

Price, Beta, and Exchange Listing

by

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I. INTRODUCTION

Recent papers [4], [7] have highlighted the central importance to many problems in finance of correctly specifying the stochastic process underlying observed returns on financial assets. Such problems include the testing of the capital asset pricing model, the evaluation of investment performance, and the selection of portfolios of risky assets. This current paper presents some new empirical results which besides being interesting in their own right, may prove useful in formulating specific return generating functions for common stocks.

The paper first shows that the price per share of a stock appears to be related to future returns even if risk as often measured is held constant. Price seems to be acting in part as an indicator of changes in the levels of risk for individual securities. In addition, there is some weak evidence that price may be a surrogate for transaction costs in the thirties.

At least one reason why price may indicate a change in future beta is that institutional factors may produce a positive correlation between price and previous rates of return. Theoretical considerations lead to the hypothesis that low previous returns might often be associated with changes in future betas, and the subsequent empirical work supports this hypothesis. This finding should allow the development of more accurate ways to assess future betas.

A more traditional way to organize this paper would have been to begin with a theoretical development of the hypotheses that historical rates of return may sometimes foreshadow changes in future betas and that stocks with higher transaction costs should yield a somewhat higher gross expected return. The empirical analyses supporting the validity of these hypotheses would then follow. These hypotheses however were developed in an attempt to explain some peculiar phenomena observed in some of our previous research. To enable the reader to determine which data were instrumental in formulating the hypotheses and which data were analyzed afterwards the empirical and theoretical analyses are presented for the most part in the chronological order in which they were done.

Finally, the paper shows that for equivalent risks the returns of stocks on the American Stock Exchange can differ by substantial amounts, either negative or positive, from the returns on the New York: an half a percent differential per month for a year or more at a time does not appear exceptional. This paper however does not examine the economic rationale for such differentials.

II. THE PRELIMINARIES

Since the articles by Clendenin in 1951 [5] and then by Heins and Allison in 1966 [10], it has generally been accepted that the price of a share of a security -- if it bore any relationship to future returns -- was only acting as a surrogate for cyclical risk.

Under the assumption that price measures risk and using data for common stocks and warrants listed on the American Stock Exchange, we constructed monthly indices for different levels of price beginning in July 1962.¹ The indices displayed the expected characteristics that lower price indices were more volatile than the higher price indices.

To assess how adequately price measured beta, the correlation coefficient between price and historically estimated beta² was calculated for American-listed stocks for each of the years from 1964 through 1968. These correlations were unexpectedly close to zero ranging from -0.08 in 1964 to 0.15 in 1968, and only the 1968 correlation was significant at the five percent level.³

To explore further the relationship among price, beta, and subsequent returns, each stock was classified into one of 24 portfolios according to its 1968 year-end price and its historically estimated beta. Table 1 gives various statistics for these portfolios as well as a similar tabulation for the NYSE. A visual examination suggests that price at the end of 1968 is positively related to realized returns in 1969 and appears to explain more of the variability in these returns than beta. Similar analyses for 1967 and 1968 again pointed to a strong relationship between price and future returns although the direction was reversed. In 1966, neither price nor beta appeared to explain much of the returns. In 1965, beta was more important on the New York, while price was more important

TABLE 1

Annual Returns in 1969 as a Function of 1968 Year-End Price and the

Historically Estimated Beta

Beta	New York Stock Exchange				American Stock Exchange				
	0-10	10-20	20-50	All	0-10	10-20	20-50	All	
0.4-down	Return	-16.5%	-20.7%	-18.7%	-19.7%	-47.7%	-37.1%	-32.5%	-33.7%
	Beta	0.23	0.25	0.19	0.24	-0.14	0.12	0.18	0.07
	Price	\$16.23	\$31.09	\$71.75	\$38.24	\$ 6.81	\$13.85	\$32.66	\$67.08
	Number	0	8	41	14	21	32	22	5
0.4-0.8	Return	-52.2%	-20.6%	-15.6%	-14.5%	-40.1%	-27.0%	-19.4%	- 3.1%
	Beta	0.52	0.63	0.62	0.63	0.56	0.60	0.62	0.65
	Price	\$ 8.63	\$15.75	\$33.64	\$42.34	\$ 7.93	\$13.93	\$31.24	\$79.03
	Number	1	10	160	227	12	44	48	5
0.8-1.2	Return	- 7.9%	-35.4%	-21.2%	-17.2%	-51.5%	-30.2%	-29.8%	-14.9%
	Beta	0.85	1.01	1.00	1.00	1.02	0.99	1.00	1.02
	Price	\$ 7.63	\$17.03	\$35.89	\$47.24	\$ 7.88	\$14.00	\$29.85	\$91.96
	Number	1	19	211	340	14	61	60	10
1.2-1.6	Return	-50.7%	-38.0%	-21.5%	-18.9%	-43.8%	-41.3%	-32.1%	-26.5%
	Beta	1.57	1.36	1.40	1.39	1.39	1.40	1.40	1.36
	Price	\$ 8.69	\$15.51	\$35.09	\$45.67	\$ 8.24	\$14.95	\$30.79	\$74.01
	Number	2	13	167	270	17	45	67	12
1.6-2.0	Return	-39.9%	-26.9%	-15.5%	-25.9%	-53.4%	-39.4%	-20.1%	-32.0%
	Beta	1.76	1.78	1.74	1.76	1.72	1.76	1.81	1.84
	Price	\$15.84	\$33.43	\$67.35	\$39.50	\$ 8.21	\$14.98	\$30.78	\$58.50
	Number	0	18	72	121	6	40	54	7
2.0-up	Return	-35.1%	-36.0%	-24.9%	-32.7%	-58.1%	-33.0%	-30.7%	- 8.2%
	Beta	2.27	2.27	2.30	2.28	2.34	2.57	2.47	2.67
	Price	\$17.00	\$34.87	\$75.29	\$44.12	\$ 8.28	\$15.08	\$32.34	\$72.70
	Number	0	9	43	74	11	51	70	20
All	Return	-40.4%	-32.9%	-21.5%	-19.2%	-48.3%	-34.1%	-27.4%	-17.9%
	Beta	1.12	1.26	1.12	1.14	0.96	1.30	1.43	1.63
	Price	\$ 8.41	\$16.24	\$34.58	\$44.25	\$ 7.77	\$14.47	\$31.14	\$74.61
	Number	4	77	694	1095	81	273	321	59

Note: The percentage change in the value of these portfolios was calculated on the assumption of an equal initial investment in each stock. Any stock delisted in 1969 was assumed sold at the last available price. The proceeds were then invested in the remaining stocks in proportion to their current weight in the portfolio.

on the American. Nonetheless, price on both exchanges in this year was negatively related to realized returns. Since price, at least on the surface, often appeared to explain relatively more of the variability of future returns, one could argue that price is in some sense a better predictor of future returns than the historically estimated beta. These results of an heuristic and admittedly non-rigorous analysis coupled with the availability of more extensive data bases and more highly developed econometric tools than were available to earlier researchers suggested that it might be worthwhile to re-examine the role played by price in the generation of returns on common stocks.

