# The Simultaneity of Systematic Stock Price Movements

bу

Jack Clark Francis\*

Working Paper No. 5-72 (Revised)

RODNEY L. WHITE CENTER FOR FINANCIAL RESEARCH

University of Pennsylvania
The Wharton School of Finance
and Commerce
Philadelphia, Pa. 19104

The contents of and the opinions expressed in this paper are the sole responsibility of the author.

### I. Introduction

Sharpe [20], [21], Treynor [22] and others have developed various capital market theories based on the characteristic regression line shown in equation (1).

$$r_{it} = a_i + b_i r_{mt} + e_t$$
 (1)

The characteristic regression line suggests that the  $\underline{ex}$  post rate of return from the  $i^{th}$  asset during the  $t^{th}$  period equals some constant,  $a_i$ , plus two stochastic elements. First, the  $b_i r_{mt}$  term represents the product of the asset's beta systematic risk index and the percentage change in the market index during period t. Changes in  $r_m$  are systematic changes in the condition of the market caused by economic, political or sociological factors which affect practically every asset in the market in some way. The second stochastic source of variation in  $r_{it}$  is the unsystematic variability, measured by  $e_t$ . This unsystematic variation is unique to each asset and tends to be statistically independent of  $r_{mt}$ .

The purpose of this paper is to examine the intertemporal relationship between the variations in  $r_{it}$  and  $r_{m}$ . The changes in  $r_{m}$  which precede, occur simultaneously, and follow the changes in  $r_{it}$  will be examined. The structure of any leads or lags which may exist between  $r_{it}$  and  $r_{m}$  are interesting for several reasons.

has probably wondered if some stocks lead or lag the market as measured by some broad based market index. If the stocks of some particular companies or industries do lead or lag the market, these tendencies could be used as a basis to formulate some profitable trading rules which could yield a rate of return after trading commissions in excess of that attainable with a naive buy and hold strategy.

An investigation of possible leads and lags in the movements of common stock prices is not only of interest to stock market traders, but is also of interest to financial economists. If some reasonably fixed lead or lag structure exists for stock prices, the capital market models developed by Sharpe, Treynor [20, 21, 22, 23] and others can be enriched by the inclusion of such factors into these models.

An investigation of leads and lags can also throw new light on studies of the efficiency of security markets (for example, [9]). Fama [8] and others have presented evidence suggesting that short-run percentage changes in stock prices are not serially correlated and are essentially random about their expected value. However if some stock prices are "trend setters" or "market laggers," they would have price movements which were not statistically independent of other market price series. This kind of dependence between otherwise random series would not only make technical analysis profitable to

perform it could yield insights into the manner in which investors' expectations about stock prices change. Facts about the length of any leads or lags, the permanency of leading or lagging characteristics which a given stock may exhibit, and measures of the strength of leading and lagging tendencies could provide the first steps toward the measurement of investor expectations.

In order to provide answers for some of the questions raised above, 770 New York Stock Exchange (NYSE) stocks which were listed continuously during a recent decade were analyzed to determine the manner and the frequency of anticipatory or delayed price movements they might have relative to the broad based market index for the NYSE developed by Lorie and Fisher [11, 12].

any reasonably fixed lead or lag from zero to six months. Pooled effects of several leads and lags will be examined later in this paper with a distributed lag model. It will be seen that most NYSE stocks usually move concurrently in the same direction as the market's movements. The stocks differ primarily with respect to their sensitivity to the market's movements. However, some statistically significant leads or lags do appear in the data.

II. Simple Lagged Regression To Isolate Fixed Leads and Lags
II A. The Model Used To Detect Fixed Leads and/or Lags

A slight adaptation of the market regression model which was

proposed by Markowitz [19, page 100], Sharpe [20], [21], and Treynor [22] furnishes the engine for the first phase of this analysis. This simple lagged regression model is shown as equation (2).

$$r_{it} = a_i + b_i r_{m,t+k} + e_t$$
 (2)

where

 $r_{\text{it}} = \frac{P_{\text{t+l}} + d_{\text{t}}}{P_{\text{t}}} - 1 = \text{a monthly price plus dividend relative for month t (adjusted for stock splits and stock dividends) less unity for the i<sup>th</sup> NYSE listed stock, that is, a monthly rate of return for some stock; 3$ 

r = the rate of return in month t for the Lorie-Fisher NYSE price relative file (including cash dividends and before taxes) [11], [12];

 $b_i = cov(r_i, r_m)/\sigma_m^2 = the regression slope coefficient for the i<sup>th</sup> stock;$ 

 $a_i = r_i - b_i r_m = the regression intercept; and,$ 

K = -6, -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5, 6 = a lag or lead stated in terms of the number of months the market's return precedes the stock's return. By varying K the market model may be fit for various leads or lags. K = 0 when there is no lead or lag.

Regression equation (2) has been examined for the zero lag case by various analysts [3], [4] [5], [10] and found to be a linear unbiased, sufficient, consistent estimator. Fama has suggested the model may not provide minimum variance estimates of the regression statistics due to infinite variance of the dependent variable [8]. However, Blume has found this to be of minor importance for recent NYSE data [3]. For

the purposes of this analysis, the model shown in equation (2) should serve quite nicely.

In order to determine if an individual stock tends to lead or lag the market, equation (2) will be fit for that stock for every integer value of K in the range from -6 to + 6 inclusive. That is, the regression model will be run for a total of 13 leads and lags for every stock examined. Then, certain regression statistics will be examined for each of the 13 regressions to determine which lead or lag (that is, which value of K) yields the best fit to the empirical data.

Five regression statistics will be calculated for each regression. These statistics measure the nature of the relation between the rate of price change in a stock and the average rate of change in the prices of all NYSE stocks [11], [12]. The 5 statistics are:

(1) the beta regression slope coefficient which is a measure of systematic risk; (2) the t statistic for the beta coefficient;  $^4$  (3) the coefficient of determination ( $\overline{R}^2$ ) for the regression;  $^5$  (4) the standard error of the regression, and, (5) the Durbin-Watson statistic.

# II B. The Results of the Tests for Fixed Leads and Lags

The first sample runs from January 1962 to December 1964 inclusive and includes one steep market drop in 1962 followed by a 2 year bull market. The second three year sample encompasses January 1965 to December 1967 inclusive, a period of stock market instability

which is often attributed to the "credit crisis of 1966." The ten year sample starts with January 1958 and ends after December 1967; it includes the two three year periods.

Exhibits 1, 2 and 3 show samples of the computer output for nine stocks from the three different sample periods. These nine stocks had better goodness-of-fit regression statistics for some lead or lag than for their zero lag regression during at least two of the three sample periods examined.

