The Error Learning Hypothesis and the Term Structure of Interest Rates in Eurodollars

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The expectations hypothesis used to explain the term structure of interest rates has as its main empirical support the error learning hypothesis first introduced by David Meiselman [1962]. Since the publication of his work, however, there has been considerable disagreement as to the robustness of his empirical estimation procedure.[1] The present study will continue that inquiry. This will be accomplished by employing the error learning approach to the term structure in a new market, the Eurodollar market. It is felt that the explicit term structure in this market is preferable to the approach used heretofore of free-hand approximations of the yield curve, and will result in the first unbiased data set.

The study may be outlined as follows. Section I will indicate the alternative explanations of the term structure in an effort to place the error learning hypothesis in proper perspective. Section II will analyse the previous studies using the Meiselman approach to indicate the problems associated with the data sets used in existing studies vis-a-vis the acceptabilty of this hypothesis. Then, Section III will offer the Eurodollar market as an interesting "testing ground" for the hypothesis. At this time this market will be discussed briefly to indicate its acceptability in the present context. Section IV will present the results of the estimations using an explicit yield curve, substantially supporting the viability of the error learning hypothesis as an empirical reality. Section V will be reserved for concluding remarks.

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[1] See the discussion in Section I below for reference.

I. The Term Structure

The theoretical discussion centering around the explanation of differing yields for identical assets with different maturity dates can be broadly characterized as the expectations hypothesis. All varieties of this theory contend that there exists some a priori relationship between short and long term rates. Proponents[2] of this theory would concede the existence of demands and supplies in each maturity, however; they contend that the cross elasticity between maturities, i.e. the existance of arbitrage, will lead to interdependent yields. Given the situation of independent markets, a risk indifferent arbitrager would borrow long to lend short if his expectations of future interest rates suggest a sufficient profit from the transactions. Proponents of the pure expectations hypothesis would, therefore, argue that in equilibrium the long term rate would equal the geometric mean of the series of short term rates expected to apply in the future.[3]

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[2] As explained below proponents of the expectations hypothesis can be segmented into two groups. The pure expectations approach has been offered by Lutz [1940] and more recently proposed by Wood [1969]. The liquidity preference argument is offered by Hicks [1946] and the recent work by Nelson [1971].

[3] The notation used throughout the article is as follows: R, the actual rate of interest prevailing in the market, and r, the forward rate of interest. The prescript refers to the time when the rate becomes applicable; the first postscript, to the length of the loan; and the second postscript, to the point of reference in time.

$$(1)\quad (1 + R_{n,t}) = (1 + R_{1,t})\ (1 + {}_{t+1}r_{1,t})\ (1 + {}_{t+2}r_{1,t})\ldots(1 + {}_{t+n-1}r_{1,t})$$

The liquidity preference variant of the expectations hypothesis allows for arbitrage but contends that due to the risk associated with expectations of future interest rates and the loss of liquidity associated with the longer term asset a premium would exist in the long yield over the expected short term yields, i.e. ${}_{t+1}r_{1,t}$ in equation (1) above is equal to the expected future rate (ρ), plus a risk or liquidity premium (L).

$$(2)\quad {}_{t+1}r_{1,t} = {}_{t+1}\rho_{1,\ t} + {}_{t+1}L_{1,t}.$$

II. <u>The Error Learning Empirical Hypothesis</u>

Until the work of Meiselman [1962] no acceptable empirical framwork had been developed to substantiate any variant of the expectation hypothesis outlined above. Due to the unobservable nature of expectations it had been thought impossible to formulate an efficient test of the hypothesis without the additional assumption of accurate expectations. Indeed, tests assuming accurate forecasting of future rates were used as evidence against the expectations hypothesis by Culbertson [1957]. However, the extreme nature of the accuracy assumption all but voids any resultant implications.

