Uncertainty Resolution and Multi-Period Investment Decisions

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Introduction

In the literature of finance, there are three commonly used methods of analyzing capital budgeting problems. It is the purpose of this paper to demonstrate that these three approaches, the single certainty equivalent (SCE) approach, the certainty equivalent (CE) approach and the risk adjusted discount rate (RAD) approach are correct under different sets of idealized assumptions. However, it will be demonstrated that none of the approaches is generalizeable, since each depends upon beliefs regarding the expected pattern of uncertainty resolution.[1] That is, each of the approaches is theoretically deficient if the implicit assumptions regarding uncertainty resolution are inappropriate to a given investment situation.

This paper consists of three interconnected but self-contained sections. In the first section, N-period results demonstrating that early uncertainty resolution is preferred to delayed uncertainty resolution are presented. The second section establishes the implicit and often unwarranted assumptions of uncertainty resolution made in using the various capital budgeting procedures. Finally, the third section considers the problem of measuring uncertainty resolution and it is shown that the "Van Horne ratio" is inherently defective.

1. The Value of Uncertainty Resolution in Investment Decisions

Consider a simple decision problem in which a decision maker (individual or firm) must decide how to allocate his wealth, W, between current consumption C_o and savings $(W-C_o)$. Future consumption C_1 is financed solely from internal sources $(W-C_o)$.

Let the decision maker's wealth be the sum of net market value of his current assets, W_o, (including labor) and the cash flows from other assets, F. Thus W and C_1 can be defined as:

(1) $W = W_o + \frac{F}{R_1}$; $C_1 = (W_o - C_o) R_1 + F$

where R_1 is a return relative to be received from time 0 to time 1.

Further, we can assume that W_o and R_1 are known before the decision maker must make his current consumption and investment decision. However, we cannot always assume that the realization of F is known, although it may involve a lottery with known probabilities and known contingent cash flows. If the individual (or firm) must decide on his current consumption and investment before F is known, the decision is made under "temporal uncertainty" (following Drèze and Modigliani [3],the resolution of the uncertainty is delayed until time 1). Alternatively, if the uncertainty is resolved before C_o is to be chosen, the decision is made under "timeless uncertainty." (Note that the timing of the actual cash flow sequences is not necessarily effected by uncertainty resolution).

Drèze and Modigliani have shown under one set of assumptions that if individuals are risk averse, are endowed with cardinal utility functions, and all probability functions have finite first and second moments, then immediate resolution of uncertainty is preferred to delayed resolution of uncertainty. We wish to extend their results from two periods to N periods.[2]

Immediate and Delayed Resolution of Uncertainty Over Two Periods

If a decision maker is endowed with a temporal uncertain lottery L(F) and resolution of uncertainty is delayed until after the choice of C_o, and therefore of W_o-C_o, his expected utility is:

$$(2) \quad (U^* \mid L(F))_{t=1} = \max_{C_o} \int U\left[C_o,\ (W_o - C_o)\ R+F\right] dL(F)$$

However, if the same uncertain lottery is timeless and the value of F is known before deciding on C_o then his expected utility is:

$$(3) \quad (U^* \mid L(F))_{t=o} = \max_{C_o} \int U\left[C_o,\ (W_o - C_o)\ R+F\right] dL(F)$$

In addition, as Marschak [7] has observed:

$$(4) \quad \int \max_{C_o} \ U\left[C_o,\ (W_o - C_o)\ R+F\right] dL\ (F) \geq$$

$$\max_{C_o} \int U\left[C_o,\ (W_o - C_o)\ R+F\right] dL\ (F) \qquad (3)$$

Thus the decision maker prefers immediate resolution.

The Extension to N Periods

The theoretical N period decision paradigm is not essentially different from the two period decision paradigm. In the N period case, we maximize a cardinal utility function $U(C_1, C_2, \ldots, C_N)$ given the uncertain prospect L(F). In order to accomplish this, we begin by establishing a 3 period problem and then generalize to an N period problem.