The sample data were all classified into one of the following three categories.

- 1) Stocks which had goodness-of-fit regression statistics for some lead or lag that were both statistically significant and more highly significant than the statistics for the zero lag regression; for example, Amalgamated Sugar in Exhibit 1 has its most highly significant regression statistics for a one-period market lag.
- 2) Stocks which had the best goodness-of-fit statistics for the zero lag regression but also had significant statistics for some lead or lag; for example, Hackensack Water Company in Exhibit 1 has a significant tendency to lag the market by 2 months (that is, the market leads Hackensack) but the zero lag statistics are the most highly significant. 7
- 3) Stocks which show no statistically significant tendency toward any lead or lag from one to six months; for example, in Exhibit 1 the Perfect Film Chemical Company has no significant goodness-of-fit statistics for any lead or lag.

ple ple
san
1962-64
market,
the
lag
or
lead
ţ
tended
have
which
stocks
· Nine
1
Exhibit

	, s-	- 5	- 4 AMA!	-3	-2 SHGAR	-1	ZERO	+1	Z+	+3			
BETA	-0.1509	-0.2098	0.4	0.039	7   I	0 6	0.2066	1.0723	-0.1849	-	-0.0522	0.4043	-0.0025
San	6600.0	) (C	000.0	0.0041	Ö	0.5	0.0173	0.0327	0.0144	0		0.0511	]
STD ERROR DURWAT	1.9868	0.0684	0.069	0.0689	0.0661	0.0	oN	0.0679	0.0686 1.8018	1:		1.8965	)
-31 FTA	-0.5173	1 7	AME 0.2475	~~	. 0	0.23	•		0	-0.153	0.2965	0.091	-0.4893
-STAT	2,7505	1	ŧ	0.3552	00	11.1	1.9471	0.4751	۳ o	0.016	1.4142	-0.3810 0.0043	$\sim$
K-SOU Sin FPRDR	0.0438 0.0438 	0.0539	0.052	053 817	p -	0.0	0.0512	0.0538	0.0525	0.0535	0.0525	0.0539	0.0507
98			8.E	JET CON	SOLIDATE	۵				X	h k		· •
ETA	0.2443	1		:O C	00	00	0.2283	0.6155 1.1879		īï	0.151	.222	⊣ ტ
-500	0.0062	0.0339		i	0.0086	0.0093	0.0053	0.0399	0.0013		0.0007	0.1378	0.1046
DURMAT	1.9164	- :	7	7	5	7	1.9374	1.9485			606 T	106	,
343 FTA	0.1175		HAC 0.1351	KENSÁČK - 0.256	MATE	ં	0.4939	-0.1737	?	-0.063	160		-0-2
1-5141	7645.0			•		<b>-</b>	2,4699	-0.8206	20	0.002	.015		000
TO ERROR	3.0555	0.0556	0 (	0.054	0.0553	0.0545	0.0513	0.0552	0.0519	6	2.3202	6.0552	0.0552
(N)KW4	7777		STAR	RETT, L.	00		,						
	-0.2659	0.0728		262 <sup>-</sup>	-0.3207	0.1094	0.5919	-0.0876	-0.2147	-0.0415 -0.6495	1.6180	0.6031	-0.7689
0.0	0.1023	6.0077	0.0287	740	0.1486	0.0169	0.0102	0.0112	0.0677	0.0123	00	0.0106	0.0171
D FRROR	0.0351	0.0369	0.0365	0.0356	0.0341	1.81.75	1.8884	1.9223	1.9762.		8618-1	1.9267	1.9717
				Σ   11:	C.H.W.	ķ	20.0	777	- 1231 -		0.7443	0325	
A	0.2092	.2120 4826	0.2656	2783 - 5329 -		-0.1613 -0.3613	5.6742	185	-0.2809		1.6889	0637	-0.1133
R-S0D	0.0000	0063		3.0116	100000	0.0038	0.4864	0.0635	0.0023	0.0069	0.0774		0.1148
STO FKROR	0.1144 2.6490	0.1144	2.6491	2.6341	2.6469	2.5574	2.2559	396	2.6764		2.6554	2.6503	2.6542
92		) 	CAMPB	$\sim$	Į.	678.	-0.0728	2036	0.3095	·	- 6271.	504	0.7517
BETA			0.0098 -		· ·	-0.1982	-0.2343	0	1.0331		- 5459 -	+26 -	2.2274
R-500	1	0.0048	0.0000		0.0336	0.0106	0.0016	0.0131	0.0304	0.0040	0.0087	0.0006	0.0743
STO ERROR Ourwat	1.7080		1.6448	u w	1.4909	1.5790	1.6737		1.6342	19	1	499	1.7278
236 acta	0.4661	TOOL	0.0522		TD 0.3005	-0.3900	-0.0590	078			- b	2090	· •
-STAT	1.8299	0.0061 -	0.1961			_	-0.2186	295		0.4503 -	0	1958	-1-3400
	0.0897	0	0.0011	0000	0.1032	0.0613	0.0692	0.0026	0.0735	0.0059	0.0692	1100.0	0.0675
STO FREER	0.04661	U. 9549	1.8374	7	1.6358	1.6392	1.8652	802		1.8650		1.8264	1.837B
362			HOWE FACT	~	02 9N1	-0.2506	-0.2956	-0.1151	0.2074	2637	0.3211		<b>.</b>
RFIA T-STAT	3.5136 -	-0.5515 -	0.4739	-0.5152		-1.0920	-1.2980	-0.5034	0.9169	1.1732 -	1.3784	1.3421 -	0.0587
1 ( )	2664		0.0066		0.0690	0.0339	0.0472	\$700°0	1420.0	V. U.507	470.04	1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1	· · · · · · · · · · · · · · · · · · ·
2000			1000		A. C.578	0.0589	0.0585	0.0597	0.6592	0.0587	.0583	0.0584	0.0581

