A Model of Capital Asset Risk

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I. Introduction

The "market model" of capital asset pricing theory posits that the one-period return on an asset is a linear function of the one-period return on a "market factor" plus the effect of factors that are unique to that asset. The coefficients of the model, estimated using realized returns, can be used for predicting asset returns conditional on market returns, and the slope or "beta" coefficient provides an estimate of the asset's risk. Though the market model has been applied to a wide variety of capital market studies, and is now being applied by practitioners for assessing asset risk, very little research has been undertaken that attempts to discover the determinants of the beta coefficient.[1]

This paper develops a model to derive and measure the underlying factors used by the market to assess an asset's beta coefficient and, thus, indicates how conditional predictions from the market model might be improved. In addition, we indicate how traditional asset valuation theory can be integrated with modern capital asset pricing theory to yield an increased understanding of the process of risk assessment. The analysis shows that the market model is an important special case of a more general theory of how asset returns are generated.

II. Risk Assessment

This section shows that the comovement of an asset's return with the return on a market factor can be expressed as a function of the asset's cash flow and capitalization rate comovement with cash flow and capitalization rate market factors. Prior to this, however, a brief description is given of the market model, its assumptions, and how it may be used to assess market risk.

Market Risk

Market risk refers to differences in risk between securities estimated using observed market price and dividend data. As mentioned, one widely accepted method for assessing market risk utilizes estimates from the market model. The market model asserts that the one period return on an asset, R_i, is a linear function of the one period return on a market factor, R_m, and factors unique to the asset, ϵ_i, or, expressed algebraically,

$$R_i = \alpha_i + \beta_i R_m + \epsilon_i \ . \tag{1}$$

The constants α_i and β_i measure the response of the asset's return to the return on the market factor.

If the error term of equation (1) is independently distributed over time and unrelated both to the market factor and to the unique factors of other assets, then, using historical market price and dividend data in a standard regression model, an estimate of the slope coefficient, $\hat{\beta}_i$, supplies an estimate of the asset's risk.[2]

Determination of Market Risk

Asset prices are determined by the actions of wealth maximizing investors in such a way that they are equal to the discounted value of expected cash flows. An asset's realized return in any finite period of time will depend on changes in investor expectations of future cash flows and changes in the rate at which these cash flows are capitalized. Mathematically, under the assumption of a discounted cash flow valuation model, the realized return on an asset, R_i, in differential form, is equal to,[3]

$$R_i = \frac{dC_i^*}{C_i^*} - \frac{dr_i}{r_i} + r_i \ , \qquad (2a)$$

where C_i^* is the perpetuity equivalent cash flow expected by investors, and r_i is the rate at which these expected cash flows are capitalized. Equivalently, the return on the market factor, R_m, can be expressed as,[4]

$$R_m = \frac{dC_m^*}{C_m^*} - \frac{dr_m}{r_m} + r_m \ . \qquad (2b)$$

If there is no change in investor's cash flow expectations and no change in the rate at which these cash flows are capitalized, then the realized return for asset i in any period will equal the return expected at the beginning of that period. The expected return, of course, is the capitalization rate, r_i.

The implication of this analysis, originally suggested by Rie,[5] is that the risk of holding any asset stems from changes in the asset's cash flow and capitalization rate that effect the investor's expected portfolio cash flow and portfolio capitalization rate. Although expected returns of an asset can be assessed using estimates from the market model of equation (1), realized returns of an asset will be determined by a multi-factor market model given by,

$$R_i = r_i + \beta_{ri}\pi_{rm} + \beta_{Ci}\pi_{Cm} + \delta_i \quad . \qquad (3)$$

In equation (3) π_{rm} is the market capitalization rate factor, and π_{Cm} is the market cash flow factor (analogous to $\frac{dr_m}{r_m}$ and $\frac{dC_m}{C_m}$ in equation (2b), respectively). The error term δ_i has an expected value equal to zero, is independently distributed and unique to asset i.[6] β_{ri} is the slope coefficient of a regression of asset i's capitalization rate on a market capitalization rate variable, and β_{Ci} is the slope coefficient of a regression of asset i's cash flow on a market cash flow variable.

