Using the Capital Asset Pricing Model and the Market Model to Predict Security Returns

by

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I. INTRODUCTION

This paper examines the validity of two widely used methods for forming conditional predicted portfolio returns. The first method, based upon the work of Sharpe (21) and Lintner (13), relies on a one-period, mean-variance theory of equilibrium expected returns, sometimes referred to as the "Capital Asset Pricing Model" (CAPM). The second method is based upon a proposal by Markowitz [14] and is called the "Market Model" (MM).

The market model bears a close resemblance to the <u>ex post</u> version of the capital asset pricing model, and both posit a linear relationship between the returns on individual securities and the returns on a portfolio of all assets.

Forming conditional predictions using one of these two models is advocated in almost every recent treatment of investment performance where conditional predictions of portfolio returns have been needed. For example, a common use has been to adjust for market wide effects in the assessment of unusual security returns arising out of an information producing event--such as dividend or earnings announcements. In addition, well-known financial institutions have been recommending common stocks with the highest (lowest) slope coefficients estimates or "beta coefficients" dependent on whether above (below) average overall market returns are anticipated.[1]

It should be noted that in several papers, Friend and Blume [9], Black, Jensen and Scholes [3], Miller and Scholes [15], and Blume and Friend [6], have found serious inadequacies in the capital asset pricing model's ability to explain the structure of actual security returns.

Also, the work of King [12] and Brennan [7] suggest serious deficiencies in the market model, though Blume's [4] analysis tends to support the empirical validity of the market model. None of these studies, however, provide analysis that directly questions the model's properties as a conditional predictor of security returns.[2]

The next section of this paper examines the general validity of using the market model and the capital asset pricing model to form expectations of security returns under the condition that the market return is assumed to be known with perfect certainty. The final section of the paper takes a closer look at performance indexes that have developed to record abnormal security return behavior.

II. THE STRUCTURE OF ONE-PERIOD ASSET RETURNS

The Capital Asset Pricing Model (CAPM)

Under a given set of assumptions, Sharpe [21] has shown that the *ex ante* expected returns on capital assets will be determined in equilibrium by

$$E(R_{it}) = R_{Ft}(1 - \beta_i) + \beta_i E(R_{Mt}), \tag{1}$$

where $E(R_{it})$ is the unconditional expected return on asset i from t-1 to t, $E(R_{Mt})$ is the expected one-period return on a "market" portfolio, R_{Ft} is a known, riskless return, and β_i, equal to $\dfrac{COV(R_{it}, R_{Mt})}{VAR\ (R_{Mt})}$, is a a coefficient which measures the response of R_{it} to the return on the market portfolio.

In empirical studies the *ex ante* equilibrium condition has been replaced by an equation for *ex post* returns, given by

$$R_{it} = R_{Ft}(1 - \beta_i) + \beta_i R_{Mt} + \varepsilon_{it},$$

with,

$$E(\varepsilon_{it}) = 0$$

$$E(\varepsilon_{it}, \varepsilon_{i,t+1}) = 0$$

$$E(\varepsilon_{it}, \varepsilon_{jt}) = \begin{cases} \sigma^2\varepsilon_{it}, & i = j \\ 0, & i \neq j \end{cases} \tag{2}$$

$$E(\varepsilon_{it}, R_{Mt}) = 0$$

where R_{it} is the *ex post* return on asset i from t-1 to t, and R_{Mt} is the *ex post* return observed for the market portfolio during the same period. R_{Mt} is commonly measured by a stock market index, such as the Fisher Link Relative Index or the Standard and Poor's 500 Composite Stock Price Index. R_{Ft} is either the observed return on a short term government security or an assumed risk-free rate of interest. The slope coefficient is usually referred to as the beta coefficient.

If equation (1) is the correct <u>ex ante</u> equilibrium that exists in the capital market, then under one set of assumptions Jensen [10, 179-181] has shown that equation (2) depicts the actual return generating process, and R_{it} is the actual observable rate of return on asset i at period t. The expected <u>ex post</u> one-period return then, conditional upon the observed market return, the risk-free return, and β_i, is given by

$$E(R_{it}|R_{Mt}, R_{Ft}, \beta_i) = (1 - \beta_i) R_{Ft} + \beta_i R_{Mt} \tag{3}$$

Therefore, if T observations are made on the returns R_{it} and R_{Mt}, if $\hat{\beta}_i$ and $\hat{\varepsilon}_{it}$ are the resulting (least squares) estimates of β_i and ε_{it}, and if β_i and R_{Ft} are constants, over time, then the one-period conditional expected <u>ex post</u> return in period T + 1 is estimated as[3]

$$\hat{E}(R_{i,T+1}|R_{M,T+1}, R_{F,T+1}, \hat{\beta}_i) = (1 - \hat{\beta}_i)R_{F,T+1} + \hat{\beta}_i R_{M,T+1} \tag{4}$$

and gives unbiased estimates of $E(R_{i,T+1}|R_{M,T+1}, R_{F,T+1}, \beta_i)$

<u>The Market Model (MM)</u>

The developments of equations (1) to (4) depend on the equilibrium property that the first term on the right hand side is equal to $(1 - \beta_i)R_{Ft}$. The market model, put forth by Markowitz [14] and Sharpe [22] and analyzed by Blume [14], is given by

$$R_{it} = \alpha_i + \beta_i R_{Mt} + \varepsilon_{it} \tag{5}$$

where $\alpha_i \gtrless (1 - \beta_i) R_{Ft}$. Obviously, the difference between the model in equation (5) and the one developed in equations (1) to (4) is that α_i is unconstrained to a specific equilibrium value. Conditional predicted returns are established by taking conditional expectations so that

$$E[R_{it}|\alpha_i, \beta_i, R_{Mt}] = \alpha_i + \beta_i R_{Mt} \tag{6}$$

Then given estimates for α_i and β_i the one-period conditional expected <u>ex post</u> return in period T + 1 is

$$\hat{E}(R_{i,T+1}|\hat{\alpha}_i, \hat{\beta}_i, R_{M,T+1}) = \hat{\alpha}_i + \hat{\beta}_i R_{M,T+1} \tag{7}$$

<u>Estimation of the Relationship Between Actual and Conditional Predicted Returns in a Cross-Section</u>

An obvious test of the conditional predictive content of the MM and the <u>ex post</u> CAPM is to determine if actual returns are approximately equal to conditional predicted returns for a large group of securities. One test we use, using the capital asset pricing model to illustrate, is to regress actual on conditional predicted returns, or

$$R_{i,T+1} = a_o + a_1\hat{E}(R_{i,T+1}|R_{M,T+1}, \hat{\beta}_i, R_{F,T+1}) + \delta_{i,T+1} \tag{8}$$

where

$$E(\delta_{i,T+1}) = E(\delta_{i,T+1}, \delta_{j,T+1}) = E(\delta_{i,T+1}, \hat{R}_{i,T+1} \ldots) = 0.$$

