The Value of Information for Investment Decisions

by

Jim Morris*

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RODNEY L. WHITE CENTER
FOR FINANCIAL RESEARCH

University of Pennsylvania

The Wharton School of Finance and Commerce

Philadelphia, Penna. 19104

1. Introduction

Investment decisions can often be improved when the investor has access to better information. Yet, it may not always be worthwhile to acquire that information when it is costly. The desideratum is a framework for analyzing the trade-off between the benefits and the costs of information relevant to investment decisions. This paper develops a simple Bayesian model of the decision of whether or not to acquire information.

In a world characterized by uncertainty a decision maker does not know with certainty the consequences of his decisions. At best, he can only make decisions which are optimal given the knowledge available at the time of the decision. While this decision can be considered optimal on an *ex ante* basis, in the sense that it is the expected utility maximizing decision given the knowledge, it may, in fact, not be optimal evaluated *ex post*, that is, after the fact with the benefit of hindsight.

If the decision maker's knowledge is characterized by his probability beliefs regarding the consequences of each decision, then we can regard information as the flow of "material" to the decision maker with which he revises his probability beliefs and thereby modifies his stock of knowledge. As more information is made available to the decision maker, his probability beliefs will be more refined and the closer his *ex ante* decisions will approach the optimal ex post decisions. It is in this sense that information decreases the risks associated with the decision.

Obviously, the investor would like to acquire as much information as possible, yet it is equally obvious that information is costly. There must be a trade off between the costs of information acquisition and the reduction of risks associated with information acquisition. There is a need for a framework for the analysis of costly information which allows us to explore the balancing of the benefits information against the costs. This paper will present such a framework and then analyze some of the implications for the investor of the costliness of information.

Some of the more interesting results of the model are summarized in Section 2. Section 3 will develop the information model and explain the reasoning behind the main results. Finally, Section 4 will present a numerical example demonstrating the information model.

2. Summary of the Results and Implications of the Model

The primary focus of the model will be upon an investor who has the opportunity to make his expected utility maximizing investment decision either without information or with information derived from a well defined information system which can be purchased at a fixed cost.[1]

If the investor does not purchase the information, he must make his investment decisions on the basis of his original probability beliefs. Of course in this case the money he saves by not buying information is available for investment. On the other hand, for a fixed cost he can acquire access to an information system which will yield messages regarding the state of the economy (the returns on his investments are assumed to be dependent upon the state of the economy). When a message is received, the investor revises his probability beliefs according to Bayesian probability laws. With the revised probability beliefs he then makes his investment decision.

Thus, the investor has two decisions, an information decision and an investment decision. This paper will focus upon the information decision. The conditions under which the investor should incur the information costs will be explored. The primary results are summarized here.

First, for an investor with a single period investment horizon and who exhibits constant relative risk aversion, the optimal information policy will be to purchase information if his initial wealth exceeds a break-even point which depends upon (a) the investor's risk attitudes, (b) the nature of the information system and the costs of information, (c) the investment return, and (d) his probability beliefs regarding those returns, both prior and posterior to receiving information.

Second, for the specific utility functions considered, the value of information to the investor can be shown to be a linearly increasing function of the investor's initial wealth. Let c^k denote the fixed cost of information system k, and let $D^k(w_1)$ denote the maximum amount the investor with wealth w_1 should be willing to pay for information system k. We show that (a) $D^k(w_1)$ is a linear increasing function of w, and (b) there exists a <u>break-even level of wealth</u>, denoted by $\underline{w}_1^k$ below which the investor should nor purchase information and above which he should purchase information. This break-even point is determined as the point where $D^k(w_1) = c^k$. Below $\underline{w}_1^k$ we have $D^k(w_1) < c^k$ and above $\underline{w}_1^k$, $D^k(w_1) > c^k$.

When the model is extended to a case where information is a continuous variable in the sense that the information system will provide "more informative" information at additional cost and the cost is a continuous variable, then we find that under certain conditions there is a unique, expected utility maximizing amount of information which should be purchased by an investor with a given amount of wealth. Furthermore, this unique optimal amount of information is an increasing function of wealth in the sense that wealthier investors will be willing to buy more informative information at a higher cost. This implies that with access to better information the wealthier investors should be able to consistently realize investment returns which dominate the returns to the less wealthy investor.

Finally, it can be shown that at every level of wealth more risk averse investors should be willing to pay more for a given information system than less risk averse investors. This means, of course, that more risk averse investors have a lower break-even level of wealth $\underline{w}_1^k$ than less risk averse investors.