Price might add to the information contained in the historically estimated beta about future returns in one or both of two ways: First, if past estimated betas are measured with error and if prices are correlated with the underlying betas, price should augment the capability of past estimated betas to explain future returns. Market folklore would argue that the relationship, if it exists, should be negative. Second, price may convey additional information about future returns if, for at least one reason to be discussed in Section IV, future values of beta tend to differ systematically from past values as a function of price. If this last point were valid, the previous results would mean that betas for low priced stocks would tend to increase in the future. Moreover, there may eventually evolve a negative correlation between price and beta.⁴

In addition, for institutional reasons, price itself might play an independent role in generating returns. Transaction costs are larger for low priced stocks than for high priced stocks: Commissions as a percentage of the price of a share traded are generally larger, and [8] gives some evidence that the bid-ask spread as a percentage of price is negatively related to price. In an efficient market, these differential transaction costs should result in higher expected rates of return in all types of markets for low-priced stocks. Even though percentage costs are a decreasing function of price, the commission schedules and spreads are such that only for extremely low priced stocks are these costs substantially greater than for other stocks.⁵ Thus, unless there are a large number of extremely low priced stocks in a sample, variability in other factors may swamp the transaction cost effect.

III. NEW EVIDENCE ON PRICE EFFECTS

The following analyses use some recently developed analytical techniques⁶ to document the statistical relationship of price to realized returns and beta. Due to data limitations, the analyses in this section are performed only for NYSE common stocks.⁷ The first step is to minimize the deleterious effects of measurement errors by assigning each stock to a group according to some specified criterion. Subsequently, one utilizes the group averages of the variables of interest rather than the original values of those variables. This averaging tends to reduce the measurement errors in the observed values, but it will not eliminate systematic effects.⁸

Following this procedure, stocks were assigned to groups or portfolios for every month from 1932 through 1971 on the basis of the previous month-end price and the beta coefficient estimated over the previous sixty months, hereafter denoted by β_{-1} .⁹ Specifically, stocks were classified into twenty-five portfolios according to the Cartesian product of the quintile ranges of these two variables in a manner analogous to Table 1.¹⁰ The average price, beta, and monthly return adjusted for dividends for each group and for each month will be used in the following analyses.

The use of the same estimated values of a variable both to place securities into portfolios and to calculate the average values would typically induce a bias into these averages. The larger values of these estimated averages would tend to overstate the true values, while the smaller values would tend to understate the true values. This bias, frequently called an order bias, will occur if the variables are measured with the usual type of normally and independently distributed measurement error.

By the way in which the portfolios were formed, the high beta portfolios intuitively are more likely to contain securities with positive as opposed to negative measurement errors in beta. The averages of these estimated betas would thus be expected to overstate the true average. The reverse would apply to low beta portfolios. There should be virtually no order bias associated with price.

A popular procedure for minimizing order bias involves using one estimate of beta to form portfolios and another estimate from different data to calculate the average. Another estimate of the beta, hereafter denoted by β_{-2} , was calculated over the sixty months prior to the sixty months from which β_{-1} had been estimated. Following the procedure outlined above, a new set of portfolios was then constructed using β_{-2} as the classifier and β_{-1} as the historical value of beta. This second set was available from 1936 on.¹¹

In addition to the estimate of beta provided by β_{-1} , an estimate of the future value of beta for each portfolio was calculated using the sixty future average monthly rates of returns from the similarly classified portfolios.¹² This estimate of beta, designated β_{+1} , can be interpreted as an estimate of the ex post beta if the same quintiles of price and beta have stationary underlying betas. Further, β_{+1} may more accurately mirror investors' ex ante expectations if investors base their assessments on more than a mere extrapolation of past betas.

An obvious way to examine the importance of price, which parallels the spirit of earlier tests [10], is to regress monthly percentage returns on an estimate of beta and previous month-end price. Such regressions were calculated separately¹³ for each month for each of the two sets of portfolios using the group averages as the variables. Beta was first measured by β_{-1} and then by β_{+1} . Following the format of [7], these cross-sectional regressions can be summarized by the time series averages of the constant, the coefficients on beta and on price, and the coefficients of determination.¹⁴ For instance, for the portfolios selected by β_{-1} and using β_{-1} as the measure of beta, the average coefficient on price of the monthly regressions from 1932 through 1966 was -0.0195%. The sample standard deviation of this average was 0.0063% implying a t-value of -3.1. This last statistic is frequently used to test whether the expected value of the process generating the time series of coefficients differs significantly from zero.¹⁵ The corresponding average using β_{+1} was -0.0125% with a t-value of -1.9.¹⁶

Due to the complexities of interpreting the average coefficients on price because of possible multicollinearities with beta, the paper will present no detailed discussion of these regressions. A more satisfactory analysis will be presented below. Nonetheless, these regressions indicate that price per share is an important variable at least statistically in explaining future returns. For the overall period, 1932 through 1966, for the portfolios selected by β_{-1} and 1936 through 1966 for the other set, monthly returns measured in percentages are positively related to either measure of beta and negatively to price. Only the relationship to price however is on average significantly different from zero. Breaking the overall periods into three subperiods, the results are the same for the first two subperiods ending April 1955, except that in the 1943 through 1955 subperiod, price loses its statistical significance. In the last subperiod, 1955 through 1966, the t-values are so extremely close to zero -- particularly for those regressions using β_{+1} to measure beta -- that any interpretation of the signs of the averages would be dubious at best.

