A New Look at the Capital Asset Pricing Model

by

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In a recent paper in the American Economic Review [6], we presented empirical evidence that the relationship between rate of return and risk implied by the market-line theory is unable to explain differential returns in the stock market. As a result, the risk-adjusted measures of portfolio performance based on this theory yield seriously biased estimates of portfolio performance.[1] We advanced but did not test several tenable reasons for these observed biases, which included the inability of investors to borrow large amounts of money at the same risk-free interest rate at which they can lend, and deficiencies in the return generating models which are required to translate *ex post* or realized into *ex ante* or expected returns and "risks". Recent papers by Fischer Black [1] and Stephen Ross [11] present theoretical models which suggest that the breakdown of the borrowing and lending mechanism would be expected to bias these measures, but not for the explicit reasons we gave.

The purpose of this paper is to examine both theoretically and empirically in greater depth than was done previously the reasons why the market-line theory does not adequately explain differential returns on financial assets. The first section of the paper briefly reviews the salient points of the market-line theory as recently modified and analyzes the implications of the theory. The second section estimates several types of risk-return tradeoffs implied by stocks on the New York Stock Exchange for three different periods after World War II and shows that the empirical results cast serious doubt on the validity of the market-line theory in either its original form or as recently modified. On the other hand, these results do confirm the linearity of the relationship for NYSE stocks. The third section suggests that the market for NYSE stocks is segmented from the bond market unless the return generating process is different from any heretofore tested. This has important implications for both the measurement of portfolio performance and the determination of optimal corporate financing.

I. The Current Theory

In the past couple of years, students of the stock market have devoted substantial effort to gaining a deeper theoretical understanding of the market-line theory. The more general theory which has evolved from these efforts might better be known as the theory of capital asset pricing to distinguish it from the pioneering but more restricted theories of [9], [12], and [13].

Under slightly different assumptions, both Fischer Black [1] and Stephen Ross [11] have recently presented different versions of this theory which lead to the same results in a mean-variance world. The theory states that in equilibrium the ex ante expected return of an asset i, $E(R_i)$, is related to the ex ante expected return of the market, $E(R_m)$, by the equation

(1) $$E(R_i) = E(R_o) + \beta_i[E(R_m) - E(R_o)]$$

where $E(R_o)$ is the expected return on an asset or portfolio uncorrelated with the market and β_i is defined as $Cov(R_i,R_m)/\sigma^2(R_m)$, the well-known measure of systematic or non-diversifiable risk.[2]
The most severe restriction in the development of (1) appears to be the assumption that the short sales mechanism works perfectly in that the total proceeds from a short sale can with no transaction costs be used to purchase additional assets and that there is no limit to the quantity of short sales allowed. The concluding section of this paper will discuss in some detail the

sensitivity of the theory to this assumption, but for the moment it will be assumed valid.

As noted above, $E(R_o)$ is the expected return on a portfolio which is uncorrelated with the market. Black has christened this portfolio the zero-beta portfolio. Such a zero-beta portfolio must possess, among others, the following characteristics: First, its expected return must be less than the expected return on the market portfolio. Otherwise, the expected premium associated with bearing risk, $E(R_m) - E(R_o)$, will be negative, which is inconsistent with the usual assumption of risk aversion. Second, of all feasible zero-beta portfolios, the actual zero-beta portfolio used by investors in constructing their own portfolios will be the one with minimum variance. This property follows immediately from observing that an individual who wishes to hold some zero-beta portfolio in conjunction with the market portfolio will only obtain an efficient combination if the zero-beta portfolio used has minimum variance.

Since the relevant zero-beta portfolio is the one with minimum variance, it must have a variance of zero if there exists a substantial quantity of assets with variances of zero. If there were to exist only one dollar of riskless assets, it would not be expected that this dollar would have much effect upon the equilibrium relationship. If returns are measured in nominal terms, government obligations with maturities identical to the horizons of individual investors would constitute such assets. If returns are measured in real terms, the only risk in holding governments of appropriate maturities would stem from unexpected changes in the price level. For most of the period following World War II, this risk for horizons of less than say three months has been very small.[2] Thus, the variance of the zero-beta portfolio would, according to this theory, approximate the variance of the unexpected changes in the level of inflation over the investor's horizon if returns are measured in real terms and would be zero if returns are measured in nominal terms.

