The Valuation of Convertible Bonds: A Further Analysis

by

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The intent of this paper is to supplement previous studies of convertible debentures ([1], [5], [13], [15]) in at least two respects. One, the specific influence of the so-called bond floor upon the risk premiums associated with convertible debentures is analyzed by reference to the market model developed by Sharpe [14] and Lintner [12]. Two, the effect of involuntary terminations upon the range of possible returns from holding convertible debentures is examined by means of a simulation model.

Under the market model, convertible debentures--like their underlying common shares--are regarded as components of a risky market portfolio. Such assets differ from their underlying stocks in that they may be less responsive to the vicissitudes of the market and may therefore feature smaller risk premiums. The essence of the bond floor in the context of the market model is that it conditions covariability with the market.

Despite the highly useful insights that it affords, the market model cannot cope effectively with the changing responsiveness of convertible debentures to the market (occasioned by varying ratios of stock conversion value to straight bond value) and with the stochastic process by which convertible debentures disappear from the scene. An alternative approach of Monte Carlo simulation is thus introduced to obtain comprehensive forecasts of rates of return on convertible debentures conditional upon the simulated behavior of the underlying stock.

In the treatment that follows, the conventional model is first described for purposes of contrast. The market model and its limitations are then treated. The remaining sections deal with the simulation model, its behavioral inputs, and simulation results.

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I. The Conventional Model

Following Poensgen [13], we can specify the expected value of a convertible bond given a straight debt value, y, as:

$$E(BV) = y\int_0^y f(x|y)\,dx + \int_y^\infty x f(x|y)\,dx \tag{1}$$

where BV = market value of the bond; x = conversion value or value of the underlying stock (equal to the product of the stock price and the conversion ratio); y = value of the bond as straight debt; and f(x/y) = conditional distribution of x given a floor, y.

To allow for bond yield variability, the summation must extend over all values of y. Let $f(y)$ be the density function of y, and rewrite equation (1), as follows:

$$E(BV) = \int_0^\infty \left[y\int_0^y f(x|y)\,dx + \int_y^\infty x f(x|Y)\,dx \right] f(y)\,dy. \tag{2}$$

However, $f(x,y) = f(x|y)\ f(y)$, and equation (2) becomes:

$$E(BV) = \int_0^\infty \left[\int_0^\infty x f(x,y)\,dx + \int_0^y (y - x)\ f(x,y)\,dx \right] dy \tag{3}$$

The first expression in the bracket under an integral is the expected stock value. The second is the expected value of the floor guarantee, with the summation ranging across all values where the straight debt value, y, exceeds the conversion value, x.

Rearranging the terms in equation (3) and knowing that $\int_0^\infty (x|y)\ dx = 1$, the equation can be transformed into:

$$E(BV) = \int_0^\infty y f(y)\ dy + \int_0^\infty \int_y^\infty (x - y)\ f(x,y)\ dx\ dy \tag{4}$$

where the first term is the expected straight debt value and the second is the expected value of the option to convert. This reduces to the expected value of a warrant if we assume that y is not stochastic, or equivalently, that bond yield variability does not affect the value of the conversion option.

This either-or characteristic of the conventional model has tended to confuse efforts at empirical analysis. Although equations (3) and (4) are mathematically equivalent, an important asymmetry in their relationship exists. Since the true bond premium is the difference between the market value of the convertible bond and the higher of the conversion or straight bond value, the proper choice of either equation as the underlying model for regression studies should depend upon whether the conversion value, x, exceeds or falls short of the straight bond value, y. If $x \geqq y$, the premium should be interpreted as the value of the floor guarantee, and equation (3) is the proper valuation model. Otherwise, the premium reflects the worth of the conversion privilege, and equation (4) is to be used.

As it is formulated, the conventional model does not consider the convertible debenture as an integral part of a portfolio of risky assets. Neither does it stress sufficiently the special features that differentiate the conversion privilege from the straight option or warrant.[1]

II. The Market Model

Risk and Return

In contrast to the conventional model, the market model stresses the relationship between the return on the i^{th} security and the return on the market. Under equilibrium conditions, the risk premium associated with the

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[1]Two important features are: (1) the investor in a convertible bond faces the risk of involuntary termination; and (2) the value of the convertible bond and the associated conversion privilege depends not only on the value of the underlying common stock but also on a base value plus a premium.

i^{th} security is:

$$E(\tilde{R}_i) - R_F = \beta_i \left[E(\tilde{R}_m) - R_F \right] \tag{5}$$

where $E(\tilde{R}_i)$ and $E(\tilde{R}_m)$ are respectively the expected returns on the i^{th} security and the market; R_F is the risk-free rate and β_i is cov $(\tilde{R}_i, \tilde{R}_m)$ divided by var $(\tilde{R}_m)$. Using ex post or historical data, we can approximate (5) with the regression equation:

$$\tilde{R}_{it} = \hat{\alpha}_i + \hat{\beta}_i \tilde{R}_{mt} + \epsilon_{it} \tag{6}$$

where ϵ_i is a residual with the properties, $E(\epsilon_i) = 0$ and $E(\epsilon_i \tilde{R}_m) = 0$.

Since the regression coefficient, $\hat{\beta}_i$ (hereafter called beta value), measures the systematic or non-diversifiable risk, the risk premiums of any paired securities should be in the ratio of their beta values.[2] It also follows that the differential in the expected rates of return for any paired securities, e.g., the i^{th} and j^{th} securities, should be equal to:

$$E(\tilde{R}_i) - E(\tilde{R}_j) = (\hat{\beta}_i - \hat{\beta}_j)\left[E(\tilde{R}_m) - R_F\right]. \tag{7}$$

To test the difference in expected returns implied by the market model, a sample of 13 convertible debentures was drawn. The principal criteria for selection were: (1) the issue must have been outstanding throughout the 1960's; and (2) the final maturity must be no earlier than December, 1979.[3]

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[2]Recent critiques of the market model have questioned both the interpretation of the intercept (Black, [2]) and the assumption of linearity (Blume and Friend, [4]). Since beta values continue to be accepted as measures of nondiversifiable risk and since Blume and Friend [4] concur that the risk-return trade-off appears "almost linear if only common stocks are analyzed," we feel, for our purposes, little need to modify the relevant conclusions of the market model.

[3]Only 15 issues reported in the November 21, 1960 Moody's Bond Survey satisfied these criteria.