Assume we maximize $U(C_1, C_2, C_3)$ given L(F) and $C_2 = (W_1 - C_1)R_2$ by the optimal choice of C_o and C_1. The decision maker's wealth, W_1, at time 1 includes investment proceeds, $(W_o - C_o)R_1$, and labor income. Let us further assume that uncertainty resolution is delayed until t=2.

$$(5) \quad (U^* \mid L(F))_{t=2} = \max_{C_o C_1} \int U\left[C_o, C_1,\ (W_1 - C_1)\ R_2+F\right] dL\ (F)$$

If resolution is delayed only one period, we have:

$$(6) \quad (U^* \mid L(F)_{t=1} = \max_{C_o} \int \max_{C_1} \ U\left[C_o,\ C_1,\ (W_1 - C_1)\ R_2+F\right] dL(F)$$

Finally, if the uncertain prospect is timeless and uncertainty resolution is immediate, then:

$$(7)\quad (U^* \mid L(F))_{t=o} = \int \max_{C_o C_1} U\left[C_o, C_1, (W_1 - C_1) R_2 + F\right] dL(F)$$

In this case maximization with respect to C_o and C_1 are both inside the integral indicating that they are chosen after F is known.

Using previous results (see equation (4)), we know that $(U^* \mid L(F))_{t=o} \geq (U^* \mid L(F))_{t=1}$. A similar procedure can demonstrate that uncertainty resolution at t=1 is preferred to t=2. Let

$$(8)\quad (U^* \mid L(F))_{t=1} = \max_{C_o} B(C_o)$$

where $B(C_o) = \int \max_{C_1} U\left[C_o, C_1, (W_1 - C_1) R_2 + F\right] dL(F)$

and let

$$(9)\quad (U^* \mid L(F))_{t=2} = \max C(C_o)$$

where $C(C_o) = \max_{C_1} \int U\left[C_o, C_1, (W_1 - C_1) R_2 + F\right] dL(F)$

and $(U^* \quad L(F))_{t=o} \geq (U^* \mid L(F))_{t=2}$

clearly, these results can be generalized to any pair of resolution time periods in an N period decision problem.

In summary, when uncertain prospects are resolved early, some consumption and investment decisions can be made under certainty and a more preferred intertemporal consumption and investment pattern can be determined. Furthermore the more risk averse the investor is, the more he will prefer early to delayed uncertainty resolution.[3] In the special case of risk neutrality, the decision maker is indifferent to uncertainty resolution.

One question that naturally arises from this analysis is how far the preceding results can be extended to N period capital budgeting problems of firms. Clearly in the case of the owner-proprietor the analysis is completely analogous. In the case of separation of ownership and management, if the managers act on the basis of a homogenous ownership utility function, the situation is again analogous.

II. Uncertainty Resolution and the Discounting of Cash Flows

The preceding section has demonstrated special conditions under which preferences arise with respect to the rate at which uncertainty is resolved. In this section the inherent assumptions regarding uncertainty resolution, associated with various schemes of discounting cash flows are developed.

In the subsequent development of uncertainty resolution and discounting of cash flows the following definitions will be employed:

$\tilde{F}_{jt}$ is a random variable corresponding to a cash flow sequence for a single project at time t and state j. F_{jt} is the analogous certain cash flow sequence.

$\bar{F}_t$ is the expectation of a project's cash flow in period t.

$\tilde{V}_{jt}$ is the value of a project <u>conditional</u> on a sequence of cash flows $(F_{j1}, F_{j2}, \ldots, F_{jN})$ and a risk free rate of discount. N is the maximum life of the project.

i is the discount rate applied to certain cash flows.

α is a certainty equivalence factor.

P_j is the probability of a particular outcome in a cash flow sequence at state j.

If we assume that an investor is presented with the opportunity to purchase now a risk-free cash receipt t periods hence, we might determine the present value of that receipt, using discrete notation, as in equation (10).