The capital asset pricing model, embodied in (1), is an <u>ex ante</u> model stated solely in terms of expectations. To test it, one must make the transition to an <u>ex post</u> model by specifying some return generating process. A simple process for generating return, $\tilde{R}_i$, is

$$(2) \qquad \tilde{R}_i = E(R_i) + \beta_i \tilde{\delta} + \tilde{\epsilon}_i$$

where $\tilde{\delta}$ and $\tilde{\epsilon}_i$ are independent random variables with expected values of zero and finite variances and β_i is a constant appropriate to asset i.[3] Let x_i^m be the proportion of the total value of the market portfolio represented by asset i. If $\Sigma x_i^m \beta_i$ equals a non-zero constant k, the random variable $\tilde{\delta}$ can be rescaled by multiplying it by 1/k so that the new sum $\Sigma x_i^m \beta_i$ will equal one.[4] With this rescaling, the return on the market portfolio is given by:

$$(3) \quad \tilde{R}_m = \sum_{i=1}^{N} x_i^m \tilde{R}_i$$

$$= E(\tilde{R}_m) + \tilde{\delta} + \Sigma x_i^m \tilde{\epsilon}_i$$

where N is the number of securities in the market portfolio. Substituting (2) and (3) into (1), assuming that R_o is the risk-free rate R_f, and dropping terms of negligible magnitude, we obtain the _ex post_ model first developed by Jensen [8]:

$$(4) \quad \tilde{R}_{it} \approx R_{ft} + \beta_i(\tilde{R}_{mt} - R_{ft}) + \tilde{\epsilon}_{it}$$

where the t subscript indicates time.

Our recent paper [6] showed that the original market-line model as translated to an _ex post_ model by the return generating mechanism of (2) was deficient. We pointed out that the basic theory might be at fault or that the generating process of (2) might be too simple.

A more complicated generating process of some intuitive appeal is

$$(5) \quad R_i = E(R_i) + \tilde{\delta}_1 + \beta_i(\tilde{\delta}_2 - \tilde{\delta}_1) + \epsilon_i$$

where there are now two factors common to all securities, $\tilde{\delta}_1$ and $\tilde{\delta}_2$, and a unique factor, all assumed to be independently distributed with zero expectations and finite variances.[5] Black, Jensen, and Scholes [2] in their recent empirical tests of the capital asset pricing model implicitly used equation (5) in translating the *ex ante* model of (1) into *ex post* observables, although they did not develop this point. With this translation, they concluded that the capital asset pricing model, given by (1), explained observed returns on the New York Stock Exchange quite well.

The next section of this paper will present new tests of the capital asset pricing model. These tests will confirm our earlier conclusions and in the process raise questions about the empirical validity of the modified market-line theory, given by equation (1). Subsequently, a new theoretical explanation of the empirical results is advanced. Before moving to this section, it will prove useful to examine some of the theoretical properties of (5).

Initially, one should note that $\sigma^2(\delta_1)$ must be non-zero; otherwise, (5) reduces to (2) which is Jensen's original model and, as [6] shows, does not provide an adequate description of returns on the stock market. In the following, it will therefore be assumed that $\sigma^2(\tilde{\delta}_1)$ is non-zero.

The return on the market, R_m, assuming that $\tilde{\delta}_2$ can be appropriately scaled so that $\Sigma x_i^m \beta_i = 1$, will be

$$(6) \quad \tilde{R}_m = E(R_m) + \delta_2 + \sum_{i=1}^{N} x_i^m \epsilon_i .$$

Since the last term of (6) will be very close to zero for large N and small values of x_i^m,[6] the variance of R_m will approximate the variance of $\tilde{\delta}_2$. Thus, $\tilde{\delta}_2$ approximates the difference between the realized value of R_m and its expected value; $\tilde{\delta}_1$ has no effect upon the return of the market portfolio.