Table 1 summarizes the results of linear regressions of monthly price relatives of the common stocks and convertible debentures of the 13 sample corporations against the monthly relatives of the Standard and Poor's 500 Index. The period covered was the decade of the sixties (a total of 119 monthly observations per security). Price relatives were used in the regressions since the conversion value is dependent on the behavior of stock prices alone.[4]

For the decade as a whole, the beta values of 12 of the 13 common stocks surpassed their convertible debenture counterparts. Similar results were obtained when the decade was split into two equal parts, despite considerable evidence of non-stationarity.

Ratios of convertible bond beta values to common stock beta values, shown in Table 2, indicate a lessening differential in the stock-convertible risk premiums for the sample companies. The median ratio was .42 for the 1960-64 period, in contrast with .87 for the 1965-69 interval. The median ratio for the decade was .70.

Under the equilibrium conditions hypothesized above, the typical risk premium for the 13 convertible securities should have been less than half that associated with the corresponding common stocks in the early sixties. The differential closed to a little over one-tenth in the latter half of the decade.

Much of the apparent non-stationarity in the beta values for the convertible securities is attributable to the changing relationship between

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[4]Price relatives are the ratios of terminal to initial prices. They are differentiated from return relatives which are ratios of terminal price plus period dividends or interest to initial price. In actual fact, however, the differences between beta values based on price relatives and those based on return relatives are negligible.

Table 1

Beta Values for Thirteen Convertible Debentures and Their Underlying Common Stock

Company	Market	Beta Values for: 1960-4	1965-9	1960-9	R^2 For: 1960-4	1965-9	1960-9	Correlation Coefficient (Bond&Stock) 1960-9
Allegheny Ludlum	N.Y.S.E.							
Stock		1.6386	.6970	1.0517	.53	.16	.30	.69
Convertible		.5307	.7419	.6598	.36	.32	.32	
American St. Gobain	O.T.C.							
Stock		.9522	1.0619	1.0083	.04	.07	.07	.22
Convertible		.3145	.2600	.2851	.04	.05	.05	
Champion Paper & Fibre**	N.Y.S.E.							
Stock		.9514	1.0580	1.0056	.22	.15	.18	.31
Convertible		.4011	1.7285	1.2044	.21	.26	.20	
Combustion Engineering	N.Y.S.E.							
Stock		1.2425	1.1266	1.1591	.37	.21	.26	.93
Convertible		.8338	1.1181	.9956	.49	.20	.23	
Bausch & Lomb	N.Y.S.E.							
Stock		1.5200	1.0837	1.2372	.31	.15	.21	.79
Convertible		.6561	1.0226	.8605	.09	.21	.16	
Consolidated Elect.Dyna.	N.Y.S.E.							
Stock*		1.5502	1.0799	1.2365	.27	.20	.22	.81
Convertible		.6514	.9407	.8200	.14	.24	.20	
Copperweld Steel	N.Y.S.E.							
Stock		1.3280	.7537	.9862	.32	.13	.21	.21
Convertible		.4444	.6577	.5817	.00	.23	.05	

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**Acquired by another company prior to 1970 and conversion terms modified.

Table 1 (continued)

Beta Values for Thirteen Convertible Debentures and Their Underlying Common Stock

Company	Market	Beta Values for: 1960-4	1965-9	1960-9	R^2 For: 1960-4	1965-9	1960-9	Correlation Coefficient Bond & Stock 1960-9
Dow Chemical	N.Y.S.E.							
Stock		.9770	.6896	.7953	.30	.23	.26	.30
Convertible		.7939	.6242	.6920	.01	.21	.03	
Richfield Oil	N.Y.S.E.							
Stock		.7747	.6275	.6639	.06	.07	.06	.86
Convertible		.6455	.5386	.5659	.19	.05	.09	
Sinclair Oil**	N.Y.S.E.							
Stock		.9235	.9678	.9405	.23	.25	.25	.72
Convertible		.1655	.6167	.4370	.08	.15	.12	
Talcott (James)								
Stock		.8881	.8147	.8266	.07	.08	.08	.59
Convertible		1.1070	.5026	.7375	.23	.04	.11	
Thriftimart	A.M.E.X.							
Stock		.7835	.3815	.5391	.18	.07	.12	.38
Convertible		.4024	.2351	.3039	.18	.02	.07	
West Coast Transmission	A.M.E.X.							
Stock		.7285	.0197	.2762	.04	.00	.00	.16
Convertible		.1885	.2351	.2221	.02	.10	.07	

Table 2

Ratios of Convertible Debenture Beta Values to Underlying Stock Beta Values and of Conversion Values to Straight Bond Values for Three Periods

Company	Ratios of Beta Values			Ratio of Conversion Value to Straight Bond Value		
	1960-4	1965-9	1960-9	1960	1965	1969
Allegheny Ludlum	0.33	1.06	0.63	.740	.980	.845
American St. Gobain	0.33	0.24	0.28	.465	.336	n.a.
Bausch and Lomb	0.43	0.95	0.70	.750	1.120	1.565
Champion Paper & Fibre	0.42	1.63	1.19	.600	.895	1.522
Combustion Engineering	0.67	0.99	0.86	.870	2.340	n.a.
Consolidated Elect. Dyna.	0.42	0.87	0.66	1.185	1.020	n.a.
Copperweld Steel	0.33	0.87	0.59	.622	1.215	.740
Dow Chemical	0.81	0.91	0.87	2.265	2.170	n.a.
Richfield Oil	0.83	0.86	0.85	1.380	2.310	n.a.
Sinclair Oil	0.18	0.64	0.46	.616	.965	1.260
Talcott (James)	1.26	0.62	0.56	1.430	.676	n.a.
Thriftimart	0.51	0.62	0.89	.855	1.060	.560
West Coast Transmission	0.26	2.19	0.81	.483	.616	.709
Median	0.42	0.87	0.70	.750	1.060	n.a.

conversion values and straight-bond values. Variations in the ratios of conversion values to straight-bond values, shown in Table 2 for November 14, 1960, and November 8, 1965, correspond with shifts in the beta values for seven of the 13 convertible issues; the remainder are ambiguous. The median conversion-bond ratios for 1960 and 1965 were, respectively, .75 and 1.06.

