. However, it is easy to conceive of situations for pollution control where social costs and benefits are not temporally independent. For example, the costs incurred for removing X tons of pollutants in our rivers during the current period may depend upon the level of pollutants contained in each gallon of water that is processed. This, in turn, may be related to the number of gallons of water processed in previous periods. Similarly, in the case we will pursue shortly, the current social value attributed to the reduction of pollution may depend upon how much pollution was reduced in earlier time periods. In our patois, the current period's social demand function for pollution reduction would be dependent upon all X(t)'s that occurred before the present period. The following social demand function is an illustration of the effects of this type of temporal interdependency.

$V(h) = V(\sum_{j=1}^{h} X(j))$ where h is the current period. This social demand function signifies that previous pollutant reduction, no matter when it occurred in the past, creates identical increases in social benefits according to the sum total of pollution reduction throughout all previous time periods. (Also, we continue to assume that V and C are stable functions, independent of time.)

The objective function for the pollution authority now will become

$$\max_{X(t),K} P = \sum_{t=1}^{n} D(t) \left[\int_0^Q V(Y(t))dY - C(X(t), K) \right] - I(K) - \sum_{t=1}^{n} \lambda(t)(X(t) - K) \qquad (13)$$

where

$Q = \sum_{j=1}^{t} X(j)$ and $t = 1, \ldots, n$

The marginal conditions for optimizing pollution control will be

$$\sum_{t=h}^{n} D(t)\, V\left(\sum_{j=1}^{t} X(j) \right) - D(h)\frac{\partial C(X(h), K)}{\partial X(h)} = \lambda(h) \qquad (14)$$

$h = 1, \ldots n$

$$\sum_{t=1}^{n} D(t)\,\frac{\partial C(X(t),\,K)}{\partial K} + I'(K) = \sum_{t=1}^{n} \lambda(t) \tag{15}$$

Equation (14) demonstrates that the true social marginal value of pollution reduction in the h^{th} period must take into account the discounted social gains created for all future periods. In equilibrium, the discounted stream of marginal benefits for the current and future periods less the social discounted marginal cost will be the shadow price for capacity for the present period.

The joint solution of Equations (14) and (15) will yield the optimal public investment for capacity, shown as equation (16).

$$\sum_{t=1}^{n} D(t)\left[t\cdot V\left(\sum_{j=1}^{t} X(j)\right) - \frac{\partial C(X(t),\,K)}{\partial X(t)}\right] = \sum_{t=1}^{n} D(t)\,\frac{\partial C(X(t),\,K)}{\partial K} + I'(K) \tag{16}$$

The interpretation of equation (16) is straight-forward. The right hand side is, as discussed earlier, the net discounted marginal costs for creating additional pollution processing capacity. The left hand side, is the net discounted marginal benefits, including intertemporal externalities, for pollution reduction for optimal capacity utilization. In a socially optimal equilibrium, these discounted marginal gains and costs must be identical.

III. POLLUTION CONTROL WITH UNCERTAIN SOCIAL DEMAND

A. The Introduction of Uncertainty

In general, the precise magnitudes of the future social demand for pollution control, the operating costs, and sometimes, the cost of the initial public investment for a pollution reduction project are not known. These values may be known in terms of likelihood or probability determined by an "uncertain" product demand. Finally, the entry and exit of potential polluters is a temporally stochastic variate, depending upon, among other things, perceived expectations about economic opportunities.

B. The Time-State Preference Model

Theoretically, it is straightforward to extend our analysis for the socially optimal pollution control project for conditions of uncertainty by utilizing a time-state preference benefit-cost model. The ability to do this analysis requires an *a priori* estimate of the probability distributions for social benefits and social costs, conditional upon their future timing and the relevant state of the world in each period. That is, uncertainty about conditions in future periods must be described by specifying a set of possible "states of the world," only one of which will actually occur in each future period. The realized values of the benefits and costs are, also, contingent on which state actually occurs.[7] We therefore require that the hypothetical decision-maker has an *a priori* notion of the social values of pollution reduction for all future periods contingent upon the state of the world in each of these periods and the appropriate social discount factors for each potential state in each future time period.