The zero-beta portfolio will consist of assets whose returns are generated according to (5). If x_i^o be the optimal proportions in this zero-beta portfolio,[7] the return R_o will be given by

$$(7) \quad R_o = E(R_o) + \delta_1 + \sum_{i=1}^{N} x_i^o \epsilon_i \, .$$

For large N, the last term will be very close to zero,[8] so that the variance of the zero-beta portfolio will approximate the variance of $\tilde{\delta}_1$.

As already pointed out, government bills with a maturity of that of the investor's time horizon are risk-free in nominal terms[9] and would have a very small variance in real terms. Thus, if returns are nominal and if (5) is to explain the observed return of all financial assets, the variance of δ_1 must be zero, which would reduce (5) to Jensen's original inadequate model. Similarly, if returns are real, the variance of $\tilde{\delta}_1$ must be no more than the variance of one's errors in anticipating inflation, a small quantity as argued above.

It thus must be concluded that (5) cannot explain the observed returns of all financial assets if the variance of $\tilde{\delta}_1$ is to take on a non-trivial value. Nonetheless, (5) may be an adequate generating process for a subset of all financial assets, such as common stocks on the NYSE (New York Stock Exchange). If this be so, the minimum variance zero-beta portfolio consisting only of common stocks would not be *the* zero-beta portfolio of the capital asset pricing model. However, equation (1) shows that the expected return on all zero-beta assets and in particular a zero-beta portfolio consisting only of common stocks must be the same, namely the risk-free asset if such an asset exists. The following section will use this observation about the expected return of a zero-beta asset to test the validity of the capital asset pricing model for common stocks on the NYSE using the generating process given by (5).

To complete the theoretical discussion of (5), it remains to show that the capital asset pricing model can be translated into a linear function of *ex post* magnitudes. This is accomplished by substituting the expected values from (5), (6) and (7) into (1), and after some algebraic manipulation and dropping terms of negligible magnitude, one obtains the *ex post* relationship:

(8) $R_{it} = R_{ot} + \beta_i [R_{mt} - R_{ot}] + \epsilon_{it}$

where the t subscript indicates time and ϵ_{it} is an independently distributed random variable with expectation of zero and finite variance.

II. The Risk-Return Tradeoff

The obvious way to estimate the risk-return tradeoff would be to collect the realized returns and beta coefficients for a large group of assets, which would typically be common stocks because of the readily available data. The next step would be to regress these realized returns on the corresponding betas to estimate the tradeoff. This procedure would be deficient for several reasons: First, the estimated betas may differ substantially from the true underlying coefficients, resulting in possibly large measurement errors and therefore biases in the regression coefficients. Second, the realized returns for individual securities will be poor estimates of the *ex ante* expected returns. Third, to preserve comparability among securities, the realized returns would have to be estimated over the same period of time. This restriction would introduce a survivorship or post-selection bias of unknown magnitude. The procedures used in this paper attempt to avoid these problems.

To cope with the measurement error problem, a grouping technique[10] was used because of its intuitive appeal and its ability to deal with the other two problems mentioned above. First, beta coefficients were estimated by regressing monthly investment relatives, properly adjusted for capital changes and cash dividends, upon the corresponding values of the Fisher Combination Link Relatives, a measure of dividend-adjusted return on the market portfolio. These beta coefficients were initially estimated for each common stock listed on the New York Stock Exchange during the entire five year period January 1950 through December 1954.

Second, twelve portfolios of roughly eighty securities apiece were formed on the basis of these estimates. No two portfolios contained any securities in common. The first portfolio consisted of those eighty or so stocks which had the lowest estimates of beta. The second portfolio consisted of those eighty or so securities with the next lowest estimates, and so on.