(10) $V_o = \frac{F_t}{(1+i)^t}$

Then multiplying both sides of equation (10) by $(1+i)^t$ reveals the implied growth in the present value of the cash receipt as discrete time passes. That is,

(11) $V_t = V_o\ (1+i)^t = F_t$

Now let us drop the assumption that the t periods hence cash flow is perceived to be risk free and allow $\bar{F}_t = F_t$ to represent the expected value of the cash receipt. Presumably, risk aversion is sufficient to lead the new present value, γV_o, to be less than V_o (i.e. $0 < \gamma < 1$). Then,

(12) $V_o = \frac{\bar{F}_t}{(1+k)^t}$

Since $\bar{F}_t = F_t$ and $\gamma V_o < V_o$, k must be greater than i. Now, the implied growth in the present value appears in equation (13).

(13) $V_t = \gamma V_o\ (1+k)^t = \bar{F}_t$.

Equation (10), which represents the situation in which no risk is perceived, implies growth in the present value at the rate i, the risk-free rate or time value of money, thus offering the rate of return i per period. Equation (13) implies growth in value in the first and all subsequent periods at the rate $k > i$. However, if the risk in the expected future cash flow $\bar{F}_t$ were perceived to be the same at the end of the first period as it was at the beginning of that period, it is not rational (assuming homogeneous expectations and risk aversion) to assume that the right to this cash receipt could be sold for any price other than γV_o $(1+i)$. The holder of the asset during that first period has borne no risk in the sense that none of the uncertainty regarding the cash receipt has been resolved during the time he held the asset. Therefore, he cannot expect to be compensated at a rate greater than the risk-free rate. Only if the risk perceived in $\bar{F}_t$ has decreased would the value at the end of the first period rationally be greater than $\gamma V_o(1+i)$. Thus, growth in value at a rate greater than i can take place only if some resolution of uncertainty occurs over time.[4]

The Single Certainty Equivalance Method (SCE) and Immediate Resolution of Uncertainty

Introduce a lottery L(F) that yields cash flows $\tilde{F}_{jt}$ contingent on state j and time t. If the lottery is played immediately and the cash flows are paid out at t=1, 2, ... N, the set of possible values of the lottery at time t=o, are

$$(14) \quad \tilde{V}_o = \sum_{j=1}^{M} \sum_{t=1}^{N} \frac{\tilde{F}_{jt}}{(1+i)^t}$$

for every possible sequence of cash flows $\tilde{F}_{j1} = F_{j1}, \tilde{F}_{j2} = F_{j2}, \ldots \tilde{F}_{jN} = F_{jN}$. The normalized random variable $\frac{\tilde{V}_j}{\bar{V}}$ will have values $\frac{\tilde{V}_1}{\bar{V}}, \frac{\tilde{V}_2}{\bar{V}}, \ldots \frac{\tilde{V}_M}{\bar{V}}$ with corresponding probabilities $P_1, P_2, \ldots, P_M$. Further assume there exists a utility function (e.g., a market valuation function) $U\left(\frac{\tilde{V}_j}{\bar{V}}\right)$ with the properties of a Von Neumann-Morgenstern utility function (as in preceding sections), except that it will be independent of scale of returns.

Combining the distribution of returns with the valuation function allows the calculation of a project's expected utility:

$$(15) \quad \bar{U} = E(U) = \sum_j U\left(\frac{\tilde{V}_j}{\bar{V}}\right) P_j$$

From this relationship the value of the asset can be determined by maximizing the expected utility of possible values $\frac{\tilde{V}_j}{\bar{V}}$.

A "certainty equivalence" factor α_o is found from an inverse transformation:

(16) $$\alpha_o = U^{-1}(\bar{U}) = U^{-1}\left[\Sigma\, U \frac{V_j}{\bar{V}}\; P_j\right]$$

and, once α_o is known, the net present value (NPV) (sometimes referred to as the net certainty equivalence value) of the project is determined by

(17)(a) $$NPV = \alpha_o \bar{V}$$

where $\bar{V} = \sum_{t=1}^{N} \frac{\bar{F}_t}{(1+i)^t}$

This procedure has been referred to as the single certainty equivalence (SCE) method of capital budgeting and was first introduced by Hillier [4]. But, note that the method <u>depends upon immediate uncertainty resolution</u>. That is, if we utilize generally accepted decision theory in a capital budgeting problem involving immediate uncertainty resolution, the solution involves using the Hillier SCE method. Moreover, it will be shown that the SCE method is correct if and only if there is immediate uncertainty resolution and is not correct if uncertainty resolution is delayed.