The multi-factor model embodied in equation (3) can be viewed as a generalized market model, and the traditional market model represents a special case[7] where

$$\beta_i = \frac{COV(\frac{dC_i}{C_i}, \frac{dC_m}{C_m}) + COV(\frac{dr_i}{r_i}, \frac{dr_m}{r_m})}{VAR(\frac{dC_m}{C_m}) + VAR(\frac{dr_m}{r_m})} \tag{4}$$

or, equivalently,

$$\beta_i = \gamma\beta_{Ci} + (1 - \gamma)\beta_{ri} \tag{5}$$

where

$$\gamma = \frac{VAR(\frac{dC_m}{C_m})}{VAR(\frac{dC_m}{C_m}) + VAR(\frac{dr_m}{r_m})} \; . \tag{6}$$

There appear to be several interesting possibilities suggested by the two factor model. For example, one implication is that the conditional predictions of the single factor market model may not be independent of the estimated market model slope coefficient used to generate the predictions. This follows if β_{Ci} is not equal to β_{Ri} and their respective importance in determining β_i (i.e., γ and $1- \gamma$) changes over time. Accordingly the usefulness of the market model to generate estimates of asset risk and to make conditional predictions may be seriously damaged.

The single factor market model appears to assume that the cash flow and capitalization rate effects on an asset's realized return are not of different magnitudes, and a given percentage variation has the same effect whether from $COV(\frac{dC_i}{C_i}, \frac{dC_m}{C_m})$ or $COV(\frac{dr_i}{r_i}, \frac{dr_m}{r_m})$.

In the next section of this paper we examine the association between the single factor market model slope coefficient, β_i, and corresponding cash flow and capitalization rate slope coefficients in equation (4). Specifically, we attempt to assess the relationship between the market model and the two factor model by estimating the regression equation.

$$\beta_i = a_o + a_1 \left[\frac{COV(\frac{dC_i}{C_i}, \frac{dC_m}{C_m}) + COV(\frac{dr_i}{r_i}, \frac{dr_m}{r_m})}{VAR(\frac{dC_m}{C_m}) + VAR(\frac{dr_m}{r_m})} \right] + u_i, \qquad (7)$$

in each of two separate time periods.

III. Estimation of Model

The accounting data used in the analysis were obtained from Standard and Poor's Annual Compustat tapes for the period January, 1947 to June, 1968. The twenty-one year period was divided into two subsamples: the first covering 120 months through December, 1956, and the second covering 138 months from January, 1957 through June, 1968. Firms were eliminated from the analysis if the required annual accounting figures were not available for at least six years during the period studied.

The market price and dividend data used were obtained from the data file constructed by the Center for Research in Security Prices at the University of Chicago, updated at the University of Pennsylvania. The market model parameters were estimated for each period using a minimum of sixty monthly observations.

A number of firms were eliminated from the analysis. In all cases these were firms that either had average earnings over the period close to zero, or which had reported earnings in one or more years that were a large number of standard deviations away from average earnings. In both of these cases the estimated cash flow slope coefficient appeared to be seriously biased in one direction or the other dependent, of course, on the level of market cash flows. The bias is the result of using reported earnings as opposed to expected earnings in determining the cash flow beta.

As an operating procedure firms were eliminated from the analysis if the cash flow slope coefficient exceeded ten or was lower than minus two. This method of selecting a sample tends to reduce the generality of the results since it eliminates firms with particular characteristics. We suspect, however, that market participants would adjust earnings for these rather unusual unique factors prior to assessing or estimating cash flow beta. Our assumption is that cash flow betas exceeding these bounds would distort the estimated relationships. After this selection procedure was performed there were 338 firms available for analysis in the 1947-1956 period, and 543 firms available in the 1957-1968 period. A total of 313 firms met the required conditions in both periods.

The Measure of Market Risk (β_{mkt})

The measure of overall market risk used in the analysis is the slope coefficient of the market model described in equation (1). This beta coefficient is obtained by regressing monthly investment relatives on investment relatives from the Fisher Index.[8]

<u>The Measure of Cash Flow Beta</u> (β_{eps})

The development of a variable to serve as a proxy for cash flow slope coefficient (the first term on the right hand side of equation (5)) was somewhat more complicated. For each firm an earnings index was created by dividing each year's annual earnings per share by the average earnings per share generated over the period studied. The same was done for a measure of market earnings using the corresponding twelve month values from the quarterly earnings per share of Standard and Poor's Composite Index.

The earnings index for the firm and the market were regressed on time to generate an arithmetic earnings growth rate. An estimate of the cash flow beta was then generated by regressing the residuals from this trend line on the residuals of the corresponding market trend line. This calculation abstracts from the differential rates of growth that exist between firms while still measuring the extent to which earnings of the firm tend to fluctuate with the earnings of the market.

A cash flow beta on operating income, β_{oi}, was also calculated in order to observe the seperate contribution of operating and financial characteristics to overall cash flow risk.