For example, if there were only one future time period with two mutually exclusive, but exhaustive possible states of the world, the appropriate estimates of the discount factors are, say, .50 and .30 for state 1 and state 2 in Period I, respectively. That is, if I pay \$.50 now, I will receive \$1 *contingent upon* state 1 occurring in the future period; similarly \$.30 now will yield \$1 in the future period contingent upon state 2. In essence, the social discount factors take into account the probability of each state occurring *and* the social time preference interest rate that will be relevant if that state occurs. This, also, can be related to the estimate of the risk-free rate. In this example, an expenditure of \$.80 now (i.e., \$.50 + \$.30) will guarantee a dollar <u>for sure</u> in the next period.

$$D(t = 1,\ s = 1) + D(t = 1,\ s = 2) = .80 = \frac{1}{1 + r}$$

where r is the risk-free rate.

Therefore, in this example the risk-free social time preference rate is 25 per cent.

1. The Model

If our policy-maker has the subsumed a priori estimates, the objective function for the net expected discounted social benefits for the pollution control project, constrained by capacity limits, will be:

$$\underset{X(s,t),K}{\text{Max}}\ P = \sum_{t=1}^{n} \sum_{s=1}^{m} D(t,s) \left(\int_{0}^{X(t,s)} V(Y(t,s),\ t,s)\ dY - C(X(t,s),\ t,\ s,\ K) \right)$$

$$- \ I(K) - \sum_{t=1}^{n} \sum_{s=1}^{m} \lambda\ (t,s) \left[X(t,s) - K \right] \qquad (17)$$

where n = the relevant future life of the project.

m = the mutually exclusive, but exhaustive possible states of the world in each future period.

D(t,s) = the discount factor for period t if state s obtains.

X(t,s) = the quantity of the pollutant reduction in period t contingent upon state s.

V(X(t,s),t,s) = the social demand function for reprocessing X(t,s) in period t if state s occurs.

C(X(t,s),t,s,K) = the operating and maintenance costs for reprocessing X(t,s) in a plant with reprocessing capacity K if state s obtains in period t.

The marginal conditions for social optimization will be

$$D(t,s)\left[V(X(t,s),t,s) - \frac{\partial C}{\partial X(t,s)}\right] - \lambda(t,s) = 0 \begin{Bmatrix} t = 1,\ldots,n \\ s = 1,\ldots,m \end{Bmatrix} \tag{18}$$

$$-\sum_{t=1}^{n}\sum_{s=1}^{m}\left(D(t,s)\frac{\partial C}{\partial K} - \lambda\ (t,s)\right) - I'(K) = 0 \tag{19}$$

$$\lambda(t,s) = 0 \text{ or } X(t,s) - K = 0;\ X(t,s) \geq 0;\ K \geq 0 \text{ for all } t,s \tag{20}$$

2. Significance of Marginal Conditions

The constrained objective function (17) and the marginal conditions (18), (19), and (20) are similar to (6)-(10), except that we have introduced states of the world for each time period. The interpretation of these marginal conditions is, as before, easy. In any time period t, for each state of the world s, the $\lambda(t,s)$ is a shadow price which equals the appropriately discounted (i.e., present social value) difference between marginal expected social gains and marginal expected social costs. If $X(t,s)$ is less than capacity K, $\lambda(t,s)$ will be zero with the net social benefits being zero. Analogous to the certainty model, capacity must limit the expected pollution reduction for at least one state in one future time period. That is, it is possible that no state will occur which actually utilizes this capacity. Moreover, the sum of the shadow prices $\lambda(t,s)$ over all states and time periods represents the marginal expected present social value for expanding capacity; and capacity should be expanded until the marginal expected social cost incurred through additional expansion equals the marginal expected social gains over all periods and states. In fact, the introduction of uncertain future states offers no new analytic and interpretative difficulties.

IV. Optimal Pollution Control with Continuous Investment Opportunities[8]

Our analysis to this point has permitted investment in the pollution control project during the initial time period only. This apparently stringent framework was adopted for illustrative reasons. We shall now briefly demonstrate through an example how we can modify our model to incorporate continuous investment activities for the pollution control plant as well as continuous depreciation of the existing pollution control plant.