The validity of the CAPM is consistent with coefficients $a_o = 0$ and $a_1 =$ 1.0. However unbiased estimation of the regression coefficients requires that no measurement errors exist in the independent variable. Rewriting (8) to allow for measurement errors in the estimation of $\hat{\beta}_i$ gives

$$R_{i,T+1} = a_o + a_1 \hat{E}[R_{i,T+1}|R_{M,T+1}, \hat{\beta}_i, R_{F,T+1}] - \{R_{M,T+1}(\hat{\beta}_i - \beta_i) - [(\hat{\beta}_i - \beta_i)R_{F,T+1}]\} + \zeta_{i,T+1} \tag{9}$$

where

$$E(\zeta_{i,T+1}) = E(\zeta_{i,T+1}, \zeta_{j,T+1}) = E\{\zeta_{i,T+1}(R_{i,T+1}| \ldots)\} = 0.$$

In order to develop consistent OLS estimates it is necessary to rely on the property that a probability limit exists for $\hat{\beta}_i$ equal to β_i and group our observations into portfolios from a rank ordering on $\hat{\beta}_i$. However, this procedure does not work since measurement errors in $\hat{\beta}_i$ will tend to be positive in high beta portfolios and negative in low beta portfolios. This problem, first mentioned by Blume [4], [5] and later by Black, Jensen and Scholes [3], is referred to as the regression phenomenon. We employ sampling procedures that adjust for this potential problem (they will be introduced in a subsequent section).

It should also be noted that the problems involved in achieving unbiased estimates of the coefficients in equation (8) are not nearly as serious as the ones encountered by Friend and Blume [9]. In their study the association between beta and risk-adjusted expected return is assessed during the <u>same time period</u> for a large cross-section of securities and non-independent measurement error potentially exists on both sides of the regression equation.[4]

The conditional predictive content of the CAPM will be embodied in the estimates of a_o, a_i and σ_ζ^2. If the conditional predictors supply unbiased estimates of actual portfolio returns on average, the slope coefficient, a_1, should equal one; a_o should equal zero. The extent to which the conditional predictors are "good" <u>ex post</u> predictors is implied by the coefficient of determination (R^2). Although our discussion has been in terms of the CAPM, equation (8), of course, can also be estimated for the market model. The only change is in the definition of the independent variable. A comparison of regressions using both the MM and CAPM to form conditional predicted returns may suggest something about the empirical validity of one vis-a-vis the other.

Data and Estimation Technique. The data used were taken from an updated version of the CRSP tapes. This file contains monthly investment relatives for all securities listed on the New York Stock Exchange any time during the period January, 1926 through June, 1968. The market model coefficients were estimated for each asset for each period by regressing investment relatives on the link relatives of Fisher's Combination Investment Performance Index. The risk-free rate, where one was necessary, was the yield on Treasury Bills with one month to maturity. For a security to be included a minimum of 60 monthly relatives were required for estimating the model coefficients.

Portfolio Selection Technique. For each of the last five periods in which conditional predicted returns were calculated, firms were combined into portfolios of one, twenty, and forty firms. In order to preserve the distribution of the beta coefficients, the portfolio selection procedure ranked on the basis of the value of the beta coefficient in the period previous to the period in which the coefficients have been estimated for conditional prediction purposes. For example, in the period July 1947 to June 1954, securities are assigned to portfolios on the basis of each security's estimate of beta coefficient, ranked from high to low. Thus, 839 securities are assigned to 21 forty firm portfolios on the basis of the security's β coefficient. Portfolio 1 contains the forty firms with the highest estimated β, and portfolio 21 contains the forty firms with the lowest β. In the next period July 1954 to June 1961, return data is used to recompute $\hat{\beta}_i$ on the securities in each portfolio, and these estimates are equally weighted across securities within portfolios to form portfolio beta coefficients, $\hat{\beta}_p$, to establish the parameters of the conditional predictors as in equations (4) and (7). The actual conditional predictions are then made for the period July 1961 to July 1968 on the basis of these coefficients estimated in the July 1954 to June 1961 period. The procedure, summarized completely in Table 1, greatly reduces the measurement error problem and eliminates the regression phenomenon tendency that is attributable to the ordering bias.

For firms that leave the New York Stock Exchange in the period for which conditional predictive returns are calculated, it is assumed their securities sold at the last closing price, and the proceeds redistributed equally among the remaining securities in the portfolio.

Tabulating Results. Summarizing, the following cross-sectional regression model was estimated for each month for both the market model and the *ex post* capital asset pricing model:

$$R_{P,T+t} = \hat{a}_{ot} + \hat{a}_{1t}\hat{E}[R_{P,T+t} | R_{M,T+t}, \hat{\beta}_p, R_{F,T+t}] + \hat{\delta}_{P,T+t} \qquad (8a)$$

$$t = 0, \ldots 83$$

where T is the calendar time representing the start of the forecasting period (e.g., from Table 1 the predicted return period 7/61 to 6/68), t refers to the month of the regression within the forecasting period, and P to the particular portfolio. The portfolio β coefficient is an average that has been calculated from estimated firm β's computed over the calendar time interval prior to T. The results (only results for forty firm portfolios are reported) are summarized in Table 2 as follows: the 84 monthly cross-sectional regressions are ordered according to the return on the Fisher index in the month and then partitioned into four equal number of monthly sections. The means of the relevant coefficients and

TABLE 1

DESCRIPTION OF SAMPLE DATA

Estimation Period for Portfolio Construction	Model Estimation Period	Number of Firms	$\bar{\hat{\beta}}$	Predicted Return Period	Average Monthly Market Return in Prediction Period
7/26 - 6/33	7/33 - 6/40	571	1.042	7/40 - 6/47	1.018
7/33 - 6/40	7/40 - 6/47	667	.999	7/47 - 6/54	1.011
7/40 - 6/47	7/47 - 6/54	753	1.016	7/54 - 6/61	1.014
7/47 - 6/54	7/54 - 6/61	839	.997	7/61 - 6/68	1.011

