Risky Corporate Debt in a Market Model Context

by

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This study attempts to fit risky debt explicitly into capital market equilibrium theory in the context of the Sharpe-Lintner-Mossin (S-L-M) capital asset pricing model. In recent years there has been voluminous theoretical and empirical research developing and testing the capital asset pricing model. However, most of this literature either explicitly or implicitly assumes that the market for capital assets includes risky equity securities and risk-free debt. The simultaneous existence of risky equity, risky debt and a risk-free asset has not been examined in any detail.

The following equilibrium relationship can be derived for any risky asset, i , in the capital market:

$$(1) \quad E(R_i) = R_f + \frac{[E(R_m) - R_f]}{\sigma^2(R_m)} \quad Cov(R_i, R_m)$$

where: $E(R_i)$ is the expected return on asset i
R_f is the rate of return on the riskless asset
$E(R_m)$ is the expected return on asset m
$\sigma^2(R_m)$ is the variance of the return on the market portfolio
$Cov(R_i, R_m)$ is the covariance of the return on asset i with the return on the market portfolio.

From this equilibrium relationship, two basically equivalent measures of the nondiversifiable risk on asset i have been developed upon which the expected return, $E(R_i)$, is conditional. One such measure, $Cov(R_i, R_m)$, is obvious from equation (1). The other is the well-traveled beta coefficent, β_i in equation (2).

$$(2) \quad E(R_i) = R_f + \frac{Cov(R_i, R_m)}{\sigma^2(R_m)} [E(R_m) - R_f]$$

$$= R_f + \beta_i [E(R_m) - R_f]$$

Nondiversifiable Risk on Corporate Debt and Equity

Modigliani and Miller (M-M) have presented a theoretical framework for equilibrium in the combined equity and debt capital market. Some of the relationships between the M-M propositions and the S-L-M capital asset pricing model have been examined elsewhere. Hamada [3] found that the M-M propositions I and II (in both the no-tax and with-taxes versions) were valid when restated in the context of the capital asset pricing model with risk-free debt. Haugen and Pappas [4], [5] and Imai and Rubinstein [7], have examined the M-M propositions in the context of the capital asset pricing model with risky debt. The M-M model was again found to be valid but a number of interesting extensions of this analysis, given the consistency of the two models, have not been examined. Stiglitz [15] found that the M-M analysis holds under conditions more general than those originally assumed by M-M. The conditions examined by Stiglitz include those consistent with the capital asset pricing model with risky debt.

Proposition II of M-M which develops the effect of increased leverage on the required return to equity, given a level of business risk to which the firm is subject, appears in equation (3).

(3) $k = \rho + (\rho - i)\lambda$

where: k is the average required rate of return on equity
ρ is the market capitalization rate for the firm's earnings stream or weighted average cost of capital.
i is the average required rate of return on debt
λ is the firm's debt/equity ratio

In the absence of corporate taxes ρ and the value of the firm are constant for any capital structure.[1] If i is also assumed to be unaffected by changing λ then a simple linear relationship exists between the required return on equity and λ.

However, it has been pointed out, reasonably enough, by Solomon [13] among others, that the firm could not expect to be able to substitute debt for equity indefinitely without an increase in the required rate of return to debtholders. The implications for an optimal capital structure of the required return on debt being a function of leverage have been examined elsewhere. From this analysis it can be demonstrated that under unrestrictive conditions, within the M-M theoretical framework, the required rate of return on equity remains a non-decreasing function of increasing leverage and the market capitalization rate remains unaffected by a change in capital structure.

The rationale behind the required return on debt increasing with increased leverage can be illustrated most clearly by imagining a probability distribution of the firm's period-by-period earnings. This distribution determines the cumulative probability of the firm's earnings being less than any amount in a given period. For small amounts of leverage, determining inconsequential levels of fixed charges, it is possible that the cumulative probability that the earnings are insufficient to meet the fixed charges is equal to zero. In this case, there is no default risk and in the absence of interest rate risk, the rate of return to debtholders is known with certainty and is equal to the promised yield on debt. However, after a point, with increasing leverage and increasing fixed charges the probability of a default increases. Under the assumption of risk aversion by debtholders, this situation would lead to an increasing required rate of return on debt for increasing leverage. This increase would continue until all equity had been replaced by debt. At that point, the firm's debtholders would be in the same situation as the equityholders in a 100% equity elsewhere. From this analysis it can be demonstrated that under unrestrictive conditions, within the M-M theoretical framework, the required rate of return on equity remains a non-decreasing function of increasing leverage and the market capitalization rate remains unaffected by a change in capital structure.