For the sake of convenience, we will return to a model with perfect foresight for costs and benefits over an infinite time horizon. Also, it will be necessary to modify the time functions from discrete to continuous forms. $X(t)$, $I(t)$, and $K(t)$ are the amounts of pollutant reduction at time t, the value of expenditures on capital to increase capacity at time t, and the total capacity for the pollution reduction at time t, respectively. X and I are considered control variables, and K is a state variable. We define $R(K,X,I)$ as the net social benefit function, and at any instant in time, t, is evaluated to be consistent with our prior models as

$$R(K,X,I) = \int_0^X V(Y)dY - C(X,K) - I(K)$$

We will utilize an instantaneous discount rate r such that $D(t) = e^{-rt}$. We continue to constrain the control variable X by the state variable K such that

$$K(t) - X(t) \geq 0 \text{ at each } t$$

Finally, we need to know the state transition or motion differential equation for the state variable:

$$\dot{K} = K + f(I) - \delta K$$

where f(I) = the amount of capacity that can be installed for an expenditure of I; δ = the instantaneous depreciation rate for the current capacity level (i.e., capital stock for pollution reduction).

In this model, the objective function we wish to maximize is

$$\int_{o}^{\infty} R(K,X,I)e^{-rt}\, dt \qquad \text{subject to}$$

$$\dot{K} = K + f(I) - \delta K \text{ and } K\text{-}X \geq 0$$

The problem can be solved by defining the Hamiltonian, H:

$$H = e^{-rt}\left(R(K,X,I) + \lambda(K-X) + u(K+f(I) - \delta K)\right) \qquad (21)$$

where $\lambda(t)$ is the shadow price for the capacity constraint in each state, and u(t) is the shadow price for the social rate of return on capacity growth at any point in time.

Our analysis, being illustrative, will only delineate the first order conditions for the maximal <u>interior</u> time path solution. Applying Pontryagin's maximum principle, we know that a maximum for the problem is achieved if we maximize the Hamiltonian and the shadow price u satisfies (assuming appropriate initial conditions):

$$\frac{du}{dt} = u \cdot r - \frac{\partial H}{\partial K} = -u(1-r-\delta) - \lambda - \frac{\partial R}{\partial K} \qquad (22)$$

$$\lim_{t \to oo}\left(e^{-rt}\ u(t)\right) \geq 0 \text{ and } \lim_{t \to oo}\left(e^{-rt}\ u(t)\ K(t)\right) = 0$$

Also, maximization requires that

$$\frac{\partial H}{\partial I} = 0 = \frac{\partial K}{\partial I} + uf'(I) \qquad (23)$$

$$\frac{\partial H}{\partial X} = 0 = \frac{\partial R}{\partial X} - \lambda \qquad (24)$$

Equation (23) signifies that in equilibrium the net social marginal gains from investment expenditures $(\frac{\partial R}{\partial I})$ must equal the investment in capacity times the social marginal rate of return for capacity (u). This is completely analogous to our earlier models. Equation (24) generates a marginal condition that also mirrors our earlier models' developments. If capacity is binding at time t, it implies that $-\lambda(t) \geq 0$ and the net social marginal gain for pollutant reduction $(\frac{\partial R}{\partial X})$ will equal $-\lambda$. If $-\lambda = 0$, which occurs if capacity is a slack variable, the net social marginal gains for additional capacity will be zero.

Equation (22) appears to be a quantum change from our previous models, but really is not. It is used to connect through time the change in the rate of return on capacity growth (i.e., the inter-temporal investment incentive) taking into account the time discount factor, the degree of capacity utilization (K-X), the depreciation rate of the current capacity, and the change in the short term cost structure with respect to changing K (i.e., $\frac{\partial R}{\partial K}$).

While we will not go through the ramifications of steady-state equilibria, smooth paths to equilibria, and boundary solutions, this type of problem form has been analyzed,[9] and yields solution counterparts to the simpler models presented earlier in this paper.