TABLE 2

MEANS OF REGRESSIONS OF ACTUAL RETURNS ON CONDITIONAL PREDICTED RETURNS

Capital Asset Pricing Model Predictions

7/61-6/68	# OF PORTFOLIOS	# OF REGRESSIONS	(1) AVERAGE CONSTANT ($\hat{a}_o$)	(2) AVERAGE SLOPE ($\hat{a}_1$)	(3) AVERAGE CORRELATION COEFFICIENT	(4) AVER MONTHLY MKT RETURN	(5) AVERAGE $\frac{\hat{\sigma}_R^2}{\sigma_R^2}$
Q_1	21	21	.25175	.73401	.45636	.95997	1.13408
Q_2	21	21	.28327	.71521	.02906	1.00137	.03918
Q_3	21	21	.31699	.68673	.33433	1.02446	.16835
Q_4	21	21	.32068	.69554	.53892	1.06066	.94404
7/54-6/61							
Q_1	19	21	.18938	.80262	.48474	.96696	.85386
Q_2	19	21	3.51951	-2.50850	-0.08154	1.00589	.04921
Q_3	19	21	1.06303	-0.03715	-0.00349	1.02623	.37797
Q_4	19	21	.62539	.40465	.31739	1.05478	1.06766
7/47-6/54							
Q_1	16	21	.07788	.91947	.64483	.95971	.73243
Q_2	16	21	-0.47546	1.48087	.17644	1.00131	.10684
Q_3	16	21	.59551	.41901	.28917	1.02550	.56631
Q_4	16	21	.13597	.87398	.71857	1.05892	1.04461
7/40-6/47							
Q_1	13	21	.15107	.83677	.64846	.94397	.99050
Q_2	13	21	-4.21929	5.22725	.25205	.99992	.10495
Q_3	13	21	.45747	.55902	.35471	1.03726	.48640
Q_4	13	21	.22624	.78864	.62204	1.08952	1.05691

TABLE 2 (CONTINUED)

Market Model Predictions

7/61-6/68	# OF PORTFOLIOS	# OF REGRESSIONS	(1) AVERAGE CONSTANT ($\hat{a}_o$)	(2) AVERAGE SLOPE ($\hat{a}_1$)	(3) AVERAGE CORRELATION	(4) AVER MONTHLY MKT RETURN	(5) AVERAGE $\frac{\hat{\sigma}_R^2}{\sigma_R^2}$
Q_1	21	21	.47866	.50166	.45431	.95997	1.60576
Q_2	21	21	1.23671	-0.23327	-0.05127	1.00137	.11693
Q_3	21	21	-0.23301	1.22123	.32817	1.02446	.05860
Q_4	21	21	.32068	.69554	.53892	1.06066	.94404
7/54-6/61							
Q_1	19	21	.38745	.59853	.47339	.96696	1.25368
Q_2	19	21	1.11862	-0.11191	-0.03002	1.00584	.06187
Q_3	19	21	1.13505	-0.10775	-0.00976	1.02623	.19005
Q_4	19	21	.55311	.47257	.32313	1.05478	.78056
7/47-6/54							
Q_1	16	21	.22050	.77073	.62138	.95971	.95348
Q_2	16	21	.94149	.06517	.06840	1.00131	.14871
Q_3	16	21	.52152	.48969	.29198	1.02550	.38080
Q_4	16	21	.13597	.87398	.71857	1.05892	1.04461
7/40-6/47							
Q_1	13	21	.15315	.83195	.65069	.94397	1.00368
Q_2	13	21	-0.08369	1.08427	.23328	.99992	.12242
Q_3	13	21	.45747	.55902	.35471	1.03726	.48640
Q_4	13	21	.22986	.78323	.61934	1.08952	1.06197

other variables are reported for each section (e.g., average constants, average slopes, etc. of equation 8a). The months within each section do not tend to be consecutive months. Section one, Q_1, represents the regression results for 21 cross-sections where there were relatively low monthly rates of return for the market (typically from -15 percent to -1 percent), section two, Q_2, represents the regression results for cross-sections where the market rates of return are relatively close to the pure riskless rate (typically -1 percent to +1.5 percent), etc.

The ability of the MM and the ex post CAPM to generate accurate conditional predictions largely depends upon whether the market return is greatly different than the risk-free return. If the realized market return is below the risk-free return (as in Q_1), high $\hat{\beta}_p$ portfolios should achieve lower realized returns than low $\hat{\beta}_p$ portfolios. In fact, both the MM and ex post CAPM are meant to predict the exact extent to which high $\hat{\beta}_p$ portfolios should do worse than low $\hat{\beta}_p$ portfolios when the market return is below the risk-free return. Moreover, the greater the difference between the market return and the risk-free return, the greater the differences in conditional predictions between high and low $\hat{\beta}_p$ portfolios. One would expect that sections one and four would be the most revealing with respect to the unbiasedness, goodness of fit, and usefulness of the MM and the ex post CAPM conditional predictions. In fact, the results contained in Q_2 are generally meaningless. Note that this procedure does not result in any attenuation bias and is a way of summarizing the results of a large number of regressions.

Interpretation of Results

There are three major conclusions that can be drawn from Table 2. First, there is no great difference in goodness of fit between forming conditional predictors assuming either the validity of the MM or the *ex post* capital asset pricing model. Second, in those periods in which the market return differed significantly from the risk-free return (Q_1, Q_3 and Q_4) the slope of the cross-sectional regression of actual portfolio returns on portfolio returns estimated from both the MM and the *ex post* CAPM on average (as measured by the mean of each section) was substantially less than the hypothesized value of one.[6] This fact suggests that in periods when the market return is less than the risk-free return the conditional predictions from both the MM and the *ex post* CAPM tends to underestimate returns on high portfolios. The opposite tendency exists when the return on the market portfolio exceeds the risk-free return. The T-values associated with the slope coefficients (the $\hat{a}_1$'s) showed the coefficients to be significantly less than one on average, particularly for categories Q_1 and Q_4.[7] Third, there were a number of cases in all categories when the estimated value of a_1 was negative or substantially greater than 2.0. For example, in the period 7/61 - 7/68, 22 of 84 estimates of a_1 were negative assuming the validity of the market model while, in the same period, 13 of 84 estimates of a_1 were greater than 2.0. Similar results exist in other periods and for regressions formed using the *ex post* CAPM. Clearly the estimates are not adequately stationary from cross-section to cross-section. These results imply a serious breakdown in the ability of the market model and the *ex post* CAPM to consistently generate valid conditional predicted returns.

Existence of More than One Market Factor. The market model and the _ex post_ capital asset pricing model depend upon a diagonal property from which _ex post_ returns of any security are generated by a single market-wide effect. The diagonal assumption specifically implies the property that $E(\varepsilon_{it}, \varepsilon_{jt}) = 0$ and $E(\varepsilon_{it}, \varepsilon_{it}) = \sigma^2(\varepsilon_{it})$. We hypothesize that the wide variation in $\hat{a}_1$ values from cross-section to cross-section can be most reasonably explained by the existence of more than one market-wide effect. As a result, the market model and the _ex post_ asset pricing model of (4) suffer from omitted variable specification bias, and the disturbance terms between portfolios are dependent.

