A Structural Study of the Income Velocity of Circulation

by

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The contents of and the opinions expressed in this paper are the sole responsibility of the author.

I. INTRODUCTION

One of the generally accepted facts of monetary theory is that the income velocity of circulation of the money supply[1] is not a constant. However, many analysts feel that movements in this measure exhibit some regularity. For example, the view is often expressed that changes in velocity tend to follow changes in the rate of growth of the money supply. Moreover, velocity rises after an expansion of the money supply and declines after a contraction [2, p. 327].

A second fact, generally accepted, is that interest rates move in the same direction that velocity moves. That is, when velocity is rising, interest rates are rising. When velocity is falling, interest rates are falling.

Much of the analysis in this area of research has fallen into one of two categories. In the first, analysis is limited to a descriptive review of the data. The second, and perhaps the more important, involves the more sophisticated use of single-equation regression techniques (See [2], [4], [5] and [10]). Regardless of the approach the conclusions have generally been the same as those mentioned above.

Both approaches, however, are subject to shortcomings. One, they lack the ability to isolate causality of movements in the velocity of circulation. Two, the single-equation approaches are under-identified so that it is impossible to explain the behavior of all the variables included in the analysis. Three, some models cannot adequately explain the behavior of velocity in crucial periods of time, i.e., the time period immediately following changes in rates of monetary growth ([3], p. 335).

I have attempted to overcome these difficulties with the use of a macro-econometric model of the United States that contains more than one equation. By this method, causality can be explained. For example, velocity is expected to rise with an increase in interest rates ([4], [5] and [10]); whereas the single-equation models provide no explanation for this increase in interest rates, a structural model can. Also, the speed of response of different variables to exogenous stimuli is important in some explanations of the behavior of velocity ([2] and [3]): a structural macro-model can a-count for the lag structure that exists in an economy and thereby isolate the various important responses. Furthermore, these lags can be used to explain the behavior of velocity at turning points in monetary growth rates.

The general conclusion of this study is that velocity behaves as it does because it is the ratio of two variables that respond differently in time to given changes in monetary aggregates.[2] This conclusion is quite similar to the theoretical conclusion of Friedman [2], [3] that the observed movements in velocity are due to changes in expected variables which adjust more slowly to monetary stimuli than do actual values. However, I believe that this result can be more clearly seen with the use of a structural model that presents the lag structure and the linkages that exist within an economy.

II. IMPLICATIONS FOR ANALYSIS

The particular model used in this study has been presented and discussed elsewhere [8]. For purposes of reference the estimated structure is included in an appendix at the end of the paper. In [8], the dynamic properties of the model are analyzed with the use of "dynamic multipliers" computed from the estimated structure. These multipliers show the response, over time, of endogenous variables that is <u>attributable exclusively</u> to a unit change in an exogenous variable.[3]

For our purposes, the most important characteristic of these multipliers is that they can actually be put into the form of an equation. For example, the following equation shows the current response of Gross National Product (Y) to current and past changes in the adjusted monetary base (B) as derived from the above mentioned structural model.[4]

$$\Delta Y_t = 0.586\Delta B_t + 0.968\Delta B_{t-1} + \Delta B_{t-2} + \ldots$$

This can be considered a reduced form of the model and thus comparable to some of the single-equation models discussed above. It can also be compared with directly estimated distributed lags such as those obtained by Anderson and Jordan [1].[5]

This form can be used for simulation purposes and for discussions of the dynamic behavior of different variables endogenous to the model. Of particular concern are the equations showing the dynamic response of Gross National Product, the money supply and interest rates to changes in the adjusted monetary base. These responses are charted in Figure 1 and are presented in tabular form in the appendix.