3. The Information - Investment Model

Assume that the investor has current wealth w_1 which he can invest in a riskless asset and a risky security for the one investment period from time 1 until time 2, at which time he will realize his wealth w_2, where the subscript refers to the discrete point in time.[2] At time 1 the investor's objective is to make his decisions so as to maximize the expected utility of wealth w_2 which will be realized at time 2. The investment decision involves the proportion of initial wealth w_1 to be invested in the risky security. For given probability beliefs regarding the investment returns obtainable from the risky asset,the investor can compute the expected utility maximizing proportions of initial wealth to be invested in the riskless asset and the risky security.

The return on the risky security is assumed to be dependent upon the state of the economy during the investment period. Depending on how finely we define the state of the economy,we may be certain of the return on the risky security if the state is known, or, we may simply have a different probability distribution over investment returns associated with each state of the economy. Let us assume for purposes of exposition that if the state of the economy is known, then the wealth relative return (1 + the rate of return) on the risky asset is known. This assumption will be relaxed in the numerical example to be presented subsequently.

Let $R_m(x)$ denote the wealth relative return obtained when the state of the economy is m, and x is the proportion of initial wealth invested in the risky asset.[3] R(x) is a random variable when the state m is not known with certainty.

Let us assume that one investment period prior to the horizon date (time 2) the investor is endowed with a given amount of costless knowledge which includes (prior) probability beliefs regarding the state of the economy and thereby the returns on the risky security. The investor's prior probability beliefs regarding the states of the economy are represented by the vector $p^o = \{p_1^o, \ldots, p_m^o, \ldots, p_M^o\}$, where p_m^o is the probability that the state of the economy is m, and the super script o denotes that these are prior probability beliefs without acquiring further information. We will call the policy of acquiring no further information beyond the initially endowed information the <u>null information policy</u>.

Using the null information policy, where $U(w_2)$ is the investor's utility function for horizon date wealth w_2, the investor's investment problem is written as

$$(1) \qquad V^o(w_1) = \max_x \sum_{m=1}^{M} p_m^o \, U(w_1 R_m(x)),$$

since $w_2 = w_1\tilde{R}(x)$, and where we now define $V^o(w_1)$ as the maximum expected horizon date utility obtainable with initial wealth w_1 using the null information policy (denoted by the superscript o). Now rewrite (1) as

$$(2) \qquad V^o(w_1) = \max_x E^o[U(w_1\tilde{R}(x))]$$

$$= \max_x E^o[U(w_2)]|_{w_1} \quad ,$$

where E^o denotes expectation with respect to probabilities, p_m^o, $m = 1, \ldots, M$, associated with the null information policy. $V_1^o(w_1)$ is a <u>function</u> which gives the maximum expected terminal date utility obtainable for each initial wealth level w_1 with the null information policy. Of course, with different probability beliefs at time 1 and different possible investment returns the function $V^o(w_1)$ will be different.

In addition to the initially endowed costless knowledge, the investor has the opportunity to acquire at a fixed cost additional information with which he can revise his probability beliefs.

Let us assume that for c^k dollars the investor can purchase the k^{th} information system. This information system consists of an observer of economic events and a message transmission system. The observer can distinguish which state of the economy currently prevails, and the degree of certainty and reliability of this observer's analytical power will be dependent on the amount paid for the system. Better and more reliable observers will cost more. Upon observing the economy, the observer transmits a message which reveals which state of the economy (or subset of states) currently prevail. We will let y_j^k denote the j^{th} message of the k^{th} information system, and typically the message y_j^k will reveal that the j^{th} state of the system currently prevails.

Not all observers are equally reliable or have the same analytical ability. In addition the message transmission system may be noisy. Thus we may receive an incorrect message. This possibility for error is reflected in the probabilities of a message being received conditional upon a given state occurring. Let $\lambda_{mj}^k \equiv pr(y_j^k|m)$ be defined as the probability that message y_j^k will be received given that state m currently prevails. We will assume that the probabilities λ_{mj}^k, $j = 1 \ldots J$, $m = 1, \ldots M$ are known and we can say that they specify the information system.

The investor will be concerned about which state of the economy currently prevails, in order to make his investment decision. Thus, given the reception of a message y_j^k,he will want to calculate the probability that a state of the economy m occurs. Given the probabilities $\{p_m^o\}$ and $\{\lambda_{mj}^k\}$, it is a simple matter via Baye's Law to calculate the probability of a state m given the message y_j^k.

Let $P(m|y_j^k)$ denote the probability that the state is m, given that message y_j^k has been received. According to Baye's Law we have

(3) $$p(m|y_j^k) = \frac{p_m^o \lambda_{mj}^k}{\sum_m p_m^o \lambda_{mj}^k}, \quad \begin{matrix} m = 1,\ldots,M \\ j = 1,\ldots,J \end{matrix}.$$

Finally, in order to make his expected utility maximizing calculations, and since the investor does not know which message will be received, he calculates the probability of a given message being received. Let q_j^k denote the probability of receiving the j^{th} message of information system k. We have

(4) $$q_j^k = \sum_m p_m^o \lambda_{mj}^k .$$

Given the message probabilities q_j^k, the state probabilities given the message received $p(m|y_j^k)$, and the prior state probabilities p_m^o, the investor can easily analyze his investment problem with information and compare it with the no information situation.