For example, the dependency in the disturbances, ε_{it}, may be represented as

$$\varepsilon_{it} = \gamma_{it}\,\phi_t + \zeta_{it} \tag{10}$$

where ϕ_t is an addttional market-wide factor that affects the returns on all securities. In essence, the existence of a second market-wide effect argues that the residuals of the market model and the _ex post_ capital asset pricing model are not distributed independently between assets (i.e., $E(\varepsilon_{it}, \varepsilon_{jt}) \neq 0$). The slope coefficient, γ_{it}, defines the relation between ε_i and δ. The error term, ζ_{it}, is independently distributed with zero expectation. This model asserts that the returns of any individual security will be a function of the market factors R_m and ϕ_t. It is clear the cross-sectional regression formulation in equation (8a) could be biased in a_1 because of the omission of the term $\gamma_{it}\delta_t$. The bias will depend upon the average realized value of ϕ_t and could be positive or negative. If the average value of ϕ_t changes from sample period to sample period, the stationarity of the estimates of

$\hat{a}_1$ will break down and may result in sign changes from period to period.

There is accumulating evidence that supports the hypothesis that serious dependencies exist in the error term of the MM and the ex post CAPM (Brennan [7]). Specifically, the results presented in Table 2 are consistent with a particular two-factor model first proposed by Rie [18], and supported by the independent developments of Pettit and Westerfield [17]. In brief, this theory shows that if asset prices are equal to a stream of discounted expected earnings, two market-wide factors will determine asset returns; namely, an earnings factor and a capitalization rate factor. The beta coefficient, β_i, can be expressed as a weighted average of the asset's earnings beta, b_{ei}, and the asset's capitalization rate beta, b_{ci}, or

$$\beta_i = \gamma b_{ei} + (1 - \gamma) b_{ci}. \qquad (11)$$

It can be shown [18], under one set of simplifying assumptions, that if the proportion γ changes from period to period the market model residuals will be a function of the market model beta coefficient. High (above 1.0) market model beta portfolios will tend to have residual values, ε_i, that are opposite in sign and magnitude from low market beta portfolios. The value of the residual for a particular portfolio depends upon the relative importance of the earnings effect and the capitalization rate effect in determining market return in a particular period. In addition, the total of the portfolio residuals over a number of periods of observation may not be zero if γ is not the same in the prediction period as in the estimation period.

If this two-factor model gives an accurate description of the return generating process, it should be expected that the correlations between portfolio residuals, that is actual returns less predicted returns, for either high beta portfolios _or_ for low beta portfolios will be positive over any particular time series. In contrast, the correlations between the residuals of high _and_ low beta portfolios will be negative over the time series. Table 3 presents the correlation matrix for the final 84 month prediction period, 7/61 to 6/68, for forty firm portfolios using the MM estimates of conditional predicted returns. The correlation matrix computed from residuals formed using the _ex post_ CAPM is similar and both support the hypothesis of the existence of a two factor return generating model, though they do not exclude alternative explanations. In any case, cross-sectional uses of the market model or the _ex post_ capital asset pricing model are likely to be quite misleading.

Alternatively, it has been proposed that errors in the measurement of R_{Ft} may cause market model and _ex post_ capital asset pricing conditional predicted return estimates to be biased in one direction or the other Friend and Blume [9] provide a discussion of the possible effects of measurement error in R_{Ft} on the validity of the market model. However, there is no suggestion from their examination that this contributes to a substantial downward bias of estimates in $\hat{a}_1$. Our results for the CAPM regressions were virtually insensitive to reasonable changes in the assumed risk-free rate of interest.

An alternative explanation for the results is provided by the recent paper, Black [2]. The Black model posits a breakdown of the borrowing and lending assumption of the market model and implicitly suggests the invalidity of equations (1) to (4). He presents an alternative linear mean-variance model in which there is no risk-free asset, but

TABLE 3

CORRELATION MATRIX OF MARKET MODEL RESIDUALS FOR 60 MONTHS, 7/61-6/68 FOR 21 PORTFOLIOS OF 40 SECURITIES EACH*

	1	2	3	4	5	6	7	8	9	10	11
1	1.00000										
2	0.69470	1.00000									
3	0.73619	0.70155	1.00000								
4	0.73391	0.70454	0.72400	1.00000							
5	0.54128	0.47128	0.45767	0.56488	1.00000						
6	0.31534	0.31523	0.35085	0.38880	0.31300	1.00000					
7	-0.08912	-0.02710	-0.11450	0.00270	-0.12179	0.02044	1.00000				
8	-0.06593	-0.01731	0.03477	0.05732	-0.19924	-0.04485	0.24519	1.00000			
9	-0.12503	-0.12188	-0.03862	-0.02229	-0.12933	0.08563	0.14365	-0.04119	1.00000		
10	0.14409	0.17611	0.19259	0.23350	0.15960	0.28423	-0.02573	-0.04773	0.10641	1.00000	
11	-0.05257	-0.15263	-0.08255	-0.11462	-0.22999	0.09408	0.06251	0.02712	0.06135	0.06311	1.00000
12	-0.23689	-0.35117	-0.17010	-0.36133	-0.32650	-0.24150	-0.06675	0.03818	0.06227	-0.00819	0.17558
13	-0.20656	-0.05614	-0.09835	-0.02635	-0.16217	-0.20002	0.20637	0.13491	-0.01129	0.01157	0.01719
14	-0.35038	-0.30533	-0.32789	-0.30702	-0.12776	-0.13389	0.14185	-0.15884	0.13029	0.11129	0.07170
15	-0.42276	-0.21967	-0.35233	-0.39784	-0.38335	-0.13878	0.05455	-0.14528	0.02717	0.07255	0.07158
16	-0.24715	-0.15091	-0.21503	-0.10628	-0.15133	-0.19154	0.32516	0.06703	0.26057	0.08492	0.00284
17	-0.34120	-0.20739	-0.40747	-0.30179	-0.21016	-0.11967	0.18658	-0.06713	0.08176	0.07357	-0.06930
18	-0.33807	-0.04809	-0.24280	-0.20804	-0.09326	-0.24169	0.19378	-0.06069	0.12981	-0.02526	-0.11794
19	-0.33993	-0.28344	-0.36730	-0.35932	-0.32020	-0.41499	0.20485	0.01446	0.09094	-0.10474	0.13036
20	-0.34916	-0.27720	-0.35131	-0.38546	-0.29985	-0.27182	0.21355	-0.07955	0.24499	-0.15313	0.14764
21	-0.35864	-0.28069	-0.31634	-0.35341	-0.15962	-0.24267	-0.02595	-0.14002	0.21869	-0.01475	0.00974