Meiselman, on the other hand, relaxed this assumption. His major contribution can be stated quite succinctly:

> "The expectations hypothesis need not be tested by relating yield curves to contemporaneous expectations. Instead <u>changes</u> in, rather than <u>levels</u> of interest rates can be related to factors which systematically cause revisions of expectations."[4]

His model, therefore, is one of interest rate adjustment. The approach is straightforward. Individuals are viewed as using previous levels of interest rates as a source of expected future rates.

$$(3) \quad {}_{t+n}r_{1,t} = \sum_{i=-1}^{-T} \omega_i R_{1,i}$$

Further, expectations are adjusted systematically to errors in forecasting future rates. Therefore if, at time t, the expected rate is not equal to the actual, i.e. ${}_tr_{1,t-1} \neq R_{1,t}$, from (3) future expected short term rates will shift so as to incorporate this new information. This results in the substantive hypothesis:

$$(4) \quad {}_{t+n}r_{1,t} - {}_{t+n}r_{1,t-1} = F\left({}_tR_{1,t} - {}_tr_{1,t-1}\right)$$

Defining the bracketed term as the error in forecasting and using a linear functional form we obtain the basic equation of the error-learning hypothesis

$$(5) \quad \Delta_{t+n} r_{1,t} = a + bE_t$$

where Meiselman's null hypotheses were:

(6) H_0: (1) $a = 0$

(2) $b > 0$

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[4] Meiselman [1962] p. 18.

Using Durand's[5] yield curve for United States corporate bonds annually, 1901-1954, the results indicated strong support for the error learning hypothesis. These results are reproduced in Table 1.

The null hypothesis concerning the constant term, a, was construed by Meiselman to be a necessary and sufficient condition for the absence of a liquidity premium. The null hypothesis concerning b was viewed as evidence in support of the expectations hypothesis of the term structure. However, a review of the study by Wood [1963] demonstrated that the null hypothesis concerning the constant term was a necessary but not a sufficient condition for the absence of a liquidity premium.

Subsequently, a paper by Grant [1964] in Economica questioned the usefulness of the Durand data set, contending that the data had been so constructed as to imply the relationship obtained by Meiselman. The Durand yield curves were originally constructed by plotting some three thousand different bond issues. Then, for each year a curve was selected using a linear interpolation to express the variability of yield given different maturities. All curves were drawn free-hand and limited to four shapes. The data was, therefore, used not to obtain direct quantitative estimates of the structure of rates but rather to obtain some general implications as to the yield curve. Durand himself referred to them as "quick and crude."[6] Grant infers from this that the resultant smoothed yield curve may have artificially imposed the relationship indicated by Meiselman.

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[5] The Durand data used by Meiselman was the result of several studies by the former author, see Durand [1942], [1958] and Durand and Winn [1947]. Reservation about this data are discussed below.

[6] Durand [1958], p. 348.

In a later article by Buse [1967] the latter demonstrates this point by using the Durand data in reverse order and randomly to determine the robustness of the Meiselman test. His results, contained in Table 2 below demonstrates that good results could be obtained even with random or reverse ordering.[7]

As a result of the unsatisfactory quality of the data Grant [1964] contended that little could be concluded without additional tests of the error learning hypothesis. Towards this end he attempted to construct a time series of yield curves on British Government Securities from 1924-1962. Yield curves were not limited to smooth constructs, rather the curves often contained several critical points. Regressing the quarterly change in expected rates of interest on the error in forecasting one year rates in both linear and cubic adjustment equations, Grant's results were not encouraging. In the linear estimation, constant terms were high and the coefficient of the forecasting error too large, with the R^2 too low. In the cubic estimation only two terms significantly differed from zero.

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[7] A subsequent comment by Wallace [1969] takes issue with the use of R^2 in Buse's work as a test of the relative goodness of fit. He contends that a more appropriate measure of exploratory power is the residual variances which indicate that the chronological order is a superior fit. Yet this appears to beg the question of whether, in fact, the Meiselman test is a satisfactory empirical test of the expectations hypothesis.

Shortly thereafter support for the expectations hypothesis through the use of error learning was forthcoming from Van Horne [1965]. Using hand-fitted curves of the Treasury yield curve, Van Horne regressed monthly changes in the forward rates against errors in the one year forecasting. The results, found in Table 5, indicate strong support for the Meiselman approach. However, the method of derivation of the data is subject to the same reservations indicated above with reference to the Durand free-hand yield curve.