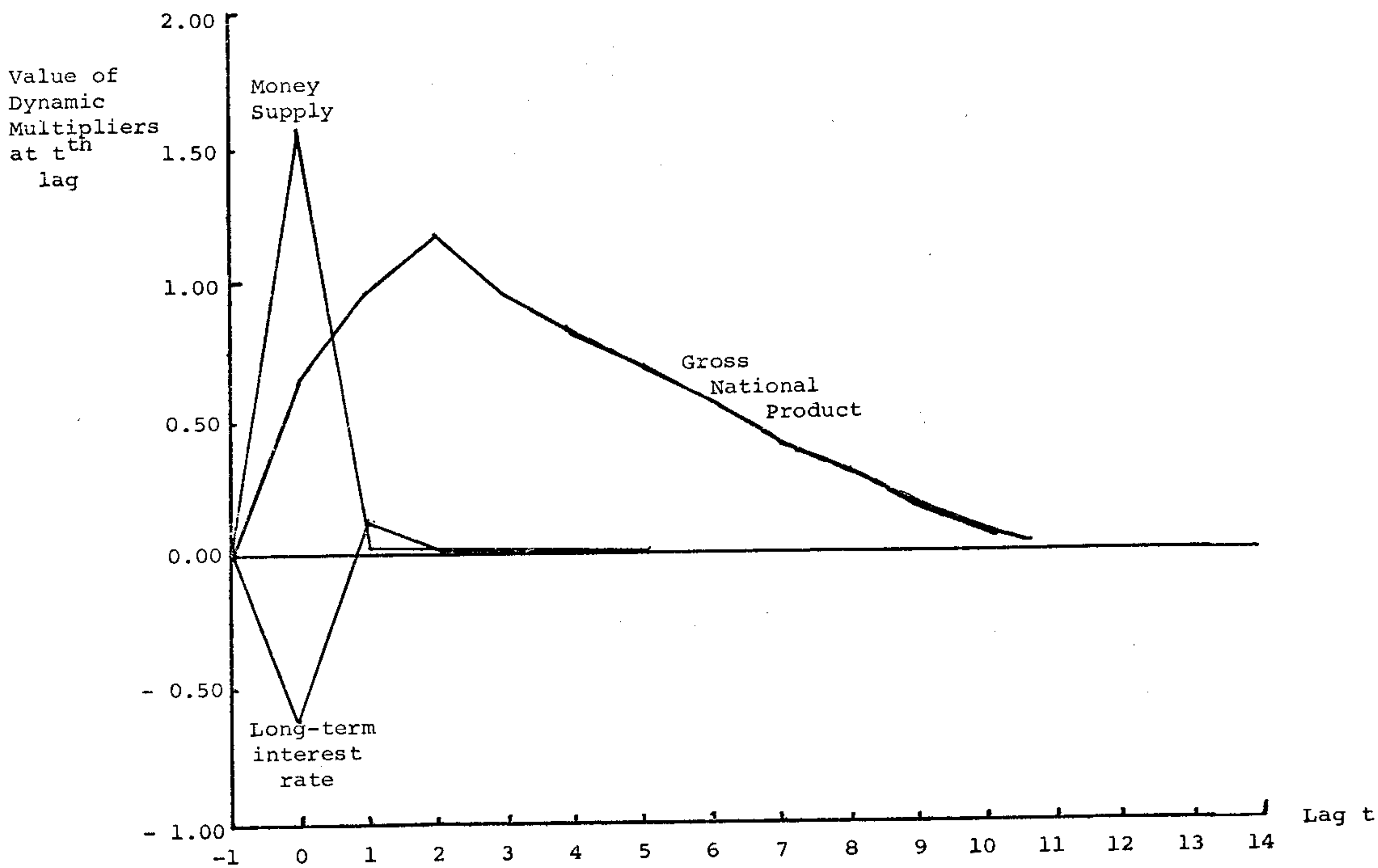

Figure 1

Time Response of Gross National Product, the Money Supply and the Long-Term Rate of Interest to a $1 Billion Change in the Monetary Base

As can be seen in the chart, the differential impacts of changes in the adjusted monetary show that each variable responds in its own way to these changes and at its own speed. It is argued below that this type of response can result in variables being "out of phase" with one another.[6] This would be particularly true at turning points in the rate of growth of the adjusted monetary base. Because velocity is the ratio of two of these endogenous variables that have different timing responses, it is plausible that it too can exhibit a behavior pattern that is seemingly out-of-phase with expected relationships.

Several caveats should be mentioned before the actual work is presented. First, it should be emphasized that the model is linear. Consequently, the initial conditions used in simulation will not affect the results. Secondly, the research was primarily interested in how velocity (and interest rates) changed in response to variations in the growth rate of a controlled (or controllable) monetary variable. Because of this, all changes in Government expenditures (the only other important policy variable in the model) are assumed to be zero. Some results of this assumption will be discussed later. Thirdly, because of the linear nature of the model and the fact that Government expenditures are estimated to have only a current quarter impact on GNP, the influence of these expenditures on velocity will be directly related to the size of the change in the variable and the direction of change. This means that the influences of Government expenditures on velocity are <u>independent</u> of the impacts of monetary policy and are solely dependent on the behavior of policy makers. This would not be true for non-linear models.[7] For this reason, simulations using actual data were not conducted because it was felt that experiments of this nature would only lead to interpretations about the policy makers and not the underlying economic relationships.