The Measure of Capitalization Rate Beta ($\beta_{e/p}$)

The variable selected as a proxy for the second term on the right hand side of equation (5) is the slope of the regression of the percentage change in the earnings-price ratio for a firm on the percentage change on the earnings/price ratio of Standard and Poor's Composite Price Index. As explained before, this variable measures the variability of the capitalization rate that is attributable to changes in the market's capitalization rate.

An observed earnings/price ratio, of course is the result not only of the capitalization rate attached to a security by the market, but also the result of the market's expectation of the growth in a security's cash flows over time. Thus, the use of earnings/price ratios as proxies for (unobservable) capitalization rates attached by the market will, in a regression, bias upward the intercept of a firm with low growth expectations and bias downward the intercept for firms with high growth expectations. The slope of the regression, however, will be left unbiased unless changes in growth expectations for the entire market cause more than proportional changes in growth expectations for some securities and less than proportional changes for other securities. In this case, if high *ex post* growth firms tend also to be high risk firms,[9] then the capitalization rate beta will be biased upward. Our empirical results, however, show that earnings growth is not very closely related to the estimated capitalization rate beta so that the degree of bias is probably not very large.

The Sample Values of the Variables

Table 1(a) summarizes the minimum and maximum number of observations used to estimate the slope coefficients for each regression, and gives the average values of the correlation coefficients from these regressions. Considering the potentially large effect of unique factors on many accounting numbers, particularly reported earnings, and the relatively small sample available, the errors in the measurement of the cash flow and capitalization rate slope coefficients can bias downward the regression coefficients between market measured risk and these determinants of market risk. In the next section we suggest a technique for overcoming this potentially serious source of bias.

If the market factor is correctly specified the mean values of all variables should be one. As shown in Table 1 (b), however, the mean value for the market model beta, β_{mkt}, is slightly less than one and the mean of the capitalization rate and cash flow betas is typically greater than one. The market model beta was estimated using the Fisher Index which equally weights all firms on the New York Stock Exchange. Our study, of necessity,

TABLE 1

(a)

Number of Observations Used in Estimation and Average Coefficients of Determination

	$\beta_{e/p}$		β_{eps}		β_{mkt}	
	1947-56	1957-68	1947-56	1957-68	1947-56	1957-68
Min. No. of Observations	5	5	6	6	60	60
Max. No. of Observations	9	10	10	11	120	138
Aver. Coef. of Determination	.37	.38	.39	.40	.32	.28

(b)

Variable Means and Standard Deviations

Variable	Mean		Standard Deviation	
	1947-56	1957-68	1947-56	1957-68
β_{mkt}^*	.948	.985	.321	.307
$\beta_{e/p}^*$	1.720	1.257	1.836	1.047
β_{eps}^*	1.063	1.276	.891	1.389
β_{oi}	.513	.872	.867	1.116
PAY	.489	.487	.141	.180
D/E	.901	.664	.882	.786
SIZE (billions)	.273	.441	.784	1.456
LIQUIDITY	2.796	2.776	1.180	2.846
g_{eps}	.050	.091	.093	.075

*The variables β_{mkt}, $\beta_{e/p}$, and β_{eps} are the identifications given to the variables used as estimates for β_i, β_{ri}, and β_{Ci}, respectively.

has eliminated firms with a short history on the exchange and these are likely to be the higher risk firms. The result will be a lower average value for the remaining firms.

The cash flow and capitalization rate betas were calculated using Standard and Poor's Composite earnings per share and earnings/price variables as market factors which are weighted by the size of the firms included. Therefore, if large firms tend to be of somewhat lower risk, then the unweighted mean betas shown in Table 1 will tend to exceed one. The operating income beta should be less than the earnings per share beta due to the average leverage of the firms studied.

<u>The Estimations</u>

The hypothesis developed in Section II is that the slope coefficient of the market model can be shown to be a function of a cash flow covariance and a capitalization rate covariance as described in equation (4). Each of these variables measures the particular association of an asset to the corresponding market factor. Our hypothesis rests on the assumptions that unexpected changes in market prices are either the result of changes in an asset's cash flow or capitalization rate and that covariances of asset factors to corresponding market factors are relevant for investment decisions. In this section, we will report preliminary empirical testing of the hypothesis embodied in equation (4). In the following section additional proxy measures of risk such as the dividend payout ratio, financial leverage, and firm size will be considered as determinants of the asset's covariability of return with the market factor return.