Therefore, despite many articles on the subject, the validity of the error learning hypothesis is still an open question. Further, any results obtained from a new test would be strongly biased in accordance with the method of constructing the complete data set, unless accurate figures can be be obtained for a full maturity range of a homogeneous asset. In the remaining sections below such a study of the Eurodollar market is conducted.

III. The Eurodollar Market As a Data Set

The development of the Eurodollar market as a substantial international financial market has been well documented in the literature.[8] Consequently a detailed description of this market at this time is hardly appropriate. However, a discussion of those elements of this market that render it amenable to the examination of its term structure seems desirable.

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[8] For studies on the development of this market see Altman [1961], [1963], [1965]; for analysis of its relation to domestic rates see Black [1971] and Rich [1972]; for the mechanism of creation see Friedman [1969].

The Eurodollar market is primarily a short term market for homogeneous assets, demand deposits at U.S. commercial banks. The leading markets are London, Paris, Zurich, Toronto, and Nassau. Funds flow into this market as corporations, and, on occasion, central banks, make time deposits at commercial banks located outside the United States to obtain interest yields higher than allowable in the domestic market. These funds are then borrowed by commercial banks as primary reserves or corporations in lieu of domestic borrowing. Traders in this market offer bid and ask quotes for different maturities with actual trading occurring within these limits. Participants usually work through brokers serving as intermediary between borrower and lender. Denominations are relatively large with minimum transactions in the area of one million dollars and single trades of fifty times this quantity not infrequent. Borrowing banks operating in this market usually accept dollar deposits from other banks as well as non-bank lenders. Lending banks will do likewise. Further, banks within this market will normally maintain balances (positive or negative) in most maturities.

Maturities, in general, are of a short duration. Virtually no transactions occur for more than one year with most transactions involving day-to-day, one and two weeks, 1, 2, 3, 4, 5, 6, 9 and 12 month maturities.

The present study will make use of the monthly rates indicated above obtained from a large United States commercial bank and a large Canadian bank. Both these institutions deal heavily in the monthly markets and their bid and ask quotes appear to be acceptable market quotes. Data has been compiled on a bi-monthly basis for the entire yield curve with explicit 1, 2, 3, 4, 5, 6, 9 and 12 month quotes. No approximations were made. The mean of the bid and asked was used as the effective rate. As such this data set appears to be the first unbiased explicit yield curve to be used in an examination of the expectations hypothesis. Data for the 1, 2, 3, 6, 12 month rates covers period January 1968 through June 1972 with observations on the first and fifteenth of each month, resulting in

108 observations. Data for 9 month rates could only be obtained for the last 56 observation periods and the 4 and 5 month data covered only 34 observations. The relative "thinness" of the last maturity dates before 1971 was thought inappropriate for a test of a hypothesis that assumes sufficient arbitrage to equate future rates with their expected values. In any case the present data set seems sufficient for the purposes at hand.

IV. Eurodollar Results with an Explicit Yield Curve

Using the above data, the Meiselman error-learning hypothesis was tested using both a linear and cubic adjustment model. The results using monthly error adjustments are contained in Table 6, while the three month adjustment results are contained in Table 7.

While the coefficient of determination, R^2, are lower than the Meiselman results (Table 1), the data substantially support the original Meiselman approach. In the monthly error adjustment regressions the constant term is not significant in any of the equations. The value for b, the regression coefficient on the short term forecasting error, differ significantly from zero in three of the four equations. However, the regression coefficient on the four month adjustment equation is not significant at the 90% level, though it is of the correct sign and relative magnitude. Correlation coefficients, as a rule, vary inversely with the maturity of the independent variable as immediate future rates adjust more quickly and with greater reliability to present forecasting errors. This is as one would expect and in agreement with the Meiselman results. The cubic adjustment process, on the other hand, is not supported by the results; signs vary; standard errors are too high and all do not significantly differ from zero.