If the k^{th} information system costs c^k dollars which must be paid prior to receiving any messages, the wealth available for investment is $w_1 - c^k$. Thus, if the state is m, then the horizon date wealth will be $w_2 = (w_1 - c^k)R_m(x)$. If the information system is purchased and message y_j^k is received then the investor makes his investment decision according to

(5) $$\max_{x} \sum_{m} p(m|y_j^k) U((w_1 - c^k) R_m(x)).$$

In order to make the information decision of whether or not to purchase the information, the investor must compute the maximum expected utility obtainable from wealth w_1 and the k^{th} information system. Since the investor does not know which message will be received, but he does know the probabilities q_j^k, $j = 1,\ldots,J$, of receiving each message y_j^k, he computes

(6) $$V^k(w_1) = \sum_j q_j^k \max_x \sum_m p(m|y_j^k) U((w_1 - c^k) R_m(x))$$

$$= \max E^k[U(w_2)]|_{w_1},$$

which is defined as the maximum expected utility of terminal date wealth obtainable with initial wealth w_1 and information system k.

For a given wealth level w_1, the investor will be willing to pay c^k for information system k only if he can obtain more expected utility (net of information costs) with the information system. Thus, the information system k will be purchased at cost c^k only if, at a given wealth level w_1, we have

$$V^k(w_1) \geq V^o(w_1)$$

where these values are calculated according to expressions (6) and (1) respectively.

Alternatively, we can say the investor should purchase the information system at cost c^k only if the dollar value of the system to him is at least as great as the cost. Define the dollar value of the k^{th} information system at given initial wealth level w_1, denoted by $D^k(w_1)$, as the maximum amount of initial wealth the investor could give up to acquire system k such that his expected utility would be at least as great with information as he could obtain without buying information. For initial wealth level w_1 in the expression

$$(7) \quad \sum_j q_j^k \max_x \sum_m p(m|y_j^k)\ U((w_1 - D)\ R_m(x)) = \max_x \sum_m p_m^o\ U(w_1 R_m(x))$$

solve for D and let $D^k(w_1) = D|_{w_1}$. In general, information system k will be worthwhile acquiring only if $D^k(w_1) \geq c^k$, and this criterion is the same as the requirement that $V_1^k(w_1) \geq V_1^o(w_1)$.

In the specialized cases where the investors exhibit decreasing risk aversion and constant relative risk aversion in the sense of Pratt,[4] the utility function for terminal wealth w_2 will be one of the forms

I. $U(w_2) = w_2^{\gamma}$, $0 < \gamma < 1$

II. $U(w_2) = -w_2^{\gamma}$, $\gamma < 0$

III. $U(w_2) = \ln(w_2)$.

When these are the appropriate utility functions, the dollar value of information will be linear, increasing function of wealth. For example, for I and III we have

(8) $$D^k(w_1) = w_1[1 - A^{1/\gamma}] \text{ for I: } U(w_2) = w_2^{\gamma},\ 0 < \gamma < 1,$$

and

(9) $$D^k(w_1) = w_1[1 - e^B] \text{ for II: } U(w_2) = \ln(w_2),$$

where

$$A = \frac{\max_x \sum_m p_m^o (R_m(x))^{\gamma}}{\sum_j q_j^k \max_x \sum_m p(m|y_j^k)(R_m(x))^{\gamma}} = \frac{E^o[\tilde{R}^{\gamma}]}{E^k[\tilde{R}^{\gamma}]}$$

and

$$B = [\max_x \sum_m p_m^o \ln(R_m(x))] - [\sum_j q_j^k \max_x \sum_m p(m|y_j^k) \ln(R_m(x))]$$

$$= E^o[\ln(\tilde{R})] - E^k[\ln(\tilde{R})],$$

where in each case $E^o[\cdot]$ and $E^k[\cdot]$ denote maximum expectation of the term in brackets based on probability beliefs associated with information systems o and k respectively.

Since an investor can always do at least as well with information as he can without information, we have $0 \leq A \leq 1$ and $B \leq 0$. It follows that

$$0 \leq [1 - A^{1/\gamma}] \leq 1 \text{ and } 0 \leq [1 - e^B] \leq 1.$$

Note that each of the above expressions is independent of wealth. Thus, for both (8) and (9) the dollar value of information is linearly increasing in wealth. From expressions (8) and (9) we can draw the dollar value of information $D^k(w_1)$ as shown in Figure 1.