The average coefficients on price of both sets of portfolios for the overall periods, though always negative, are smaller in absolute value using β_{+1} than using β_{-1} to measure beta. The same behavior is observed for the first subperiods ending in 1943, while the results are mixed for the second two subperiods. For the portfolios selected by β_{-2} , the average coefficients on beta are positive for the overall period and the first two subperiods ending in 1955 but are greater using β_{+1} than using β_{-1} . Because of the order bias associated with portfolios selected by β_{-1} , it is inappropriate in the case of these portfolios to compare the levels of the average coefficients on β_{+1} with those on β_{-1} .

There are numerous ways to explain the changes in the average coefficients in using β_{+1} rather than β_{-1} to measure beta. For instance, if the variance of the error¹⁷ in measuring the true beta is smaller with β_{+1} than with β_{-1} , these movements are consistent with the proposition that price indicates a change in beta which is picked up by β_{+1} .¹⁸ Rather than dwell on possible explanations, the text now turns to a more satisfactory analysis.

The next analysis regressed separately for each month and for each type of portfolio the twenty-five portfolio returns upon one of the estimates of beta. The residuals from these regressions were in turn regressed upon the corresponding average price for the portfolio measured as a deviation from the average cross-sectional price. As pointed out in footnote 16, the order bias should not greatly affect the explanatory power of the first stage, so that the residuals for the portfolios selected by β_{-1} should be unaffected. With the exception of a column headed "modified c" to be discussed below, Table 2 summarizes these monthly regressions with the same type of statistics as those already discussed.

The coefficients in the second stage of these cross-sectional regressions yield to a simple and intuitive interpretation.¹⁹ At the outset, note that if beta is measured with no error and if price has no independent effect, the coefficient in the second stage would be zero even if price and beta were perfectly negatively correlated.

Now consider the case in which the underlying betas are stationary but measured with error, in which price is negatively correlated with beta but not with the error, and in which price has no independent effect. If realized returns were positively related to the true beta, the coefficient on price would tend to be negative. The reason is that the measurement error in beta would tend to produce an understatement of the slope coefficient and an overstatement of the intercept. Thus, the residual errors for high beta stocks would tend to be positive and for low beta stocks negative resulting in a negative coefficient on price in the second stage. Likewise, if the realized returns in a cross-section were negatively related to beta, the coefficient on price would tend to be positive. If on average realized returns are positively related to beta because of risk aversion, the average coefficient on price would tend to be negative.

Table 2

Summary of Cross-Sectional Regressions of the Form

$$R_{it} = a + b\beta_{it} + \epsilon_{it}, \epsilon_{it} = c(P - \bar{P}) + \mu_{it}$$

Time Period	Beta in Regression Defined as	Portfolios Selected by β_{-1}				Time Period	Beta in Regression Defined as	Portfolios Selected by β_{-2}					
		a	b	c	Avg. R^2			First Stage Average Regression Coefficients (t-values)	a	b	c	Avg. R^2	Second Stage Average Regression Coefficients (t-values)
/32-12/66	β_{-1}	1.074 (5.2)	0.5654 (1.8)	-0.0144 (-2.9)	0.35	1/36-12/66	β_{-1}	0.547 (2.7)	0.7074 (2.0)	-0.0020 (-1.3)	0.32	-0.0056 (-3.9)	0.10
	β_{+1}	0.567 (2.4)	1.0716 (2.4)	-0.0051 (-3.0)	0.35		β_{+1}	0.510 (2.2)	0.7487 (2.0)	-0.0014 (-1.2)	0.30	-0.0050 (-4.1)	0.10
/32- 8/43	β_{-1}	1.085 (2.0)	1.2987 (1.5)	-0.0410 (-2.9)	0.35	1/36- 8/43	β_{-1}	-0.136 (-0.2)	1.3253 (1.1)	-0.0092 (-2.3)	0.34	-0.0148 (-3.8)	0.05
	β_{+1}	-0.234 (-0.4)	2.6152 (2.2)	-0.0142 (-3.7)	0.36		β_{+1}	-0.220 (-0.3)	1.4208 (1.3)	-0.0060 (-2.2)	0.34	-0.0093 (-3.6)	0.04
/43- 4/55	β_{-1}	1.141 (5.3)	0.3963 (1.3)	-0.0017 (-0.7)	0.37	9/43- 4/55	β_{-1}	0.924 (3.9)	0.6059 (1.6)	0.0000 (0.0)	0.33	-0.0049 (-2.8)	0.09
	β_{+1}	0.870 (3.5)	0.6709 (1.5)	-0.0006 (-0.3)	0.38		β_{+1}	0.736 (2.6)	0.7933 (1.6)	0.0001 (0.1)	0.32	-0.0078 (-4.2)	0.10
/55-12/66	β_{-1}	0.996 (5.3)	0.0011 (0.0)	-0.0006 (-0.2)	0.33	5/55-12/66	β_{-1}	0.618 (2.4)	0.4030 (1.1)	0.0008 (0.4)	0.28	-0.0003 (-0.1)	0.15
	β_{+1}	1.064 (4.5)	-0.0710 (-0.2)	-0.0005 (-0.2)	0.31		β_{+1}	0.764 (2.4)	0.2624 (0.6)	0.0000 (0.0)	0.26	0.0006 (0.3)	0.13

Note: Although the format suggests that the numbers in this table are the output of a standard regression analysis, the numbers actually represent summaries of cross-sectional regressions run separately for each month. For example, the number 1.074 in the upper left is an average of 420 regression coefficients, one for each monthly regression. The t-value is calculated by dividing 1.074 by the standard error of the average calculated from the 420 coefficients. For a further discussion, the reader is referred to the text.

Similarly, if price indicates that the measured beta tends to be smaller than the true beta, it can be shown that the coefficient on price in the second stage would also tend to be negative if realized returns were positively correlated to the true betas. If however realized returns were negatively correlated to the true betas, the coefficient on price would tend to be negative. On average, one would expect a negative coefficient from this source due to risk aversion. Any independent effect price has on returns would merely be added to the bias effects associated with price.

The first stage regressions presented in Table 2 are interesting in their own right in that they can be compared to many of the tests of the capital asset pricing model, e.g., [1], [4], and [7].²⁰ The average coefficients on price in the second stage are almost always negative and for the period ending in 1943 significantly different from zero. For the overall period, the average coefficients on price are significantly different from zero for the portfolios selected by β_{-1} but not by β_{-2} . Such negative coefficients mean that on average realized rates of return are negatively related to price. Such a relationship however is consistent with a transaction cost effect as well as the possibility that price is acting in some way as a surrogate for the true beta.