Third, monthly returns for each portfolio for the January 1955 through December 1959 period were calculated under two different assumptions concerning the initial investment in each security: (1) an equal investment in each security and (2) an amount in each security proportional to the market value of the shares outstanding (issued less treasury) on December 31 1954. Cash dividends were assumed reinvested in the security which paid them. Securities which were delisted were treated as follows: The security was assumed to be sold at the closing price of the last trading day of the month preceding the delisting or the closing price of the month if the stock were delisted on the last day.[11] At the end of each month, the total investment in all securities in the portfolio including stocks which were delisted was redistributed to maintain the two initial weighting schemes, involving equal and proportional investment in each security.[12]

Fourth, these monthly returns for 1955 through 1959 were averaged for each portfolio to obtain the portfolio monthly returns[13] and were then regressed upon the corresponding values of the Fisher Combination Link Relatives to yield an estimate of the beta for the portfolio. Finally, these arithmetic average returns were regressed on the beta coefficients in both linear and quadratic forms.

For the weighted regressions, i.e., those based on portfolios containing investments in stocks proportional to the market value of outstanding shares, each observation representing the average return and beta for a particular portfolio was weighted by the market value of that portfolio. This is equivalent to weighting the observation for each stock by its market value which gives equal weight to every dollar invested. Such a weighting procedure is conceptually preferable to the equal weighting of all stocks since the market mechanism equilibrates the market value of each stock to its expected dollar return and dollar risk.[14] In spite of the conceptual superiority of the value-weighted regressions, it is possible that they may provide less efficient estimators and they probably are more subject to the type of measurement errors discussed below since our portfolio grouping technique is likely to be more effective in reducing measurement errors in equal-weighted portfolios.

All of these steps were repeated to yield similar regressions for January 1960 through December 1964 and January 1965 through June 1968. In both cases, the covariations with the market of the monthly returns for the five years previous to the initial date were used to assign the securities to portfolios classified by their beta coefficients. These regressions are presented in Table 1 and will be discussed below.

These rather complicated procedures are an attempt to minimize the statistical problems of merely regressing realized returns of individual securities on the corresponding betas. The use of portfolios has several purposes: Although the estimated betas for individual securities may contain big measurement errors, the estimated betas for portfolios, which are merely weighted averages of the estimates of the betas for the individual securities, will tend to have substantially smaller measurement errors.[15] If the measurement errors for individual securities are independent and an equal investment is assumed in each of 80 securities, the variance of the measurement errors for portfolios will be one-eightieth of the variance for individual securities. Further, the realized returns for portfolios will tend to be less affected by the vagaries of individual securities and

Table 1

Average Monthly Returns as a Function of Risk

Period and Portfolio							
A. 1/55-12/59							
Equal Weight	(1)	R = 1.0128	-	0.0005 β			$\bar{R}^2$ = 0.00
		(6.73)[1]		(-0.4)			
	(2)	R = 1.0042	+	0.0181 β	-	0.0092 β^2	$\bar{R}^2$ = 0.37
		(0.65)		(2.8)		(-2.9)	
Value Weight	(1)	R = 1.0109	+	0.0014 β			$\bar{R}^2$ = 0.00
		(5.40)		(0.8)			
	(2)	R = 1.0083	+	0.0083 β	-	0.0041 β^2	$\bar{R}^2$ = 0.00
		(1.49)		(0.8)		(-0.7)	
B. 1/60-12/64							
Equal Weight	(1)	R = 1.0140	-	0.0051 β			$\bar{R}^2$ = 0.54
		(8.21)		(-3.8)			
	(2)	R = 1.0199	-	0.0168 β	+	0.0057 β^2	$\bar{R}^2$ = 0.52
		(2.47)		(-1.2)		(0.8)	
Value Weight	(1)	R = 1.0226	-	0.0157 β			$\bar{R}^2$ = 0.55
		(5.64)		(-3.8)			
	(2)	R = 1.0405	-	0.0557 β	+	0.0218 β^2	$\bar{R}^2$ = 0.54
		(1.81)		(-1.2)		(0.9)	
C. 1/65-6/68							
Equal Weight	(1)	R = 0.9977	+	0.0202 β			$\bar{R}^2$ = 0.97
		(-5.71)		(19.4)			
	(2)	R = 0.9944	+	0.0272 β	-	0.0034 β^2	$\bar{R}^2$ = 0.97
		(-2.20)		(3.1)		(-0.8)	
Value Weight	(1)	R = 0.9919	+	0.0229 β			$\bar{R}^2$ = 0.48
		(-2.63)		(3.5)			
	(2)	R = 0.9945	+	0.0148 β	+	0.0057 β^2	$\bar{R}^2$ = 0.43
		(-0.79)		(0.4)		(0.3)	