For our capital asset risk model, the following empirical specification is estimated for the time periods 1947 to 1956 and 1957 to 1968:

$$\beta_{mkt,i} = a_o + a_1 \frac{COV_{eps} + COV_{e/p}}{VAR_{eps} + VAR_{e/p}} + \delta_i \qquad (8)$$

where COV_{eps} and $COV_{e/p}$ represent the covariance values associated with β_{eps} and $\beta_{e/p}$, and VAR_{eps} and $VAR_{e/p}$ represent the market variance figures. For strict conformation of our hypothesis the intercept term should equal zero, and a_1 should be equal to one. The results of the ordinary least squares estimation for individual firms are presented in part I of Table 2. For these regressions the values of the variables were standardized by dividing through by the sample means. In both cross-sectional samples the slope coefficient is substantially less than one, while the intercept is significantly greater than zero. In general, however, the estimations tend to confirm our hypothesis that a significant association exists between the market model slope coefficient and the corresponding cash flow and capitalization rate covariance terms.

TABLE 2

REGRESSION ESTIMATES FROM THE MODEL

Time		Intercept	$\frac{COV_{eps} + COV_{e/p}}{VAR_{eps} + VAR_{e/p}}$	R^{2*}
I. Individual Firms				
1947-1956	Regression Estimates	0.947	0.064	0.057
	T Values	(2.85)	(4.74)	
	Sample Size = 338			
1957-1968	Regression Estimates	0.892	0.118	0.080
	T Values	(2.94)	(7.05)	
	Sample Size = 534			
II. Five Firm Portfolios				
1947-1956	Regression Estimates	0.769	0.235	0.244
	T Values	(2.65)	(4.87)	
	Sample Size = 67			
1957-1968	Regression Estimates	0.488	0.516	0.372
	T Values	(2.00)	(8.12)	
	Sample Size = 108			
III. Twenty Firm Portfolios				
1947-1956	Regression Estimates	0.456	0.515	0.604
	T Values	(2.34)	(5.14)	
	Sample Size = 16			
1957-1968	Regression Estimates	0.119	0.882	0.642
	T Values	(0.63)	(7.09)	
	Sample Size = 27			

*Adjusted for degrees of freedom.

It's also clear that there are a number of specification problems, both theoretical and empirical, that have affected the estimations given in the first part of Table 2. First, omission of an explicit growth variable in our theoretical model results in an inability to separate the effects of a change in reported cash flows on the growth of expected cash flows and the level of expected cash flows. In the following section, we explore in more detail the possibilities of adding a growth variable. Second, errors in the measurement of earnings -- due in part to accounting inaccuracies -- used to compute the slope coefficients cause the cash flow and capitalization rate variables to be only approximations for the underlying "true" values. Third, it is impossible to unravel the *ex post* observation from the expected value in the measurement of earnings.

One probable effect of these measurement errors on ordinary least squares estimates is to yield downward biased estimates of the model's parameters. Unfortunately, this bias does not disappear as sample size is increased.[10] To overcome the measurement errors, and thus achieve less biased relationships than those shown in part I of Table 2, we make use of a technique based upon grouping the observations.[11] Firms were ranked in descending order of market model betas, β_{mkt}, in each period and partitioned into equally weighted portfolios of five each and twenty each. The results of these regressions are given in parts II and III of Table 2. As expected, one effect of using portfolios in the estimations is to dramatically increase the coefficients of determination. More importantly, much of the estimation bias disappears. The intercept term decreases in the second period from 0.891 for individual firms to 0.488 for portfolios of five firms each and 0.119 for portfolios of twenty firms each. Similar decreases are evident in the first period. The regression coefficients on the independent variables increase, but still fall somewhat short of the hypothesized values.

From these preliminary empirical results we conclude the following:

1. There is some indication that the observed market model slope estimate for an asset may be determined by corresponding covariance values for cash flows and capitalization rates. Investors appear to act as if changes in corporate earnings and capitalization rates, in part at least, determine observed market price changes and that covariances of cash flows and capitalization rates to corresponding market factors are relevant in their pricing decisions.

2. There are important empirical specification problems, some of which have been treated in this paper, some of which have not, that interfere in the estimation procedure. For example, we found that grouping observations when error in variables exist give better ordinary least squares estimates.

3. Finally, the market model predictions may not be independent of a cash flow variable and capitalization rate variable.

The remaining portion of our empirical investigation is conducted in two distinct parts: (1) a study of the degree of association that exists between a number of variables that have often been suggested as measures or determinants of risk; (2) the testing of an alternative structural hypothesis suggested by the theoretical arguments made in sections II and III. This second part assumes a particular model of risk determination and, once again, applies regression analysis to cross-section data.