If the underlying betas were stationary over time, there is no reason to believe that the average coefficients on price should differ whether β_{-1} or β_{+1} is used to measure the true beta. Both estimates are subject to the same grouping techniques to minimize measurement error. Yet, with only an occasional exception, the average coefficient on price is less negative with β_{+1} than with β_{-1} . Such a consistent tendency suggests that beta does change over time as a function of price.

To distinguish between the transaction cost effect and the effects due to errors in measuring beta, a modified average price coefficient was calculated by multiplying each of the coefficients in the second stage by the sign of the slope coefficient in the first stage. An average of such modified coefficients would tend to be less negative if the transaction cost effect dominated the bias effects and more negative if the reverse were true. Only in the 1932 through 1943 subperiod using β_{+1}

is there any suggestion in the empirical results that transaction costs may dominate the bias effects. However, it was during this period and particularly during 1932 through 1934 when the abundance of low price stocks²¹ made differential transaction costs more important.²² For the overall period and the other subperiods, transaction cost effects appear to be less important than the informational effects of price in explaining future returns.

That the average coefficients differ on the second stage whether a prior or a future beta is used in the first stage suggests quite strongly that price is an indicator of changes in beta. To provide a picture of the magnitude of these price-associated changes in beta, Table 3 presents estimated values of β_{+1} for the portfolios selected by β_{-1} for every sixtieth month beginning with January 1932. With the exception of 1942 and, to a lesser extent, 1947 and 1952, the estimated values of β_{+1} for any five portfolios constructed from securities falling in the same quintile of prior estimated betas, β_{-1} , tend to be a function of price in the anticipated direction. Thus, for the first month in the table, the values of β_{+1} for the five portfolios from the lowest quintile of β_{-1} range from 1.04 for the lowest priced stocks to 0.36 for the highest. Similar results were obtained for the portfolios selected by β_{-2} , but are not presented because of space limitations.

IV. PRICE AND HISTORICAL RATES OF RETURN

The previous analyses have shown that price is an important variable, at least statistically, in explaining returns or future betas. To this point, however, we have not suggested any justification for this apparent ability of price to explain returns except for transaction costs which found only a modicum of support in the thirties. This section proposes an economic hypothesis of why price might help to explain future returns or betas and finds some empirical support for this hypothesis.

Table 3

Future Estimated Values of Beta Classified by Prior Price and Previously Estimated Betas

Date	β_{-1} Quintile	Average Value of β_{+1} Price Quintile					Number of Securities Price Quintile				
		Low	2	3	4	High	Low	2	3	4	High
1/32	Low	1.04	0.96	0.73	0.56	0.36	11	11	11	26	55
	2	1.68	1.13	0.96	0.72	0.60	9	11	24	42	42
	3	1.45	1.28	1.07	0.84	0.76	11	30	28	33	21
	4	1.57	1.32	1.11	1.09	0.82	31	35	30	18	8
	High	1.53	1.39	1.33	1.07	0.71	49	42	21	7	4
1/37	Low	0.66	0.61	0.64	0.49	0.38	5	15	21	38	52
	2	0.98	0.84	0.79	0.83	0.74	16	23	34	35	22
	3	1.23	0.99	1.02	0.93	0.90	22	25	27	27	35
	4	1.37	1.34	1.24	1.20	0.95	36	33	24	19	22
	High	1.59	1.46	1.43	1.26	1.12	61	33	20	12	5
1/42	Low	0.88	0.74	0.68	0.60	0.53	5	12	23	46	61
	2	1.04	0.87	0.85	0.70	0.62	18	24	34	27	45
	3	1.30	1.07	0.92	0.84	0.74	26	25	36	34	29
	4	1.40	1.15	0.96	0.92	0.83	39	37	29	22	15
	High	1.90	1.39	1.23	1.04	0.93	56	57	18	16	4
1/47	Low	0.67	0.63	0.54	0.56	0.57	3	5	21	42	83
	2	0.87	0.80	0.86	0.84	0.81	4	25	27	48	52
	3	1.11	0.97	0.98	0.98	0.97	18	34	45	42	24
	4	1.27	1.15	1.13	1.12	1.06	29	43	50	26	13
	High	1.58	1.39	1.35	1.38	1.32	94	45	15	6	1
1/52	Low	0.57	0.58	0.53	0.60	0.56	25	25	36	58	43
	2	0.91	0.84	0.77	0.89	0.91	25	36	31	39	56
	3	0.98	0.98	1.05	1.06	1.06	35	38	31	47	44
	4	1.15	1.05	1.16	1.17	1.23	39	41	41	32	40
	High	1.43	1.39	1.37	1.40	1.45	67	51	37	21	16
1/57	Low	0.71	0.61	0.60	0.53	0.50	37	49	46	37	24
	2	1.02	0.83	0.82	0.76	0.80	27	41	44	51	34
	3	1.17	0.99	1.02	0.86	0.94	45	24	41	46	37
	4	1.23	1.21	1.09	1.06	1.06	34	43	30	35	62
	High	1.58	1.39	1.32	1.29	1.20	49	36	32	33	47
1/62	Low	0.80	0.61	0.62	0.56	0.61	19	31	37	46	62
	2	0.90	0.81	0.81	0.77	0.76	25	37	44	54	43
	3	1.10	0.95	0.96	0.86	0.82	31	27	47	40	52
	4	1.34	1.22	1.07	1.01	1.07	45	44	35	44	34
	High	1.51	1.43	1.34	1.24	1.36	80	44	39	24	18
1/67	Low	0.52	0.53	0.56	0.60	0.58	28	43	55	50	41
	2	0.85	0.84	0.84	0.77	0.79	30	43	47	47	48
	3	1.11	1.06	1.03	0.95	0.82	36	41	49	49	50
	4	1.31	1.19	1.14	1.03	1.03	46	43	42	43	52
	High	1.49	1.49	1.36	1.25	1.24	69	40	29	41	41

After careful consideration, we could think of no persuasive hypothesis of why price itself should foreshadow changes in future betas from those estimated in the past. We therefore asked ourselves whether there was some other variable for which price might be acting as a surrogate. The market folklore that there is an optimal price range at which a stock should trade suggested that a stock might become low priced not because of splits but because of poor historical returns. Indeed, monthly correlations between annual historical returns on individual securities and subsequent prices are on balance positive. From January 1932 to December 1966, 420 months in total, 350 months showed positive correlations. The majority of the negative correlations occurred from 1942 through 1946.