The model presumes that the floor characteristic of convertible debentures operates through its effect upon systematic risk, as reflected in the beta values. If the median values for the sample are indicative, the beneficial influence of the bond floor upon the systematic risk of the convertible security relative to the associated common stock, diminishes rapidly as the conversion value approaches and exceeds the straight bond value.

Limitations of the Market Model

The adequacy of the market model for predictive purposes is circumscribed by the changing relationship between conversion values and straight bond values, by the finite probability that a given convertible bond issue will be called in whole or in part in any period prior to maturity, and perhaps by other factors. The changing stock-bond relationship influences comparative returns through its effect upon convertible bond premiums. The probability of involuntary termination through a call either for redemption or sinking fund purposes in any period adds a temporal aspect to realizable returns; it is no longer sufficient to express the valuation model in terms of the distribution of possible bond values at some future period.

III. Determinants of Convertible Bond Premiums

Investors who choose to liquidate holdings prior to involuntary termination by selling the convertible bond in the market receive a dollar amount equal to the higher of the straight bond value, y, or the conversion value, x, plus a premium.[5] The bond premium, defined as the excess of the convertible bond price over the higher of the straight bond or conversion value, recognizes the fact that investors can always elect the higher of the two less-than-perfectly-correlated values.

As stipulated in Section I, the bond premium measures the value of a non-detachable warrant if $y > x$, and is equal to the convertible bond price minus y.[6] Where $x \geqq y$, the investor in a convertible bond has a combination stock holding plus an insurance--a floor guarantee--that he will be able to sell the security at its straight debt value should x fall below y in the future.[7]

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[5]As long as the premium exceeds zero, no question of voluntary conversion arises. The rational investor will always liquidate his position by selling the bond.

[6]Poensgen /13/ used equation (4) as the basis of his regression studies. This is appropriate to his sample which consisted of a cross-section of all convertible bonds between 1948 and 1963 at their issue date. At this point in time, the major component of the bond price is the straight debt value. The difference between this and the issue price represents the value of the conversion privilege (the non-detachable warrant), whose existence allows the issuing firm to pay a coupon below that of its own straight bond issue or that of another firm in the same risk class.

[7]Weil, Segall and Green, Jr. /15/ in effect, use equation (3) as their valuation model, their sample being drawn from those convertible bonds selling in the market whose stock prices exceeded their respective conversion prices. In this event, it is more than likely that x would exceed y, and that the premium represents the difference between convertible bond price and the conversion value.

Cretien, Jr. /6/ and Duvell /7/, in their comments, fault Weil, Segall and Green, Jr. on their seemingly biased sample, without realizing that the approach taken implicitly recognizes the asymmetry in the valuation models which calls for stratifying sample observations on the basis of whether x or y dominates. Brigham /5/ has anticipated this difficulty but his empirical work did not extend far enough to identify explicitly the basic asymmetry.

The regression model of convertible bond premiums presented here, utilizes a comparatively standard set of explanatory variables drawn partly from the conventional model and partly from the market model. In functional form, this is:

$$Pr_i = f(M_i, M_i^2, l_{10}T_i, \hat{\beta}_i, Q_i, C_i) \qquad (8)$$

where Pr_i = premium on the i^{th} convertible; M_i = ratio of the conversion value, x, to the straight debt value, y, if $x \geqq y$, or its reciprocal, if $y > x$; M_i^2 = square of the ratio M_i; $l_{10}T_i$ = logarithm to the base 10 of the number of months to final maturity; $\hat{\beta}_i$ = the slope calculated from the market model; Q_i = a dummy variable set at 1 for issues rated Baa or better by Moody's Investment Service, and 0 for lower rated issues; and C_i = bond coupon minus cash dividends per share, D_i, adjusted for the conversion ratio, ENS_i, which is equal to par divided by the conversion price, CVP_i.[8]

Equation (8) does not presume that all of the variables enter simultaneously into the regression model as independent variables. The variable M_i recognizes the fact that the most important determinant of convertible bond premiums is the relationship between the straight debt and conversion values, the major components of the convertible bond price. Since the contribution of the dominated variable to the premium varies inversely with the relative distance between x and y, if $x \geqq y$, and with the reciprocal of

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[8]On the presumption that the periodic cash differential should be in current terms, i.e., that the dividend stream to be considered should be that obtainable from the number of shares an investor can acquire for the price of one convertible bond, we experimented with C_i = coupon minus the product of cash dividends per share and the quantity (BV_{it}/SP_{it}), where BV_{it} = current bond price and SP_{it} = current stock price.

The adjusted dividend stream would approach that used in the model asymptotically as stock price approached conversion price, and diverge from it as stock price exceeded conversion price. The results using this formulation were rather poor.

the distance, if $y > x$, the coefficient of M_i or $1/M_i$ should always be negative. The possibility of nonlinearities is recognized by including M_i^2.

The variables T_i and $\hat{\beta}_i$ follow directly from the random walk literature that shows the expected value of an option, the conversion privilege in this instance, to be a positive function of the square root of time to maturity and the standard deviation of price changes. The inclusion of $\hat{\beta}_i$, as opposed to the square root of the total variance of price changes, is consistent with the hypothesis that investors in convertible bonds are principally concerned with systematic risk. As determinants of the expected value of the conversion option, these variables are prime candidates for inclusion in regression models whenever the straight bond value dominates (that is, $y > x$). Subject to their potential association with Q_i and their information content as to the applicability of the bond floor, no such basis exists for the inclusion of T_i and $\hat{\beta}_i$, whenever the conversion value dominates ($x \geqq y$).

The remaining two variables, Q_i and C_i, respectively measure the quality of the issue and issuer, and the periodic income differential between the convertible bond and the underlying common stock. Our hypothesis is that Q_i and C_i influence the size of the bond premium in the event of dominance by conversion values ($x \geqq y$), but not when the reverse obtains.

Table 3 presents regression results for two sets of cross-section data, differentiated on the basis of whether the stock conversion value exceeds or falls short of the straight bond value, as of three separate dates. The diverse dates (1970, 1968, and 1965) chosen for analysis recognize the possible influence upon regression parameters of (1) different market environments and (2) the imposition of margin requirements in early 1968. The regressions shown represent the <u>best fits</u> of the various combinations tested.