IV. Further Empirical Estimates

Cash flow and capitalization rate covariance estimates made by investors are subject to a considerable degree of error because of errors in the measurement of observations used in assessing the betas, and because historical values may not perfectly reflect current relationships. As a result, market participants may make use of other accounting variables to assess the risk associated with a security in addition to, or possibly instead of, those variables hypothesized to be the determinants of risk. The rationale for the use of these variables in assessing risk is that directly or indirectly they imply something about the uncertainty associated with the shareholders expected cash flow or the market determined discount rate. A list of these variables is given below:

a. Dividend Payment (Pay): the ratio of the sum of all dividend payments made to the sum of net income earned over the period studied.

b. Leverage (D/E): the ratio of total book value of liabilities and preferred stock to the market value of equity averaged over the period studied.

c. Firm Size (Size): total assets averaged over the period studied.

d. Liquidity (Liq): the ratio of current assets to current liabilities averaged over the period studied.

e. Growth (g_{eps}): the slope coefficient of the arithmetic least squares trend line of earnings per share.

Because of space limitations we have not given any justification of why these particular variables should or should not be related to asset risk. In some cases we think a relationship is expected, in other cases we feel that any observed relationship is a spurious one. Nevertheless, each variable was included in the analysis because at some time someone proposed that the variable suggested something about the risk associated with an asset.

Correlation Analysis

The objective of the simple linear correlation analysis carried out in this section is to test and discover the pattern of interdependency that exists between the various measures of (or proxies for) security risk. Table 3 presents the results of the correlation analysis for each of two periods for each of three different sized portfolios. In addition, the entries made in the diagonal elements of each matrix in the upper-half of the table represent the inter-period correlation of that variable with itself. For this analysis the cash flow beta and capitalization rate beta were seperately correlated with the other factors assumed to influence risk.

A number of conclusions can be made from these results. As was implied by the results presented in the previous section, the capitalization rate beta, $\beta_{e/p}$, and cash flow beta, β_{eps}, are rather closely associated with the market model beta.

TABLE 3

CORRELATION ANALYSIS

INDIVIDUAL FIRMS

Variable Designations

Period		β_{mkt}	$\beta_{e/p}$	β_{eps}	β_{oI}	PAY	D/E	SIZE	Liq	δ_{eps}
1947 - 1956	β_{mkt}	.537*	.329	.259	.197	-.481	.049	-.074	-.068	.215
	$\beta_{e/p}$		.162	.290	.177	-.074	.073	-.010	.012	.009
	β_{eps}			.032	.740	-.075	.003	-.059	.064	.145
	β_{oI}				.166	-.105	.021	-.048	.098	.044
	PAY					.426	-.285	.093	.152	-.254
	D/E						.846	.004	-.167	.098
	SIZE							.984	-.163	.078
	Liq		N = 338						.697	-.284
	δ_{eps}									-.013
1957 - June, 1968	β_{mkt}		.292	.164	.147	-.394	.069	-.182	.013	.249
	$\beta_{e/p}$			.176	.207	-.032	.013	-.048	.111	.092
	β_{eps}				.811	-.088	.005	-.059	.030	.147
	β_{oI}					-.022	-.056	-.047	.160	.086
	PAY						-.119	.099	.115	-.434
	D/E							.010	-.113	.001
	SIZE								-.077	-.059
	Liq		N = 543							-.030
	δ_{eps}									

FIVE FIRM PORTFOLIOS

Variable Designations

Period		β_{mkt}	$\beta_{e/p}$	β_{eps}	β_{oI}	PAY	D/E	SIZE	Liq	δ_{eps}
1947 - 1956	β_{mkt}	.824	.630	.455	.307	-.766	.092	-.161	-.204	.409
	$\beta_{e/p}$		.420	.502	.354	-.467	.068	-.007	-.099	.151
	β_{eps}			.199	.767	-.345	-.123	-.138	.155	.147
	β_{oI}				.020	-.304	-.028	-.217	.166	-.060
	PAY					.531	-.204	.125	.192	-.460
	D/E						.849	.006	-.543	.294
	SIZE							.987	.086	.020
	Liq		N = 67						.722	-.247
	δ_{eps}									.194
1957 - June, 1968	β_{mkt}		.621	.389	.261	-.719	.154	-.400	.035	.481
	$\beta_{e/p}$			.335	.324	-.365	.013	-.226	.009	.351
	β_{eps}				.812	-.172	.070	-.211	.091	.269
	β_{oI}					-.115	-.081	-.173	.223	.157
	PAY						-.099	.246	-.047	-.574
	D/E		N = 108					.159	-.202	-.015
	SIZE								-.058	-.231
	Liq									.025
	δ_{eps}									