The simplest type of evaluation model for security prices is the ratio of expected earnings to a risk adjusted discount rate. Consider now a case in which the future discount rate increases from some previous level. Insofar as beta measures risk, the value of beta would tend to increase in the future. If the expected earnings were to increase by the same percentage as the increase in the future discount rate, this model would indicate that the historical rates of return would be neither higher nor lower than normal. A merger with no synergistic effects of two properly priced companies but of different risks provides an example here. If expected earnings were to increase by more than the percentage increase in the discount rate, the adjustment to the new equilibrium would result in greater than normal rates of return. The same merger as above but accompanied by synergisms illustrates this adjustment. If expected earnings were to drop, remain constant, or as a minimum not increase as much as the discount rate, historical rates of return would be lower than normal. This situation would occur if for some reason, investors' assessments of the risk of future earnings increases with no corresponding change in their assessments of expected earnings.

More generally, if the discount rate changes either up or down, and if expected earnings change by the same percentage, the historical rates of return would tend to be what were expected. If earnings change by a greater percentage, the historical rates

of return would tend to be larger than expected. If by a lesser percentage, the historical rates would tend to be smaller than expected. The power of abnormal historical rates of returns to predict changes in future discount rates and thereby future beta centers upon the following: When expected earnings and the discount rate change by different percentages, is it more likely that expected earnings change by a larger or smaller percentage than the changes in the discount rate? The empirical results in the last section would suggest that historical rates of return lower than expected would on average foreshadow increases in future betas. The paper now examines this empirical question.

The empirical results in Table 4 confirm that an extremely simple measure of historical rates of return, which does not even adjust for normal market movements, is an indicator of changes in future levels of beta. The format of this table is identical to Table 2, except that historical rates of return have replaced price. These historical rates of returns are measured arbitrarily by the percentage changes in value adjusted for capital changes and dividends over the previous twelve months.²³

If low historical rates of return are associated with increases in beta, the logic of the previous section suggests that the unmodified average coefficients in the second stage should be negative using β_{-1} . Moreover, the modified average coefficients should be even more negative. However, if historical rates of return act only as an indicator of change in beta, the average or modified coefficient on historical rates of return should be insignificantly different from zero using β_{+1} .

The results in Table 4 are consistent with the interpretation in at least two important ways. First, the average R^2 's in the second stage are higher than those for the second stage using price and this in spite of the fact that there are little systematic differences among the average R^2 's in the first stage regressions in the two tables.²⁴ Second, in all but the latest subperiod the modified average coefficients are negative and significant using β_{-1} to measure beta but, although still negative using β_{+1} , are usually smaller in absolute value and insignificantly different from zero. This would suggest that historical rates are associated with changes in the underlying beta in conformity with economic theory.

Table 4

Summary of the Cross-Sectional Regressions of the Form

$$R_{it} = a + b\beta_{it} + \epsilon_{it}, \quad \epsilon_{it} = c(H - \bar{H}) + \mu_{it}$$

		Portfolios Selected by β_{-1}						Portfolios Selected by β_{-2}											
Time Period	Beta Regression Defined as	First Stage Average Regression Coefficients (t-values)		Avg. R^2	Time Period	Beta in Regression Defined as	First Stage Average Regression Coefficients (t-values)		Avg. R^2	Time Period	Beta in Regression Defined as	Second Stage Average Regression Coefficients (t-values)		Avg. R^2					
		a	b				a	b				a	b	c					
1/32-12/66	β_{-1}	1.098 (5.3)	0.5437 (1.8)	0.35	1/36-12/66	β_{-1}	0.740 (4.2)	0.5194 (1.5)	0.31	1/36-8/43	β_{-1}	0.237 (0.5)	0.9753 (0.9)	0.34					
	β_{+1}	0.593 (2.5)	1.0422 (2.2)	0.34		β_{+1}	0.667 (3.1)	0.5908 (1.6)	0.25		β_{+1}	-0.028 (-0.1)	1.2284 (1.1)	0.27	0.2571 (1.4)	-0.3547 (-1.9)	0.17		
1/32-8/43	β_{-1}	1.158 (2.1)	1.2339 (1.5)	0.37	1/36-8/43	β_{-1}	0.237 (0.5)	0.9753 (0.9)	0.34	9/43-4/55	β_{-1}	1.076 (5.0)	0.4547 (1.2)	0.32	5/55-12/66	β_{-1}	0.735 (2.8)	0.2845 (0.7)	0.28
	β_{+1}	-0.169 (-0.3)	2.5360 (2.0)	0.37		β_{+1}	-0.028 (-0.1)	1.2284 (1.1)	0.27		β_{+1}	0.840 (3.0)	0.6925 (1.4)	0.28		β_{+1}	0.950 (3.2)	0.0701 (0.2)	0.21
9/43-4/55	β_{-1}	1.154 (5.5)	0.3824 (1.2)	0.38	9/43-4/55	β_{-1}	1.076 (5.0)	0.4547 (1.2)	0.32	5/55-12/66	β_{-1}	0.735 (2.8)	0.2845 (0.7)	0.28	5/55-12/66	β_{-1}	0.735 (2.8)	0.2845 (0.7)	0.28
	β_{+1}	0.836 (3.1)	0.7072 (1.4)	0.37		β_{+1}	0.3813 (1.3)	-0.5966 (-2.0)	0.21		β_{+1}	0.4113 (1.5)	-0.1712 (-0.6)	0.21		β_{+1}	0.840 (3.0)	0.6925 (1.4)	0.28
5/55-12/66	β_{-1}	0.982 (5.2)	0.0150 (0.1)	0.31	5/55-12/66	β_{-1}	0.982 (5.2)	0.0150 (0.1)	0.31	5/55-12/66	β_{-1}	0.735 (2.8)	0.2845 (0.7)	0.28	5/55-12/66	β_{-1}	0.735 (2.8)	0.2845 (0.7)	0.28
	β_{+1}	1.112 (4.5)	-0.1164 (-0.3)	0.29		β_{+1}	0.8203 (2.8)	0.5175 (1.7)	0.24		β_{+1}	0.8203 (2.8)	0.5175 (1.7)	0.24		β_{+1}	0.6481 (2.0)	-0.0630 (-0.2)	0.22

Note: Cf. note for Table 2.