III. THE SIMULATED BEHAVIOR OF INCOME VELOCITY

A. Simulation of a Growing Economy

In order to observe the behavior of a model within a short-run dynamic framework it is necessary to get over some start-up problems. Thus, before experiments could begin the model had to be simulated over several years time in order to free itself from the inertia of starting in a "no-growth" state. The model used in this study has been shown to be stable, i.e., that it does not explode after an initial stimulus; therefore, it was expected that, over time, a steady growth rate in the endogenous variables would be achieved, given a steady growth rate in the exogenous variables. Once these long-run growth paths were achieved the short-run experiments could be performed.

The model was projected into the future assuming three different growth rates for the adjusted monetary base. The choice of rates of growth was based on an estimate of the elasticity of the money supply with respect to the adjusted monetary base obtained from the money supply equation. Since this elasticity was estimated to be 0.68 (at sample means), growth rates in the monetary base of 3%, 6% and 9% were chosen for the experiments because these figures imply growth rates in the money supply of approximately 2%, 4% and 6%, respectively. This result carries over to the dynamic case, since the major impact of changes in the monetary base on the money supply occur in the quarter in which the change occurs. Initial values of the variables were used as a base for the simulations. As mentioned earlier, however, different initial values would not be expected to alter the results due to the linearity of the model.

Projections of the variables were extended until GNP attained a steady long-run growth path. Also, the growth rates of all endogenous variables stabilized after a period of time. 40 quarters, or 10 years, was more than enough time to achieve this. In Figure 2, the long-run growth rates of GNP are presented for different rates of growth in the money supply. The derived velocity of circulation figures are shown in Figure 3. It is obvious that the different rates of growth produce similar results; again a result of the linearity of the model.

In all cases, the rate of growth of the money supply stabilizes at a level slightly above two-thirds of the rate of growth of the adjusted monetary base due to the relationship of the two mentioned above. However, Gross National Product grows at an even faster rate than the money supply, implying that a secular rise in velocity is incorporated into the model.[8] This result coincides with post-World II history.

Interest rates, however, do not perform, over time, in the way one would expect them to. They fall initially, as does velocity, reflecting the liquidity effect of growth in the monetary base, but the income effect incorporated into the model does not seem to be strong enough to return them to their original level. The reason for this behavior is easily found: the experiments were aimed solely at the influence of monetary disturbances on velocity behavior. The simplest assumption one could make concerning the behavior of Government expenditures, therefore, was to assume them to be constant at their initial level throughout all simulations. However, taxes are implicitly included in this model as an endogenous variable positively related to GNP; thus, an increase in GNP will raise Government tax receipts.[9] This has the effect of creating greater supluses (or smaller deficits) as econmic growth is induced. Thus, if Government expenditures were assumed to grow (say to keep a constant surplus or deficit) over time, they would exert