Table 3

Cross-Section Regression Analyses with Convertible Bond Premium as Dependent Variable, for five Periods

Explanatory Variables

Month & Year		M_i	M_i^2	$\ell_{10}T_i$	$\hat{\beta}_i$	Q_i	C_i	CONS	R^2	S.E.	F-Ratio	No. of Obser-vation
Conversion Value Exceeds Straight Bond Value												
May, 1970	(3.1)	-8.898				5.623	2.274	15.385	0.466	4.899	8.270	26
		(-1.899) *				(2.237)	(2.483)	(2.532)			(3,22)+	
Nov. 1968	(3.2)	-46.691	8.024					58.804	0.563	5.118	47.367	73
		(-8.628)	(7.188)					(11.187)			(2,70)	
	(3.3)	-44.731	7.866	18.625			0.835	9.808	0.618	4.784	30.144	
		(-8.554)	(7.494)	(2.673)			(1.813)	(0.572)			(4,69)	
Oct., 1965	(3.4)	-54.017	11.816	19.801			0.826	14.165	0.745	4.494	25.82	35
		(-3.340)	(2.579)	(3.954)			(1.773)	(0.751)			(4,30)	
	(3.5)	-76.364	17.954					78.463	0.612	5.541	27.82	
		(-4.229)	(3.379)					(5.516)			(2,32)	
Straight Bond Value Exceeds Conversion Value												
May, 1970	(3.6)	-18.218	3.442					23.523	0.370	3.514	11.881	38
		(-2.745)	(1.944)					(4.080)			(2,35)	
Nov., 1968	(3.7)	-13.589		21.859			2.128	-26.058	0.486	5.173	6.68	19
		(-3.037)		(2.728)			(1.702)	(-1.338)			(3,15)	
Oct., 1965	(3.8)	-21.298	2.949					34.194	0.443	5.834	8.862	25
		(-3.099)	(2.509)					(4.953)			(2,22)	

* Figures in parentheses below coefficients are t-values.

+ Figures in parentheses below F-Ratio are degrees of freedom.

As anticipated, the coefficient for M_i was negative and significant everywhere. Similarly, the coefficients for Q_i and C_i were generally positive and significant for regressions employing cross-section data with $x \geqq y$. The lone exception occurred in October, 1965, when the market environment apparently failed to reward bond quality.

Contrary to expectations, the variables T_i and $\hat{\beta}_i$ failed to contribute notably to the explanation of the bond premiums in situations where $y > x$. Only in 1968 were the regression results moderately consistent with the model hypothesized.

Inclusion of a squared term, that is

$$Pr_i = \hat{a}_i - \hat{b}_i M_i + \hat{c}_i M_i^2 + \varepsilon, \qquad (8a)$$

proved beneficial. With $x > y$, the simple quadratic model outperformed all regression models tested without the squared term, for November, 1968, and was one of the best fits for October, 1965. More important, the coefficients had the predicted signs and magnitudes, with $|b_i| > |c_i|$ considerably. With $y > x$, the quadratic form gave <u>best</u> results for May, 1970, and October, 1965, and unsatisfactory results for November, 1968. In both cases, the signs and sizes of the coefficients were as anticipated.

Regression values shown in Table 3, as expected, point to asymmetrical behavior between issues featuring $y > x$ and those with $x \geqq y$. Regression coefficients also seem to be somewhat affected by the market environment. The apparent nonstationarity of regression coefficients poses problems for the choice of behavioral input to the simulation model.

IV. Involuntary Termination

Calls for Conversion and Sinking Fund

Investors who choose to hold convertible bonds run the risk of involuntary termination in each period. Of 108 convertible debentures with final maturities after 1975, reported in Moody's Bond Survey of November 21, 1960, the number still listed in the Survey had declined to 65 by the end of 1965, to 22 by the end of 1969, and to 8 by mid-1970.

The possibility of premature termination stems from the omnipresence of the call privilege, the prevalence of management decision rules that activate the call provision conditional upon the positive behavior of stock prices, and other factors that cause management to exercise the call privilege. The call premium, the usual penalty for calling an issue, is simply too small to be of consequence, as evidenced by the following breakdown of 1960 call prices for the sample of 108 convertible debentures:

Call Price	Percent of Total
107 and over	3.7%
105 to 107	16.7
103 to 105	69.4
Under 103	10.2
	100.0%

The relative frequency of calls to force conversion in any period, conditional upon the ratio of stock conversion value to par, is shown below for the base sample of 108 convertible issues.

Ratio of Conversion Value to Par (End of Prior Year)	Percentage Called	Number of Observations
1.75 and over	20.5	68
1.50 to 1.75	30.0	30
1.10 to 1.50	25.1	103
1.00 to 1.10	14.3	42
Under 1.00	1.8	390
		633

Even when the entire issue remains uncalled, the individual investor may be subjected to sinking-fund call. More than three-fifths of the convertible debentures sampled had sinking-fund provisions that commenced 10 or 11 years subsequent to date of issue and called for the periodic redemption of a stipulated percentage or dollar amount at par (or at market, if less than par). In excess of one-fourth had payments that began 5 or 6 years after date of issue.

Once commenced, required sinking fund payments--as a percentage of either total issue or amount outstanding at the time payments commence--vary substantially. About 70% of the convertible debentures sampled provided for annual payments ranging from three to seven per cent.[9] An additional one-fourth fell between seven and ten per cent. About three-fifths of the sample also provided for optional payments of the same magnitude as required payments.

Termination of the Conversion Privilege

A significant proportion (31.5%) of the convertible debentures sampled terminated conversion privileges prior to maturity. With two exceptions (15 years), the duration of the conversion privilege was uniformly 10 years. For analytic purposes, it appears that the maturity of the

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[9]Of these convertible debentures, about one-sixth provided for substantially higher subsequent payments.

debenture should be equated with that of the conversion privilege.

Sampling of recent issues of convertible debentures reveals that few--if any--new issues provide for the early termination of conversion privileges.

V. The Simulation Model

The simulation model presented below mitigates the inadequacies of the market model, and other analytic models as well, by explicitly accounting for the possibility of involuntary termination and thereby assessing its influence on the distribution of expected returns to the investor. In addition, it makes no special assumptions about the nature of the capital markets. It accounts for transactions costs, differing risk characteristics among securities, and stipulates only that investors be rational decision makers.[10]

As shown in Figure 1, the model involves a period-by-period analysis to determine whether the debenture is called for (a) conversion, or (b) sinking fund purposes. If either call is activated in any period, the sequential process terminates, and rates of return on the convertible bond and the common stock, as well as present values, are calculated. If neither is activated, the process continues to the horizon and rates of return and present values are computed.