TWENTY FIRM PORTFOLIOS

Variable Designations

Period		β_{mkt}	$\beta_{e/p}$	β_{eps}	β_{oI}	PAY	D/E	SIZE	Liq	δ_{eps}
1947 - 1956	β_{mkt}	.943*	.821	.734	.519	-.855	.038	-.296	-.274	.656
	$\beta_{e/p}$		.689	.614	.440	-.734	.098	-.206	-.256	.377
	β_{eps}			.623	.882	-.762	-.084	-.238	-.042	.296
	β_{oI}				.342	-.534	.039	-.316	-.155	-.018
	PAY					.895	-.107	.394	.299	-.724
	D/E						.900	-.275	-.713	.276
	SIZE							.984	.331	-.346
	Liq		N = 16						.873	-.344
	δ_{eps}									.613
1957 - June, 1968	β_{mkt}		.615	.665	.489	-.904	.321	-.677	.074	.701
	$\beta_{e/p}$			.574	.466	-.723	.245	-.472	-.057	.167
	β_{eps}				.848	-.506	.093	-.526	.064	.403
	β_{oI}					-.330	.042	-.492	.275	.183
	PAY						-.297	.583	-.006	-.738
	D/E							.065	-.204	.095
	SIZE								-.177	-.467
	Liq		N = 27							.050
	δ_{eps}									

*Diagonal elements in upper half are inter-period correlation coefficients between that variable and itself. The correlations are based on 313, 62, and 15 observations for one, five and twenty firm portfolios respectively.

The stationarity of $\beta_{e/p}$ and β_{eps} over time, as measured by the interperiod correlation coefficients, is quite low for individual firms, but the pooling introduced by combining firms into portfolios causes the expected positive correlation to rise quite rapidly. The contemporaneous degree of association between the cash flow and capitalization rate beta is quite high.

As might be expected from the simple correlation analysis both leverage and liquidity, being partially endogenous variables, are for the most part not correlated with the market model beta. To accurately measure the separate effects of these variables on market model beta, holding all other values constant, a much more intricate and complex interdependent model would have to be developed than that implied in the correlation analysis.

Perhaps the most interesting bit of information to be gleaned from the correlation analysis is the remarkably strong negative relationship that exists between the payout ratio and market model beta, and, particularly, as portfolio size increases, between payout and the cash flow and capitalization rate betas. We interpret this to mean that the manager's decision as to how much of the firm's earnings should be paid out as dividends is very much dependent on the uncertainty of the cash flows as reflected in the cash flow beta. But, in addition, we think there is reason to believe that payout ratio adds information concerning management's assessment of overall risk because its level is based on managerial expectations whereas the cash flow beta is estimated by historical values.

Finally, the simple relationship between size and growth and all other variables is as expected. We suggest however, that the relatively close relationship of size and market model beta may be a spurious one since size may simply be a proxy for another variable that measures degree to which a firm is a public utility -- and therefore is large and of low risk.

Regression Analysis

The relationship between the growth in the cash flows accruing to stockholders of a particular security and market growth was not explicitly considered in the derivation that led to equation (3). Therefore, the model analyzed (equation (4)) may have omitted a variable that at least in part determines market beta.

To overcome this possible misspecification two variables, the payout ratio and earnings per share growth, have been added to equation (8) to attempt to increase the explanatory power of the equation. We hypothesize the combination of the two variables may serve as a reasonable proxy for the omitted and unobservable growth rate covariance. A firm with a growth rate that is highly volatile with respect to the market growth rate will tend to maintain a lower payout than a firm with the same expected level of but less volatile growth. But, clearly, the level of the payout ratio is also a function of the expected rate of growth, thus, growth is included in the model so that the real effect of the payout ratio can be ascertained.[12]

The results of this regression using the single firm, five firm portfolios, and twenty firm portfolios as the unit of observation are given in Table 4.[13]

A comparison of Tables 2 and 4 indicates that the addition of the payout and growth rate variables increases the percentage of total market risk variance explained by approximately 200 per cent, 100 per cent, and 30 per cent for one, five, and twenty firm portfolios respectively. The growth rate variable is generally insignificant which argues that, once the small effect of growth is taken into account through the cash flow beta and capitalization rate beta, the level of growth is not directly related to risk.[14] There is some tendency for the coefficients on the covariance variable to decline which suggests some possible misspecification of the model, possibly related to an omitted growth rate variable.