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If we designate the expected rate of return on a chosen firm's equity as $E(R_e)$ and the expected return on its debt as $E(R_d)$, we can rewrite equation (1) as:

$$(4a) \quad E(R_e) = R_f + \frac{[E(R_m) - R_f]}{\sigma^2(R_m)} \, Cov(R_e, R_m)$$

$$= R_f + \beta_e [E(R_m) - R_f]$$

and:

$$(4b) \quad E(R_d) = R_f + \frac{[E(R_m) - R_f]}{\sigma^2(R_m)} \, Cov(R_d, R_m)$$

$$= R_f + \beta_d [E(R_m) - R_f]$$

Given the consistency of the M-M and S-L-M models a simultaneous consideration of equations (3), (4a) and (4b) should lead to some insights concerning the theoretical relationships between the nondiversiable risks of equity and debt instrum nts and the determinants of nondiversifiable risk to debtholders.

For k and therefore (R_e) to be minimized for a given level of variability in the firm's earnings at a capital structure of 100 percent equity, $Cov(R_e, R_m)$ and therefore β_e

must also be a minimum at that point. From that point, substituting debt for equity causes the well known "leverage effect" on the variability of return to stockholders. That is, adding leverage causes the $\sigma(R_e)$ in equation (5) to increase without, under unrestrictive assumptions, any concomitant change in $Corr(R_e,R_m)$.

$$(5) \quad Cov(R_e,R_m) = Corr(R_e,R_m)\ \sigma(R_e)\ \sigma(R_m)$$

When the required return on debt is assumed constant with respect to leverage, both $Cov(R_e,R_m)$ and β_e are linear functions of λ, the debt/equity ratio. However, when we assume that increased leverage implies increased cumulative probability of default, $Cov(R_d,R_m)$, β_d and the required return on debt increase with leverage. Thus we can let the cost of debt be a function of the firm's debt/equity ratio, $f(\lambda)$. M-M proposition II then requires that:

$$(6) \quad k = \rho + (\rho - f(\lambda))\lambda$$

It has been pointed out several times (e.g. see Stapelton [14], chapter 2) that risk aversion and the preferred risk position of debtholders over equityholders is sufficient to keep the sign of the first partial derivative of (6) with respect to λ, $\frac{\alpha k}{\alpha \lambda}$, non-negative at all points.

$$(7)\quad \frac{\partial k}{\partial \lambda} = \rho - \lambda\frac{\partial f(\lambda)}{\partial \lambda} - f(\lambda)$$

That is, if we assume that $f(\lambda)$ is assymptotic to ρ, the cost of equity when $\lambda=0$, the last two terms on the right of equation (7) cannot exceed ρ.

It can be further shown that if $f(\lambda)$ is assymptotic to ρ, then the second partial of (6) is negative. That is, in equation (8), the first term, which has a positive sign, must be smaller than the absolute value of the second term.

$$(8)\quad \frac{\partial^2 k}{\partial \lambda^2} = -\lambda\frac{\partial^2 f(\lambda)}{\partial^2 \lambda} - 2\,\frac{\partial f(\lambda)}{\partial \lambda}$$

Thus, the linear relationship between the required return on equity and the debt/equity ratio has been lost.

Given the linear relationship bewteen the required return on equity and the risk measure in (4a) and (4b), it must be the case that the covariances and beta coefficients must exhibit analogous assymptotic relationships when expressed in terms of the debt/equity ratio.

The foregoing analysis suffers from its having ignored an important element of the risk inherent in an investment in a bond. Traditionally, analysis of bond market risk-return relationships have dealt with both default risk and interest rate risk. This segmentation of total risk into cash flow variability and capitalization rate variability factors corresponds to the recent introduction of two-factor market models for equity securities. The models segment total nondiversifiable risk into a non-diversifiable cash flow risk factor and a non-diversifiable capitalization rate risk factor. Some empirical work utilizing two factor models has been done including attempts at isolating the firm characteristics which are determinants of these factors.

Roll [12] has examined interest rate risk in a market model context for default risk free securities. He used a capital asset pricing framework combined with a dynamic, efficient-markets theory of spot and forward rates to estimate a "market horizon" for investors in Treasury Bills. He noted that if the term to maturity of a Treasury Bill matches the investment horizon of the investor, he is not concerned about movements in prices and interest rates. Then, expressing the "liquidity premium" that the market appears to be requiring on securities which do not match this investment horizon in terms of the CAPM risk measure,he iteratively examined the strength of the association between the liquidity premiums and beta coefficients from Treasury Bill data produced by different market horizons.

Corporate Bond Beta Coefficients

There are a number of empirical problems which present themselves when one attempts to estimate beta coefficients on corporate bonds. Probably the most important problem concerns the yield data. In order to accumulate a meaningful sample of bonds it is necessary to include securities which are traded very infrequently. In the absence of actual market prices, it is necessary to resort to bid and ask quotations. There appear to be bonds in which there is very little interest on the part of investors on which even the bid and ask quotations change very infrequently. Because of these effects it is necessary to choose wide time intervals in order to detect any meaningful movement in prices and yields. This is a very important shortcoming of the empirical results presented here but one that is avoidable only by introducing other biases.