LEAD			- 8 -	
9+	-0.5671 -2.2383 0.1284 0.0669 2.3523 -1.01873 -1.0677 0.0463 2.2253	-0.2429 -0.3943 0.0046 0.1628 1.8294 -0.3480 -2.1421 0.1189 0.0429	0.7546 1.9147 0.0973 0.1041 1.9387 3.7314 0.1714 0.1714	5.5146 5.0529 5.0987 2.2382 5.066 4119 6.054 6.0948 2.408 1.1818 6.947 6.103 6.103
+5	-0.0830 -0.3242 0.0031 0.0716 2.1706 -0.3751 -2.4103 0.0435	3523 6075 0107 1623 7604 1935 1935 1412 1418	0.2210 - 0.5668 - 0.0094 0.1091 1.8801 - 0.5984 0.0104 0.4063 2.3425	0.2484 0.6902 0.0138 0.1007 2.3748 0.0560 0.0975 0.0975 0.3348 2.2141 0.060 0.0805 0.0805
<b>*</b>	0.02793 -1.1005 0.0344 0.0705 0.1460 0.1460 0.0218 0.0468	-0.2393 -0.4082 0.0049 0.1627 1.7774 0.1201 0.7348 0.0156	0.2641 0.6734 0.0132 0.1089 1.8765 0.5934 0.4044 0.0048	0.0913 0.02501 0.0018 0.1013 2.2717 0.0470 0.0975 0.0975 0.0975 0.0975 0.0123 0.0123 0.0123
+3	8 0.2614 5 0.9595 7 0.0264 6 0.0708 4 2.0806 -0.0419 -0.2312 0.0016	-0.4929 -0.7918 0.0181 0.1617 1.9090 0.1770 1.0201 0.0297 0.0450	0.6466 1.5720 0.0678 0.1058 1.7968 0.2471 0.1572 0.0007 0.4082	1.0191 2.9186 2.9186 0.2003 0.0907 2.3899 3.3796 0.1428 0.4862 0.4862 0.4862 0.4862 0.4862 0.1747 0.1747 0.0737 0.0737
+5	6 0.083 4 0.303 1 0.002 5 0.071 0 2.134 1 -0.4115 1 -2.4573 2.04034	-1.103 -1.838 0.090 0.155 1.731 0.3834 2.3424 0.1390	0.0474 - 0.1122 - 0.0304 0.1096 1.9131 0.0237 0.0151 0.0000	0.0574 0.1467 0.1014 2.2639 0.1343 0.0376 0.0974 0.0974 0.0534 0.0534 0.0534 0.0554 0.0554 0.0554 0.0554
+1	0.128 0.455 0.006 0.0071 2.178 0.0123 0.0662 0.0671	N-000	0.3050 0.7112 0.0147 0.1088 1.9652 -2.6813 -1.7379 0.0816 0.3914	-0.0721 -0.1804 0.0010 0.1013 2.2918 0.3159 - 0.8298 - 0.0199 0.0199 0.0966 2.2981 0.2051 0.6449 0.0121 0.0807 2.1208
ZERO	0.6877 2.7068 0.1773 0.0650 2.3789 -0.0233 -0.1265 0.0055	98 59 06 92 14 14 14 14 14 14	0.4957 1.1814 0.0394 0.1074 2.0663 0.5700 0.3580 0.0038	-0.3264 -0.8324 0.0200 0.1004 2.3055 -0.1592 -0.4189 0.0051 0.0051 0.0051 0.0051 0.0051 0.0051 0.0051 0.0051
71	0.4036 1.4749 0.0601 0.0695 2.4390 0.1190 0.6456 0.0121 0.0468	0.7777 1.2377 0.043 0.159 1.837 0.4376 2.6740 0.07138	0.8759 2.1658 0.1212 0.1027 2.3421 0.0801 0.0001 0.4084 2.4391	0.0688 0.1725 0.0009 0.1014 2.3064 2.3064 0.0018 0.0018 0.2000 0.2651 0.2651 0.2651 0.2651 0.2651 0.2651 0.2651 0.2651
-2	52 0.3309 22 1.1981 93 0.0405 77 0.0702 72 2.3475 NOTE 8 0.1822 0 0.0977 0 0.0284	O.052 0.052 0.082 0.000 0.163 1.773 1.773 47ER CC 0.1139 0.0455	S. CO 0.9527 2.3892 0.1438 0.1014 2.0575 CHEM -2.2215 - -1.4242 - 0.0563 0.3967 2.3677	D LAKE -0.2434 -0.6139 -0.6139 -0.1100 0.1008 0.1008 0.1008 0.1008 0.0036 0.0036 0.00374 0.00062 0.000
-3	ALGAMAT 0.555 2.04 2.06 7.0.06 7.0.06 8.003 0.000 0.000 0.000 0.000	GUET CON: 0.7174 1.1145 0.0352 0.1602 1.8605 1.8605 0.0495 0.2699 0.0021	ETT, L. 0.3653 0.8383 0.0203 0.1085 1.8010 CT FILM 1.5949 0.9859 0.0278 0.4027	ELL RED 0.2399 0.5919 0.0102 0.1009 2.3179 41NES, L 0.0297 0.0297 0.0297 0.0297 0.0297 0.0297 0.0297 0.0297 0.0297 0.0297
4	AME 0.1386 0.1386 0.1386 0.1386 0.058	0.0008 0.0454 0.034 0.161 1.871 1.871 0.0008 0.0008	STARR -0.0309 - 0.0702 - 0.0001 1.9189 PERFE -1.1459 -0.7034 0.0143	CAMPE 8326 8326 1004 1004 1094 0004 0007 1029 1029 1029 1029 1029 1029 1029 1029
	0 -0.2552 0 -0.8994 2 0.0232 3 0.0709 0 2.1844 0 2.202 0 0.0400 0 0.0462	1.0 0.0 0.0 1.9 1.9 1.8 1.8 2.2	-0.0474 -0.1079 0.0003 0.1096 1.9182 0.5933 0.3635 0.0039	0.7069 0 1.8241 0 0.0891 0 0.0958 0 0.0926 0 0.0926 0 0.5575 0 0.5575 0 0.0865 0
LAG -6	0.0339 0.039 0.070 0.070 0.070 0.070 0.3502 0.0036 0.0036	71000	-0.2362 -0.5237 0.0082 0.1091 1.9493 0.8579 0.5151 0.0077	0.1407 0.3395 0.0034 0.1612 2.3188 0.6209 0.05209 0.0970 2.3201 0.0643 0.0643 0.0643 0.0643 0.0643
<b>-</b>	26 RFTA T-STAT R-S00 STD ERROR OURWAT 31 BFTA T-STAT R-S00 STD ERROR DURWAT	PETAT RETAT T-STAT R-SOD STD ERROR DURWAT 343 RETA T-STAT R-SOD STD ERROR DURWAT	664  BFTA T-STAT P-SOD STO FRROR DURWAT 554 BETA T-STAT R-SOD STD FRROR DURWAT	126 BETA T-STAT R-SOD STD ERROR DURWAT 236 3FTA T-STAT R-SOD STD ERROR DURWAT R-SOD STD ERROR DURWAT T-STAT R-SOD

3510 3504 0511 0534
9 FNG1 703 109 004 473
-C. 0947 -0. 7871 -0. 7453 -0. 7594 -0.
Die of contract
C.3083 U.777 2.0 5.005 V.1534 0.0 0.0164 0.1534 0.0 0.0176 0.0072 0.0 0.0798 0.0798 0.0 2.1014 0.1528 0.0 -0.1243 0.4748 0.0 -0.7243 0.4748 0.0 0.0045 0.0081 0.0084 0.0084 0.00884 0.00884 0.0088 0.