The one troubling result is the relatively low coefficients of determination, as compared to the previous studies. Two rationales can be given for this. Firstly, the Eurodollar market is subject to considerably more forces than merely the term structure. Exchange rate variability alters net return to non-dollar participants that will result in variability in the rate structure independent of the forecasting error problem analyzed here.[9] Secondly, the explicit yield curve here with its implied uneven shape may well indicate that the superior fit obtained from smoothed curves in previous studies was in some degree accountable by the nature of the yield curve construction, as Buse and Grant had contended. However, the results contained in this study are sufficiently good to support the error-learning hypothesis without reference to the smooth curve bias that appears to be present in former data sets.

Examining the quarterly error adjustment regressions obtained from the 3, 6, 9, and 12 month data in Table 7[10] further support of the basic approach is evidenced. However, the constant term in the first equation suggests the existence of a liquidity premium,[11] supporting the work of Van Horne over the

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[9] This is not to say, however, that equilibrating forces will not equate the term structure of implied yields in an exchange crisis. In fact, variability of exchange rates that effect demands and supplies within the Eurodollar market will call into play equilibrating forces that will cross maturities within this market to realign the rate structure.

[10] Only the linear model is presented due to lack of significance of the cubic specification in earlier results.

[11] It will be noted that a necessary condition for the absence of a liquidity premium is an insignificant constant term. This is not, however, sufficient. Therefore the results obtained in Table 6 can not be said to imply <u>no</u> liquidity premium in the market in general.

Meiselman results of no significant constant term.

Finally, one may examine the degree of synchronization of the sign of the actual change in the future implied rate with the estimated change in this rate over the sample period, as in Table 8.[12] These results indicate that with an average of over four to one the direction of change as indicated by the model was the same as the actual direction of movement.

V. Concluding Remarks

The error learning hypothesis of David Meiselman has been used to support the expectations view of the term structure of interest rates. However, the empirical support for this approach has been questioned due to the nature of the data used in its use. It has been contended that the free hand approximation used to derive a yield curve over not perfectly homogeneous assets biases the results obtained. This study rejects this contention and empirically supports the error learning approach with an explicit unbiased yield curve, and without reference to approximations. Its results suggest that, while the earlier studies may have been aided by the use of derived yield curves, their results were not totally dependent upon its use. Essentially error-learning emerges relatively unscathed.

An additional outcome of the study is the results obtained for the Euro-dollar term structure. The analysis indicates that this market too can be substantially explained by the economically meaningful expectations theory.

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[12] An interesting extension of the analysis to forecasting adjustments in future rates due to errors in prediction of actual short rates in the period following the sample period is precluded due to insufficient data since the end of the sample set used in the estimation of the actual regression results. However, the possibility of future analysis of this nature seems clearly warranted.

Table 1[*]

David Meiselman Results

$$\Delta_{t+n} r_{1,t} = a + bE_t$$

Durand Data on Corporate Bond Yields

n^*	Constant Term (standard error)	Regression Coefficient	R^2
1	.00 (.02)	.703	.906
2	.00 (.03)	.526	.752
3	-.01 (.04)	.403	.590
4	-.03 (.04)	.326	.465
5	-.02 (.04)	.277	.412
6	-.01 (.03)	.233	.391
7	-.02 (.03)	.239	.398
8	.01 (.03)	.208	.348

[*] Source: Meiselman <u>op</u> <u>cit</u> p. 22, Table 1, with correlation coefficients converted to coefficients of determination.

Table 2 [*]

Buse Results

Durand Data for U.S. Corporate Bonds

$$\Delta_r = a + bE_t$$

n	Reverse Order			Average Over Four Random Orderings		
	a	b	R^2	a	b	R^2
1......	-.004	.663	.832	.014	.842	.970
2......	-.011	.476	.579	.015	.718	.914
3......	-.020	.329	.351	.013	.624	.857
4......	-.035	.296	.286	-.013	.542	.808
5......	-.018	.217	.184	.001	.478	.775
6......	-.008	.231	.253	.013	.434	.758
7......	-.012	.208	.218	-.003	.394	.738
8......	.009	.190	.172	.021	.357	.704

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[*] Source Buse [1969], p. 60