The results presented in Table 4 also suggest that the

TABLE 4

REGRESSION ESTIMATES WITH PROXY FOR GROWTH RATE BETA

Time	Constant	$\frac{COV_{eps} + COV_{e/p}}{VAR_{eps} + VAR_{e/p}}$	PAY	g_{eps}	R^2
I. Individual Firms					
1947 - 56 Regression Estimates	1.417	0.052	-1.000	0.555	0.275
T Values		(4.54)	(-8.45)	(3.08)	
Sample Size = 338					
1957 - 68 Regression Estimates	1.215	0.108	-0.671	0.133	0.260
T Values		(6.98)	(-9.69)	(0.76)	
Sample Size = 534					
II. Five Firm Portfolios					
1947 - 56 Regression Estimates	1.990	0.140	-2.628	1.397	0.668
T Values		(3.81)	(-6.81)	(2.06)	
Sample Size = 67					
1957 - 68 Regression Estimates	1.671	0.302	-2.009	0.131	0.701
T Values		(6.03)	(-9.37)	(0.23)	
Sample Size = 108					
III. Twenty Firm Portfolios					
1947 - 56 Regression Estimates	2.353	0.230	-3.117	-1.131	0.754
T Values		(1.89)	(-2.51)	(-0.27)	
Sample Size = 16					
1957 - 68 Regression Estimates	1.866	0.340	-2.555	0.372	0.877
T Values		(3.12)	(-5.97)	(0.27)	
Sample Size = 27					

payout and growth variables working in combination may to some extent serve as a proxy for the cash flow and capitalization rate beta variables. Because of the large estimating errors in the beta values for one firm portfolios, the payout ratio, in particular, shows up to be relatively important. As the errors in the measurement of observations of the betas is reduced through the portfolio pooling process the dividend payout and growth variables decline in significance and offer less and less to the total explanatory power of the equation.

Finally, as described in Table 5, market model beta was regressed on those accounting variables that have often been used to explain variations in price-earnings ratios between firms (that is, variables that are often felt to be proxies for firm risk). The results using these variables are generally poorer than those using the derived cash flow and capitalization rate covariance terms in the sense that the explanatory power of the set of variables (measured by the R^2) tends to average ten to twenty per cent less than the explanatory power of the variables listed in Table 4. This offers some evidence that the particular way in which the risk determinants were measured improves our understanding of the factors that the market uses to assess capital asset risk.

TABLE 5

REGRESSION ESTIMATES WITH TRADITIONAL RISK PROXIES

Time		Constant	Pay	D/E	g_{eps}	Size	R^2
I. Individual Firms							
1947-56	Regression Estimates	1.596	-1.159	-.037	.389	-.020	.240
	T Values		(-9.32)	(-1.96)	(2.16)	(-.89)	
	Sample Size = 338						
1957-68	Regression Estimates	1.257	- .285	.012	.398	-.030	.176
	T Values		(-7.65)	(.78)	(2.20)	(-3.66)	
	Sample Size = 534						
II. Five Firm Portfolios							
1947-56	Regression Estimates	2.485	-2.894	-.066	.914	-.070	.570
	T Values		(-7.56)	(-1.01)	(1.06)	(-3.85)	
	Sample Size = 67						
1957-68	Regression Estimates	1.840	-1.911	.130	.661	-.120	.571
	T Values		(-7.56)	(2.12)	(1.06)	(-3.85)	
	Sample Size = 108						
III. Twenty Firm Portfolios							
1947-56	Regression Estimates	2.921	-3.982	-.057	1.140	.091	.695
	T Values		(-4.14)	(-.386)	(.29)	(.33)	
	Sample Size = 16						
1957-68	Regression Estimates	2.045	-2.589	.312	1.123	-.190	.839
	T Values		(-4.59)	(1.64)	(.78)	(-2.74)	
	Sample Size = 27						

FOOTNOTES

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[1] Beaver, Kettler and Scholes, "Tha Association Between Market Determined and Accounting Determined Risk Measures," The Accounting Review (October, 1970) examined the degree of association between various accounting ratios and the beta coefficient of the market model.

[2] See M. Blume, "Portfolio Theory: A Step Towards Its Practical Application," Journal of Business (April, 1970), and Blume, "On the Assessment of Risk," Journal of Finance (March, 1971).

[3] To show how equation (2) was derived consider the following perpetuity discounted cash flow valuation model:

$$P_i = \frac{C_i^*}{r_i} \qquad \text{(a)}$$

P_i is the price of the i^{th} asset; in this case a share of common stock issued by the i^{th} firm. Although equation (a) is a simplification of a more complex valuation model, it incorporates two important variables always considered in asset price determination; namely, expected cash flows and the rate at which the expected cash flows are capitalized. A small, unanticipated change in the asset's market price, dP_i, can be represented as the differential equation:

$$dP_i = \frac{\partial P_i}{\partial C_i^*} dC_i^* + \frac{\partial P_i}{\partial r_i} dr_i \qquad \text{(b)}$$

or, evaluating the partial derivatives,

$$dP_i = \frac{dC_i^*}{C_i^*} \frac{C_i^*}{r_i} - \frac{dr_i}{r_i} \frac{C_i^*}{r_i}. \qquad \text{(c)}$$

The realized rate of return from holding asset i is defined as,

$$R_i = \frac{dP_i + C_i}{P_i} \qquad \text{(d)}$$

where, from the investor's viewpoint, C_i is the anticipated dividends plus capital gains generated, and will equal expected cash flows C_i^*. The value of dP_i is the realized but unanticipated (windfall) gains attributable to changes in actual cash flows generated by the firm, changes in expected future cash flows, and changes in the rate at which whese cash flows are discounted.