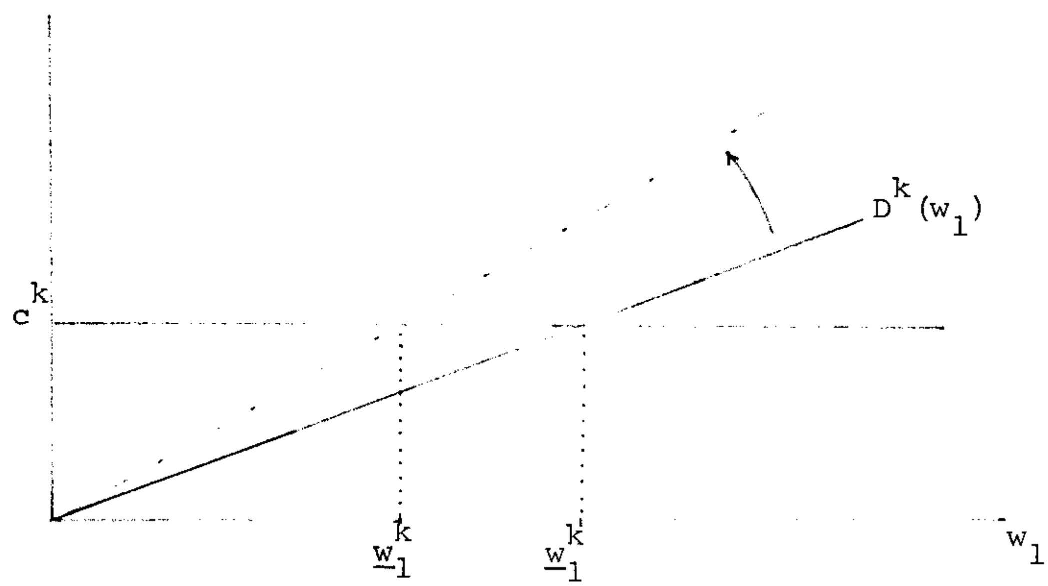

FIGURE 1

THE VALUE AND COST
FOR A SINGLE INFORMATION
SYSTEM

As the informativeness of the information system increases the difference between the maximum expected utility of wealth relative returns obtainable with information and the maximum expected utility obtainable without information will increase. For the power utility function $U(w) = w^{\gamma}$, the fraction A measures this difference since the denominator represents the maximum expected utility of wealth relative returns obtainable with information and the numerator represents that obtainable without information. Thus, for that utility function, as informativeness increases the fraction A will approach zero. Similarly, for the logarithmic utility function, the difference B will increase in the negative direction as the informativeness of the information system increases. The result of this is that, for both utility functions, as the informativeness of the information system increases the slope ($[1 - A^{1/\gamma}]$ and $[1 - e^{B}]$ respectively) of the $D_k(w_1)$ line will increase. This means that as informativeness increases then $D^k(w_1)$ will intersect c^k at a lower level of initial wealth w_1. Thus, more informative information systems are worth more and therefore less initial wealth is required to justify the purchase of information. In Figure 1 we can see that increasing the slope of $D^k(w_1)$ decreases the minimum wealth required to justify the purchase of information.

When we consider that there may be numerous information systems available to the investor,then we find that there may be numerous break-even points for the purchase of information, one for each information system available. For a finite number of discrete information systems we would get a unique break-even point for each system for a given investor who exhibits constant relative risk aversion.

Assume we have available K information systems numbered in ascending order of informativeness k = 0, 1, ..., K, and that more informative information systems involve greater cost. Then, as is shown in Figure 3, where K = 3, for each information system k there will be a unique break-even point $\underline{w}_1^k$ such that for $w_1 \geq \underline{w}_1^k$ we have $D^k(w_1) \geq c^k$. Note that there is an interval of initial wealth within which the investor would purchase a given information system since it would be the most informative information system for which the value of the information exceeded its cost (for his given wealth level). At either end of this interval the optimal information system would change abruptly. At the lower end of the interval he would shift to the less informative and less expensive information system and at the upper end he would shift to the more informative, more expensive information system.