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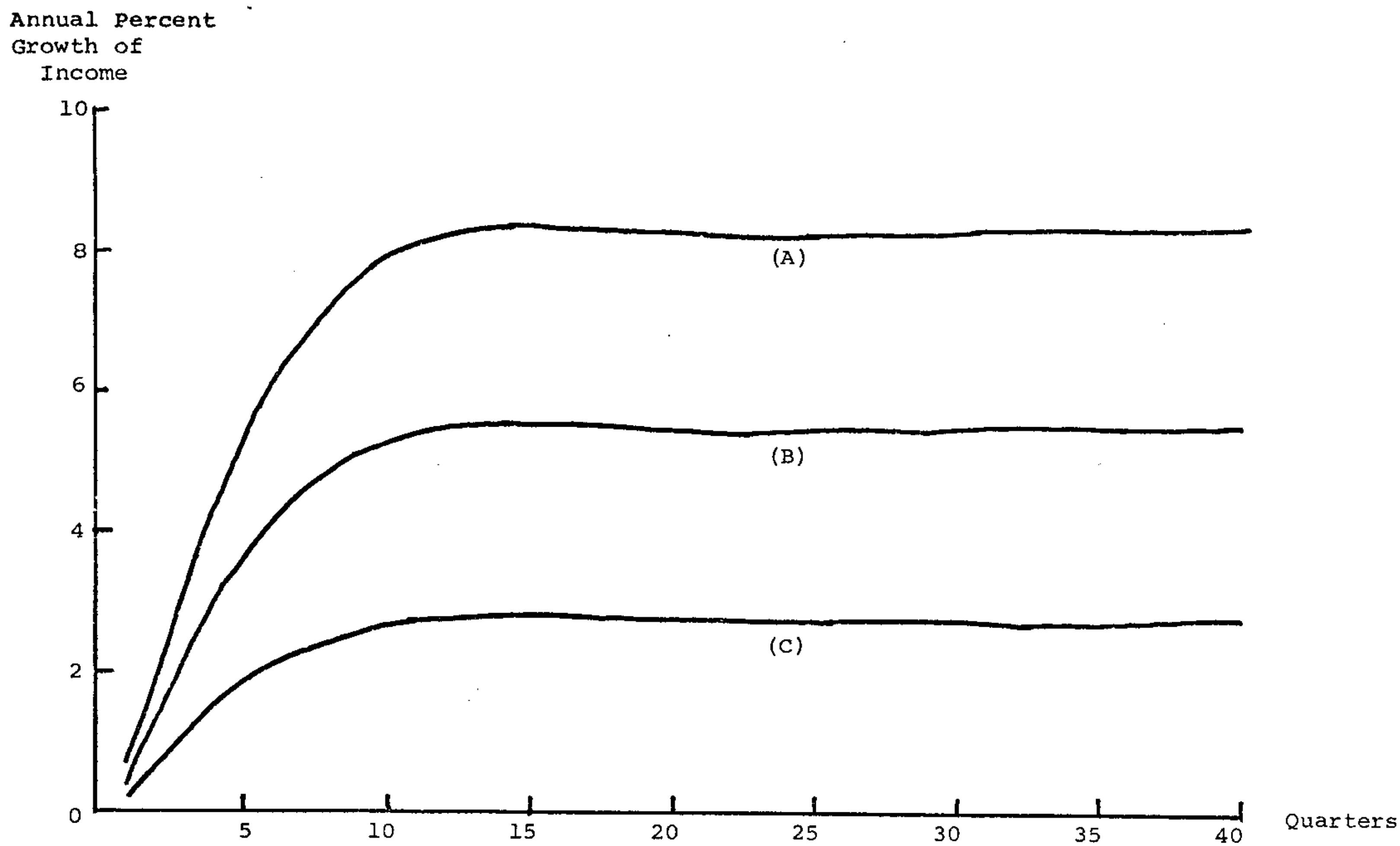