A population of 175 bonds was selected from the industrial, utility and railroad bonds listed in the Standard and Poor's Bond Guide. All of the bonds which satisfied the following criteria were included: (1) the bond was on the market for the entire 1953-1967 time period,[1] (2) Standard and Poors did not change its default risk classification during this period, (3) the bond was not convertible, (4) it was not a serial issue, (5) the bond was rated BBB or better by Standard and Poors, (6) no bond of the same rating of that firm had already been included. The 175 bonds which met these criteria are listed in Appendix A.

In all empirical work involving the CAPM, two common measurement problems arise. One involves arriving at the empirical equivalent of the market portfolio and the other involves how to measure the risk-free rate. This study copes with the second problem by assuming that the investor in long term corporates, consistent with Roll's analysis [12], has some indeterminate long-term investment horizon over which the risk-free rate should be measured. Empirically, the closest thing to such a rate is the rate promised on insured savings deposits. This rate as published periodically in the Federal Reserve Bulletin was used in this study.

The first problem mentioned above was handled first by using the Standard and Poors composite 500 stock index which is consistent with the way the market portfolio has been empirically defined in previous studies.

Annual holding-period yields were calculated for each of the 175 bonds using the 1953-1967 year-ending prices and the coupon rates as published by Standard and Poors. Then rewriting the equilibrium relationship in equation (2) as:

$$(9) \quad E(R_i) - R_f = \beta_i [E(R_m) - R_f]$$

and expressing its empirical analog as:

$$(10) \quad Y_{it} = \alpha_i + \beta_{1i} X_{1t} + e_{it}$$

where: Y_{it} is the difference between the return on the i^{th} bond and the risk-free return in period t

X_{1t} is the difference between the return on the market portfolio and the risk-free return in period t

the beta coefficients were regression estimated. These coefficients appear in Appendix B and are labeled as β_1's. They are mixed in sign (137 positive and 38 negative) and range from -.126 to .406. However, only 25 of the 175 β_1's are significantly different from zero at the 95 percent confidence level.

These β_1's were then used to try to explain realized returns for the 175 bonds. For this purpose it would be ideal to be able to estimate the beta coefficients using one time span and use them to try to explain returns realized over a subsequent interval. However, because of the problems involving the data that were mentioned above, it was necessary to use the full time span over which the data was available in order to estimate the β_1's. Lengthening the time span would have involved decreasing the sample size and including the time period of post-war administered interest rates.

Therefore, realized returns, defined to be the geometric mean holding-period yields, were calculated for the 175 bonds. Then, a regression of the form given in equation (11) was run.

(11) $R_i = a_i + b_i\beta_{1i} + e_i$

R_i is the geometric mean return on bond i during 1953-1967 where: Y_{it} is the difference between the return on the i^{th} bond and the risk-free return in period t

X_{1t} is the difference between the return on the market portfolio and the risk-free return in period t

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$$(11) \quad R_i = a_i + b_i\beta_{1i} + e_i$$

R_i is the geometric mean return on bond i during 1953-1967

β_{1i} is the regression estimated β_1 for bond i

The results of this test appear in Table I.

Table I

R^2	a	b	S.E.	t-value
.02	.02058	.01301	(.00698)	1.865

In addition to the problems associated with the lack of statistical significance of the majority of the β_1's, there is a further shortcoming in their use as risk measures. Since the index that was used in estimating the β_1's is an infinite maturity index (using common stocks) and the bonds themselves are of finite and decreasing maturity over time, the estimated β_1's could not be expected to be stationary over time. It is an attribute of the mathematics of bond yields-to-maturity, demonstrated by Malkiel [8], that a given change in yield-to-maturity will have a greater effect on the price of a bond the greater in the term-to-maturity of the bond. One might expect then that if yields-to-maturity on long and short maturity bonds change by the same amounts, long-term bonds would have more variable holding-period yields than short-term bonds. The evidence from Culbertson [2] and others indicates that short-term yields-to-maturity tend to be more variable than long term yields but that even so, holding-period yields on long-term maturity bonds are more variable than those on short-term bonds.

In order to minimize the problems arising from a lack of stationarity in the beta coefficients a market portfolio with changes in maturity parallel to maturity changes in the individual bonds was chosen. A single portfolio comprising all 175 of the corporate bonds equally weighted was constituted as this representation of the market portfolio.

β_2's for the 175 bonds were regression-estimated according to equation (12) in a fashion analogous to that for the β_1's above.