When we confine our attention to power utility functions

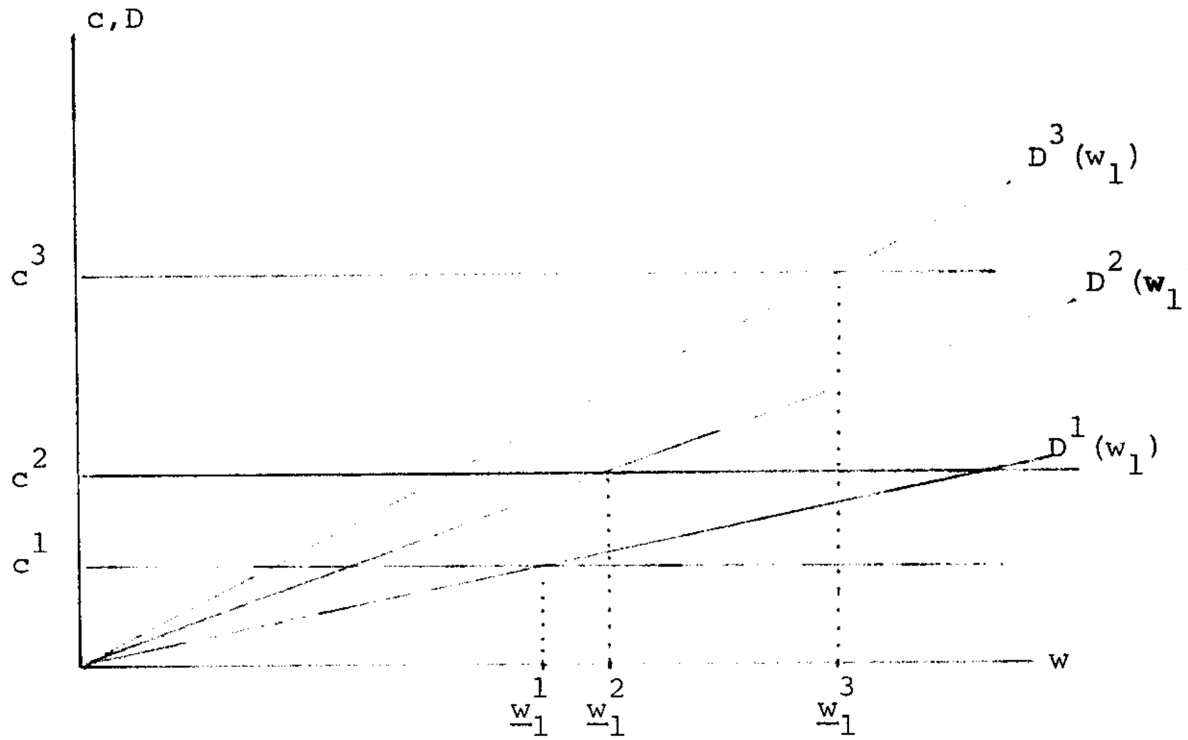
c,D
$D^3(w_1)$
c^3
$D^2(w_1)$
c^2
$D^1(w_1)$
c^1
w
$\underline{w}_1^1$
$\underline{w}_1^2$
$\underline{w}_1^3$

FIGURE 2

of the form $U(w_2) = w_2^{\gamma}$, $0 < \gamma < 1$, we can explain the relationship between risk aversion and the value of information. Note for $U(w) = w^{\gamma}$, $0 < \gamma < 1$, that as γ approaches 1 the utility function approaches linearity. A linear utility function $U(w) = w$ represents an investor who is indifferent toward risk since he will always choose the investment with the greatest expected return regardless of the dispersion of the return. On the other hand, for $U(w) = w^{\gamma}$, as γ approaches zero, the utility function increases in concavity and represents an investor who is more risk averse in the sense that he will pay a larger premium to avoid a given risk.

Now consider the expression (8) representing the value of information for this investor with the power utility function:

$$(8) \qquad D^k(w_1) = w_1[1 - A^{1/\gamma}].$$

Assuming that the value of the fraction A does not change substantially as we change the value of γ, we can analyze $D^k(w_1)$ for various levels of risk aversion as we vary the value of γ, Since $0 < \gamma < 1$ and $0 < A \leq 1$ we have the following conditions

$$\lim_{\gamma\to 0} \frac{1}{\gamma} = \infty , \quad \lim_{\gamma\to 0} A^{1/\gamma} = 0 \quad \text{(except if } A = 1\text{)}$$

$$\lim_{\gamma\to 1} \frac{1}{\gamma} = 1, \quad \lim_{\gamma\to 1} A^{1/\gamma} = A$$

which imply

$$\lim_{\gamma \to 0} D^k(w_1) = \lim_{\gamma \to 0} (w_1[1 - A^{1/\gamma}]) = w_1$$

$$\lim_{\gamma \to 1} D^k(w_1) = \lim_{\gamma \to 1} (w_1[1 - A^{1/\gamma}]) = w_1(1 - A),$$

where of course $0 < 1 - A < 1$ except if $A = 1$. Thus, as risk aversion increases (as γ approaches 0) the value of information to the investor will approach his initial wealth. That is, a very risk averse investor will pay almost all his initial wealth for information if that information is in fact better than no information.