Substituting equation (c) into (d) and assuming that the elasticity of expected cash flows with respect to reported cash flows is unity (a much more restrictive assumption than is necessary), the realized rate of return, R_i, on asset i can be expressed in terms of changes in its realized cash flows and changes in its capitalization rate as shown in equation (2). It's important to remember that, given our assumption about the elasticity of expected cash flows, $r_i = C_i^*/P_i = C_i/P_i$.

[4]The assumption is made that there is a valuation equation for the market asset given by

$$P_m = \frac{C_m^*}{r_m},$$

and that $r_m = \frac{1}{\sum_{i=1}^{N} P_i} \left(\sum_{i=1}^{N} C_i^*\right)$.

Then the derivation of R_m can proceed in exactly the same way as R_i was derived in footnote 3.

[5]Daniel Rie, "Preliminary Test of a Theory of Market Returns When Capitalization Rates are Variable," (Unpublished manuscript, University of Pennsylvania, Wharton School of Finance and Commerce, 1970).

[6]Equation (3) appears to be equivalent to a version of a model developed by Rie, "Preliminary Test . . .," op. cit. Rie shows that prediction error of the market model can be traced to two kinds of macro-economic variables. Rie was the first to introduce the terminology of a cash flow variable and a capitalization rate variable. Also, Michael Brennan, "Capital Asset Pricing and the Structure of Security Returns," (Unpublished manuscript, University of British Columbia, 1970) has shown that the prediction error of the market model is associated with two factors, using a factor analytic technique.

[7]Equation (4) can easily be derived by expressing the market

model beta coefficient as

$$\beta_i = \frac{COV(R_i, R_m)}{VAR(R_m)} \qquad (a)$$

where $COV(R_i, R_m)$ is the covariance between the returns on asset i and the market factor, and $VAR(R_m)$ is the variance of the returns on the market factor. If the definitions (2a) and (2b) are substituted into equation (a) and if all cross-product terms have zero expectation, then,

$$\beta_i = \frac{E\left[\left(\frac{dC_i}{C_i}\right)\left(\frac{dC_m}{C_m}\right) \quad \left(\frac{dr_i}{r_i}\right)\left(\frac{dr_m}{r_m}\right)\right]}{E\left(\frac{dC_m}{C_m}\right)^2 + E\left(\frac{dr_m}{r_m}\right)^2}, \qquad (b)$$

and equation (b) can be rewritten as equation (4) in the text.

[8] The Fisher Index is an equally weighted index of returns on all New York Stock Exchange securities during that period. An investment relative is defined as an assets end of period price plus dividends in the period divided by the price at the beginning of the period.

[9] Writers in the theory of finance have often assumed that growth and risk are positively related. For examples, see E.M. Lerner and W.T. Carleton, A Theory of Financial Analysis (New York: Harcourt, Brace & World, Inc., 1966), p. 114, and Myron Gordon, "Optimal Investment and Financing Policy," Journal of Finance (May, 1963), p. 267.

[10] The estimates for OLS will be biased downwards and inconsistent. See J. Johnston, Econometric Methods (New York: McGraw Hill Book Co., 1963), Chapter 6 for a proof. The one necessary condition is that the measurement errors be independently distributed.

[11] A similar kind of grouping technique has been used in Beaver, Kettler and Scholes, op. cit. and Brennan, op. cit.

[12]The discussion implies our belief that the payout ratio is an endogenous variable at least partially dependent on growth expectations. Thus the model as developed here is not correctly identified resulting in a biased coefficient for the payout variable. Time has prevented the two stage approach that would give an unbiased estimate of coefficient and its standard error.

[13]The economic interpretation of the constant term given in the previous section becomes irrelevant when payout and growth are added as explanatory variables.

[14]The result is not inconsistent with those studies that have found a strong relation between growth and the price-earnings ratio. However, a strong relation between P/E and growth confounds the affect of growth expectations with the affect of risk on the P/E ratio. What our results say is that the relation between growth and P/E is unrelated to risk because growth and risk are unrelated.