(12) $Y_{it} = \alpha_i + \beta_{2i} X_{2t} + e_{it}$

where: X_{2t} is the realized market premium in period t based on the 175 bond market portfolio

The estimated β_2's were all positive and ranged from .137 to 1.968. A complete list of the β_2's appears in Appendix B. One hundred fifty five of these β_2's were significantly different from zero at the 95 percent confidence level.

The β_2's were used in an attempt to explain the mean returns on the 175 bonds over the 1953-1967 time period in a manner analogous to the procedure used for the β_1's and depicted in equation (11) above. The results of this test appear in Table II.

Table II

R^2	a	b	S.E.	t-value
.15	.03089	-.00931*	(.00165)	-5.629

*significant at the 95 percent confidence level

The negative sign on the regression coefficient should be noted. This negative sign will be discussed as an interest-rate risk phenomenon below.

In order to test for a lack of stationarity in the β_2's, they were regression estimated again for the 175 bonds using two sub-periods of seven years each, 1953-1959 and 1960-1966. The β_2 for the second period was then compared with that for the first period for each bond. There did not seem to be any marked tendency for the β_2's to decrease from the first sub-period to the second. Of the 175 bonds, 91 had β_2's which decreased, while 84 increased. The corelation between the β_2's for the two sub-periods was .27. In addition, it was found that the second period β_2 was within one standard error of the first sub-period β_2 for 54 percent of the bonds, and within two standard errors for 77 percent of the bonds.

An empirical problem involving the use of the 175 bond market portfolio and in fact involving the selection procedure for the 175 bonds can be seen by first rewriting equation (12) as:

$$(13)\quad \frac{P_{it+1} - P_{it} + C_i}{P_{it}} = \alpha_i + \beta_{2i}\left[\frac{P_{Mt+1} - P_{Mt} + C_{Mt}}{P_{Mt}}\right] + e_{it}$$

where: P_{it} is the market price of bond i at point in time t

P_{Mt} is the market value of the 175 bond market portfolio at point in time t

C_i is the annual interest paid on bond i

C_M is the annual interest paid on the 175 bond market portfolio

The fact that C_i and C_M need no time subscript indicates that default risk is actually not taken into account in the beta coefficient of equation (13). The selection criteria utilized for the 175 bond sample systematically eliminated bonds which defaulted during the 1953-1967 time period. Thus, in the context of two factor market models, only one factor (the capitalization rate or interest rate factor) is reflected in equation (13). In order for the regression estimated beta coefficient to reflect default risk, possible cash flow variability would have to represented in (13).

If the β_2's measure only interest rate risk, a possible explanation for the negative sign on the regression coefficient in Table II is provided. 1953-1967 was a period of generally falling bond prices and generally rising interest rates. One might expect those bonds with the highest β_2's to have suffered the greatest price depreciation and thus to have realized the lowest average holding-period yields during this period of time.

During a period of rising bond prices and falling interest rates, one might expect there to be a positive ex-post relationship between realized holding-period yields and β_2's.

Therefore, a sub-period during which interest rates were generally falling, 1960-63, was chosen and the geometric mean holding-period yields for the 175 bonds during this sub-period were computed. A regression of the form given in equation (11) was then run and the results appear in Table III.

Table III

R^2	a	b	S.E.	t-value
.14	.04195	.01388*	(.00252)	5.30556

*significant at the 95 percent confidence level

The regression coefficient is now positive and significant. This result is consistent with the explanation of the negative risk-return relationship in Table II having resulted from the upward secular trend in interest rates during the period 1953-1967.

In order to incorporate cash flow variability risk into the analysis of bond risk-return relationships, the Standard and Poors default risk ratings of the 175 bonds were noted. Confidence in such ratings as indicators of default risk can be based upon Hickman's [6] results on the accuracy of such ratings in predicting default rates. In addition, Pogue and Soldofsky [10] have isolated firm traits that can be used to explain such ratings and these traits are consistent with the analysis of default risk presented previously in this paper.

Furthermore, Hickman's study and that of Atkinson [1] dealing with default experience indicate that the timing and degree of default are closely related to swings in the business cycle. This would tend to indicate that such risk is largely nondiversifiable.

The default risk classifications and β_2's were combined in a multiple regression model which attempts to explain the realized 1953-1967 holding-period yields for the 175 corporate bonds. In order to include default risk in this model, two sets of dummy variables were used. One set was the Standard and Poor's rating with BBB as the excluded class. On the assumption that default ratings may not be constant across industries, a second set, specifying the industry of the issuer (railroad, utility or industrial) with industrial as the excluded class, was used. The results of testing this model appear in Table IV.