The results above imply that the two stage regression techniques can detect changes in beta. Thus, if the average modified coefficient of price in the second stage is significant using a future beta, such a variable may well be conveying information about an effect different from a change in beta. Of the two possible types of information, a surrogate for the underlying beta or for transaction costs, the transaction cost effect is somewhat more supportable since (1) there is at least an economic rationale for such an effect and (2) empirical evidence indicates that price effect is really only important in the thirties when the existence of many low priced stocks would suggest it should be.

As with price, the estimated values of β_{+1} provide a picture of the magnitude of the changes in beta associated with historical returns. These values presented in Table 5 for the same months as for price, display similar behavior. For any quintile of previous beta, the range of changes in beta are sometimes larger with historical returns and sometimes smaller. That the range is not always larger with historical returns suggests that price may be acting as a surrogate for additional variables responsible for changes in beta or that the arbitrary measure of historical rate of return used here does not capture all the information contained in the past sequence of returns.

V. EXCHANGE LISTING

It is sometimes alleged that American-listed stocks behave differently from New York-listed stocks. The portfolios already constructed for the New York coupled with additional ones for the American provide some insight into this proposition. The average coefficients of monthly cross-sectional regressions of the portfolio returns on beta, price and a dummy variable for exchange listing disclosed that from 1965 through 1971, American-listed stocks for the same level of beta and price averaged 0.27 percent per month more than New York-listed stocks. From 1965 through June 1969, the differential was on average 0.51 percent per month in favor of the American, while from February 1970 through 1971, New York-listed stocks outperformed equivalent stocks on the American by 0.82 percent per month.²⁵

Table 5

Future Estimated Values of Beta Classified by Historical Rate of Return and Previously Estimated Betas

Date	β_{-1} Quintile	Average Value of β_{+1} Return Quintile					Number of Securities Return Quintile				
		Low	2	3	4	High	Low	2	3	4	High
1/32	Low	1.06	0.66	0.53	0.48	0.45	9	18	14	27	45
	2	1.38	0.94	0.82	0.71	0.64	15	27	23	32	30
	3	1.46	1.22	1.14	0.89	0.78	19	23	26	34	21
	4	1.57	1.57	1.25	1.00	0.81	33	33	26	15	14
	High	1.72	1.38	1.39	1.25	0.99	35	34	27	14	13
1/37	Low	0.78	0.75	0.59	0.51	0.43	45	37	27	14	8
	2	1.07	0.90	0.82	0.72	0.72	24	23	31	31	21
	3	1.27	1.05	1.01	0.87	0.79	19	33	31	29	24
	4	1.48	1.32	1.14	1.06	1.01	22	16	24	32	40
	High	1.52	1.50	1.35	1.36	1.22	18	23	21	28	41
1/42	Low	0.65	0.60	0.60	0.66	0.70	10	33	37	47	20
	2	0.78	0.69	0.75	0.78	0.88	17	19	36	35	41
	3	1.01	0.88	0.99	0.88	0.99	23	32	29	31	35
	4	0.97	1.06	1.14	1.14	1.12	36	25	23	30	30
	High	1.50	1.46	1.68	1.34	1.71	61	33	22	9	28
1/47	Low	0.64	0.64	0.55	0.62	0.70	1	13	32	55	53
	2	0.88	0.82	0.80	0.81	0.94	7	31	35	44	39
	3	1.10	0.98	0.88	1.00	0.99	29	37	38	33	26
	4	1.25	1.18	1.11	1.09	1.09	39	46	31	21	24
	High	1.62	1.35	1.41	1.42	1.36	73	38	16	16	18
1/52	Low	0.78	0.46	0.57	0.64	0.48	26	53	46	38	24
	2	0.99	0.76	0.83	0.86	0.91	23	43	42	40	39
	3	1.15	0.98	0.98	1.01	1.03	39	42	27	46	41
	4	1.24	1.16	1.04	1.16	1.14	36	30	39	37	51
	High	1.45	1.31	1.43	1.43	1.32	66	33	29	25	39
1/57	Low	1.12	0.74	0.55	0.57	0.64	21	47	66	41	18
	2	1.21	0.91	0.77	0.77	0.86	41	41	49	33	33
	3	1.15	1.03	0.88	0.84	1.07	41	34	30	40	48
	4	1.23	1.15	1.05	1.00	1.11	45	38	30	39	52
	High	1.50	1.28	1.23	1.05	1.38	47	33	30	45	42
1/62	Low	0.77	0.78	0.64	0.60	0.68	24	33	58	50	30
	2	0.92	0.78	0.77	0.78	0.87	32	41	40	49	41
	3	1.12	0.92	0.86	0.87	0.91	45	37	38	45	32
	4	1.22	1.15	1.09	1.05	1.10	41	52	42	29	38
	High	1.47	1.37	1.23	1.33	1.30	52	32	34	32	55
1/67	Low	0.85	0.50	0.55	0.59	0.73	24	42	56	68	27
	2	0.95	0.91	0.79	0.79	0.81	32	50	57	51	24
	3	1.14	1.01	0.93	0.90	0.97	46	46	49	37	47
	4	1.28	1.16	1.12	1.09	1.05	48	46	26	47	58
	High	1.52	1.40	1.30	1.29	1.31	64	36	30	27	63

Although only one of these average monthly differential returns appears to be significantly different from zero, these differentials are nonetheless of a magnitude which would be important to an investor. In view of the question of statistical significance and the rather limited period of analysis, there is little basis for projecting which exchange is likely to show better returns in the future for the same level of beta and price although the differences may well be large.

VI. CONCLUSION

Besides documenting the various roles played by price, by historical rates of return, and by exchange listing in explaining future rates of return, the results in this paper suggest that the return generating function may be considerably more complex than many investigators have previously assumed. In testing the capital asset pricing model or in evaluating investment performance, it is necessary to assume the validity of some return generating function in order to translate the ex ante magnitudes of an equilibrium model into ex post realizations. For any ex ante model and ex post realizations, there is almost certainly some generating function, although possibly devoid of economic meaning, which will link those realizations with the model. It is therefore imperative that the return generating function have economic and empirical content. This paper has attempted to make some modest contribution toward the proper specification of such functions.