The Certainty Equivalent Method (CE) and Delayed Uncertainty Resolution

Consider, again, the lottery L(F) described previously. Now assume the lottery is played at t=1, i.e., delayed one time period. All other things remain the same so that the cash flows are paid out at t=1, 2, ..., N.

The net present value of the lottery is generally expressed as

(17)(b) $$NPV = \sum_{t=i}^{N} \frac{\alpha_t \bar{F}_t}{(1+i)^t}$$

or, explicitly recognizing that all uncertainty is resolved at t=1.

(17)(c) $$NPV = \frac{\alpha_1 \bar{F}_1}{1+i} + \sum_{t=2}^{N} \frac{F_t}{(1+i)^t}$$

where $\alpha_1 = U^{-1}(\bar{U}) = U^{-1}\left[\sum_j U\left(\frac{\tilde{F}_{1j}}{\bar{F}_1}\right) P_j\right]$

and, equivalently,

$$(17)(d) \quad NPV = \alpha_1 \frac{\bar{V}_1}{1+i} .$$

This procedure is commonly referred to as the certainty equivalent (CE) capital budgeting method. It is the consequence of endowing the cash flow sequences with delayed uncertainty resolution. It would have been incorrect to have applied the SCE method to the case of delayed uncertainty since it is obviously incapable of discriminating between cash flow sequences having delayed uncertainty resolution and those having immediate uncertainty resolution.[5]

In contrast the CE implicitly assumes uncertainty resolution is delayed until the end of the period. However, since no uncertainty is resolved between t=o and t=1 the present value of the asset increases at the riskless rate over time. Thus, the CE procedure is only correct if uncertainty resolution is delayed and takes place at one point in time. If some early uncertainty resolution is expected to take place, the CE cannot yield correct value and its implications about how value increases over time would also be erroneous.

Using the pure rate of interest is appropriate if either all uncertainty resolution is delayed or if all uncertainty resolution is immediate; that is, if the investor accepts no risk betwen points in time (i.e., during the time interval).

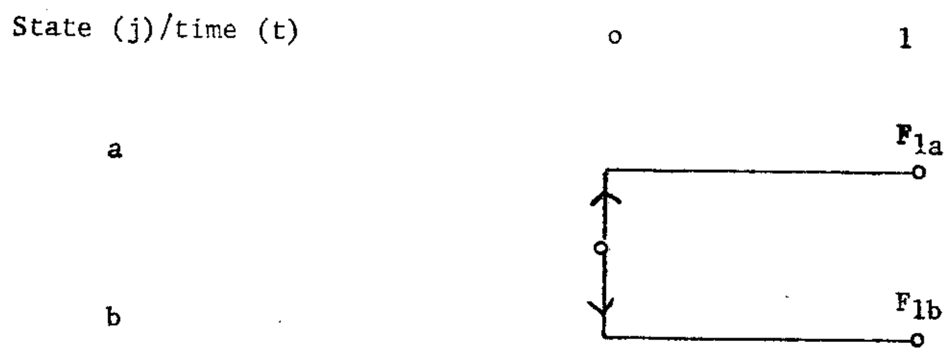
Comparing the SCE Method with the CE Method

Determining the exact conditions under which the SCE method and CE method of capital budgeting are equivalent (i.e., yield the same valuations) is analogous to determining the instances when the expected uncertainty resolution involved in cash flow sequences is irrelevent. Consider two projects I and II that differ only in their expected uncertainty resolution and are represented in Figure 1.