Table IV

Variable	Reg. Coeff.	t-value	Partial Corr.
β_2	-.00983*	-7.23260	-.48609
RR	.00287	1.90368	.14486
Utility	.00027	.21610	.02659
AAA	-.01299*	-7.81376	-.51508
AA	-.00898*	-6.12497	-.42613
A	-.00484*	-3.50251	-.26903

$\bar{R}^2$ = .4680, Intercept = .03720

*significant at 95 percent confidence level

If the β_2's estimated above are measures of interest rate risk, then the mathematics of interest rates suggests that they should be negatively related to coupon rates and positively related to term-to-maturity. In addition, the literature on two factor market models suggests that the factors should be independent of each other. In order to test for these relationships a multiple regression model with the estimated β_2's as values of the dependent variable was set up. The independent variables included the industry and default rating dummy variables described above. In addition, the coupon rate and a maturity measure were used as independent variables. The maturity variable was defined to be equal to the number of years-to-maturity of the bond as of January 1953.

The results of this test appear in Table V. It appears that when both are considered together, the primary determinate of a bond's β_2's involve attributes of the bond rather than the issuer.

Table V

Variable	Reg. Coeff.	t-value	Partial Corr.
RR	.15615*	2.02466	.15476
Utility	.05014	.85818	.06844
AAA	-.17812	-1.92164	-.14706
AA	-.11759	-1.42777	-.10980
A	.02467	.33048	.02475
Coupon	-.27153*	-4.69422	-.34138
Maturity	.02793*	7.41529	.49765

$\bar{R}^2$ = .3345, Intercept = 1.19605

*significant at 95 percent confidence level

In addition to the factors listed in Table V, three additional attributes of the issue were tested as determinants of the β_2's. Marketability was measured in three different ways and each was added to the multiple regression depicted in Table V. The three marketability measures were the total funded debt of the issuing company as of January 1954, measured in millions of dollars (book value), the size of the particular bond issue measured in millions of dollars (book value), and a dummy variable used to indicate whether or not the issue was listed on either of the two major exchanges (New York or American). The coefficients on the three measures were all insignificant when entered separately. In addition, a variable used to reflect sinking fund provisions was used. This variable is a dummy variable equal to one if the bond had a sinking fund provision and equal to zero if it did not. The coefficient on this variable was not significant. Finally, a dummy variable equal to one if the bond was callable,and equal to zero if it was not, was employed. This variable was found to be significantly positively related to the β_2's given the other independent variables. This implies that bonds which are callable demonstrate more price volatility when market interest rates change than non-callable bonds do.

The foregoing analysis has attempted to place risky corporate debt explicitly in the theoretical and empirical framework of the capital asset pricing model. The empirical evidence suggest that bond beta coefficients which are regression estimated on a bond portfolio are useful measures of interest rate risk. However, such beta coefficients must be combined in the context of the two factor market models with a nondiversifiable-default-risk measure in order to more fully explain realized returns in the bond market.

APPENDIX A

No.	Company	Rating	Coupon	Maturity
1	Shell Union Oil Corp.	AAA	2.500	1971
2	Socony Vacuum Oil Co.	AAA	2.500	1976
3	Standard Oil Co. (N.J.)	AAA	2.375	1971
4	Bethlehem Steel	AA	2.750	1970
5	Borden	AA	2.875	1981
6	Inland Steel	AA	3.200	1982
7	May Department Stores	AA	2.625	1972
8	National Steel Corp.	AA	3.125	1982
9	Ralston Purina Co.	AA	3.125	1977
10	Union Oil of Cal.	AA	2.750	1970
11	Westinghouse Elec. Corp.	AA	2.625	1971
12	Aluminum Company of Canada	A	3.875	1970
13	Anheuser-Busch, Inc.	A	3.375	1977
14	Burroughs Adding Machine	A	3.375	1977
15	Cities Service Co.	A	3.000	1977
16	Continental Can	A	3.250	1976
17	Household Finance Corp.	A	2.750	1970
18	Lorillard Co.	A	3.000	1976
19	Macy & Co.	A	2.875	1972
20	Pillsbury Mills	A	3.125	1972
21	Thompson Products	A	3.250	1971
22	United Biscuit	A	3.375	1977
23	U. S. Rubber Co.	A	2.625	1976
24	West Va. Pulp and Paper	A	3.250	1971
25	Glenmore Distilleries	BBB	4.000	1972
26	Sylvania Elec. Prods.	BBB	3.750	1971
27	Boston Edison Co.	AAA	2.750	1970
28	Cincinnati Gas & Elec.	AAA	2.750	1975
29	Cleveland Elec. Illum. Co.	AAA	3.000	1970
30	Commonwealth Edison	AAA	3.000	1977
31	Con. Gas El. Lt. & Pwr.	AAA	2.875	1976
32	Consumers Power Co.	AAA	2.875	1975
33	Duke Power Co.	AAA	2.875	1979
34	Duquesne Light Co.	AAA	2.750	1977
35	Ill. Bell Tele. Co.	AAA	3.000	1978
36	K.C. Pwr. & Light	AAA	2.750	1976
37	Louisville Gas & Elec.	AAA	2.750	1979
38	Mich. Bell Tel.	AAA	3.500	1988
39	N. J. Bell Tel.	AAA	3.125	1988
40	N. Y. Tel. Co.	AAA	3.125	1978
41	Northwestern Bell Tel.	AAA	2.750	1984
42	Phila. Elec. Co.	AAA	2.750	1971
43	Southern Bell Tel. & Tel.	AAA	3.000	1979
44	Southwestern Bell Tel.	AAA	2.750	1985
45	Atlantic City Elec.	AA	2.875	1980