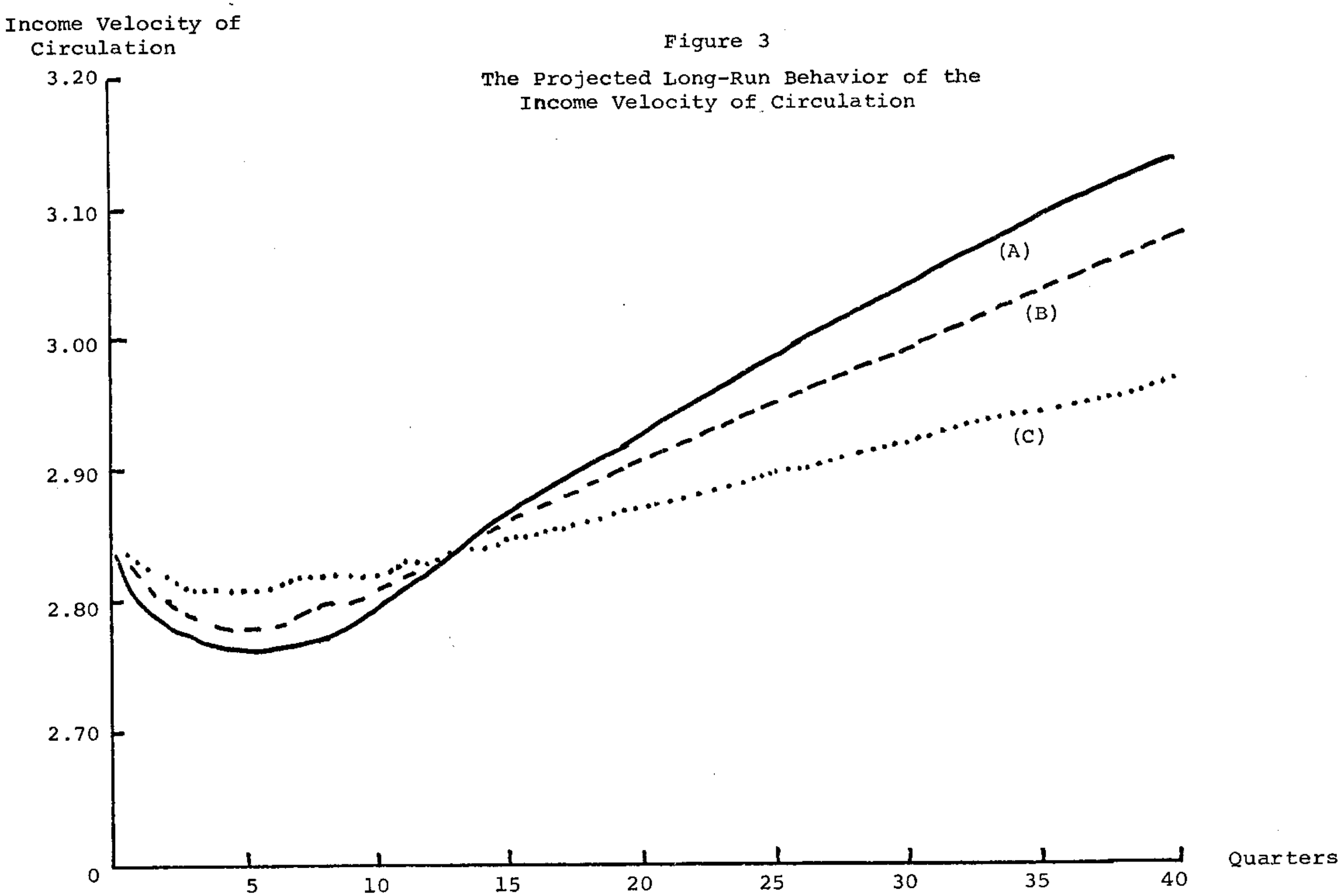
Figure 2

Projected Annual Rates of Growth of Income

(A) Assuming 9% Annual Rate of Growth in the Adjusted Monetary Base
(B) Assuming 6% Annual Rate of Growth in the Adjusted Monetary Base
(C) Assuming 3% Annual Rate of Growth in the Adjusted Monetary Base

Figure 3

The Projected Long-Run Behavior of the Income Velocity of Circulation



(A) Assuming 9% Annual Rate of Growth in the Adjusted Monetary Base
(B) Assuming 6% Annual Rate of Growth in the Adjusted Monetary Base
(C) Assuming 3% Annual Rate of Growth in the Adjusted Monetary Base

some upward pressure on interest rates.

Interest rates do stop declining in all the simulations performed at the very time velocity returns to its initial level. It is clear, therefore, that if Government expenditures were allowed to increase, interest rates would increase as velocity rose.[10]

B. The Short-Run Behavior of Velocity

Various shocks were introduced to the system once GNP and other endogenous variables stabilized at their long-run growth rates. That is, the rate of growth of the adjusted monetary base was altered for several quarters before it could resume its former advance.

Several different combinations of growth rates were used. Initially, the adjusted monetary base was expanded at annual rates of 3% and 6% for 8 quarters and for 20 quarters. In both cases, the growth was either halted or caused to decline at a 3% annual rate for several quarters. In one experiment, it was restrained for 4 quarters, while in another case it was restricted for 6 quarters. The base was then allowed to grow again at either a 3% or 6% annual rate. Thirty-two combinations were tried. Due to the linearity of the model, the results were very similar; therefore, only one example will be presented. Some slight differences were recorded due to the different magnitudes of change in the monetary variable. For example, a larger growth rate in the monetary variable coupled with a smaller reduction resulted in a longer lag in effect because the lagged responses of GNP built up from past monetary growth was greater in this case relative to the slowdown, than in cases where the initial growth was less and the restraint was more severe.