	12	13	14	15	16	17	18	19	20	21
12	1.00000									
13	0.11208	1.00000								
14	0.01098	0.13905	1.00000							
15	0.23700	0.17163	0.34343	1.00000						
16	0.21690	0.20980	0.35185	0.06331	1.00000					
17	-0.03353	0.11517	0.30765	0.40819	0.20259	1.00000				
18	-0.01518	0.08515	0.23418	0.13265	0.41514	0.31211	1.00000			
19	-0.04666	0.11849	0.35959	0.18749	0.36195	0.26119	0.31399	1.00000		
20	0.07487	0.18872	0.35918	0.23151	0.38179	0.35605	0.54000	0.54893	1.00000	
21	-0.07441	0.01719	0.23867	0.02387	0.37316	0.27638	0.33546	0.46325	0.44800	1.00000

*Portfolio 1 is made of the 40 highest estimated beta coefficients and portfolio 21 of the lowest 40 estimated betas.

rather a zero beta asset developed by allowing a perfect short sale mechanism (i.e., the investor has use of all the proceeds from the short sale). If the theory is correct in its extreme it could explain some of the results evident in Table 2. But because it depends on a perfect short sale mechanism for common stocks and the total absence of short selling for bonds, we think that it is the least likely rationale for the apparent poor predictive ability of the MM and the *ex post* CAPM. However, the Black theory can explain the long run bias against high beta portfolios constructed using the *ex post* CAPM. This bias would occur if the return on the zero beta portfolio was greater than the risk-free return. The theory does not adequately explain the long run bias against high beta portfolios constructed using the market model, without employing some peculiar assumptions about the secular trend in the return on the zero beta portfolio.

The recent empirical findings of Black, Jensen and Scholes [3] and Blume and Friend [6] suggest a long run bias against the return performance of high $\hat{\beta}_p$ portfolios. Our results are fully compatible with these studies even though our approach is different. However, our results are different in at least one substantive way. We find that in bear markets, when the market return is below the risk-free returns, high $\hat{\beta}_p$ portfolios generally do better than expected.

III. INDEXES OF INVESTMENT PERFORMANCE

One problem in the assessment of performance for groups of assets has been to properly abstract individual asset returns from market-wide effects on the returns of all assets. Indexes have been constructed that purportedly trace out the abnormal return that an investor would have achieved over time had he invested in a particular set of securities. Although these indexes rely on the *ex post* CAPM or MM, it is interesting to observe that no examination of the empirical validity or reliability of these indexes has been reported.

Prediction Error

The estimated conditional prediction error for asset i over an arbitrary N period can be expressed as

$$PE_{i,T+N} = \prod_{\tau=T+N}^{T+N} R_{i\tau} - \prod_{\tau=T+1}^{T+N} \hat{E}(R_{i\tau}|I_{i\tau}) \qquad (12)$$

and for a portfolio of S assets

$$PE_{T+N} = \frac{1}{S}\sum_{i=1}^{S} PE_{i,T+N}$$

where $\hat{E}(R_{i\tau}|I_{i\tau})$ is the one-period predictor (developed in preceding pages), conditional upon information set $I_{i\tau}$.[8] The contents of $I_{i\tau}$ will depend upon whether the validity of the market model or of the *ex post* capital asset pricing model is being assumed. The expectation is that the prediction error, PE_τ, will be equal to zero. If it is either significantly positive or negative, then unusual or abnormal returns are being generated. The prediction error of equation (12) forms the basis for our discussion of the multi-period applications of the MM and the *ex post* CAPM. However, most investment performance indexes have relied on the validity of the MM, rather than the CAPM; so most of our discussion in this section will be in terms of the market model.

Ball and Brown Performance Index

Ball and Brown [1] made use of the market model in developing their "Abnormal Performance Index" in order to assess the price impact of changes in firms' reported earnings. The value of the Ball and Brown Performance Index is equal to

$$BBPI_{T+N} = \frac{1}{S} \sum_{i=1}^{S} \prod_{\tau=T+1}^{T+N} (1 + U_{i\tau}) \tag{13}$$

where

$$\prod_{\tau=T+1}^{T+N} (1 + U_{i\tau})$$

is defined as the prediction error relative for asset i in each time period, τ, compounded to the end of period N. $U_{i\tau}$ is defined as the difference between actual turns for asset i in period τ, $R_{i\tau}$, and conditional predicted returns, $\hat{E}(R_{i\tau})$(see equation 4 and 7). Specifically, $BBPI_{T+N}$ records the value of one dollar invested equally in all S securities in a portfolio at a time T+1 and held to the end of period N, after abstracting from market-wide effects. In addition to the Ball and Brown study of reported earnings, this index has been used by Scholes [20] to assess the impact of secondary common stock distributions on market prices. In both of these studies it was assumed that if there were no effect of earnings announcements or secondary distributions on stock prices then the value of the index at any time, T+N, should be unity.[9]

The Pettit Index

Pettit [16] in identifying the price impact of dividend announcements has used an index similar to $BBPI_{T+N}$ except that it reallocates the prediction error to equality after each period to achieve constant proportions invested in each asset. The Pettit version of the BBPI is the following,

$$PPI_{T+N} = \prod_{\tau=T+1}^{T+N} \left(1 + \frac{1}{S} \sum_{i=1}^{S} U_{i\tau}\right) \tag{14}$$

The Fama, Fisher, Jensen, and Roll Performance Index

Fama (et al.) [8] evaluated the adjustment of stock market prices to the new information contained in the announcement of stock splits. As part of their examination the market model was used as a conditional predictor to assess unusual price movements around stock split dates. Fama (et al.) assumed the validity of the market model and the specific index used by Fama (et al.) is similar to the one discussed in the preceding sections, except that an apparent concern about the horizon problem prompted a specification of the market model predictor in natural logarithms.

The prediction errors for any portfolio after N periods is reformulated as the following,[10]

$$FPI_{T+N} = \frac{1}{S} \sum_{i=1}^{S} \sum_{\tau=T+1}^{T+N} [Log_e R_{i\tau} - (\hat{\alpha}_i + \hat{\beta}_i Log_e R_{M\tau})] \tag{15}$$

Methods of Analysis and Empirical Reuults

Four different performance indexes were computed (PE_{T+N}, $BBPI_{T+N}$, PPI_{T+N}, and FPI_{T+N}) according to the definitions given in equations (12), (13), (14), and (15). These values at six month intervals for 40 firm

TABLE 4

SEMI-ANNUAL PERFORMANCE INDEX VALUES OF 40 FIRM PORTFOLIOS FOR 60 MONTHS IN EACH OF 4 PERIODS USING THE MARKET MODEL TO FORM CONDITIONAL PREDICTIONS*