The data in Exhibit 4 seems to show that some stocks temporarily lead or lag the market. Over 40% of the stocks in each of the three sample periods showed some statistically significant tendency to lead or lag the market. This is a surprising finding to an economist. Economic theory suggests that it is unlikely that leads or lags would actually occur. If some stocks lead or lag the market consistently investors would tend to buy non-leading stocks after the leader's prices rose and sell lagging stocks when non-lagging stocks prices fell in anticipation of the subsequent gains and losses, respectively. This profit maximizing activity would tend to narrow the leading stock's leads and the lagging stock's lags. For this reason, the lead and lag findings were analyzed critically from several different view points.

### II C. Fixed Leads and Lags In the Long-Run

Analyzing a sample of 120 monthly observations tends to wash out temporary phenomena which might appear in a short-term sample. Therefore, it is not surprising to find that the data in Exhibits 3 and 4 do not reveal many stocks which consistently lead or lag for 120 consecutive months. Out of 770 stocks only the first 6 stocks listed in Exhibit 3 showed a statistically significant (that is, t-value above +2 or below -2) tendency to lead or lag which was more pronounced than their

See footnote 4.

EXHIBIT 4

Breakdwon of Lead and/or Lag Statistics On 770 Stocks From Three Different Sample Periods

Tendency To	1962-64 Sample	Sample	1965-67 Sample	Sample	1958-67 Sample	Sample
Lead or Lag	frequency	percentage	frequency	percentage	frequency	percentage
<ol> <li>Lead or lag yields most significant statistics</li> </ol>	44	5.6%	96	12.3%	9	%8*0
<ol> <li>Lead or lag yields significant but second best statistics</li> </ol>	276	36.0%	283	36.7%	339	44.0%
3) No significant tendency to lead or lag evident	450	58.4%	391	51.0%	425	55.0%
TOTALS	770	100.0%	770	100.0%	770	100.0%

tendency to move concurrently with the market. The only noteworthy characteristic which all six of these securities seem to have in common is their low coefficients of determination. These values of R<sup>2</sup> occur for all 13 leads and lags with the six stocks. This is an indication that these stocks do not show any truly consistent pattern of movement relative to the market.

In order to gain more information relevant to the hypothesis that leaders or laggers exist temporarily the 44 stocks from the 1962-64 sample and the 96 stocks from the 1965-67 sample which were in the first category in Exhibit 4 were compared. Although these two groups of 44 and 96 stocks which showed the strongest tendencies to lead or lag during their respective three year sample periods were from the same sample of 770 NYSE stocks, the two groups only had six stocks in common.

More specifically, only the first six stocks in Exhibit 3 were in both the group of 44 and in the group of 96 more significant leading and lagging stocks. The suprisingly large number of leading and lagging stocks found in the two contiguous three year samples are a transient group.

The few stocks which did tend to lead or lag in both of the two short periods had lead and lag times were not constant. That is, of the first six stocks shown in both Exhibits 1 and 2, five of the six stocks had a different lead or lag in one sample than in the other.

#### II D. The Frequency of Particular Leads or Lags

Regardless of whether the total number of stocks which lead

or lag the market is significant, there may be one or a few leads or lags which predominate. Exhibit 5 was prepared to reveal whether some leads or lags occurred more often than would tend to occur if the length of the leads or lags were purely the result of sampling errors. To make this determination, every regression which had a t statistic larger than + 2 or smaller than -2 was tabulated; thus, a stock which had 3 leads or lags with significant t statistics was counted 3 times.

Normally distributed regression slope coefficients will have t statistics above + 2 or below -2 about 5% of the time if the true underlying regression slope coefficient is not significantly different from zero; this would be due to sampling error. Therefore, if more than 5% of the 770 regressions for any given lead or lag (that is, more than 38.5 out of 770 stocks) have significant t statistics, this tends to indicate that that lead or lag is occurring more frequently than it would if lead and lag times were random.

The data in Exhibit 5 shows that market leads and lags of three to four months were the most frequent over all three sample periods. In particular, the number of stocks which followed the market movements by 3 to 4 months was statistically significant. \* Over 17% of the stocks trailed the market by either 3 or 4 months in each sample period. The scarcity of 1, 2, 5 and 6 month leads and lags combined with the disproportionately large number of 3 and 4 month leads

<sup>\*</sup> See footnote 4.

EXHIBIT 5

The Frequency of Leading and Lagging Regressions which had t Statistics Larger Than + 2 or Smaller Than - 2

			Mc	onths By	' Which	Months By Which Market Leads the Stocks	ads the	Stocks				
	9-	-5	4-	-3	-2	-1 0	F-1	2	3	4	5	9
1962-64, frequency	67	36 months, most 12 67	hs, most 67	tly bullish 18 25	lish 25	16	17	14	21	115	59	37
percent	8.7%		1.6% 8.7%	2.4%	3.2%	2.0%	2.2%	1.8%	2.7%	2.7% 15.0%	3.7%	4.8%
1965-67,		36 mont	hs, mar	ket disr	upted ]	36 months, market disrupted by "credit	G	of 196	6"	C	ç	7.
frequency percent	26 3.4%	3/ 4.8%	7.7%	5 / 7.4%	34 4.4%	%8. 8.8%	2.5%	2.7%		112 38 14.6% 4.9%	3.0%	4.8%
1958-67,	ر بر	120 mon <sup>1</sup>	ths, mos	120 months, mostly bullish	lish 52	0,	09	25	102	54	12	10
percent	3.2%	2.6%		5.6%	6.8%	3.8%	7.8%	3.3%	13.1%	13.1% 7.0%	1.6%	1.3%
		Market Lags =		stock leads	ads		Ma	rket L	Market Leads =	stock lags	lags	
					į							

and lags suggests that sometimes some secondary reaction may be occuring 3 or 4 months before or after—and more usually after—movements in the market index. Since all NYSE listed firms are required to give quarterly reports to all shareholders, one possible hypothesis is that information disclosed at the time the first quarterly reports are prepared and released after a change in the condition of the market (for example,  $r_{\rm mt}$  changes sign) is used by some investors as a basis for buy and sell decisions made in response to the market's change. This is highly conjectural.