Thus far it has been noted that (a) for a given information system the value of information is a linearly increasing function of wealth for the logarithmic and power utility functions, which implies (b) for a given fixed cost of information c^k there is a unique level of wealth $\underline{w}_1^k$ below which it does not pay to buy the information and above which the information should be purchased and (c) at any given level of wealth the value of a more informative information system exceeds the value of a less informative information system: and finally, (d) for an investor with the utility function $U(w) = w^\gamma$, $0 < \gamma < 1$ the value of information increases as the investor's risk aversion increases.

When we consider the possibility that information may be a continuous variable, then we obtain some further interesting conclusions relating investor wealth to the price of information.

Let information be a continuous variable in the sense that we let the number of information systems increase to an arbitrarily large number. We can think of this as analogous to making the transmission capacity of an information system a continuous variable. As we increase the transmission capacity of the system we increase the informativeness of the system, where we say that one information system is <u>more informative</u> than another if the maximum expected utility obtainable with the first system is greater than that obtainable with the second system <u>without</u> deducting the costs of information.

If there are an arbitrarily large number of information systems, let us assume that a more informative system costs more, and assume that an investor will want to purchase the most informative information available subject to the condition that the dollar value of information $D^k(w_1)$ exceeds the cost c^k. Should the investor with wealth w_1 attempt to bid an amount greater than $D^k(w_1)$, he will decrease his expected utility below that which he could obtain from purchasing no information. Should the investor attempt to obtain a marginally more informative information system, he must outbid an investor who has greater wealth. But we already noted that the investor with greater wealth should be willing to pay more for information. Thus, our investor could not be acting optimally if he attempts to outbid a more wealthy investor for a more informative information system.

In general, for an information market composed of investors exhibiting constant relative risk aversion, at each wealth level w_1 there will be a unique information system k such that $D^k(w_1) = c^k$. An investor with wealth level w_1 would be acting optimally if he purchases that information system.

Consider for example, Figure 3. Points labeled $\underline{w}_1^k$, k = 1, ..., 5 represent the minimum wealth levels such that $D^k(w_1) > c^k$. An investor with wealth $w_1 = \underline{w}_1^3$ would find it worthwhile to purchase systems 1, 2 and 3, but given his wealth level, he would not purchase systems 4 or 5, since for these systems $D^k(w) < c^k$. Assuming the investor purchases the maximum amount of information that he can afford, it is obvious that he would purchase system # 3. When we consider information as a continuous variable, then there will be an information system associated with each wealth level such that D(w) = c, represented by the intersection of the D(w) and c lines in the diagram. As we increase the number of information systems in the diagram we will obtain a locus of points (represented by the dashed line) where D(w) = c. This locus will be monotonic and increasing and would represent the unique optimal expenditure for information at each level of wealth. (It should be noted that this uniqueness depends upon there not being increasing marginal returns to information expenditures.)