Figure 1

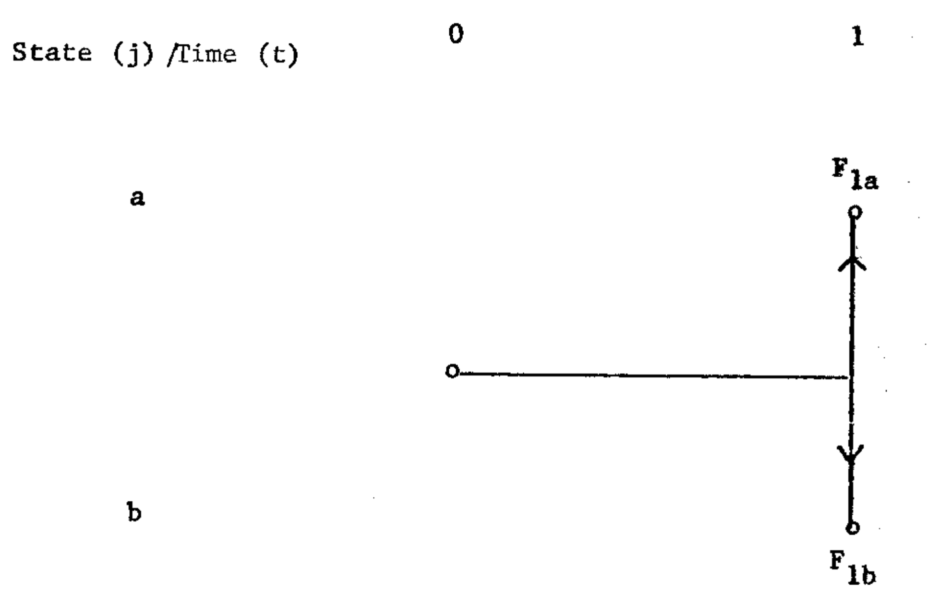
I Immediate Resolution of Uncertainty

Asset I



II. Delayed Resolution of Uncertainty

Asset II



$F_{1a} = 50$ $F_{1b} = 25$ $i = .05$

The net present value of the two assets is solved for by the following equations:

$$(18)\quad \text{Asset 1: } NPV_o = \alpha_o \sum_{j=a}^{b} P_j \tilde{V}_j = \alpha_o \sum_{j=a}^{b} P_j \frac{\tilde{F}_{1j}}{1+i}$$

and

$$(19)\quad \text{Asset 11: } NPV_o = \frac{\alpha_1 \bar{F}_1}{(1+i)} = \sum_{j=a}^{b} \frac{\alpha_1 P_j \tilde{F}_{1j}}{(1+i)}$$

Taking the difference yields

$$1 \ NPV_o - 11 \ NPV_o = \alpha_o \sum_{j=a}^{b} \frac{P_j \tilde{F}_{1j}}{(1+i)} - \sum_{j=a}^{b} \frac{\alpha_1 P_j \tilde{F}_{1j}}{(1+i)}$$

$$= \alpha_o \bar{V} - \alpha_1 \bar{V}$$

Accordingly, if the net present values are equal it is necessary that $\alpha_o = \alpha_1$. However, this will be true only if there is no risk adjustment (only time preference adjustments) after period 0. The cash flow sequences of Asset 1 are endowed with immediate uncertainty resolution and the SCE method is used to compute the asset's net present value. In contrast, the cash flow sequences of Asset 11 are endowed with delayed uncertainty resolution and its net present value is solved for by utilizing the CE method. Since the same set of cash flows are embodied in both assets and uncertainty is resolved earlier in asset than asset II, asset I must be worth more than asset II if we assume risk aversion. Using either the SCE method or the CE method (but not both) will yield the same net present value for both assets. Hence, a decision maker cannot rely solely on either method.[6]

The Risk Adjusted Discount Method and Continuous Uncertainty Resolution

If a discount rate higher than the risk-free rate is employed for the time interval Δt, the implicit assumption is that some uncertainty resolution continuously takes place. The risk adjusted discount rate (RAD) approach to the discounting of cash flows makes use of a rate of return equal to the risk-free rate plus a premium for bearing risk. In its most common form the RAD approach uses only one such risk adjusted rate, k, and thus involves the assumption that uncertainty is resolved at a constant and continuous rate between points in time.