V. Welfare Implications of Public Pollution Control Projects

Our analysis has demonstrated that for a given stream of generated pol-utants over time, known social demands for pollutant reduction and known clean-up costs functions, there exists an optimal set of public policy actions designed for reducing pollutants. In our models, the government's cleaning-up of the environment is essentially an after-the-fact activity for pollution con-trol rather than a set of actions affecting the immediate pecuniary incentives of the polluters, and was justified as frequently representing the only type of available, feasible public policy alternative. It is, therefore, possible that the public sector optimization under this constrained feasibility set for public actions may not yield the societal optimum optimorum. In this section of the paper, we will now identify the nature and the extent of this potential societal loss created by a necessarily "second best" public policy solution for pollution control.

A. Finding the Optimum Optimorum

As previously noted, it is convenient to view pollution as an externality resulting from the economic market-oriented activities of the polluters. In this scenario, the environment is being used as a relatively costless disposal bin for unwanted by-products of the economic activity of society. However, the aggregate social costs of pollution created by the operation of the economy are considered to be non-trivial, and need to be explicitly measured in the accounting of social welfare.

For simplicity, assume a one time period context for our analysis. The mar-ket demand for good Q, the economic good, is D(Q). The private costs of production (i.e., excluding all pollution costs elements) are C(Q). From society's outlook, the value, V, of reducing the pollutant X, the by-product of Q, is functionally related to the quantity of pollutant reduction X and the level of Q in each period: V = V(X,Q). In terms of our earlier discussion above, V is the social demand for pollution reduction, except it now explicitly is related to the level of private market activity, Q. Finally, the social cost of directly cleaning-up the environment depends upon the quantity of pollutant to be reduced and the total level of pollutant that has been generated, X^M, where X^M is a function of Q. Therefore, S(X,Q) is the function for social costs of pollution reduction. S is the envelope curve for the discounted least cost combination of inputs for each (X,Q), and hypothetically is derived from the optimization solutions in our analysis above.

1. The Social Welfare Measure

If public policy decision-makers were capable of instructing each industry or firm to behave in order to maximize society's well-being, economic market and pollution related activities would be coordinated such that W would be maximized.

$$W = \left[\int_0^Q D(Q)\,dQ - C(Q) - \int_0^{X^M} V(X,Q)\,dX \right] - \left[S(X,Q) - \int_0^X V(X,Q)\,dX \right] \tag{26}$$

$$W = \left[\int_0^Q D(Q)\,dQ - C(Q) \right] - \left[\int_X^{X^M} V(X,Q)\,dX + S(X,Q) \right] \tag{27}$$

In the first form, equation (26), the two sets of large brackets separate the market activity pollution effects from governmental clean-up effects on the social welfare of society. The first set of bracketed terms represents the net consumers' surplus created by the consumption of Q, including the total social loss generated by the total pollutant, X^M, created as the by-product of Q without any environmental clean-up. The second bracketed terms reflect the net social gain from reducing pollution levels created as the by-product of Q by an amount X, where $X \leq X^M$.

Also, the social welfare measure can be written as equation (27). This latter regrouping of terms separates market decision variables in the first set of brackets and pollution reduction decision-making variables in the second set of brackets. The strictly market variable in our analysis is the choice of output levels, excluding the social costs of pollution, by the firms or the industry. The set of terms in the second brackets is construed as the societal loss created by the cleaning up of pollutants, X, once X^M has been created. That is, if society decides to remove $X \leq X^M$, pollution will engender a social loss in terms of the social value of the residual, unremoved pollutant (X^M-X) and the social costs for the removal of X.

2. Maximizing Welfare

In other words, W is the net social benefit to society, taking into account the value of economic goods, residual pollutants and pollution clean-up costs. It is perhaps intuitive but noteworthy that the social optimum may require that some level of pollutant should not be reprocessed. That is, if we denote * as the superscripts for social optimizing values for variables, $X^* \leq X^M$. Then, social welfare maximization will be W*, and is embedded in the simultaneous solution of the marginal conditions for W:

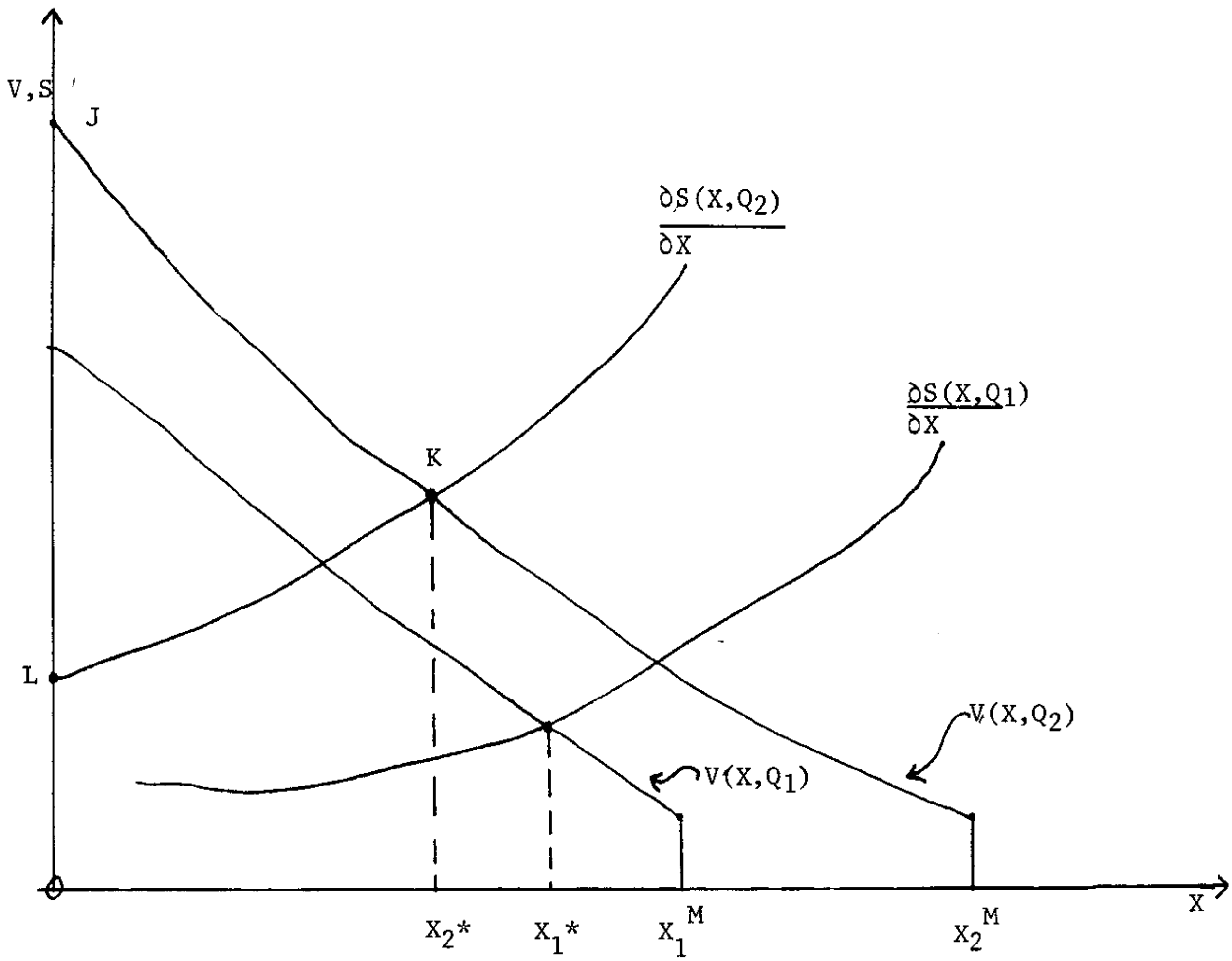
$$\frac{\partial W^*}{\partial X^*} = 0 = V(X^*,Q^*) - \frac{\partial S(X^*,Q^*)}{\partial X^*} \tag{28}$$

$$\frac{\partial W^*}{\partial Q^*} = 0 = D(Q^*) - C'(Q^*) - \frac{\partial}{\partial Q^*}\left[\int_{X^*}^{X^M} V(X,Q)dX + S(X^*,Q^*)\right] \tag{29}$$

The first of these marginal conditions for the maximizing of social welfare is analogous to our general pollutant reduction models presented above. In equilibrium, given the socially optimal level of output, Q*, and therefore its concomitant level of total pollutant X^M, the optimal quantity of pollutant reduction X* will occur when the sum of the social marginal costs for pollution reduction and the social marginal benefits derived from pollution reduction are zero. This marginal condition solution maximizes social welfare and differs from our earlier models in an important way: it is part of the co-determined solution with the social marginal condition for finding the socially optimal levels for Q* and X* simultaneously.