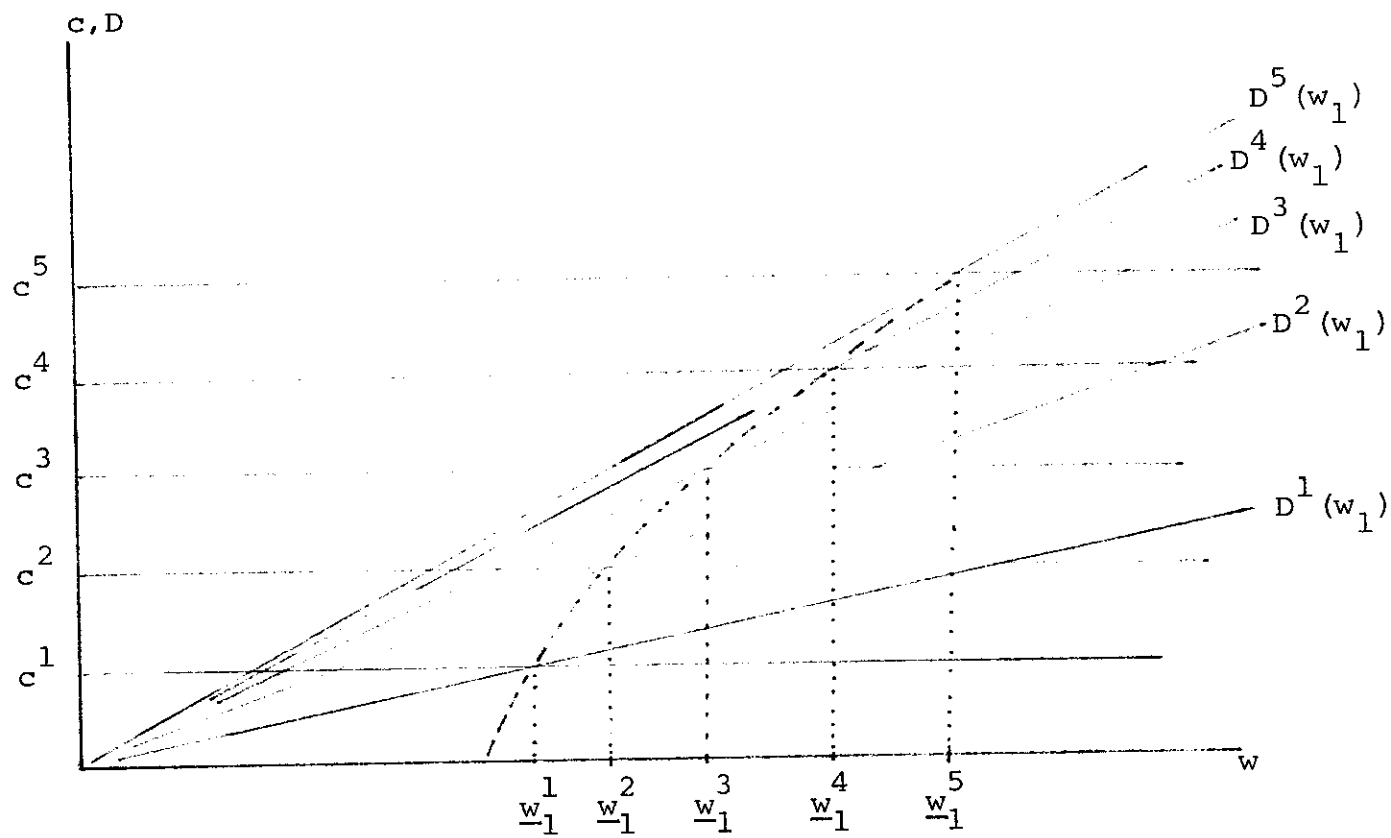

FIGURE 3

THE VALUE OF INFORMATION
FOR VARIOUS INFORMATION SYSTEMS

4. A Brief Numerical Example

An investor whose utility function for terminal wealth is of the form $U(w_2) = \ln(w_2)$ has available for investment a riskless asset and a risky security. His investment problem is to decide the expected utility maximizing proportion of initial wealth to invest in the risky security.

The riskless asset will, at the end of one investment period, return, with certainty, one dollar for each dollar invested. The returns on the risky security are dependent upon the state of the economy. The states of the economy are very broadly defined as recession (state # 1), normal (state # 2) and prosperity (state # 3). The states are broadly defined so that even if the investor knows the state of the economy, he will still not be certain of the investment returns. Rather, there will be a probability distribution over wealth relative returns on the risky security associated with each state of the economy. Let p_{im} represent the probability that the i^{th} wealth relative return will occur given that the current state of the economy is m. The wealth relative returns and their probabilities of occurrence (given the state) are shown in the following table.

TABLE 1

i	d_i wealth relative return on the risky security	(p_{im}) Probability of Return i given state of the economy m			π_i^o prior probability of return #1 with no information
		m = 1 (recession)	m = 2 (normal)	m = 3 (prosperity)	
1	0.8	.56	.20		.3780
2	0.9	.05	.48	.09	.2045
3	1.0	.04		.72	.0940
4	1.1	.11			.0605
5	1.2	.24			.1320
6	1.3		.20		.0700
7	1.4		.12		.0420
8	1.6			.18	.0180
9	1.8			.01	.0010
	Total:	1.00	1.00	1.00	1.00

Prior to purchasing any information the investor does not know the state of the economy, although he does have estimates of the probabilities of each state. Letting p_m^o denote the investor's (no information) estimate of the probability of state m occurring, we have for our example:

$$p_1^o = .55,\ p_2^o = .35,\ \text{and}\ p_3^o = .10.$$

Given no information regarding the state, the investor's estimate of the probability of return i being realized (denoted by π_i^o) is derived as

$$\pi_i^o = \sum_m p_m^o \, p_{im} \, .$$

For example, with no information, the investor believes the probability of earning a wealth relative return of 0.9 (i = 2) to be

$$\pi_2^o = .55(.56) + .35(.20) + .10(0) = .2045,$$

which is shown in the last column of Table 1.

If the investor acquires no further information concerning the state of the economy, he must make his investment decision on the basis of the prior probability distribution over investment returns which is represented by π_i^o, i = 1, ... , 9 and is shown in the last column of Table 1. In this particular case, with no information, the situation is quite risky, with, for example, a .5825 (.3780 + .2045) chance of losing money. Our investor with a logarithmic utility function is quite conservative, so that the expected utility maximizing investment is to put all his wealth in the riskless asset.

The investor has the opportunity to purchase for $100 an information system which will provide messages revealing the state of the economy. There are three possible messages y_j, j = 1, 2, 3, one for each state. Assume that the probability of receiving each message is equal to the probability of the state associated with that message.