$$(20) \quad V_o = \sum_{t=1}^{N} \frac{\bar{F}_t}{(1+k)^t}$$

However, equation (20) is only one of several possible formulations for the RAD approach. Equation (20) is at one extreme of a hierarchy of relationships including those in equations (21) and (22).

$$(21) \quad V_o = \sum_{t=1}^{N} \frac{\bar{F}_t}{\prod_{T=1}^{t} (1+k_t)}$$

$$(22) \quad V_o = \sum_{t=1}^{N} \frac{\bar{F}_t}{(1+i)^{t-1}(1+Y_t)}$$

Equation (21) is the most general case. That is, for each cash flow, and in each time period, a different rate of uncertainty resolution is assumed. Since any particular $\bar{F}_t$ can be equal to zero, it is not necessary in the RAD approach to define the time intervals over which uncertainty is resolved as equal to the intervals at which cash flows are expected to be received.

With time intervals sufficiently small and returns defined in terms of those intervals, the formulation in equation (22) approaches equivalency with the certainty equivalent approach. According to (22) there is a final time period in which the cash flow is received when all uncertainty is resolved and all compensation for bearing risk takes place.

The difficulty with the RAD approach involves specifying how the risk adjusted discount rates are to be objectively and consistently determined.

III. Measuring Uncertainty Resolution

It appears that the generally accepted capital budgeting procedures are deficient because they ignore or make unwarranted assumptions concerning uncertainty resolution. Moreover, the existing practical techniques for measuring uncertainty resolution are insufficient.

Van Horne's Measure

Van Horne [11] proposes a measure of uncertainty resolution analogous to the coefficient of variation. In his measure,

$$CV_t = \frac{S_t}{\overline{NPV}}$$

where: CV_t is the average coefficient of variation of cash flows at time period t, S_t is the square root of the weighted average variance about the mean of all branches of the probability tree at time period t and $\overline{NPV}$ is the expected net present value of the investment.

Thus, Van Horne measures the degree of uncertainty resolved by the reduction in the "average standard deviation" of the investment expected through time.

Comparing the Van Horne Ratio With a More General Procedure

Assuming that uncertainty resolution takes place at discrete points in time, using the certainty equivalence method, the investor specifies the cash flow which, with certainty, he requires at time t such that he is indifferent between this guaranteed cash flow (denoted F_t) and the expected value of the uncertain cash flow (i.e., $\bar{F}_t$).

We have assumed that α_t in the model for the cash flow expected in period t is set by the investor at t = o. Define ${}_{\varphi}\alpha_t$ as the α perceived at point in time φ for the cash flow at point t (where $\varphi < t$), assuming all uncertainty is resolved at point t. Thus the α_t which has been discussed above would be termed ${}_{0}\alpha_t$ in this notation. During the course of the investment, as φ goes from 0 towards t, it would be expected that ${}_{\varphi}\alpha_t$ would increase if uncertainty is actually being resolved over time. That is, as the particular cash flow approaches in time the investor may feel he knows the cash flow he will receive with greater certainty.

Theoretically it is possible for an investor to identify at the outset his expectations of ${}_{\varphi}\alpha_t$ for each cash flow and all points in time. This is equivalent to allowing him to estimate the expected uncertainty of the cash flow in period t as viewed from each period from 0 to t-1. The measures might involve the expectation of no resolution (${}_{0}\alpha_t = {}_{1}\alpha_t = \dots {}_{t-1}\alpha_t$) or expecting most of the uncertainty to resolve in t = 1 (implying ${}_{0}\varphi_t < {}_{1}\alpha_t$).

If uncertainty resolution matters to the investor (and we have shown previously when it does), then this measure of resolution of uncertainty could join the expected return and the distribution of returns as decision variables in capital budgeting analysis.

We will proceed to solve for the expected certainty equivalence factors, $E({}_{\varphi}\alpha_t)$, for assets A, B and C described in Figure 2. Note, all assets have equal end of period mean-variance properties.

Computation of the Proposed Coefficient

The calculation of α flows directly from the investor's utility function. This is the primary reason for justifying it as a fundamental measure of uncertainty resolution since it incorporates all of the investor's preferences as described by his utility function - not merely the standard deviation of the distribution.