The specific example to be discussed allowed for a 3% rate of growth in the adjusted monetary base for 20 quarters, a 3% rate of decline for 4 quarters, and a consequent resumption of a 6% rate of growth.

Extending the experiments in time to incorporate additional monetary cycles did not alter the results. That is, the model indicates no cumulative effects of past monetary cycles on the behavior of velocity or interest rates. This, of course, assumes that a relatively consistent monetary policy is followed for several quarters, i.e., we did not alternate quarterly between 6% growth and 3% decline.

In Figure 4, the results of the experiment under review are presented. As is obvious from the model, growth in the money supply should stop, and does, as the base stops growing. GNP continues to grow for several quarters, although its rate of growth begins to slow in the quarter the base stops growing, and starts to decline in the fourth quarter after the shock is introduced. Income, is, in the sense developed above, "out of phase" for a time with the monetary aggregates. It continues to grow because of the lagged response of expenditure variables to past monetary growth. As can be seen, this slow adjustment of spending accounts for the observed changes in velocity.

Although the rate of growth of GNP began to slow in the quarter the base stopped growing, it did not lose one percentage point of its growth rate until the third quarter after the base had stopped growing. This is not inconsistent with other reported results. In fact, the results achieved here are remarkably similar to those reported by Friedman. He states, for example, that the rate of change in the money supply will not have any "appreciable effect" on the rate of change of income for six to nine months [3, p. 335].

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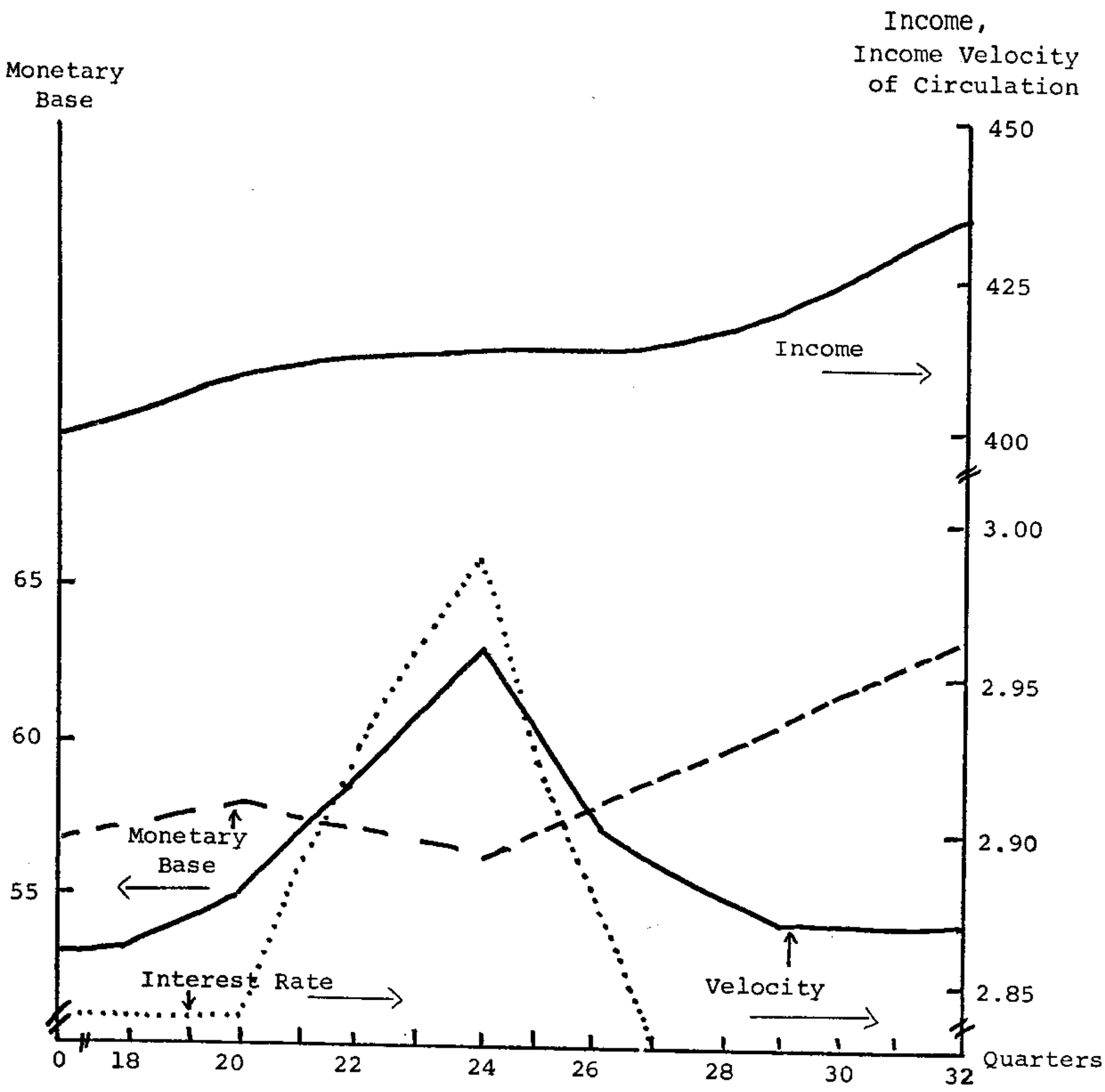