Thus, letting $q_j \equiv pr(y_j) = p_m^o$, we have

$q_1 = .55 \quad q_2 = .35 \quad q_3 = .10.$

If the investor receives message #1 (y_1), he is certain that the economy will be in a state of recession and he will make his investment decision on the basis of the probabilities represented by column 3 of Table 1. Of course in that case he invests all his wealth in the riskless asset. The optimal investment strategy associated with each message is calculated to be as shown in the following table.

message j =	#1	#2	#3
optimal proportion of wealth invested in the risky security	.0	.435	1.00

Prior to receiving the message the investor, in order to evaluate the desirability of purchasing information, calculates the maximum expected utility obtainable with and without information. In this case we have, analogous to expressions (1) and (5), for initial wealth w_1

$$(1') \qquad V^o(w_1) = \max_x \sum_i \pi_i^o \ln[w_1((d_i - r)x + r)]$$

for no information, and

(5') $V^k(w_1) = \sum_j q_j \max_x \sum_m (p(m|y_i)\sum_i p_{im} \ln[(w_1-100)((d_i-r)x+r)])$

with information system k, where d_i represents the i^{th} wealth relative return on the risky asset and r represents the wealth relative return on the riskless asset.

In this case, the investor finds for initial wealth below \$10,488 that his expected utility is greater if he purchases no information. If initial wealth exceeds \$10,488, than his expected utility is maximized by purchasing information, and at w_1 = \$10,488 he is indifferent between no information and paying \$100 for the information system.

Calculating the value of information according to expression (9) we have for various wealth levels the following values $D^k(w_1)$.

TABLE 2

w_1 = initial wealth	$D^k(w_1)$ = The value of the information system to the investor with wealth w_1
\$1000	\$ 9.53
5000	47.67
9000	85.81
10000	95.34
10488	100.00
11000	104.88
15000	143.02
20000	190.69

In this case it is easy to see that the value of information to the investor increases with wealth and does not exceed the cost of information until wealth exceeds \$10,488.

5. Summary and Conclusions

The simple model of the information decision which has been developed here represents an attempt to explicitly consider the costs and benefits of information. While there has been considerable work analyzing the economics of information, there has been relatively little work which correctly allows for the costs of information.

In the investment context, the relation between the value of information and the investor's wealth may hold some important implications for, for example, mutual funds and other institutional investors. We found, for the specific utility functions considered, that the value of information is an increasing function of investor wealth. Where there are numerous information systems available, which provide more and better information at a greater cost, then wealthier investors will tend to purchase more information than poor investors. This may mean that it is suboptimal for small mutual funds to spend much for information gathering and processing, while large funds can easily justify the purchase of more expensive information systems. In the very specialized case where there is just one information system available at a fixed cost we get the result that there is a break-even point such that with wealth below that point it is not optimal to purchase information. For mutual funds with a single information system available this implies that the small funds would be better off purchasing no information.

These conclusions are, of course, consistent with some of the empirical work on the performance of mutual funds. For example, Sharpe[5] noted that good fund performance appeared to be related to low expense ratios, which in the present context can be interpreted to mean that some funds were operating sub-optimally by incurring excessive information costs.

We noted, for a particular type of utility function, that the value of information increases as the investor's risk aversion increases. This, of course, implies that conservative mutual funds and other institutional investors would be justified in making larger information expenditures than less risk averse investors.

While the present model would be difficult to apply to a realistic situation, it does represent the type of analysis which investment managers should apply in order to consider whether or not it is worthwhile to acquire information relevant to their investment decisions.

FOOTNOTES

[*] Lecturer in Finance, University of Pennsylvania. The author wishes to thank the Rodney L. White Center for Financial Research of the Wharton School for financial support.

[1] The present analysis will consider only the value of information to a decision maker. No reference will be made to the supply of information. It is implicitly assumed that the decision maker will have exclusive access to the particular information system.

[2] We could assume any number of risky securities, but with no loss in generality we can assume that there is just one risky security which we can view as a market portfolio consisting of each security held in proportion to its total market value relative to the total value of all risky securities in the market.

[3] Letting d_m denote the wealth relative return on the risky security when the state of the economy is m, and r wealth relative return on the riskless asset, we have

$$R_m(x) = d_m x + (1 - x)\, r = (d_m - r)\, x + r.$$

In a more general case when knowledge of the state implies a given probability distribution over d, let d_i represent the i^{th} wealth relative return on the risky security and p_{im} the probability that the d_i will occur given the state m. Then we have for example, given that state m will occur, $E(R_m(x)) = \sum_i p_{im}((d_i - r)x + r)$.

[4] John W. Pratt, "Risk Aversion in the Small and in the Large," Econometrica, Vol. 32, No. 1-2 (January-April 1964), pp. 122-136.

[5] William F. Sharpe, "Mutual Fund Performance," Journal of Business, XXXIX, No. 1, Part II (January 1966), pp. 119-138.

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