	+1	+6	+12	+18	+24	+30	+36	+42	+48	+54	+60
Fama Performance Index - FPI_{T+N}											
7/61-6/66											
ALL β	.00274	.00428	.01656	.01521	.02158	.03094	.04334	.04538	.04128	.03422	.02905
LOW β	.00546	.04103	- .04452	- .03634	- .05240	- .07968	- .09631	- .09193	- .12233	- .19470	- .25851
MED β	.00186	- .00631	.01366	.00619	.01387	.02818	.03753	.04760	.05763	.05667	.07086
HIGH β	.00092	- .02192	.09663	.07578	.10329	.14431	.18881	.18046	.18853	.24068	.27480
7/54-6/59											
ALL β	- .00217	.00522	.00385	.00577	.00777	.01101	.01246	.00024	- .00443	- .01165	- .01787
LOW β	.00215	.00766	.00610	.00554	.01129	.00949	.01305	.00648	.02113	.02288	.01623
MED β	- .00348	.00473	.00473	.00347	.00238	- .00698	- .01592	- .03613	- .04892	- .06252	- .05421
HIGH β	- .00591	.00285	.00035	.00835	.00905	.03079	.04016	.02932	.01025	- .00096	- .02129
7/47-6/52											
ALL β	.00173	.00787	.00874	.00840	.01580	.02205	.01946	.02447	.02665	.02956	.03671
LOW β	- .00034	- .01588	- .04070	- .03673	.00289	.02706	.00104	- .01657	- .00648	- .00696	- .00998
MED β	.00116	.01895	.00819	- .02543	- .03636	- .02050	- .04340	- .03427	- .02065	- .01283	- .02059
HIGH β	.00479	.02528	.06862	.09638	.08343	.05859	.10443	.13245	.11370	.11579	.15005
7/40-6/45											
ALL β	- .00055	- .00391	.00624	- .00790	- .00407	- .00413	.00649	- .00340	.01778	.02471	.03950
LOW β	.00072	- .00431	- .02499	- .03527	- .06358	- .03000	- .01626	- .00266	.01240	.01801	.02560
MED β	- .00077	- .00347	.02201	.01971	.02205	.01036	.01326	.00778	.02177	.04255	.05214
HIGH β	- .00935	- .00396	.02558	- .00820	.03765	.01011	.02647	- .01830	.01954	.01079	.04107
Ball and Brown Performance Index - $BBPI_{T+N}$											
7/61-6/66											
ALL β	1.00251	1.00349	1.02850	1.02993	1.05060	1.08115	1.12854	1.14674	1.16143	1.22697	1.31850
LOW β	1.00566	1.04355	.96459	.97449	.96102	.94439	.93453	.94634	.91739	.86834	.86403
MED β	1.00162	.99236	1.02593	1.02287	1.04095	1.07053	1.09779	1.10880	1.13790	1.19828	1.27245
HIGH β	1.00027	.97457	1.09499	1.09243	1.14984	1.22855	1.35330	1.38509	1.42900	1.61430	1.81902
7/54-6/59											
ALL β	.99847	1.00496	1.01039	1.01393	1.02442	1.04244	1.04921	1.04504	1.05242	1.08771	1.12998
LOW β	1.00294	1.00848	1.01025	1.01481	1.03488	1.04660	1.04752	1.03802	1.05949	1.09415	1.13533
MED β	.99630	1.00347	1.00339	1.00412	1.01373	1.02195	1.03595	1.03383	1.03671	1.07209	1.13904
HIGH β	.99542	1.00235	1.01755	1.02270	1.02291	1.05807	1.06445	1.06444	1.05988	1.09583	1.11469
7/47-6/52											
ALL β	1.00028	1.00137	1.00242	1.00967	1.00682	1.01069	1.01501	1.01900	1.02082	1.04662	1.07061
LOW β	.99997	.98167	.95685	.96167	.99741	1.02927	1.00618	.98966	1.00064	1.01232	1.02203
MED β	.99998	1.01658	1.00939	.97324	.94554	.96094	.95050	.96686	.97362	1.00602	1.00796
HIGH β	1.00096	1.00979	1.04772	1.10369	1.07938	1.03815	1.09010	1.10636	1.09223	1.12838	1.19220
7/40-6/45											
ALL β	.99549	.97784	.96935	.96755	.94633	.94808	.99577	.97974	1.04194	1.09529	1.17316
LOW β	1.00552	.98088	.96304	.96595	.93341	.97145	1.03841	1.05451	1.11863	1.17252	1.25285
MED β	.99463	.97755	.98624	.99557	.98111	.96333	.97957	.96393	1.02922	1.07789	1.13886
HIGH β	.98404	.96442	.95613	.93451	.91899	.89981	.96273	.90605	.96198	1.02051	1.11643

TABLE 4 (cont'd)