Figure 4

Experiment Using a -3% Rate of Growth in the Adjusted Monetary Base After 20 Quarters of a 3% Rate of Growth

The velocity of circulation continues to rise (in fact the rate of increase accelerates as Friedman has described in [3]) after the halt in the rate of growth of the base until the fourth quarter after the shock, and then begins to decline. Thus, an exogenous reduction in the rate of growth of the adjusted monetary base causes a decline, after a lag, in the velocity of circulation. In the 25th quarter, the adjusted monetary base begins to grow again; this time at a 6% annual rate. GNP and the money supply also begin to grow again. Velocity does not turn up, however, until the 31st quarter. As described above, this is due to the lagged response of the expenditure sectors.

Interest rate behavior conforms to the movement in income velocity just as expected (see Figure 4).[11] Although the interest rate had not been rising, for reasons mentioned above, the restriction in the growth of the monetary base causes a definite jump in the interest rate in the quarter the base ceases to grow. This jump is due entirely to the liquidity effect. Obviously, if interest rates had been rising, the jump would have resulted in an acceleration of the rise in interest rates rather than in just an increase.

The interest rate continues to increase until velocity hits its peak and then quickly drops off as velocity beings to decline. These results were duplicated in all thirty-two experiments performed. As mentioned above, the interest rate won't begin to rise again because Government expenditures still remain constant. However, it does stabilize at a constant level just at the time velocity begins to rise.

IV. SUMMARY

The above analysis demonstrates that the behavior of the income velocity of circulation can be explained quite well with a dynamic quarterly macro-econometric model. In the reported experiment, it was seen that exogenous monetary changes can cause changes in the velocity of circulation commonly associated with variations in rates of growth of economic activity. This pattern results from the dynamic nature of the model which allows for lagged adjustments within many sectors of the economy. Also, because the model is a more completely specified model than those used in the single-equation approaches, it is able to describe the complementary behavior of interest rates.

Although the specific model used in this paper is consistent with "Keynesian" type models and possesses attributes similar to other, large scale, macro-models, the conclusions depend solely on the model used. In another paper [7], however, I have shown that the results can be extended to cover a more generalized representation of economic behavior.

FOOTNOTES

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[1]The particular measure most commonly referred to is the average velocity which is obtained by dividing current dollar Gross National Product by the money supply.

[2]This is true of other ratios commonly used in economics and finance. For example, see [6] for a discussion of price-earnings ratios.

[3]For a fuller discussion of dynamic multipliers, refer to [8], pages 805-808.

[4]These are just derived estimates from the estimated structural model. As such, we do not know the standard errors of the coefficients.

[5]In other words, David A. Pierce and I have referred to an equation of this type as a Fundamental Dynamic Equation (FDE) and have attempted to show how directly estimated FDE's can be used to improve estimates of the lags incorporated in the structure itself [9].

[6]Out of phase in this case means that even, though GNP and the monetary base are positively related, in a dynamic situation the monetary base could be declining and GNP could be increasing or vice versa. Furthermore, they could be growing at quite different rates for a time due to the past behavior of the monetary base.

[7]Non-linear models use actual data to perform simulation studies due to the dependence of the simulation on initial conditions. In a sense, then, the results of simulations using non-linear models are not as general as those using linear models. This abstracts, however, from the question of whether the world is approximated better by a linear or non-linear model.