The probability of a call to force conversion in any period depends upon the behavior of the stock price and managerial policies toward conversion. The model ascertains whether or not the bond is callable within any period prior to the horizon, through inputs of call dates contained in the bond indenture.

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[10] Rationality here means obtaining the highest possible return for some level of risk. Cf. footnote 5.

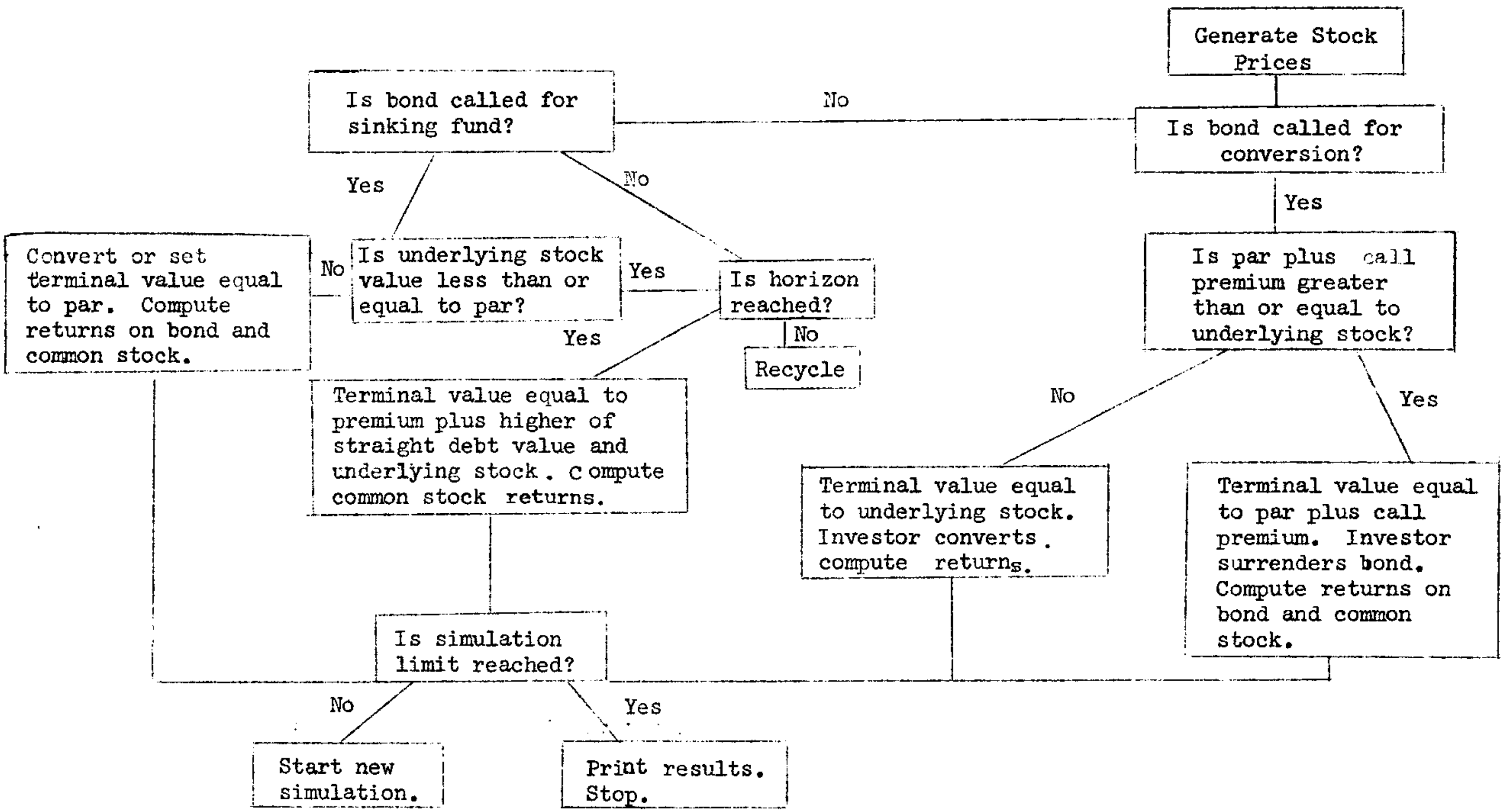

Figure 1. <u>Simulation Model</u>

If the bond is callable, checks for calls are made on a stochastic basis using empirically derived probabilities.[11] It is presumed that in the event of a call, the investor will always liquidate his position by converting if the value of the underlying stock exceeds par plus the call premium, if any.

The conditions under which the sinking fund is operative, i.e., the periodic retirement of a fraction of the outstanding securities to reduce the cash outlay at maturity, are stipulated in the bond indenture. Checks for sinking fund calls are made on a similar stochastic basis as those for conversion calls, given that the bond is callable within the period.

In the event of a call for sinking fund purposes, two possibilities obtain. One, the conversion value may equal or exceed par, in which case the trustee issues a call at par and the investor chooses the higher of the conversion value and par. Two, the market price of the convertible bond may fall below par, in which case the sinking fund requirement will be satisfied in the open market at a lower cost to the issuing corporation, and no involuntary termination occurs.

The stochastic process that generates stock prices uses as a base the actual monthly price relatives of a given stock for the postwar period, January, 1946, to June, 1968.[12] The price of the i^{th} stock at time t, P_{it}, is calculated as:[13]

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[11]See the previous section for the actual values used.

[12]The monthly data were obtained from updated University of Chicago tapes. See Fisher and Lorie /10/ for a description of the original tapes.

[13]The process was originally based on the market model. The price relatives of the i^{th} stock were regressed against the price relatives of the Fisher Index /9/. The actual Fisher Index relatives for the postwar period, 1946-1968, in turn served as the market environment base.

The price relative for the i^{th} stock in period t was found by:

$$R_{it} = a_i + b_i R_{mt} + s_{u_i} z_t$$

which is identical to equation (5) above except for our replacement of ε_{it} by the product of the standard error of the regression equation, s_{u_i}, and a

$$P_{it} = P_{i,t-n} \prod_{t=1}^{n} (R_{it}) \tag{9}$$

where R_{it} = monthly price relative. R_{i1}, the starting point of the string of relatives for n periods, is selected at random from the array of post-war price relatives.