### III. Negative Betas

The three gold mining stocks listed at the bottom of Exhibits 1, 2 and 3 demonstrated a weak tendency to have negative beta coefficients for the zero lag. These are the only stocks in the sample of 770 which had negative beta coefficients for a zero lag from the 120 month sample period. Although these three negative betas were not significantly different from zero (at the .05 level of significance), they are nevertheless interesting because of the homogeneity of their operations Apparently, the three firms comprise the entire U.S. gold mining industry.

The data suggests that some investors may react to a bear market by liquidating their holdings and seeking refuge in gold mining stocks thus driving the prices of these issues up in a bear market. Of course, other rationalizations of these negative betas are also possible. This meager group provides a piece of evidence in support of the hypo-

thesis that certain groups of stocks tend to move independently of the market.

Stocks from some other industrial categories (for example, utility stocks) had a significant proportion of firms which tended to move independently of the rest of the market (that is, had zero correlations for K = 0) in both the long and short run. However, in those few industrial categories which tended not to be market dependent, the number of highly independent stocks always comprised fewer than half of the stocks in that particular industrial category. Thus, the gold mining industry appears to be the only group which is statistically independent of the market.

After completing the regressions, the five regression statistics for 13 different leads and lags were cumulated over all 770 stocks. These 65 (= 5 x 13) totals for each sample period were then divided by 770 to obtain average regression statistics for each sample period.

Exhibit 6 shows these average regression statistics. These statistics represent objective mathematical expectations of the statistics an investor would have encountered by selecting stocks from the NYSE randomly with replacement.

Of the thirteen average beta coefficients for each sample period, twelve are not significantly different than zero. Only the

Exhibit 6

Average Statistics Over 770 Equally Weighted NYSE Stocks From Three Different Sample Periods

ri T	1962–64	1962-64, 36 months, mostly bullish	is, mostl	y bullis	ų								٠	
<b>ن</b>	LAG 6		<u> </u>	-3	-2	-1	ZERO	T+	+2	+3	7+	+5	+6 LEAD	AD
i ;	7.00	72 to 0 = 877		AVERAGE STATISTICS	CS -0.0434	0.0680	0.9725	0.0782	-0.0361	0.0037	0.2732	0.0535	-0.0930	
1001	4 . (	- CO 70 - CO 7	ı	0.0256 -0.168	-0.1683	0.3093	4.5691		-0.1590	0.0267	1.0229	C.2138	-0.3398	
4 - 2 - 0	0.40	770 0 0475			0.0222	0.0204	0.3684		0.0200	0.0175	0,0503	0.0227	0.0268	
TOTO FRESOR	?	ļ		0.0725 0.0724	0.0724	0.0725	0.0577	0.0725	0.0725	0.0725	\$12C-0	0.0724	0.0722	
DURWAT		809 2,0315			2.0275	2.0814	2,1686	2.0384	2.0277	2.0341	2.0430	2.0277	2.0466	1
	:													
	1965-67	1965-67, 36 months, market disrupted	ıs, marke	t disrur		by "credit	crisis of	1966"						
ز	LAG6	tr I	<b>†</b>	rs 1	-2	7	ZERO	+1	+5	+3	++	+5	+6 LEA0	40
4 1 0	ć	000	AVERAGE		CS 0903	6000	20.80	0.1191	9090	0.0338	**************************************	-0.1079	0.0839	
7-1-A	0.0544	544 10.5063	7602-0-	0.7122	0.1477	0.2359	3.6463	0.3822	0.1759	0.9318		-0.3234	-0.1977	
R-SOD	0.0254		0.0340		0.0286	0.0259	0.2800	0.0238	0.0209	0.0475	0.0260	0.0219	0.0262	
STU ERROR				0.0811	0.0814	0.0816	96 90 0	0.0816	0.0317	0.0809		0.0816	0.0814	
DURWAT	2,0036	036 2.0474	2.0301	2.0225	2.0618	2.0966	2.1451	2.1679	2.0482	2.0267		2.0550	2.0560	
1	1928-67	1958-67, 120 months, mostly bullish	hs, most	lv bulli	ls.									
-	9 T	ر ا	7)	۲,	->	Ī	7590	+1	+2	+3	+4	+5	+6 LEAD	<b>©</b>
			3000HAV	5	\$									
AFTA	0.0187	187 - A. 070E	!	+560°0	0.03	C. 0773	-6450°C	0.1255	0.0750	0.1553	0.061.6	-0.0453	١.	
T-STAT	0.0725				C. 4337	00270	6.4537	0,7523	0.3986	- 1	0.4613	-3.2042	וי	
2-50D	0.0078			l	0.0110	0.0077	0152*3	0.0110	0.0073	0.7149	00100	\$900°C	\$ \$ \$ \$ \$ \$ \$ \$ \$ \$ \$ \$ \$ \$ \$ \$ \$ \$ \$	
STO FRRER			7970,7	992000	0.0765	C.0766	0.0663	6.0745	992000	- 1	0.0766	0.0767	- 1	1
DURZAT			5 2.6337	2,0396	2.0429	2.0720	2.1372	2.1045	2.0444		2.0404	2,0323		

zero lag average betas are significant, on average.

The average coefficients of determination in Exhibit 6 for the zero lag regressions are statistically significant. But, the R-squared values are not significantly different from zero for any other lead or lag. The three R-squared average statistics for the zero lag regressions range in value from .26 to .36. This implies that about one-fourth to one-third of the average NYSE stocks' variance may be attributed to factors common to all stocks on the NYSE which occur concurrently. These statistics are similar to those found in other studies [4], [17].

All of the average Durbin-Watson statistics are well within the range that indicates a lack of serial correlation in the regression errors. And, an examination of all 770 individual Durbin-Watson statistics for the zero lag regressions showed that virtually none of them has serially correlated errors.

# V . Distributed Lags Analysis

# V A. Almon Distributed Lags

In order to make an intertemporal estimate of the impact on security prices caused by investor's expectations and reactions to systematic changes the Almon distributed lag technique was employed [1]. Various models of the form shown in equation (3) were esti-