Appendix A (continued)

No.	Company	Rating	Coupon	Maturity
46	Brockton Edison Co.	AA	3.000	1978
47	Buffalo Niagara Elec.	AA	2.750	1975
48	Cambridge Elec. Light	AA	2.875	1974
49	Central N.Y. Pwr.	AA	3.000	1974
50	Commonwealth Edison	AA	3.000	1999
51	Consol. Nat. Gas	AA	3.250	1976
52	Dayton Power and Light	AA	2.750	1976
53	Detroit Edison Co.	AA	3.000	1970
54	El Paso Elec. Co.	AA	2.750	1976
55	Gulf States Utils.	AA	2.625	1976
56	Ill. Power Co.	AA	2.875	1976
57	Ind. & Mich. Elec.	AA	3.000	1978
58	Iowa-Ill. Gas & Elec.	AA	2.750	1977
59	Iowa Pwr. & Light Co.	AA	3.250	1973
60	Madison Gas & Elec.	AA	2.500	1976
61	National Fuel Gas Co.	AA	3.000	1973
62	New Bedford Gas & Edison Lt.	AA	3.000	1973
63	New England Pwr. Co.	AA	3.000	1978
64	N.Y. Pwr. & Lt. Corp.	AA	2.750	1975
65	Niagara Mohawk Pwr.	AA	2.750	1980
66	No. States Pwr. Co.	AA	2.750	1974
67	Ohio Edison Co.	AA	3.000	1974
68	Ohio Pwr. Co.	AA	3.000	1971
69	Pac. Gas & Elec.	AA	3.000	1977
70	Penn. Pwr. Co.	AA	2.875	1975
71	Pennsylvania Wtr. & Pwr.	AA	3.250	1970
72	Phila. Elec. Pwr. Co.	AA	2.625	1975
73	Public Svce. of Colo.	AA	3.125	1978
74	Pub. Svce. Co. of Ind.	AA	3.125	1977
75	Pub. Svce. Co. of Okla.	AA	2.750	1975
76	San Diego Gas & Elec. Co.	AA	3.375	1970
77	Svce. Pipe Line Co.	AA	3.200	1982
78	So. Cal. Edison Co.	AA	2.875	1976
79	Union Elec. Co. of Mo.	AA	3.375	1971
80	Va. Elec. & Pwr.	AA	2.750	1975
81	West Penn. Pwr. Co.	AA	3.000	1974
82	Ala. Pwr. Co.	A	3.500	1972
83	Amer. Gas & Elec.	A	3.375	1977
84	Ark. Pwr. & Light	A	3.125	1974
85	Assoc. Tele. Co.	A	3.125	1977
86	Birmingham Elec.	A	3.000	1974
87	Cal. Elec. Pwr. Co.	A	3.000	1976
88	Cal. Water Svce.	A	3.250	1975
89	Carolina Pwr. & Light	A	2.875	1981
90	Central Ind. Gas	A	2.875	1971

Appendix A (continued)