This formulation assumes that any historical sequence of market price relatives is equally likely to recur and implies that investors react to new information in the same manner as they had done in the past. Whether the past distribution of stock price changes follows a random walk or a sub-martingale pattern is unknown to us. What is important is that the distributional characteristics are preserved.

The convertible bond value in any period is taken to be the higher of the conversion value or the straight debt value, plus the convertible bond premium, i.e.,

$$BV_{it} = \text{Max}\ (x_{it}, y_{it}) + Pr_{it}. \tag{10}$$

The straight debt value, y_{it}, is estimated as:

$$y_{it} = \frac{c_i}{r_t} \left[1 - (1 + r_t)^{-n} \right] + P(1 + r_t)^{-n} \tag{11}$$

where c_i = coupon rate; n = remaining period to maturity; P = par; and r_t = the effective market interest for the risk class to which the i^{th} security belongs. The effective market rate is assigned a value equal to:

$$r_t = r_{t-(n/12)} (1+d) \prod_{t=1}^{(n/12)} (I_t) \tag{12}$$

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standard normal deviate, z_t, with mean zero and unit variance. The value of R_{mt} is randomly selected from the table of Fisher Index price relatives which is an input.

Experimentation, however, has led us to modify the basis to that presently used in the model.

I_t = annual Moody's AAA bond yield relative;

where d = a drift variable equal to the sum of the expected increase or decrease in interest rates and $E(i)tz_t$, the product of the standard error of the regression of market rates against time, the number of holding periods and a normal random deviate.

To calculate the premium, Pr_{it}, equation (13a) below is used if $x \geqq y$; if $y > x$, equation (13b) is used. Since we calculate bond and stock values in terms of thousands, and since the regressions were in hundreds, the computed premiums are scaled by ten. The coefficients of these two equations are from regression equations 3.3 and 3.8, respectively, in Table 3. The variables are defined in Section III.

$$Pr_{it} = 9.308 - 44.731 M_{it} + 7.866 M_i^2 + 18.625\ {}_{10}T_i + 0.835 C_{it} + 4.784 z_t; \qquad (13a)$$

$$Pr_{it} = 34.194 - 21.298(1/M_{it}) + 2.949(1/M_{it})^2 + 5.834 z_t \qquad (13b)$$

From a sample of premium values, it was found that Pr_{it} did not exceed \$270.00. This value has therefore been used as an upper bound to the premium calculated with equation (13a) or (13b). A lower bound of zero was set in cognizance of the fact that a negative premium is not plausible, or at worst is a short-term aberration immediately rectified by market movements or the action of arbitragers.

The terminal value of the convertible bond, TV_{it}, is calculated as:

$$\begin{aligned} TV_{it} &= \text{Max}\ (P + CP_{it},\ x_{it}), \text{ if called for conversion;} \\ &= \text{Max}(P,\ x_{it}), \text{ if called for sinking fund;} \qquad (14) \\ &= BV_{it}, \text{ in the absence of calls.} \end{aligned}$$

CP_{it} is the call premium, if any, on the bond.

Rates of return from holding the convertible bond and from holding the underlying stock, i_b and i_s, respectively, are calculated through an iterative process from the following standard equations:

$$BV_{i0} + TC_0 = \sum_{t=0}^{n} c_{it}(1+i_b)^{-t} + (TV_{in} - TC_n)(1 + i_b)^{-n} \qquad (16a)$$

and

$$SP_{i0} + TC_o = \sum_{t=0}^{n} D_{it}(1+i_s)^{-t} + (SP_{in} - TC_n)(1 + i_s)^{-n} \qquad (16b)$$

where D_{it} = cash dividend per share, and TC_t = transactions costs.

For comparability, i_b and i_s are always calculated at the same point in time, and annualized.

The investor's horizon is relevant only when there is no involuntary termination. When the horizon is reached, the model generates the convertible bond value by equations (10) and (11) above, and the period-by-period analysis is terminated.

VI. Simulation Results

As evidenced by Table 4 and the Appendix, the 25 convertible issues chosen for simulation featured well diversified industry representation, beta values for the underlying common stocks that ranged from 0.6473 to 1.9143, and median ratios of (a) conversion to straight bond value between 0.9 and 1.1, (b) initial stock price to conversion price between 0.6 and 0.8, and (c) bond premium to bond price between 0.10 and 0.15. Almost three-fifths had maturities between 1990 and 1995. One-fourth had sinking funds in effect.

Simulation results, summarized in Table 4, ran the gamut from virtually identical cumulative distributions of rates of return for both the convertible bond and the underlying stock to largely unrelated distributions. Appendix Table 1 shows the mean, standard deviation, skewness, kurtosis, median and interquantile range for each convertible bond and underlying stock

Table 4

Selected Statistics for 25 Simulations, Grouped by Similarity of Relationships Between Bond and Stock Cumulative Distributions

Simulation[1]	$E(R_B)/E(R_S)$	σ_B/σ_S	$\hat{\beta}_i$	Premium	Premium(%)	Conv.Value/ St. Bond Value	Initial Price/Conv. Price	Periodic Cash Differential[2]	% Unprof- itable
				Virtually Identical					
2	1.22	1.04	0.647	(36.19)+	(1.7)	3.45	2.20	30.36	0
3	1.17	0.93	1.261	(11.34)+	(1.2)	1.09	0.95	13.78	8
5	1.18	1.01	1.168	(17.48)+	(1.1)	2.12	1.61	17.59	0
17	1.30	0.95	0.901	(6.22)+	(0.4)	2.32	1.72	20.36	0
				Closely Related					
7	1.03	0.67	1.355	113.70+	15.4	1.11	0.63	19.17	44
9	0.96	0.75	0.739	130.00+	17.8	1.05	0.60	9.60	48
10	0.91	0.82	0.832	76.87+	8.5	1.15	0.82	10.58	28
14	1.08	0.65	1.104	120.51+	13.9	1.16	0.75	35.54	24
16	1.67	0.85	1.486	(0.51)+	(0.1)	1.41	0.82	45.00	32
22	1.34	0.73	1.400	124.81+	13.9	1.23	0.78	50.00	38
				Moderately Related					
1	0.75	0.55	1.059	40.00	5.7	0.74	0.49	(6.15)	72
15	1.24	1.13	0.688	89.22+	12.2	1.13	0.64	10.05	74
18	1.16	0.72	1.065	97.50	13.9	0.88	0.53	38.28	46
19	0.84	0.67	0.955	133.96+	13.5	1.27	0.86	24.04	6