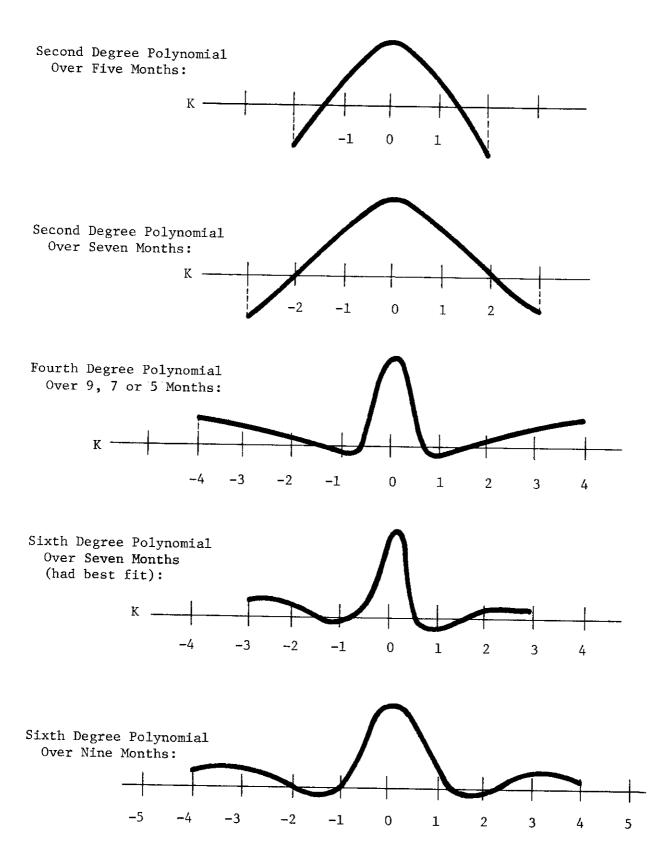
Each of these models was fit with a constraint that the regression coefficients (or weights) conform to a polynomial of degree two, four or six. The degree of the polynomial must be less than the number of regression coefficients. So, it was not possible, for example, to fit equation (4) for anything other than the second order polynomial. The tails of the polynomial distribution were left free to fluctuate to obtain the best fit.

The Almon distributed lag was fit to each of the 30 stocks in the Dow-Jones Industrial Average (DJIA). These are all large, old New York Stock Exchange (NYSE) listed stocks for which ample financial information is widely available. This sample was selected because these stocks are closely followed by many professional security analysts and their markets have sufficient depth to minimize fluctuations due to problems with market liquidity. The price behavior of seven heterogeneous small NYSE listed stocks was also analyzed along with the 30 DJIA stocks to increase the heterogeneity of the sample. Ten years of monthly data was used to fit all the distributed lag models for all 37 stocks.

# V B. Interpretation of the Results

ed with each of the three polynomials which were estimated. Regression equations (4), (5) and (7) and the second and fourth degree polynomials almost consistently yielded lower coefficients of determination than the simple no lag regression equation (1). Only equation (6) with

Typical Distributed Lag Structures Obtained With Second, Fourth, and Sixth Degree Polynomials



the weights distributed according to a sixth degree polynomial was able to increase the coefficients of determination above those obtained with equation (1) with much success. This improvement in the  $R^2$  only occured in slightly over half the 37 stocks used in the distributed lag analysis. And, this increase in the  $R^2$  averaged only approximately 8% above the  $R^2$  attained using equation (1). Exhibit 8 shows the polynomial distributions for the weights for a few of the stocks.

American Telephone and Telegraph (ATT). It is typical of almost half the sample of 37 stocks in that the simple zero lag regression model (1) yielded a slightly higher coefficient of determination than the distributed lag regression. The lower  $R^2$  from the distributed lag regression resulted from the additional constraint that the weights conform to some polynomial which was imposed in using the Almon technique. This added constraint limited the range of values the regression coefficients could assume to minimize the residual variance. An unconstrained multiple regression equation (6) would yield  $R^2$  values at least as large as those obtained from the simple zero lag regression (1).

The three distributions of weights shown in the lower portion of Exhibit 8 are typical of the majority of the 37 stocks for which Almon distributed lags were fit. The Almon distributed lag of best fit  $^9$  is essentially an inverted V centered over K = 0 with tiny long tails on each side.

### EXHIBIT 8

# Selected Almon Distributed Lags For Sixth Degree

## Polynomial Over Seven Months

	Coeff. of Dete	ermination
American Telephone and Telegraph	Eqn. (1)	.24
·	Eqn. (6)	.23
Months lead	1 1	
-3 -2 -1 0 1	2 3	
	Coeff. of Dete	ermination
$\wedge$	Eqn. (1)	.51
U.S. Steel	Eq <b>n.</b> (6)	<b>.</b> 55
/ \		
Months lead		
-3 -2 -1 0 1	2 3	
	Cooff of Dot	amminotion
	Coeff. of Dete	
Standard Oil of New Jersey	Eqn. (1) Eqn. (6)	.10
_	Equ. (0)	•10
Months lead		
-3 -2 -1 0 1	2 3	
	Coeff. of Dete	ermination
	Eqn. (1)	.17
Chrysler	Eqn. (6)	.20
Months lead	ŀ	
		<del></del>
-3 -2 -1 0 1 2	. 3	

#### VI. Conclusions

# VI A. Implications For Technical Analysis

According to the efficient markets hypothesis security prices at any given point in time should reflect all publicly available information [9]. Stock prices may over-react or under-react to new information in an efficient market. However, prices should react immediately, continuously and in an unbiased fashion until all information is fully reflected in the market price. This implies that the investors in certain stocks shouldn't react faster or slower than the investors in other stocks to information which systematically affects all stocks traded in a market.

The data in Exhibits 1,2,4 and 5 for the shorter-term sample periods reveals some leads and lags which may be deviations from the efficient markets hypothesis. But, these brief periods of apparent leading and lagging activity exist for a minority of stocks listed on the NYSE. The existence of these stocks is consistent with the chartists' belief that certain stocks temporarily lead the market [18]. However, it would be extremely difficult to find and profit from these temporarily leading or lagging stocks because they are such a small transient group. Furthermore, it is not clear that the temporarily significant leads and lags which appear in the data are not just sampling errors caused by unsystematic changes in the firm which lead or lag similar changes in the market merely by coincidence.

exhibit 5 shows that the three and four month leads and lags occur more frequently than would result from sampling error in a normally distributed population. However, this tendency is not highly pronounced and could be due to inefficient statistics from an open-ended distribution [8] or sampling errors. More importantly, even if 20% of the stocks in the NYSE have a second direct reaction to systematic changes or 4 months after the market's reaction, this secondary reaction is so diminutive that it cannot be expected to yield trading profits large enough to cover the transactions costs involved in buying and selling the stocks. So, the preparation of charts to detect such secondary reactions would not be worthwhile.

A company by company examination of the 770 stocks reveals that those few firms which do appear to lead or lag the stock market have nothing in common. And, the data shown in Exhibits 4, 6 and 8 imply that the efficient markets hypothesis is an accurate description of reality in the long-run. That is, over a ten year period virtually no stocks in the sample of 770 showed any tendency of any kind to either anticipate or to react latently to systematic fluctuations in the market index. Thus, although charting leading and lagging stock prices may appear to be profitable over periods of a few years, there is no reason to expect chartists to outperform a naive buy-and-sell strategy.