Table 3*

Grant Study

Linear Regression Model

British Government Securities 1924-62

n	a	b	R^2
1	.86	1.49	.64
2	1.23	1.93	.44
3	.57	1.35	.62
4	1.17	2.20	.43

* Grant [1964], p.62

Table 4*

Grant Study

Cubic Adjustment Function

British Government Securities, 1924-62

n	a	b	c	d	R^2
1	.34 (.09)	.77 (.08)	.01 (.05)	-.01 (.01)	.446
2	.15 (.09)	.50 (.08)	.13 (.05)	.01 (.01)	.233
3	-.09 (.08)	.45 (.07)	.10 (.05)	.02 (.01)	.437
4	-.15 (.10)	.55 (.09)	.23 (.06)	.03 (.01)	.264
5	.12 (.11)	.34 (.10)	.04 (.06)	.01 (.01)	.121
6	.16 (.12)	.25 (.11)	.13 (.07)	.02 (.01)	.062
7	.10 (.10)	.29 (.09)	.06 (.06)	.01 (.01)	.085
8	.27 (.08)	.23 (.07)	-.03 (.04)	-.00 (.01)	.120

*Grant [1964], p. 63

Table 5*

Van Horne Study

(Treasury Yield-Curve Data)

n	Constant Term (Standard Error)	Regression Coefficient (Standard Error)	R^2
	January, 1958 - September, 1963 (57 Observations)		
1......	0.220 (0.030)	.829 (.027)	.947
2......	.264 (.033)	.615 (.030)	.885
3......	.261 (.028)	.461 (.025)	.859
4......	.260 (.028)	.349 (.025)	.783
5......	.238 (.029)	.293 (.026)	.702
6......	.218 (.024)	.231 (.022)	.674
7......	.209 (.022)	.206 (.020)	.662
8......	.189 (.019)	.174 (.017)	.658
9......	.166 (.018)	.175 (.016)	.686
10......	.142 (.020)	.154 (.018)	.575
11......	0.135 (0.020)	.149 (.017)	.573

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*Van Horne [1965], p.346, with correlation coefficients converted to coefficients of determination.

Table 6

Eurodollar Data

Bimonthly With Monthly Error Adjustment

$$\Delta_{t+n} r_{1,t} = a + bE_t + c(E_t)^2 + d(E_t)^3$$

n		a	b	c	d	R^2
1		.094	.675	-	-	.5141
	SE =	(.065)	(.064)	-	-	
	t =	(1.44)	(10.49)	-	-	
1		.069	.667	.006	.009	.5174
		(.090)	(.075)	(.157)	(.053)	
		(.77)	(8.82)	(.04)	(.159)	
2		-.035	.532			.5556
		(.098)	(.087)			
		(-.35)	(6.12)			
2		-.181	.553	.167	-.025	.6099
		(.163)	(.085)	(.324)	(.132)	
		(-1.11)	(6.45)	(.51)	(-.19)	
3		-.110	.252			.2148
		(.098)	(.088)			
		(-1.11)	(2.86)			
3		.095	.223	-.239	.039	.3935
		(.154)	(.081)	(.308)	(.125)	
		(.62)	(2.74)	(-.78)	(.308)	
4		-.080	.193			.0814
		(.133)	(.118)			
		(-.60)	(1.63)			
4		.052	.175	-.198	.051	.1134
		(.233)	(.122)	(.463)	(.189)	
		(.23)	(1.43)	(-.43)	(.27)	

Table 7

Eurodollar Data

Bimonthly With Quarterly Error Adjustment

$$\Delta_{t+n}\, r_{1,t} = a + bE_t + c(E_t)^2 + d(E_t)^3$$

n		a	b	c	d	R^2
3		.525	.776			.7722
	SE =	(.083)	(.061)			
	t =	(6.32)	(12.75)			
6		-.012	.309			.3541
		(.081	(.58)			
		(-.14)	(5.34)			

Table 8

Eurodollar Data

Synchronization Results

Linear Model

	1	2	3	4	3	6
Same Sign	90	26	20	22	46	42
Different Sign	16	6	12	10	4	12
No. of Observ.	106	32	32	32	50	54

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