Table 4 (continued)

Simulation[1]	$E(R_B)/E(R_S)$	σ_B/σ_S	$\hat{\beta}_i$	Premium	Premium(%)	Conv.Value/ St. Bond Value	Initial Price/Conv. Price	Periodic Cash Differential[2]	% Unprof-itable
			Related-Consistently Lower Bond Returns						
20	0.87	1.00	0.635	111.82+	13.9	1.02	0.69	(2.39)	46
23	0.81	0.78	0.704	175.59+	20.0	1.14	0.70	12.50	56
			Limited Relationship						
4	0.68	0.28	1.452	0.0	0	0.38	0.21	49.44	100
12	1.58	1.54	1.158	0.0	0	0.64	0.36	33.89	94
			Significant Floor Characteristic						
6	0.94	0.68	1.245	88.75	11.1	0.81	0.58	44.28	34
8	1.21	0.56	1.914	185.00	24.1	0.92	0.54	55.00	66
11	1.08	0.67	1.181	150.00	22.9	0.97	0.49	27.74	62
13	1.88	0.98	1.221	0.0	0	0.64	0.35	50.00	92
21	0.80	0.51	1.284	174.25	20.1	0.92	0.64	39.00	38
24	0.92	0.47	0.651	88.75	15.0	0.70	0.35	27.00	62
25	1.23	0.66	1.914	60.00	9.6	0.77	0.43	40.90	70

[1] Data obtained from "Moody's Convertible Bonds," in the December 21, 1970 issue of Moody's Bond Survey.

* See the Appendix for the names of the securities. The simulation number corresponds to the numbering there.

[2] Calculated as coupon minus the product of cash dividends per share and the conversion ratio.

+ Conversion value exceeds straight debt value.

simulation. The number of Monte Carlo runs per simulation was universally set at 50; and the investor's horizon at 60 months.

Highly similar bond and stock distributions (simulations numbered 2, 3, 5 and 17) arose in situations where (1) the initial conversion value exceeded the straight bond value, (2) the initial stock price was high in relation to the conversion price, and (3) the bond premium was negligible. The differential between coupon rates and cash dividends gave rise to expected returns on convertible bonds slightly in excess of those on the underlying stocks.

Closely related bond and stock cumulative distributions (simulations numbered 7, 9, 10, 14, 16 and 22) featured (1) initial conversion values greater than straight bond values, and (2) initial stock prices slightly less than conversion prices. The association between expected returns on convertible bonds and underlying stocks was mixed.

Third in order of closeness of association between the convertible bond and common stock distributions was a set of four simulations (numbered 1, 15, 18 and 19). Conversion to straight bond values averaged about one for this group, but there was an apparent trade-off between the level of such ratios and the beta values. Once again, no consistent relationship was visible for paired rates of return.

From a comparative return standpoint, the least attractive convertible bond situation (simulations numbered 20 and 23) was one characterized by (1) low beta values, (2) initial conversion values somewhat in excess of straight bond values, (3) initial stock prices, as a fraction of conversion prices, in the neighborhood of two-thirds, and (4) bond premiums in the median range. The comparatively poor convertible bond performance stemmed from the fact that the bond premium tended to decline whichever direction the underlying stock happened to move.

The classic floor characteristic of convertible bonds (simulations numbered 6, 8, 11, 13, 21, 24 and 25) appeared in instances featuring (1) high beta values and (2) conversion to straight bond values slightly less than unity. Comparative bond-stock returns were dependent upon the size of the initial bond premium and the frequency with which the bond premium was forced to zero by calls to force conversion.

Table 5 shows the specific influence of initial bond premiums upon realized rates of return. Given that the stock price is growing at 10% per year and that the convertible bond is called at the end of three years, for instance, an initial bond premium of 15% (over conversion value) halves the realized rate of return on the convertible bond to 5%. Other combinations produce other results.

As indicated in the foregoing application of the market model, the interdependence between the convertible bond and its underlying common stock hinges largely upon the relationship between the conversion value and straight bond value. The extreme case simulated (AVCO, numbered 4) featured the lowest conversion-to-straight-bond ratio and a non-call provision (precluding forced conversion) that extended past the 60-month horizon.

VII. Concluding Remarks

Notwithstanding the limitations of the market model, the conclusion seems inescapable that--in the case where conversion values equal or exceed straight bond values--the bond floor contributes less to the worth of the convertible bond than is normally believed. As the ratio of conversion value to straight bond value approached one for the sample issues, the median differential in the observed beta values declined to .13.

Table 5

Effect of Bond Premium on Realized Rate of Return

Growth in Stock Price and Years to Conversion	Rate of Return on Convertible Bond if Bond Premium over Stock is: 10%	15%	20%
5%			
1 year	*	*	*
3 years	1.7%	.2%	*
5 years	3.0	2.1	1.2%
10%			
1 year	.0	*	*
3 years	6.6	5.0	3.5
5 years	7.9	7.0	6.0
20%			
1 year	9.1	4.5	.0
3 years	16.2	14.5	12.9
5 years	17.7	16.7	15.7

* Negative return

Other things being equal, the significance of the floor characteristic varies directly with the volatility of the underlying stock. Simulations in which the bond floor assumed importance generally had common stock beta values in excess of unity and ratios of conversion to straight bond values of less than one, but roughly two-thirds or more.

Behavioral input, derived for the simulation model, attested to the powerful influence of the relationship between conversion values and straight bond values upon convertible bond premiums and to the asymmetry of regression models designed to explain bond premiums, depending upon whether conversion values or straight bond values dominated. Also in evidence was the substantial risk of involuntary termination through calls to force conversion.

The simulation model, designed to compensate for the deficiencies of both the market model and the conventional model, offers good potential for improved forecasts of the possible rates of return on convertible debentures <u>conditional</u> on the behavior of the underlying stock. It is limited only by our ability to derive realistic behavior input. Since both convertible bond and stock returns are generated simultaneously, it is possible to establish the degree of correlation between the two sets of returns and thus to improve on the market model estimates.