The analysis for determining the appropriate value for X is shown graphically in figure 4. The contribution of pollution reduction to the social welfare varies according to the level of output, Q, and the costs of pollution reduction, X. In figure 4, if Q_1 is the output, $V(X, Q_1)$ is the social demand function for pollution reduction, with $0 \leq X_1 \leq X_1^M$ where the subscripts correspond to the output levels. Similarly, if Q_2 is the output level, $V(X,Q_2)$ is the social demand function with $0 \leq X_2 \leq X_2^M$ being the appropriate range of pullutant reduction.[11] The marginal social costs of pollutant reduction varies according to the output level as well as the quantity of pollutants treated. Two examples of the social marginal costs functions are represented in figure 4 for output levels Q_1 and Q_2.

Figure 4: Optimal Pollution Reduction



X_i = pollutant reduction corresponding to Q_i

* = social optimizing value (superscript)

M = total or maximum possible value (superscript)

X_i^M corresponds to Q_i

X_i* corresponds to Q_i

If $Q = Q_1$, the optimal X is X_1*, and if $Q = Q_2$, the optimal X is X_2*. At these values for X, the net social marginal gains are zero, which is the optimization rule denoted by equation (28). The contribution to the social welfare function is a quasi-consumers' surplus, represented for Q_2 by the area JKL. For $Q = Q_2$, it is the net gain to society of reprocessing $X_2^* < X_2^M$. In other words, pollutant reduction yields a quasi-consumers' surplus that is the difference between the social value attributed to reducing pollution, J 0 X_2* K, and the social cost of reducing pollution, LOX_2*K, and is the <u>net</u> social welfare recaptured by society through its direct pollution reduction. Similar analysis could be applied for any other level of output. Also, note that optimal pollution reduction policy requires that $X_2^M - X_2^*$ pollutant not be reprocessed. The social disutility of this residue of pollution is the area K X_2^M X_2*. In figure 4, it is not economically feasible to reprocess this residue because of the high marginal social costs of further pollutant reduction relative to the potential social benefits. Mathematically, the social cost of pollution residue in the environment, considered uneconomic to reprocess, is