FOOTNOTES

*Associate Professor of Finance, University of Pennsylvania, and Donaldson, Lufkin, and Jenrette, Inc., respectively. The authors wish to thank Professors Fisher Black, Eugene Fama, Irwin Friend, Stephen Ross, and Randolph Westerfield for their much appreciated comments, and the Rodney L. White Center for Financial Research for financial support.

¹In constructing indices for different levels of risk, it is desirable to include new listings immediately. The historically estimated beta coefficient would permit the inclusion of a stock in an index only after some period of time following listing. In addition, the data files for the American available to these researchers were limited and the use of price as a measure of risk would allow the construction of indices over a longer period. Finally, the changing characteristics of many companies on the American might make the beta coefficient considerably less stationary than it is on the New York.

²These coefficients were estimated by regressing from 24 to 30 previous monthly relatives upon the New York Stock Exchange Index adjusted for dividends. Virtually all stocks listed over the previous two years were included.

³Similar correlations for the NYSE ranged from -0.06 to 0.01, none of which was significant. The use of betas for individual securities means that no attempt was made to minimize measurement errors, so that the correlation between the true beta and price may be substantially different.

⁴Stock dividends, splits, delistings, and new listings may blunt this tendency.

⁵For instance in 1932, the commission on a fifty cent share was six percent, while on a \$100 share the commission was one fifth of one percent. This pattern of commissions has persisted through the current schedule. Further, Rule 62 of the New York Stock Exchange prescribes except in special cases that the minimum bid-ask spread be 12.5 cents on any stock over \$1, 6.25 cent on any stock priced between 50 cents and \$1, and 3.125 cents on any stock less than 50 cents.

⁶Numerous articles have contributed to the development of these techniques: a few include [1], [2], [3], [4], [7], and [9]. For reasons of space, this article will generally not attempt to attribute to any author the development of any particular econometric technique. Fama and MacBeth [7] give a chronology.

⁷The data files at the Rodney L. White Center for Financial Research currently contain monthly relatives adjusted for dividends for each common stock listed on the NYSE from February 1926 through December 1971. The files contain similar relatives for common stocks and warrants on the AMEX from July 1962 through December 1971.

⁸Insofar as the measurement errors of individual observations are not perfectly correlated, these errors will tend to offset each other so that the error in the average is likely to be of smaller magnitude. More precisely, if successive observations on the same number x are measured with error ϵ_i , if all ϵ_i 's have the same variance σ^2 , and if the correlation between any pair of errors is ρ , the variance of an average of n of these measured values will be $[\sigma^2 + (n - 1)\rho\sigma^2]/n$. As n approaches infinity, the variance of the error will approach $\rho\sigma^2$.

⁹The beta coefficients were estimated by regressing sixty previous monthly returns on the Fisher Combination Link Relatives. To conserve computer time, the beta coefficients were revised only every six months for the January and July portfolios. The January values were used for the first half of each year, and the July values for the second half of each year.

¹⁰If price and beta for individual securities were highly correlated many of the portfolios might contain no securities. In fact, this did not happen. The distribution of securities in Table 1, although based upon a slightly different grouping technique, is typical of other periods. For sixteen months it was possible to form only 24 portfolios; all the other months had 25 portfolios.

¹¹For the first month, the available data permitted only fifty-nine months of data to be used in calculating the beta coefficient used in classifying a security.

¹²The monthly return immediately following the time of classification is included in these sixty months. The reader should also note that since the portfolios are constructed anew each period, it is theoretically possible that none of the stocks contained in a portfolio at the beginning of a sixty month period will be in at the end. Further, any survivorship bias is held to a minimum.

¹³These regressions as well as all other regressions in this paper were weighted by giving each squared deviation in the sum of squares to be minimized a weight proportional to the number of stocks in the portfolio. Such weighting helps to avoid possible distortionary effects associated with a widely disparate number of stocks in the different portfolios. In previous work, securities were classified into portfolios according to only one variable. The number of stocks in each were thus approximately equal, so that there were no distortionary effects.

¹⁴The coefficients of determination were calculated using weighted sums as outlined above.

¹⁵One should however interpret these t-values with caution since, for one reason, there is substantial evidence that normal distributions do not describe the distribution of returns on common stocks as well as other types of distribution such as non-normal stable process [3], [6].

¹⁶Because of the order bias associated with the portfolios selected by β_{-1} , the magnitude of the average coefficient on β_{-1} would be expected to be misstated. Nonetheless, the order bias as it is usually interpreted should not affect the coefficient on price nor the significance level of the average coefficient on β_{-1} . The reason is that if the grouping technique has eliminated extraneous noise in the betas leaving only systematic biases, the value of beta without order bias would be a linear function of the value with order bias. A mathematical proof of this statement follows:

Assume that the true betas are distributed by a stationary normal distribution with mean $E(\beta)$ and standard deviation $\sigma(\beta)$ and that the estimated betas, $\hat{\beta}$, are measured with an independent, normally distributed error, η , the estimated betas will be distributed by a normal distribution with mean $E(\beta)$ and $\sigma(\beta + \eta)$. Further,

assume that Z is distributed as a unit normal variate and let $E(Z|Z_{.i} \leq Z < Z_{.j}) = E_{ij}(Z)$ be the expectation of Z conditional on Z falling between the variates corresponding to the i^{th} and j^{th} fractiles. Then, the expected value of $E(\hat{\beta}|\hat{\beta}_{.i} \leq \hat{\beta} < \hat{\beta}_{.j}) = E_{ij}(\hat{\beta})$ will be $E_{ij}(\hat{\beta}) = E(\beta) + E_{ij}(Z) \sigma(\beta + \eta)$. The portfolio approach associates $E_{ij}(\beta)$ with $E_{ij}(\beta) = E(\beta) + E_{ij}(Z)\sigma(\beta)$. Eliminating $E_{ij}(Z)$ from these two expressions yields

$$E_{ij}(\beta) = E(\beta) \left[1 - \frac{\sigma(\beta)}{\sigma(\beta + \eta)} \right] + \frac{\sigma(\beta)}{\sigma(\beta + \eta)} E_{ij}(\hat{\beta}),$$

which established the linear relationship. The reader should note that this proof does not really require normality, but only that both β and η be distributed by symmetric stable distributions of the same characteristic exponent.