No.	Company	Rating	Coupon	Maturity
91	Central Maine Power Co.	A	3.500	1970
92	Cent. Vermont Pub. Svce.	A	2.750	1975
93	Columbia Gas System	A	3.375	1977
94	Equitable Gas Co.	A	3.250	1973
95	Georgia Pwr. Co.	A	3.375	1978
96	Gulf States Utilities	A	3.000	1969
97	Idaho Pwr. Co.	A	3.250	1981
98	Jamaica Water Supply	A	2.875	1975
99	Jersey Central Pwr. & Light	A	2.875	1976
100	Ky. Util.	A	3.000	1977
101	Ky. & W. Va. Pwr.	A	3.000	1979
102	Lake Superior Dist. Pwr.	A	3.000	1975
103	La. Pwr. and Lt.	A	3.125	1978
104	Mich. Consol. Gas Co.	A	3.500	1976
105	Minn. Pwr. & Light	A	3.125	1975
106	Miss. Pwr. Co.	A	3.125	1971
107	Miss. Pwr. & Light	A	2.875	1977
108	Mo. Pwr. & Light	A	2.750	1976
109	Mountain Fuel Supply	A	3.500	1971
110	New Orleans Pub. Svce.	A	3.125	1974
111	Panhandle East Pipe Line	A	3.250	1973
112	Penn. Elec. Co.	A	2.750	1976
113	Penn. Tel. Corp.	A	2.875	1975
114	Plantation Pipe Line	A	2.750	1970
115	Potomac Edison Co.	A	3.000	1974
116	Rochester Tel. Corp.	A	2.500	1981
117	Rockland Light & Pwr.	A	3.125	1978
118	Safe Harbor Water Pwr.	A	3.000	1981
119	Saguenay Pwr. Co. Ltd.	A	3.000	1971
120	St. Joseph Lt. & Pwr.	A	2.625	1976
121	Scranton Spg. Brk. Wtr.	A	2.875	1976
122	So. Counties Gas Co.	A	3.000	1976
123	So. Nat. Gas	A	2.875	1970
124	United Gas Corp.	A	2.750	1970
125	Utah Pwr. & Light	A	2.750	1976
126	Wash. Gas Light	A	3.500	1976
127	West. Texas Utils.	A	3.125	1973
128	Western Light & Tel.	A	3.000	1975
129	Wisc. Mich. Pwr. Co.	A	3.000	1978
130	Equitable Gas Co.	BBB	3.375	1970
131	Milwaukee Gas Light Co.	BBB	3.125	1975
132	New Eng. Elec. System	BBB	3.250	1977
133	New Eng. Gas & Elec. Assn.	BBB	4.000	1971
134	Portland Gen. Elec.	BBB	3.125	1975
135	Pub. Svce. Co-ord. Trspt.	BBB	4.000	1990

Appendix A (continued)

No.	Company	Rating	Coupon	Maturity
136	United Gas Corp.	BBB	4.375	1972
137	Upper Peninsula Power	BBB	3.250	1977
138	West Penn. Elec.	BBB	3.500	1974
139	Atchison, Topeka & S.F. Ry.	AAA	4.000	1995
140	Atlanta, Knoxville & N. Ry.	AAA	4.000	2002
141	K.C. Terminal Ry.	AAA	2.750	1974
142	Union Pac. RR	AAA	2.500	1991
143	Det. & Tol. Shore Line	AA	3.250	1982
144	Elgin, Joliet & Eastern Ry.	AA	3.250	1970
145	Ky. Cent. Ry.	AA	4.000	1987
146	St. Paul Union Depot	AA	3.125	1971
147	Virginian Ry.	AA	3.000	1995
148	Wheeling & Lake Erie	AA	2.750	1974
149	Chicago, Burlington & Quincy	A	2.875	1970
150	Chi. & West. Ind. RR	A	4.375	1982
151	Connecting Ry. & Co.	A	3.125	1976
152	Det., Tol. & Ironton RR	A	2.750	1976
153	Great Northern Ry. Co.	A	4.500	1976
154	N.Y., Chi. & St. L. RR	A	3.250	1980
155	N.Y. Connecting RR	A	2.875	1975
156	Northern Cent. Ry.	A	4.500	1977
157	Peoria & Pekin Union Ry.	A	3.125	1975
158	Phila., Balt. & Wash. RR	A	4.500	1977
159	Southern Pac. RR	A	2.875	1986
160	Texas & N.O. RR	A	3.250	1970
161	Texas Pac.-Mo. Pac. Trm. RR	A	3.275	1974
162	Western Pac. RR	A	3.125	1981
163	Akron Union Pass. Depot	BBB	4.500	1974
164	Ala. & Vicksburg	BBB	5.000	1974
165	Atlantic Coast Line RR	BBB	4.250	1972
166	Fort Worth & Denver Ry.	BBB	4.375	1982
167	Kanawha & Mich. Ry.	BBB	4.000	1990
168	Kansas, Okla. & Gulf Ry.	BBB	3.625	1980
169	Lake Shore & Mich. So. Ry.	BBB	3.500	1997
170	N.J. Junct. RR	BBB	4.000	1986
171	Northern Pac. Ry.	BBB	4.000	1997
172	Pennsylvania RR Co.	BBB	4.250	1981
173	Pitts., Cin., Chi. & St. L. RR	BBB	3.375	1975
174	St. Louis-San Francisco	BBB	4.000	1997
175	Seaboard Air Line RR	BBB	3.875	1977