For the analysis, the following simple and commonly used utility functions of the risk averting investors have been selected:

(23) $$E(U) = \bar{F} - a\sigma_f^{\,2}$$

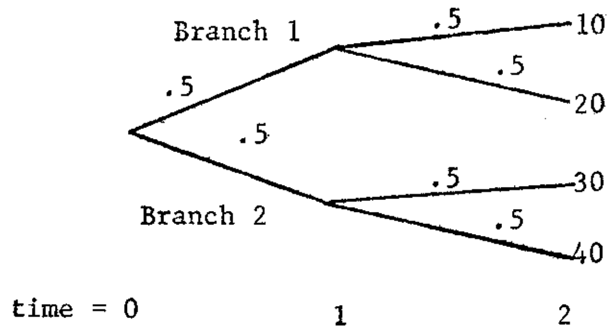
(24) $$U = F - b\,F^2; \quad F \leq \tfrac{1}{2}\,b$$

(25) $$U = \ln F$$

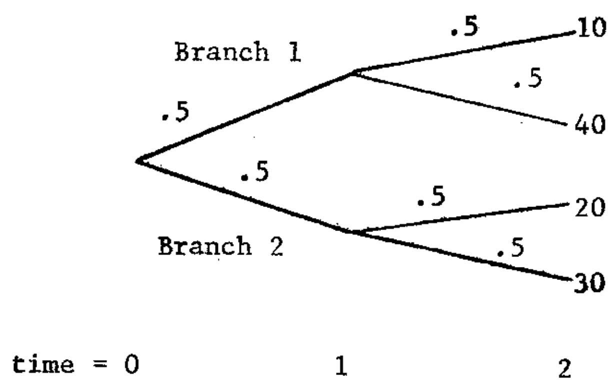
(26) $$U = F^{\gamma}, \; 0 < \gamma < 1$$

a, b, γ are constants greater than 0.

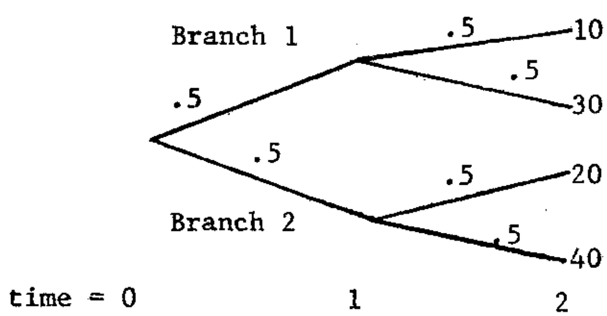
All of the above functions imply risk aversion and except for (23) all imply decreasing marginal utility for cash flow. The function represented by (23) can lead to the conclusion that expected utility increases linearly with increasing cash flow.



Asset A



Asset B



Asset C

Figure 2

Table 1

Asset A Utility Functions		${}_0\alpha_2$	$E({}_1\alpha_2)$	Uncertainty resolved from t=0 to t=1
(1)	$E(U) = F - a\sigma^2$	.95000	.986095	76.19%
(2)	$U = F$	.90455	.97656	75.44%
(3)	$U = \ln(F)$	.88534	.96627	70.58%
(4)	$U = F^{.2}$	.90924	.97299	72.32%
	$= F^{.8}$	.97831	.99328	69.02%
(5)	Van Horne	$CV_o = .4472$	$CV_1 = .2000$	55.28%
Asset B				
(1)		.95	.95	No resolution
(2)		.90455	.90700	2.57%
(3)		.88534	.889898	3.97%
(4)		.90924	.911538	9.36%
		.97831	.978343	0.15%
(5)		$CV_o = .4472$	$CV_1 = .4472$	No resolution
Asset C				
(1)		.95	.95833	16.67%
(2)		.90455	.92008	16.27%
(3)		.88534	.90441	16.63%
(4)		.90924	.92335	21.46%
		.97831	.98107	12.72%
(5)		$CV_o = .4472$	$CV_1 = .4$	10.55%

These computations generally reveal that investment A resolves its uncertainty most rapidly. Asset C also shows unambiguous uncertainty resolution, although at a slower rate. All measures including Van Horne's indicate that uncertainty has been resolved in these two cases, and all lead to the conclusion that A resolves relatively faster than C. The percentage of uncertainty resolved in the first period for a particular asset differs among utility functions. However, Van Horne's measure generally understates the true expected uncertainty resolution.