Subject to the usual qualifications, the simulation results point to certain imperfections in the arbitrage process. At the upper end of the conversion value scale, the difference between the coupon rate and adjusted cash dividends was not reflected in the bond premium. For eleven of 25 simulations, the expected return on the convertible bond exceeded that on the corresponding common stock, even though the standard deviation of returns was lower for the bond than for the stock. With but two exceptions, ratios of expected return to standard deviation of returns for convertible bonds surpassed those for the underlying stocks.

The magnitude of these imperfections can only be speculated. Other moments of the distribution, as shown in Appendix Table 1, may be pertinent to the decision calculus; the behavioral input may be suspect; and so on.

APPENDIX

Securities in the Sample

(1) Allegheny Ludlum 4s, 10/1/81*
(2) Amerada Hess $4\frac{1}{2}$s, 5/1/87
(3) Armour $4\frac{1}{2}$s, 9/1/83*
(4) Avco Corp. $5\frac{1}{2}$s, 11/30/93
(5) Bausch & Lomb 4 3/4 s, 7/1/80
(6) Bulova 6s, 2/1/90
(7) Celanese 4s, 4/1/90*
(8) Duplan $5\frac{1}{2}$s, 2/1/94
(9) General Tel. 4s, 3/15/90
(10) Greyhound $6\frac{1}{2}$s, 1/15/90
(11) Howmet $4\frac{1}{2}$s, 8/1/92
(12) MSL Ind. $4\frac{1}{2}$s, 10/1/84*
(13) Metro-Goldwyn 5s, 7/1/93
(14) National Can 5s, 10/1/93*
(15) Nat'l Distillers $4\frac{1}{2}$s, 8/1/92
(16) Pan. Am. Air. $4\frac{1}{2}$s, 1/15/84*
(17) Phillip Morris 6s, 9/1/94
(18) Phillips-Van Heusen $5\frac{1}{4}$s, 5/15/94
(19) Pillsbury 4 3/4s , 2/15/89
(20) So. Cal. Edison 3 1/8s, 8/15/80
(21) Sunshine Mining $6\frac{1}{2}$s, 4/15/89
(22) Tandy Corp. 5s, 5/1/89
(23) Union Pacific 4 3/4s, 4/1/99
(24) United Aircraft $4\frac{1}{2}$s, 10/1/92
(25) White Cons. $5\frac{1}{2}$s, 10/23/92

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*Sinking fund operative within the 1971-1975 period. Except for Avco Corp. $5\frac{1}{2}$s, 11/30/93, all the securities were callable for conversion during the same period.

Appendix Table 1

Distributional Characteristics of Simulated Rates of Return

	Mean		Std. Dev.		Skewness*		Kurtosis+		Median		Inter-Quartile Range	
	B	S	B	S	B	S	B	S	B	S	B	S
1	12.8	17.1	5.3	9.7	112.6	526.4	362.2	4993.0	10.3	16.6	7.2	15.4
2	17.5	14.3	25.5	24.5	205.0	176.1	953.8	789.3	13.1	10.7	22.9	23.5
3	23.0	19.6	15.3	16.5	93.8	54.2	345.5	453.2	18.6	16.5	19.6	19.4
4	13.6	19.9	3.2	11.4	136.4	332.8	391.4	30024.7	12.6	21.5	2.6	18.7
5	18.4	15.6	30.9	30.5	64.9	55.7	327.9	297.0	20.7	18.7	32.7	40.9
6	18.2	19.3	8.6	12.7	76.4	78.2	356.0	1612.7	17.9	21.5	8.8	17.6
7	17.1	16.6	14.1	21.1	166.5	383.5	753.9	3403.3	16.7	18.0	20.4	26.3
8	16.3	13.5	15.4	27.4	178.4	850.1	581.1	5904.8	10.2	7.6	17.0	31.2
9	15.7	16.3	9.1	12.1	149.6	50.9	546.2	1263.9	13.7	16.0	9.3	11.4
10	16.7	18.3	9.8	11.9	147.4	258.2	508.8	1111.3	12.8	13.6	11.6	14.5
11	23.5	21.8	24.9	37.0	146.5	471.0	472.0	2385.9	10.9	4.5	36.7	48.1
12	16.6	10.5	12.5	8.1	496.0	11.4	2980.3	68.4	13.3	10.4	3.2	10.1
13	17.9	9.5	14.7	15.2	429.7	189.3	2424.9	989.7	12.9	6.7	9.2	13.4
14	17.7	16.4	9.3	14.2	(3.1)	(150.5)	184.4	1546.3	17.5	17.6	15.6	18.3
15	11.0	8.9	7.6	6.7	275.7	(18.7)	1406.8	170.9	9.4	7.8	8.6	10.4
16	23.0	13.8	19.2	22.5	188.3	136.7	621.9	799.7	17.9	12.2	14.4	16.4
17	10.5	8.1	15.0	15.7	211.0	191.1	1070.9	1003.7	9.8	7.9	14.3	14.8
18	20.0	17.2	10.6	14.8	185.3	(45.6)	696.8	1807.8	18.1	17.4	9.9	14.2
19	14.2	17.0	6.6	9.8	154.4	418.2	606.3	1972.8	13.2	13.7	7.2	8.7
20	11.6	13.3	4.8	4.8	223.7	139.3	1055.5	575.3	10.4	12.5	3.7	6.2

Table 1 (continued)

	Mean		Std. Dev.		Skewness*		Kurtosis+		Median		Inter-Quartile Range	
	B	S	B	S	B	S	B	S	B	S	B	S
21	20.5	25.7	14.8	28.9	90.8	726.1	322.6	5737.1	17.6	24.2	20.0	31.3
22	18.4	13.7	19.6	26.7	258.9	506.7	1074.4	3033.5	12.9	10.5	14.1	32.7
23	8.7	10.7	4.5	5.8	(191.1)	(267.7)	879.0	1494.7	9.8	11.7	4.9	6.8
24	15.4	16.7	7.7	16.3	52.0	(408.7)	298.4	4118.8	16.2	21.4	12.7	23.1
25	27.9	22.7	24.7	37.2	96.1	384.7	229.8	1846.9	13.8	8.2	32.3	45.8

*Calculated as the third central moment divided by the second central moment.

+Calculated as the fourth central moment divided by the second central moment.

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