The six stocks in a sample of 770 which show some tendency to lead or lag over a ten year period comprise slightly less than one percent of 770 stocks in the sample. This small number of non-systematic stocks could occur in a market characterized by systematic price movements due purely to random chance. The comments of a Wall Street portfolio manager 10 were helpful in gaining an insight into the possible causes of the first six stocks listed in Exhibits 1, 2 and and 3. At least 4 of these 6 stocks underwent financial reorganizations, management shake-ups, shifts in their product lines, and/or other significant changes which could induce price movements which were not related to the markets movements but could yield spurious lead and lag statistics. As a result, it is possible that not all of the 6 stocks which appear to consistently lead or lag the market were actually leading or lagging the market in any meaningful sense.

VI B. The Implications for One Period Capital Market Theories

Sharpe [20] has shown that under a given set of assumptions the <u>ex ante</u> expected returns on capital assets in equilibrium will be determined in accordance with the one-period model shown in equation (8)

$$E(r_{it}) = R_t(1 - b_i) + b_i E(r_{Mt})$$
 (8)

where  $R_t$  is a riskless rate and  $E(r_{it})$  and  $E(r_{Mt})$  are the expected rates of return for the  $i^{th}$  asset and the market portfolio [13,pp.

ll7-8], respectively, during time period t, and b is an index of market sensitivity equal to  $(\frac{\text{Cov}(r_{\text{it}}, r_{\text{Mt}})}{\text{Var}(r_{\text{Mt}})})$ . And, Treynor [22]

developed a portfolio performance measure which, like Sharpe's model, utilizes the simple "one-period" model shown in equation (1).

For empirical tests the <u>ex ante</u> model of equation (8) is replaced with the <u>ex post</u> characteristic regression model shown in equation (1). Jensen [16] has shown that under reasonable assumptions it is possible to move from the unobservable equation (8) to the observable return generating process measured by equation (1). The question here is: What is the significance of the one-period models shown in equations (1) and (8) in light of the <u>multiperiod</u> distributed lag model shown in equation (6) and the lagged regression equation (2) which sometimes fits the data better than the one-period model of equation (1)?

The average statistics shown in Exhibit 6 are relevant for evaluating the propriety of the "one-period" aspect of Treynor's portfolio performance model. Most of the statistics are unbiased estimates of the statistics which would be obtained from a large portfolio constructed of many stocks. On average, the leading or lagging statistics are all statistically insignificant. The statistically significant leads and lags which occur in some individual stocks are washed out when they are averaged into a large portfolio. This means that restating Treynor's portfolio performance measure in terms of a distributed lag

regression model is pointless. A multi-period portfolio characteristic regression would not only be clumsy to work with, but would not yield any better measure of the portfolio's systematic risk.

Sharpe's one-period model shown in equation (8) represents a market equilibrium. In order to obtain the best empirical estimate of the long-run equilibrium tendencies of the market, the 10 year sample data shown in Exhibits 3, 4 and 5 are probably the most appropriate. These data were used to evaluate different fixed leads and lags in Sharpe's model. As explained above, only 6 stock out of 770 showed any discernable sign of consistently leading or lagging the market. The comments of an experienced portfolio manager suggested that fundamental changes in these 6 stocks possibly induced spurious leads or lags. Considering these factors, there is no reason to suspect Sharpe's model would conform to the empirical data better if it were restated in terms of some fixed lead or lag model analogous to equation (2).

On the other hand, the data in Exhibit 8 showed that the predictive power of the empirical analogue to Sharpe's equilibrium model (8) could usually be improved slightly by using a complicated distributed lag. The majority of the stocks in the sample had higher coefficients of determination from equation (6) than from equation (1). But, these improvements in the goodness-of-fit statistics were both small and unreliable. The distributed lag model failed to yield a better

fit than the simple regression equation (1) in almost half the stocks tested. This is evidence that there is no prevalent lead or lag structure in the market. In view of the high degree of explanatory power and robustness which equation (1) possess, and, the lack of convincing evidence that any lead or lag structure exists, it seems best not to complicate the elegant simplicity of equations (1) and (8) by appending leading and lagging independent variables.

VI C. The Market's Efficiency in Dealing With Systematic Changes

Defining an efficient market to be one in which prices reflect all public information is popular -- but, this definition isn't entirely satisfactory. An efficient market should also channel resources into their most productive uses. In order for the prices in a market to reflect all information and allocate resources efficiently, they must adjust to new information soon after it becomes available-the sooner, the better. Furthermore, an efficient market should be able to discriminate between information which is relevant in asset pricing and irrelevant information. In essence, at every point in time the assets in an efficient market should be priced in such a manner that comparable investments yield equivalent rates of return. Comparable investments may be defined to be investments which have comparable levels of systematic risk, as measured by their beta regression slope coefficients in equation (1).

Equations (1) and (8) presume that assets' prices adjust simultaneously as systematic changes in the market occur so that their expected returns at any point in time are proportional to their systematic risks. The goodness - of - fit statistics for regression equation (1) tend to validate it as a description of reality. This is a strong piece of evidence that stock prices reflect all publicly available information immediately after it becomes available. In contrast to the ability of stock prices to react months in advance to changes in unsystematic factors [2] [10], their inability to anticipate systematic factors is noteworthy.

### VII. Summary

The analyses above revealed no significant tendency for certain stocks or groups of stocks to lead or lag the market. This implies that (1) all stock prices react immediately to new news which systematically affects the security markets and (2) investors quickly develop similar expectations about the impact of new factors which affect all stocks. This type of market efficiency implies several things. First, technical analysis of stocks which lead or lag the market indices may not be expected to yield long-run profits in excess of a naive buy and hold strategy. And, the one-period capital market theories developed by Sharpe, Treynor and others would not be sufficiently enriched by the inclusion of leads and lags to justify the associated complications.

#### FOOTNOTES

\*Assistant Professor of Finance, Wharton School of
Finance, University of Pennsylvania. The Rodney L. White Center
for Financial Research at the Wharton School provided financial
support for this research. Messrs. John Percival, A. Walter Temple,
Leon Levy and Irwin Friend made contributions which improved
this paper.

This project did not delve into leads or lags of less than one month. Of course, fairly consistent leads or lags of a few days or weeks may exist whether or not longer ones were present. Nevertheless, leads and lags of less than one month were not investigated for the reasons listed below. First, stocks which lead or lag the market by only a few days or weeks would not appreciably distort the market models which are based on the concept that securities have systematic risk [20], uninsurable risk [22], or, concurrent covariability of returns -- as it is variously called. Second, leads or lags of only one or a few days length would constitute a very minor imperfection in the efficient markets hypothesis [9]. A third reason that differencing intervals of less than one month were not analyzed is because of the measurement errors which are bound to be present when measuring short-term changes in security prices [8]. These errors in the variables would downbias the statistics [15] which are used in this analysis to such an extent that it would be difficult to detect small lead or lag tendencies.