17 The reader should note that the differences between the estimates β_{-1} and the underlying betas appropriate to explaining next month's returns are not measurement errors in the usual sense. It is logically possible for β_{-1} to be measured without error and not reflect the beta for the next month if the underlying betas have changed. If the underlying betas are stationary, there is no need to distinguish between these two types of errors.

18 Such an explanation would be consistent with a model in which β is measured with error η . Assuming that the residuals from the regression of R on the true or estimated beta and price, P , are independent, the conclusion in the text follows if $\rho(R, \beta)$ and $\rho(\eta, P)$ are positive and $\rho(R, P)$ and $\rho(\beta, P)$ are negative.

19 If β is measured without error, the first stage in deviate form is $R = b\beta + \epsilon$ where return R and β are measured from their expected values, and the second stage is $\epsilon = cP + \mu$. If β is measured with error η , the first stage estimated regression was $R = b'(\beta + \eta) + \epsilon'$ and the second stage $\epsilon' = c'P + \mu'$. It can be assumed that η is uncorrelated with β and R .

the probability limit of c is

$$\text{plim } c = \text{plim } \frac{\sum \epsilon P}{\sum P^2} = \text{plim } \frac{\sum (R - b\beta)P}{\sum P^2} = b_{RP} - b_{R\beta} b_{\beta P}$$

where b_{ij} is the slope coefficient in the regression of variable i on variable j . If price has no independent effect on returns, $\text{plim } c$ will equal zero. Notice now that if price is negatively correlated to beta, such a correlation will induce a correlation between return and price.

The probability limit of the estimate c' after some manipulation takes the form

$$\text{plim } c' = (b_{RP} - b_{R\beta} b_{\beta P}) + b_{R\beta} b_{\beta P} \left(1 - \frac{1}{1 + \frac{\sigma^2(\eta)}{\sigma^2(\beta)}} \right) + \frac{-b_{R\beta} b_{\eta P}}{1 + \frac{\sigma^2(\eta)}{\sigma^2(\beta)}}$$

The first term is c ; the second is a bias stemming from the correlation between β and P ; and the third is a bias stemming from the correlation between changes in beta and price. If $\sigma^2(\eta)$ is non-zero and if $b_{\beta P}$ is negative, the sign of the second term will be opposite that of $b_{R\beta}$. The slope $b_{\eta P}$ would be expected to be positive if past betas for low priced stocks are underestimated so that the third term takes on a sign opposite of $b_{R\beta}$.

20 The first stage regressions using β_{-1} to select portfolios imply an unreasonably large order bias. The ratio of the average coefficients on the regressions using β_{-1} to the average using β_{+1} can be interpreted as an estimate of the slope coefficients in the equation for the order bias in footnote 16. For the overall period, this ratio is roughly 0.5 which serves as an estimate of $\sigma(\beta)$ to $\sigma(\beta + \eta)$. Thus, $\sigma^2(\beta)/[\sigma^2(\beta) + \sigma^2(\eta)]$ is roughly 0.25, so that $\sigma^2(\eta)$ must be roughly three times as large as $\sigma^2(\beta)$. That the standard error of the estimate of β for individual securities, $\sigma(\eta)$, is implied to be roughly 75 percent larger than the standard deviation of the underlying betas of all securities is inconsistent with empirical estimates of these numbers. For the data used in [2], the values of $\sigma(\eta)$ were much smaller than $\sigma(\beta)$ -- often only one-eighth as large.

One possible explanation is that there is a true regression tendency or non-stationarity in the betas, but it would have to be a peculiar one in that the ratio of the average coefficients for the portfolios selected by β_{-2} are extremely close to one implying very little non-stationarity on average. Another explanation is that the portfolio returns are distributed by non-normal stable processes in conformity with the evidence in [3], [6]. Yet, a characteristic exponent as small as 1.0 would only equalize the values of $\sigma(\eta)$ and $\sigma(\beta)$, where " σ " is interpreted as a dispersion parameter. In any case, there appears to be something in these results which cannot be explained solely by the type of order bias discussed in footnote 16.

21 Previous to World War II, the NYSE listed many stocks of extremely low prices. After the War, there were much fewer low priced stocks. For instance, the average price of stocks in the lowest quintile was 0.55 cents in June 1932. This average gradually increased to slightly under seven dollars in 1937 and then decreased to around two dollars before the start of the War. During the War, the average increased rapidly so that by the end of 1945, it was \$12.54. From 1946 through 1966, this average ranged from \$5.33 in July 1949 to \$14.43 in February 1966.

22 Professor Scholes in his comments on this paper suggested that the empirical results in the 1932 through 1943 subperiod might be explained alternatively by errors in estimating betas for low priced stocks. He speculates that the quoted prices for low priced stocks in the early thirties might be because of lack of trading in these stocks differ by large amounts from the actual price if there had been a trade. Granting for the moment the validity of this speculation, it can be demonstrated that the estimates of beta coefficients for low priced stocks will be biased downwards providing that the errors in measuring the true returns are negatively correlated to the market return. Thus, the error η in the estimated beta ($\beta + \eta$) would tend to be positively correlated with price, but then contrary to the empirical results for 1932 through 1943 the modified coefficient should be more negative than the unmodified coefficient in the second stage.

Professor Scholes' alternative explanation for the 1932 through 1943 subperiod therefore does not seem to explain the empirical results. Space limitations do not allow us to answer his other criticisms, which we judge to be less important.

²³A more refined way to measure the previous rate of return related to changes in beta is to use a return which abstracts from normal market movements. If such an adjustment is made the results in the text should only be improved.

²⁴An unexplained but potentially important phenomenon is that the average R^2 's in the second stage frequently are larger for the β_{+1} regressions whether measured by the percentage of variation to be explained by the second stage or by both stages.

²⁵For the period from 1965 through 1971, the portfolios were selected by β_{-1} using however only thirty months of data on the American and beta was measured by β_{-1} . The t-value on the average coefficient of the dummy variable was 1.0. For the period from 1965 through June 1969, beta was measured by β_{+1} and t-value was 2.0. For the last periods, the portfolios were selected by β_{-2} , beta in the regressions was defined as β_{-1} , and the t-value was -1.5.

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