All three assets have some intermediate uncertainty resolution, and, thus, neither the SCE method or the CE method will yield the theoretically correct answer. The SCE method will tend to overstate the values and the CE method will understate the values, and since the uncertainty resolution for all assets takes place at discrete points in time, the RAD method is inappropriate.

IV. Conclusions

We have demonstrated that when comparing sets of uncertain cash flow sequences in an N-period problem, early uncertainty resolution is preferred to delayed uncertainty resolution. The importance of this conclusion stems from the fact that investments may have the same mean-variance properties but different uncertainty resolution properties. Currently developed capital market theories of asset selection (see [5]) ignore uncertainty resolution and concentrate soley on the mean-variance properties.

It has not been generally recognized that the three widely used methods in capital budgeting problems make implicit, and often unwarranted, assumptions regarding uncertainty resolution. The current debate (see Chen [2]) over which of the methods is most appropriate has not properly included an analysis of uncertainty resolution. We illustrate the exact dependence of each method on particular patterns of expected uncertainty resolution.

Finally, we demonstrate that the generally accepted procedure (i.e., the Van Horne ratio) for measuring uncertainty resolution is insufficient for the task.

Footnotes

[1] Recent articles on uncertainty resolutions include [1] and [11].

[2] Using the capital asset pricing model as discussed in Fama [5], there would be no way to distinguish between assets that differ only in expected uncertainty resolutions.

[3] In an extension of Marschak's results it can be shown that

$$\text{(a)} \quad \int \left\{ \max_{C_o} \left[C_o,\ (W_o - C_o)\ R+F\right] - U\left[C_o^*,\ (W_o - C_o^*)\ R+F\right] \right\} d(F) = 0$$

where C_o^* refers to the optimal amount of consumption in time 0,

and,

$$\text{(b)} \quad \max_{C_o} U\left[C_o,\ (W_o - C_o)\ R+F\right] = U\left[C_o^*,\ (W_o - C_o)\ R+F\right]$$

if

$$\text{(c)} \quad \frac{dU}{dC_o} = 0$$

This last condition states that the indifference curves are linear and the investor is risk neutral.

[4] The fact that a project has a discount rate k greater than i usually has meant that its cash flows are riskier than a certain cash flow sequence. A corresponding interpretation is that the uncertainty is being resolved at the k-i rate over time. Thus a discount rate for any project measures the magnitude of the project's risk and the rate at which uncertainty is being resolved over time.

[5] The SCE method of capital budgeting assumes all uncertainty is resolved at t=o and none is resolved thereafter.

[6] Consider the two assets in Figure 1. The cash flow sequences are the same except that the uncertainty in cash flow shows sequence I is resolved immediately, whereas, the uncertainty in cash flow sequence II is delayed. Furthermore, assume that a mean-variance utility function $U = \bar{F} - a\sigma_f^2$, $a = \frac{1}{2}$ adequately describes a market valuation function. The value of the two assets using the SCE method is

$$NPV = \alpha_o\ \bar{V} = 32.87 = \frac{32.87}{35.7}\ (35.7) = 32.87$$

Similarly, the value of the two assets using the certainty equivalent method is

$$NPV = \sum_{t=o}^{1} \sum_{\alpha=o}^{b} \frac{\alpha_t\ \bar{F}_{tj}}{(1+i)} = 32.69$$

where $i = .05$

$$\alpha_o = \frac{U(\bar{V} - a\sigma_v^2)}{\bar{V}}$$

$$\alpha_1 = \frac{U(\bar{F} - a\sigma_f^2)}{\bar{F}}$$

The value of immediate uncertainty resolution is 32.87 - 32.69 = .18.

References

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