[8]In [7] I have developed sufficient conditions for determining the long-run velocity behavior implied in a model from the model's dynamic multipliers. Thus, the velocity behavior exhibited by the model under review could have been determined without performing simulation experiments.

[9]See equation A-4 in appendix. It is assumed that $Y^d = Y - T$, where T represents Government tax receipts. A-4 could therefore be rewritten as $T = 0.33Y$.

[10] Interest rate behavior is not charted in Figure 3, since the paper is primarily interested in velocity and interest rate changes caused by variations in the growth rate of the controlled monetary variable. Interest rate results, however, will be presented below.

[11] Just the long-term interest rate is charted in Figures 4 and 5 in order to make the chart compact. The behavior of the short-term interest rate mirrors that of the long term rate.

APPENDIX

The Moroney-Mason model used in this article. The figures in parentheses are the asymptotic standard errors of the estimated structural coefficients.

(A-1) $$Y_t = C_t + I_t + G_t + E_t - O_t$$

(A-2) $$C_t = -37.449 + \underset{(.007)}{.242Y_t} + \underset{(.007)}{.702C_{t-1}} + \underset{(.104)}{.315M_t} + \underset{(.119)}{.046M_{t-1}}$$

$R^2 = .999$ D.W. = 2.028

(A-3) $$I_t = 6.116 + \underset{(.018)}{.060Y_t} + \underset{(.249)}{.502(C_{t-1} - C_{t-2})} - \underset{(1.815)}{3.301r^L_{t-2}} + \underset{(.097)}{.6631I_{t-1}}$$

$R^2 = .947$ D.W. = 1.950

(A-4) $$Y^d_t = .67Y_t$$

(A-5) $$O_t = -1.203 + \underset{(.000008)}{.048Y_t}$$

$R^2 = .863$ D.W. = 2.282

(A-6) $$r^L_t = 1.650 + \underset{(.001)}{201r^s_t} + \underset{(.000002)}{.003Y_t}$$

$R^2 = .5431$ D.W. = 2.063

(A-7) $$M^s_t = 43.219 + \underset{(.082)}{.890r^s_t} + \underset{(.159)}{.208R^d_t} + \underset{(.007)}{1.832B_t}$$

$R^2 = .932$ D.W. = 1.307

(A-8) $$r_t^s = 15.569 + 0.40Y_t - .230M_t^d$$
$$\quad\quad\quad\quad (.00004) \quad (.013)$$

$R^2 = .456$ D.W. = 1.578

(A-9) $$M_t = M_t^s = M_t^d$$

The model and its properties, the data and the estimation procedures used are presented in [8].

The variables of the model are as follows:

Y = gross national product

Y^d = disposable income

C = consumption expenditures

I = gross private domestic investment

G = government purchases of goods and services

E = exports

O = imports

M = money stock (coin and currency in the hands of the nonbank public plus adjusted demand deposits in commercial banks). The superscript s indicates the supply function; the superscript d indicates the demand function.

r^S = short-term interest rate (average quarterly rate on 90-day Treasury bills)

r^L = long-term interest rate (average quarterly rate on U. S. Treasury bonds)

B = adjusted monetary base (quarterly average)

R^d = discount rate

The endogenous variables are Y_t, Y_t^d, C_t, O_t, M_t^s, M_t^D, r_t^s, I_t, and r_t^L. The predetermined variables are C_{t-1}, C_{t-2}, M_{t-1}, r_{t-2}^L, G_t, E_t, R_t^d, and B_t, the last four being exogenous.

APPENDIX TABLE

Dynamic Multipliers for the Time Path of Various Endogenous Variables Due to a Unit Increase in the Adjusted Monetary Base

Lag, s	Gross National Product	Long-Term Interest Rate	Money Supply
0	+0.586	-0.065	+1.538
1	0.968	+0.091	0.029
2	1.156	0.011	0.034
3	0.983	0.009	0.029
4	0.808	0.007	0.024
5	0.650	0.006	0.019
6	0.511	0.004	0.014
7	0.392	0.003	0.012
8	0.292	0.002	0.009
9	0.211	0.002	0.006
10	0.145	0.001	0.004
11	0.094	--	--
12	0.055	--	--
13	0.026	--	--
14	0.002	--	--
15	-0.014	--	--
16	-0.025	--	--
17	-0.030	--	--
18	-0.033	--	--
19	-0.033	--	--
20	-0.031	--	--

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