 $^2$ Multi-index models of the following form have been analyzed  $r_{it} = b_o + b_1 I_{lt} + b_2 I_{2t} + \dots + e_t$  where  $I_{jt}$  is the value of the j<sup>th</sup> market index during period t in this multiple regression model seeking to explain  $r_{it}$  -- the <u>concurrent</u> rate of return from the i<sup>th</sup> asset [6]. Curvi-linear models of the following form have been tested [23]:  $r_{it} = b_o + b_1 r_{nt} + b_2 r_{nt}^2 + e_t$  where  $r_{mt}^2$  is the squared percentage change in some <u>concurrent</u> market index. And, the following linear regression model underlying serial correlation tests has been used by several analysts (for example, [8, pp. 72-3]:  $r_{it} = a + br_{i,t+k} + e$  where k is a lag. However, there is a paucity in research dealing with

<sup>3</sup>The data was obtained from the University of Chicago Price Relative Magnetic Tape File [11], [12]. Cash dividends were included in the monthly rate of return calculations in order to avoid having the price dropoffs which accompany the payment of cash dividends [7] introduce "noise" into the data.

lagged index models of the following general form:  $r_{it}$  + a + b  $r_{m,t+k}$ .

The first 770 firms in the file which met the following conditions were included in the sample: (a) the stock was listed on the NYSE every month from July 1956 to June 1968 inclusive; and, (b) the stock's record had no zero price relatives (which are so unlikely to actually occur that they were presumed to be erroneous entries).

lead and lag was added to the 36 months under analysis, a total of 4 years of data were used for the three year analyses.

<sup>7</sup>The small autocorrelation coefficients reported by Fama [8, pp. 69-74] could cause the regression statistics which were significant for some lead or lag but less significant than the statistics for the zero lag regression.

<sup>8</sup>Very little analytical research dealing with leads and lags has been published. In 1936 Fritzemeier concluded that "there is no tendency for any price group to lead or lag" the market [14,p.153].

 $^9$ Goodness-of-fit is judged on two main criteria, following Shirley Almon's suggestion [1]. First, the  $\overline{R}^2$ ; and second, "close similarity of the weights among distributed lags of optimal and longer length . . . " [1, p. 185].

Comments made by Mr. Leon Levy, Partner, Oppenheimer and Company, New York City, were enlightening.

<sup>11</sup>For proof that the beta coefficient of a portfolio is the average of the betas for the individual assets in the portfolio see page 173 of [13].

#### REFERENCES

- 1. Almon, Shirley, "The Distributéd Lag Between Capital Appropriations And Expenditures," <u>Econometrica</u>, Vo. 33, No. 1, January 1965, pp. 178-196.
- Brown, Phillip and Ball, Ray, "An Empirical Evaluation of Accounting Income Numbers," <u>Journal of Accounting Research</u>, Autumn 1968, pp. 159-178.
- 3. Blume, M.E., <u>The Assessment of Portfolio Performance: An Application of Portfolio Theory</u>. Unpublished Ph. D. dissertation, University of Chicago, 1968.
- 4. \_\_\_\_\_, "On the Assessment of Risk," <u>Journal of Business</u>,
  March 1971.
- 5. \_\_\_\_\_, "Portfolio Theory: A Step Toward Its Practical Application," <u>Journal of Business</u>, April 1970.
- 6. Cohen, Kalman J. and Pogue, Jerry A., "An Empirical Evaluation of Alternative Portfolio Selection Models," <u>Journal of Business</u>, April 1967, pp. 166-193.
- 7. Durand, D. and Alan M. May, "The Ex-Dividend Behavior of American Telephone and Telegraph Stock," <u>Journal of Finance</u>, 15, No. 1 (March 1960), pp. 19-31.
- 8. Fama, E., "The Behavior of Stock Market Prices," <u>Journal of Business</u>, 38, No. 1 (January 1965), pp. 34-105.
- 9. \_\_\_\_\_\_, "Efficient Capital Markets: A Review of Theory and Empirical Work," <u>Journal of Finance</u>, May 1970, pp. 383-423.
- 10. \_\_\_\_\_, L. Fisher, M. Jensen and R. Roll, "The Adjustment of Stock Prices to New Information," <u>International Economic Review</u>, 10, No. 1 (February 1969), pp. 1-21.
- 11. Fisher, L., Lorie, J.H., "Rates of Return on Investment in Common Stock," Journal of Business, 37, No. 1 (January 1964), pp. 1-21.
- 12. \_\_\_\_, and \_\_\_\_, "Rate of Return on Investments in Common Stock: The Year by Year Record, 1926-65," <u>Journal of Business</u>, 39, No. 1, Part II (January 1966), pp. 291-316.
  - 13. Francis, J.C. and Archer, S.H., <u>Portfolio Analysis</u>. Englewood-Cliffs, New Jersey: Prentice-Hall, Inc. 1971.

- 14. Fritzemeier, Louis H., "Relative Price Fluctuations of Industrial Stock in Different Price Groups," <u>Journal of Business</u>, April 1936, pp. 133-155.
- 15. Johnston, J., Econometric Methods, McGraw-Hill, Chapter 6, 1960.
- 16. Jensen, M., "Risk, the Pricing of Capital Assets, and the Evaluation of Investment Portfolios," <u>Journal of Business</u>, April 1969.
- 17. King, B.E., "Market and Industry Factors in Stock Price Behavior,"

  <u>Journal of Business</u>, January 1966, pp. 139-190.
- 18. Leffler, G.E., and Farwell, L.C., <u>The Stock Market</u>. Third Edition. New York: The Ronald Press Co., 1963, pp. 55-56.
- 19. Markowitz, H., <u>Portfolio Selection</u>. New York: John Wiley & Sons, Inc., 1959, p. 100.
- 20. Sharpe, W.F., "Capital Asset Prices: A Theory of Market Equilibrium Under Conditions of Risk," <u>Journal of Finance</u>, Sept. 1964, pp. 425-442.
- 21. \_\_\_\_\_, "A Simplified Model for Portfolio Analysis," <u>Management Science</u>, January 1963, pp. 277-93.
- 22. Treynor, Jack L., "How to Rate Management of Investment Funds,"

  Harvard Business Review, January/February 1965, pp. 63-75.
- 23. \_\_\_\_\_, and Kay K. Mazuy, "Can Mutual Funds Outguess the Market?" <u>Harvard Business Review</u>, July-Aug., 1966, pp. 131-136.