Appendix B

No.	β_1	β_2
1	- .03656	.70302
2	- .01524	.94877
3	- .03224	.82416
4	- .04007	.61363
5	.03041	.90328
6	.01643	.98979
7	.07560	.86144
8	.02738	1.01163
9	.08172	1.02471
10	- .04224	.60983
11	.04434	.77414
12	- .01343	.39889
13	- .00529	1.06914
14	- .05286	.91617
15	- .01825	1.31927
16	- .01579	1.09483
17	.14326	1.23819
18	- .01058	.72969
19	.03894	.70221
20	.05300	.83986
21	.00716	.68444
22	.06346	.80641
23	.15606	1.26754
24	.05216	1.03714
25	.00083	1.45307
26	.12571	.90749
27	- .02899	.77566
28	.00275	.97756
29	- .05584	.72187
30	- .08465	.92964
31	.06405	1.24438
32	- .05998	.90982
33	- .00127	.96305
34	- .06134	.96355
35	- .05459	1.05575
36	- .04766	1.18741
37	.13103	.87905
38	.15433	1.25353
39	.09384	1.44363
40	- .00933	1.04310
41	.09515	1.24396
42	.01699	.91717
43	- .04616	1.13108
44	- .12594	1.31286
45	.06182	1.22757

Appendix B (continued)

No.	β_1	β_2
46	- .02393	.91551
47	- .03962	1.00933
48	.06489	.68963
49	- .05101	.93787
50	.09258	1.73817
51	.02367	.90142
52	- .09614	1.03547
53	- .02516	.71527
54	.12650	.86848
55	.10659	.98578
56	.07915	1.00639
57	.04449	1.07668
58	.06435	1.30958
59	.05374	.75226
60	.11357	.87073
61	.10644	.93655
62	.11521	1.22621
63	.14501	1.33001
64	- .04572	.98578
65	.06951	.86373
66	.06036	.78184
67	- .07182	.85039
68	.03960	.70170
69	- .09086	1.18731
70	.09382	1.10002
71	.01967	.57719
72	.02441	.94017
73	.01465	.89970
74	.00869	.97044
75	.06426	1.01341
76	.11246	.61761
77	.00243	1.08136
78	- .00166	1.13186
79	- .01072	.83884
80	- .04327	1.06299
81	.04181	.80932
82	- .02065	.92703
83	.04688	1.22155
84	.06033	1.02474
85	.14070	1.33130
86	.09944	.97702
87	.14786	1.54899
88	.12706	.87842
89	.19410	1.20268
90	.09035	.25466

Appendix B (continued)

No.	β_1	β_2
91	.04972	.79650
92	.04225	1.17808
93	.05200	1.11097
94	.09775	1.13234
95	.17599	1.41911
96	.03632	.54539
97	.02405	.90346
98	.15781	.64468
99	- .01103	1.28948
100	.08197	1.40236
101	.11239	1.29068
102	.09372	.96712
103	.13467	1.21161
104	.19172	1.05901
105	.08996	1.17608
106	.11134	.57683
107	.13691	.98008
108	.15359	1.14419
109	.08142	.89062
110	.11348	1.20059
111	.12579	.97494
112	.08421	1.13851
113	.09562	.71502
114	.11698	.55899
115	.05410	1.24371
116	.21286	1.69918
117	.14828	1.28320
118	.18356	1.03353
119	.06404	.60706
120	.07757	1.23159
121	.08837	1.27115
122	.04765	.77692
123	.06636	.62446
124	.08538	.45462
125	.11890	1.29050
126	.18362	1.19599
127	.10330	1.19400
128	.14147	1.09008
129	.17455	1.38400
130	.07675	.74151
131	.09407	1.07403
132	.24668	1.45632
133	.06291	.20804
134	.13012	1.29632
135	- .00516	.13691

Appendix B (continued)

No.	β_1	β_2
136	.01397	.45065
137	.19113	1.16007
138	.02309	.87538
139	- .00175	1.09916
140	.27008	1.21547
141	.03509	.65642
142	- .00491	1.23675
143	.21962	1.43219
144	.03394	.86120
145	.10608	1.31080
146	.04532	.75197
147	.11768	1.68493
148	.16509	1.18675
149	.04780	1.15214
150	.06845	.81503
151	- .09393	.91448
152	.14872	1.05912
153	.10396	1.35905
154	.07175	1.16270
155	.04482	1.11423
156	.14533	.45697
157	.18657	1.40656
158	.06291	.90058
159	.12065	1.96789
160	.10468	1.30819
161	.26469	1.10656
162	.13166	1.09082
163	.03382	.63452
164	.15477	.41940
165	.11974	.78229
166	.20856	.29369
167	.04690	.51470
168	.06617	1.14547
169	.05933	1.88845
170	.14367	.41193
171	.04257	1.50519
172	.16810	1.54886
173	.40621	1.76857
174	.20681	1.94044
175	.21672	.89008

FOOTNOTES

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[1] Of course, modifications of the M-M theoretical framework lead to the possible existence of optimal capital structures. In particular, the possibility of bankruptcy may result in an optimal debt level for the firm (see Stiglitz [16]).

[2] This introduces a bias since bonds which fall into default are systematically eliminated from the sample.

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