$$\int_{X^*}^{X^M} V(X,Q)\ dX.$$

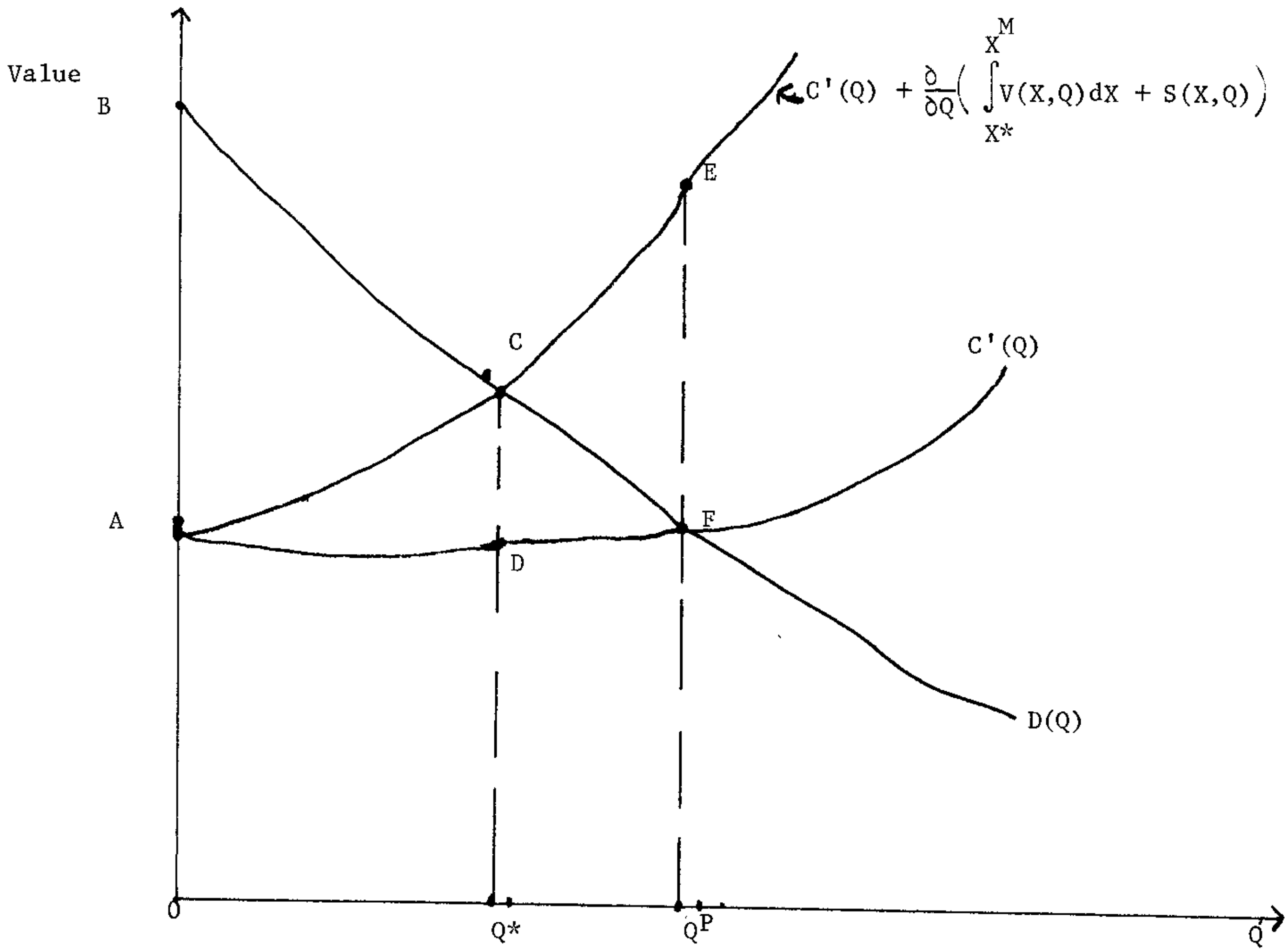
In a strictly private competitive market system, with social costs of pollution not considered by producers, private optimizing behavior of the economy would lead to $D(Q) = C'(Q)$. In figure 5, the private market solution would be $Q = Q^P$. However, it is argued that private marginal costs do not reflect social marginal costs because of pullution externalities. Therefore, in figure 5, the social marginal costs for the production of Q have been added to the strictly private marginal costs. The social marginal cost function takes into account, in accordance with our analysis for figure 4, the optimal X_i^* for each Q_i. That is, the social marginal costs incorporates the appropriate change in X* for changes in Q.

The socially optimal level of Q* is consistent with the marginal condition represented by equation (29). At that point, Q*, the sum of marginal costs to society and the marginal benefits are zero. The net social consumers' surplus is graphically shown as ABC, and is the difference between the gross value of demand for economic output (BOQ*C) and the gross social costs of production and pollution (AOQ*C). Note that after optimal pollution reduction has been conducted, the costs of the residue pollution and the costs of the partial cleaning-up of the pollutants reduce society's well-being by ADC. Of course, a public policy of <u>no</u> pollution reduction for Q = Q* would necessarily reduce the level of well-being more than ADC.