	7/61-6/66	+1	+6	+12	+18	+24	+30	+36	+42	+48	+54	+60
Prediction Error - PE_{T+N}	ALL β	.00252	.00318	.00941	.00704	.00810	.00631	.01715	- .00841	- .05278	- .06662	- .09197
	LOW β	.00567	.04480	- .04713	- .04801	- .08152	- .12519	- .16845	- .19950	- .29025	- .48909	- .63520
	MED β	.00162	- .00888	.01183	.00504	.00295	.00129	.00197	- .02247	- .05137	- .08445	- .11958
	HIGH β	.00027	- .02638	.06353	.06409	.10286	.14284	.21794	.19674	.18327	.37368	.47887
	7/54-6/59											
	ALL β	- .00153	.00381	.00333	- .00168	- .00390	.00631	- .01906	- .05197	- .12500	- .20079	- .24183
	LOW β	.00294	.00596	.00252	.00194	.01820	.02588	.00656	- .03892	- .05813	- .08592	- .11433
	MED β	- .00374	.00366	- .00233	- .01072	- .01540	- .02103	- .04246	- .06722	- .15554	- .24293	- .22396
	HIGH β	- .00457	.00144	.00992	.00312	- .01819	.01082	- .02558	- .05195	-0.17245	- .29267	- .40645
	7/47-6/52											
	ALL β	.00028	.00015	- .00598	- .00961	- .02752	- .05223	- .08590	- .18506	- .23818	- .37313	- .46376
	LOW β	- .00003	- .01967	- .05407	- .05513	- .02720	- .00709	- .05809	- .11457	- .12503	- .16329	- .21113
	MED β	- .00002	.01530	- .00441	- .04539	- .08799	- .12840	- .19621	- .37071	- .44648	- .71818	- .96950
	HIGH β	.00096	.00879	.05048	.08078	.03256	- .03024	- .00895	- .08401	- .16566	- .27990	- .26120
	7/40-6/45											
	ALL β	- .00451	- .02654	- .04841	- .05437	- .13184	- .24658	- .68296	- .82204	-1.35930	-1.91290	-3.18516
	LOW β	.00552	- .01377	- .05175	- .06009	- .13863	- .17029	- .35705	- .39875	- .66350	- .95442	-1.62913
	MED β	- .00537	- .02585	- .02527	- .02091	- .09409	- .21930	- .68633	- .82823	-1.36324	-1.89321	-3.20399
	HIGH β	- .01596	- .04335	- .07315	- .08905	- .17052	- .37605	-1.08613	-1.34343	-2.22412	-3.13563	-5.10667
Pettit Performance Index - PPI_{T+N}	7/61-6/66											
	ALL β	1.00252	1.00531	1.02170	1.01924	1.02720	1.03988	1.05788	1.05619	1.05535	1.05416	1.06571
	LOW β	1.00566	1.04526	.96550	.96923	.95189	.92936	.91968	.91986	.88767	.82367	.78063
	MED β	1.00162	.99364	1.01661	1.00994	1.01580	1.02714	1.02803	1.03032	1.03908	1.03672	1.05262
	HIGH β	1.00027	.97702	1.06300	1.07854	1.11391	1.16316	1.22594	1.21841	1.23932	1.30210	1.36389
	7/54-6/59											
	ALL β	.99847	1.00676	1.00880	1.01082	1.00902	1.01382	1.01347	1.00727	1.00841	1.00894	1.00670
	LOW β	1.00294	1.01293	1.01453	1.01607	1.02292	1.02341	1.02354	1.01602	1.03191	1.04065	1.03580
	MED β	.99630	1.00537	1.01220	1.01323	1.01045	1.00478	.99935	.99330	.98740	.98621	1.00404
	HIGH β	.99542	1.00095	.99874	1.00227	.99138	1.01168	1.01586	1.01102	1.00200	.99468	.97542
	7/47-6/52											
	ALL β	1.00028	1.00074	.99654	.99500	.99681	.99447	.98643	.98016	.97067	.96400	.95891
	LOW β	.99997	.98296	.95628	.95487	.98962	1.01210	.97983	.95990	.96484	.96037	.95217
	MED β	.99998	1.01356	1.00040	.96614	.94944	.95926	.93597	.93819	.93671	.93666	.91517
	HIGH β	1.00096	1.00926	1.04099	1.07202	1.05281	1.00853	1.04480	1.04643	1.01163	.99571	1.01075
	7/40-6/45											
	ALL β	.99549	.98138	.98962	.97552	.99135	.98279	1.02335	.99852	.99802	.98459	.97854
	LOW β	1.00552	.98934	.97299	.96495	.94278	.97403	.99519	1.01615	1.02530	1.02536	1.02633
	MED β	.99463	.97904	.99266	.99333	.98948	.97489	.99532	.96911	.96386	.96073	.95268
	HIGH β	.98404	.97436	1.00661	.96648	1.05439	1.00362	1.09361	1.01324	1.00663	.96346	.95114

*For the all Beta group there were 21 forty firm portfolios in the 7/61 to 6/66 period, 19 in the 7/54 to 6/59 period, 16 in the 7/47 to 6/52 period, and 13 in the 7/40 to 6/45 period. The portfolios were divided equally for the high, medium, and low β classes.

portfolios are given in Table 4. The portfolios were divided into three different classes of equal size according to their estimated beta coefficient (high $\hat{\beta}$, medium $\hat{\beta}$, and low $\hat{\beta}$). All the grouping procedures used to eliminate the error-in-variables problems described in the preceding section have been employed in this section. Note that the indexes have been computed assuming the validity of the market model.

Results

Generally, the results cast doubt on the efficacy of indexing techniques that rely on the validity of the market model (similar results were obtained when the validity of the CAPM was assumed). First, there are large differences between the expected values and the actual values of the indexes. The expected values of the BBPI and the PPI are unity, and the expected values of the PE and the FPI are zero. The practical value of each of these indexes depends upon the property that large, random samples of common stock should result in realized values of each index that are close to the expected values. However, examination of Table 4 reveals serious drifts in each of the index values in each time period. Consider the most recently used of the indexes, the BBPI. In period 5, 7/61-6/66, it had achieved a value of 1.02850 at 6/62, 1.05060, at 6/63, 1.12854 at 6/64, etc. Although it is difficult to test the statistical significance of these drifts away from unity, they can be quite large for periods beyond six months.

Another peculiar, and more serious, problem is that the drifts in the different index values are contradictory. During the period 7/54 to 6/59 the BBPI had achieved a value of 1.12998. This fact implies that the 21 forty firm portfolios securities in our sample has achieved superior returns over the period. In contrast, the FPI had achieved a value of -.01787 for the same set of securities and the same period--implying inferior returns. Similar inconsistencies appeared between PPI, FPI and BBPI in particular intervals in each of the periods.

Finally, we find the seriousness of these drifts is a function of the level of the beta estimate. For example, the low beta portfolios (seven portfolios of 40 firms each) had achieved a value of .86403 during the period 7/61-6/66 on the BBPI; whereas, the high beta portfolios had achieved an index value of 1.81902. In other periods the direction of the differential effects was reversed. Clearly, the farther the beta of a particular group of securities from the beta of the market as a whole the greater the problem of using indexing procedures of the type being analyzed. These results are consistent with those given in Table 2 and strongly imply that the measurement of investment performance may give completely erroneous results when the groups being studied have different risk characteristics than the overall market.

The variability of the values shown in Table 4 can be easily explained by appealing to the two-factor model described completely in Section II. The correlation of the changes in the performance values for high and low beta portfolios in the final 84 month period, 7/61 to 6/68, for each of the indexes (using both the MM and CAPM) varied between -.85 and -.75. These correlations show that, whatever is causing the change in the performance values, the effect is different depending on the value of the portfolios beta coefficient. This result, so far as we can determine, is only consistent with a two (or more)-factor model.

It can reasonably be asked whether the assessment of investment performance is improved by employing the market model or the capital asset pricing model to adjust for market-wide effects. Although our study is not conclusive, the evidence presented does cast serious doubt upon the overall effectiveness of the several performance indexes that have been relied upon in recent studies. At a minimum, we conclude, our results suggest that any use of an index such as the BBPI, PPI or FPI on a particular class or group of securities should be accompanied by index values of a random group of securities calculated over equivalent time periods.

Conclusions

This paper has developed statistical methods in order to assess the validity of the *ex post* capital asset pricing model and market model as representing stationary, stochastic return generating processes. To test these models we developed cross-sectional regressions of actual security returns on their conditional predicted values. The conditional predicted security returns were constructed under the assumption of the validity of the MM and the *ex post* CAPM. In addition, we examined the validity of the most widely used application of the MM, abstracting from market returns in studies of abnormal investment performance. In both parts of the empirical analysis our examinations were done using NYSE securities and monthly return data. We employed a grouping technique that eliminates an ordering bias that causes successive beta estimates to regress towards unity.

The principal findings of the study are: (1) the predictions from the *ex post* CAPM and MM include a bias towards zero in providing estimates of actual asset returns for portfolios of common stocks over successive periods on one month. Thus high risk portfolios perform better than expected when the market return is lower than the risk-free return and better than expected when the market return is higher than risk-free returns. The opposite is true for low risk portfolios.

(2) The conditional predictions of the MM and CAPM provide non-stationary, biased estimates of actual returns. The relationship between predicted portfolio returns and actual portfolio returns fluctuates from sample period to sample period. These non-stationarities in the estimates (i.e., in $\hat{a}_i$) suggests a breakdown in the diagonal assumption of the MM and *ex post* CAPM. We hypothesize that a substantial part of the non-stationarity in the estimates is traceable to non-zero realized values of additional market-wide factors. In particular, our evidence is consistent with a two-factor model suggested by Rie [18] and supported by Pettit and Westerfield [17].

(3) The single-factor market model does not properly adjust for market-wide effects in assessing security performance. Performance indexes that depend upon the validity of the market model have serious biases that appear to be a function of the level of risk of the particular portfolio being studied. As a minimum such applications of the MM and *ex post* CAPM should present index values for a randomly selected sample of firms with comparable betas over comparable time periods.

FOOTNOTES

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[1] Empirical studies that have made use of the market model or the capital asset pricing model to formulate expected returns include Ball and Brown [1], Fama, Fisher, Jensen and Roll [6], Pettit [16], and Scholes [15]. All of these studies were specific attempts to judge stock market efficiency. An example of an application of the market model to security selection is a manual, TMA developed by Thomson and McKennon Auchincloss, Inc. (members of the New York Stock Exchange). See, also, Security Risk-Evaluation Service and Security Risk Evaluation: Beta Coefficients, Merrill Lynch, Pierce, Fenner and Smith, Inc. Securities Research Division, March 1972.

[2] Black-Jensen-Scholes conclude that there are two common factors determining security returns. It follows that one would want to take into account the second common factor in developing conditional expectations. However, Blume and Friend [6] assert that Black-Jensen-Scholes have misinterpreted the magnitude of the second factor; whereas Rie [18] and Pettit-Westerfield [17] argue that Black-Jensen-Scholes have misspecified the *ex post* return generating equation entirely. The Rie and Pettit and Westerfield results suggest the existence of a second market factor that is difficult to measure and thus difficult to take into consideration its effect.

[3] Unbiased estimates of β_i and ε_{it} are obtained from the regression model

$$R_{it} - R_{Ft} = \hat{\alpha}_i + \hat{\beta}_i (R_{mt} - R_{Ft}) + \varepsilon_{it}.$$

The intercept term $\hat{\alpha}_i$ is not constrained to its expected value of zero so that $\hat{\beta}_i$ can be estimated relatively free of the influence of unique factors associated with the asset i during the estimation period. R_{Ft} in this study is estimated using thirty day (approximately) Treasury Bill returns. It is interesting to point out that our results were completely independent of the value of R_{Ft} used. When a long term rate (approximately) was used instead none of our results or conclusions were significantly changed.

[4] For a discussion of this problem see Jump [11] and Black [2].

[5] We realize that expecting the market model to be a good predictor 84 months past the end of the estimation period is demanding of the model. However, our results suggest that the model does no better or no worse in the last few months than in the first few months of the estimation period.

[6] The mean coefficients do not tend to increase with portfolio size which suggests, given the way the portfolios were constructed, that the measurement error in the estimated returns (i.e., the errors in estimating beta coefficients) is not biasing the coefficient, $\hat{a}_1$, downward.

[7] For example, in the low market return quartile for forty firm portfolios over the five periods, there were 62 negative T-values averaging -2.2, and 7 positive T-values averaging 1.3 out of 80 observations.

[8] We refer to equation (12) as the multi-period prediction error only in the sense that if and only if $PE_{i,T+N} \geq 0$ the i^{th} asset has achieved returns greater than or equal to those predicted. There are probably more desirable specifications of the prediction error from an empirical standpoint. One important property of $PE_{i,T+N}$ is that changes in it over time between assets depend upon the levels of the prediction $\hat{R}_{i,T+N}$.

[9] A mathematical expansion of $\sum_{\tau=T}^{N} (1 + U_{i\tau})$ reveals a series of cross-product terms. There doesn't appear to be any obvious way of summarizing the cross-product terms that result. However, for the two-period case, they are, after rearranging terms:

$$\prod_{\tau=T+1}^{T+2} (1 + U_{i\tau}),$$

let $\hat{\alpha}_i$ equal $(1 - \beta_i)R_F$, and after rearranging terms,

$$BBPI_{T+2} = \prod_{\tau=T+1}^{T+2} R_i - \prod_{\tau=T+1}^{T+2} (\hat{\alpha}_i + \hat{\beta}_i R_M) - (\hat{\alpha}_i + \hat{\beta}_i R_{M,T+1} - 1)$$

$$U_{i,T+2} - (\hat{\alpha}_i + \hat{\beta}_i R_{M,T+1} - 1)\, U_{i,T+2} \qquad \text{(A)}$$

or

$$BBPI_{T+2} = PE_{i,T+2} - (\hat{\alpha}_i + \hat{\beta}_i R_{M,T+1} - 1)\, U_{i,T+2} -$$

$$(\hat{\alpha}_i + \hat{\beta}_i R_{M,T+2} - 1)\, U_{i,T+1} + 1 \qquad \text{(B)}$$

The first two terms of the expression (A) are the prediction error of equation (8) and the whole expression is equal to the Ball and Brown

Performance Index. From equation (8) we can make the following generalizations: (1) If the expected *ex post* return, $\hat{\alpha}_i + \hat{\beta}_i R_m$, is on the average greater than (less than) unity over the period studies, and if the expected value of $U_{i\tau}$ is greater than (less than) zero over the period, the Ball and Brown Performance Index will be less than one plus the multiperiod prediction error given by equation (8); (2) If the expected *ex post* return $\hat{\alpha}_i + \hat{\beta}_i R_{m\tau}$ is greater than (less than) unity, the multiperiod prediction error, $PE_{i,T+N}$, will have a higher (lower) variance than the Ball and Brown Performance Index.

[10]Using the fact that $\ln(1 + U) \cong U$, it can be shown that

$$\ln BBPI_{T+N} \cong FPI_{T+N}$$

for each i^{th} portfolio. Natural logarithms of the observed end of period abnormal returns are the "nominal" abnormal returns given any particular differencing interval. Therefore, specification of the market model in natural logarithms should generally be superior to other specifications because the model will retain its linearity property regardless of the differencing interval actually used in observing returns. Note that natural logarithms imply an instantaneous compounding interval for returns. In addition, since the logarithmic specification is additive over successive compounding intervals, no cross-product terms arise in the computation of the Fama Performance Index.

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