B. The Social Loss Created by Public Pollution Control

As a final point, we will demonstrate that a public pollution control project that reduced pollutants after they are generated, and does not alter the level of outputs of polluters will yield sub-optimal social welfare. Returning to figure 5, we can calculate the net social consumers' surplus[11] (i.e., the social welfare) when the competitive market is permitted to act unimpeded. We have already shown that output in this case will be Q^P, and private marginal costs are unequated with private marginal benefits (i.e., revenue). The total costs to society for an output of Q^P are $OAEQ^P$, and the total market produced benefits are $OBFQ^P$. The net gain to society is ABC-CEF, which is less than the net social consumers' surplus for Q* by an amount CEF. Assuming non-pathological

Figure 5: Social Welfare Maximization



Q^P = private market level of output, without taking into account net social costs of pollution.

Q* = Social optimum level of output

Net Social Consumers' Surplus for Q* = ABC

Net Social Consumers' Surplus for Q^P = ABC - CFE cases (i.e., the second order conditions for maximization are satisfied), this result is true in general. In other words, the inability to control pollution by taking appropriate economic sanctions that alter polluters' incentives will necessarily lead to socially sub-optimal welfare levels. Therefore, CEF is the social cost of non-control of polluters, including subsequent public policy for direct pollutant reduction.

Summary

Environmental pollution created as an externality of either private market or public economic activities has become an issue of public concern. From this concern has emerged a wide range of policies and penalties that are designed to curb pollution by curbing polluters. Because most of these policies have proved only partially effective, this paper focused upon an alternative strategy: that of the government going directly into the business of "reprocessing" the polluted environment, thereby restoring it to desirable levels rather than solely relying on economic sanctions against polluters to protect its constituents. Our study utilized a flexible time-state-preference framework, permitting the analysis of the effects of uncertainty in future social demand for pollution control, depreciation and reinvestment alternatives on optimal governmental policies. The government's "cleaning-up" of the environment is essentially an after-the-fact activity for pollution control rather than a set of actions affecting the immediate pecuniary incentives of the polluters, and has been justified as frequently representing the only type of available, *feasible* public policy alternative. However, we show that public sector direct pollution reduction *by itself* may result in social sub-optimization. In general, the social optimum optimorum requires both direct government clean-up *and* policies affecting the incentives of polluters.

Footnotes

[1]Excellent, recent examples of theoretically oriented articles about pollution problems are Plourde (1972), Smith (1972), and Zeckhauser, Spence, and Keeler (1972).

[2]While there are many articles on public investment analysis, such as Steiner (1959), in general the literature has not been directed towards the problem inherent in pollution control projects as discussed in this paper.

[3]An informative discussion about recent anti-pollution policies can be found in Boulding, Stahr, Fabricant, and Gainsburgh (1971), particularly pp. 100-129.

[4]The pollutant as a social "bad" is, in general, a stock rather than a flow problem. The pollutant as a stock problem either may affect consumption opportunities adversely or impair production. It is usually produced as a by-product of economic activity; but the pollutant, once generated, decays or depreciates very slowly, thereby leaving an undesired residual stock. For a further discussion on this point see Edel (1970) and Zeckhauser, Spence, and Keeler (1972).

[5]Clearly, the social value ascribed to the amount of pollutant reduction may depend, among other things, upon the total size of the existing stock of the pollutant. Of course, this stock depends upon the rate of past creation of the pollutant and its natural decay or depreciation rate, if any, in all subsequent time periods. Also, we tacitly assume that pollution reduction does not produce an income effect, or, roughly speaking the marginal utility of income for society is constant.

[6]This is a simplification for expositional reasons; it does not impair the analysis.

[7]It may be true that the value of benefits and costs may depend upon the <u>sequence</u> of states that obtain over time. That is, the social values for benefits or costs may have temporal dependencies. Analytically, this complication can be handled as in a similar fashion to part II-D above; but it is needless complication for our discussion in this part of the paper.

[8]The development of this section is strongly influenced by the paper by Arrow (1968).

[9]See Arrow (1968) for the discussion of the methods necessary for the analysis.

[10] We are assuming that second order conditions for social welfare maximization obtain. This still can permit the shape of the V and S functions to do "unusual" things by microeconomic analysis standards.

[11] I remind the reader that we assume that the marginal utility of income is constant (e.g., there are no income effects produced by pollution control activities) and, therefore, these areas can be used for consumers' surplus analysis.

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