[1] t-value is measured from the risk-free rate.

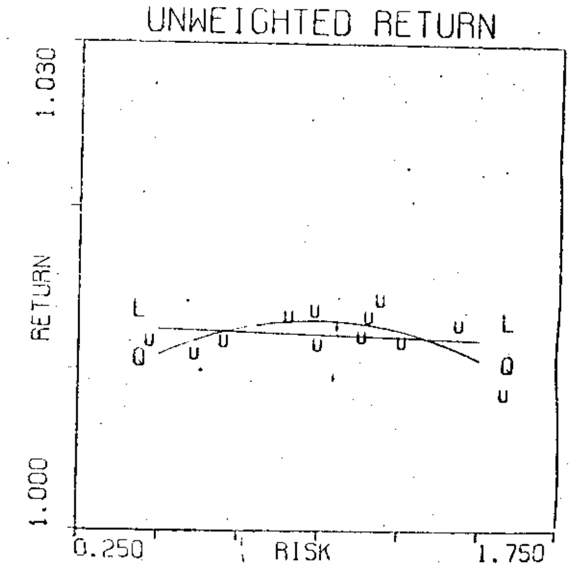
therefore may give a more efficient _ex post_ estimate of the _ex ante_ conditional expected return. Finally the use of portfolios provides a convenient way of adjusting for delistings.

The regression results (Table 1) are presented in the usual format with t-values and coefficients of determination adjusted for degrees of freedom.[16] Scatter plots of the actual data[17] superimposed upon the graph of the regression lines are presented in Charts 1 and 2.

In only one of the six comparisons shown in these charts is there any discernible difference between the quadratic and linear models over the observed range of betas. During the 1955-59 period, the quadratic model appears somewhat superior to the linear for the equal-weighted portfolios. However, the comparisons as a whole suggest that a linear model is a tenable approximation of the empirical relationship between return and risk for NYSE stocks over the three periods covered.

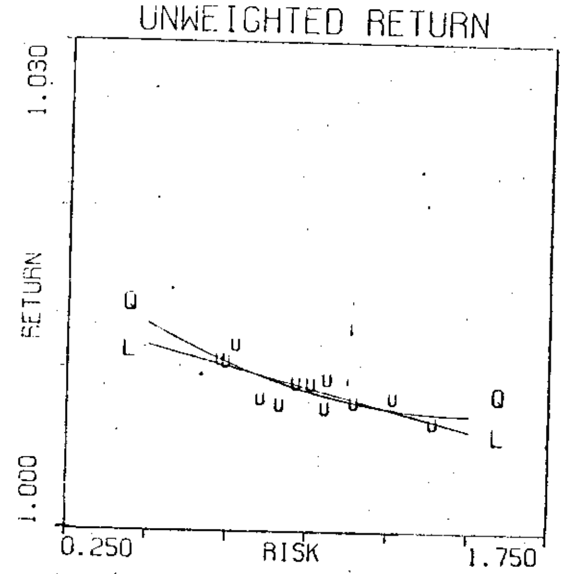
A comparison of the equal-weighted and value-weighted regressions indicates that the former provide somewhat higher correlations and, as noted earlier, they may be less subject to statistical error. However, in view of the conceptual superiority of the value-weighted portfolios, both sets of results must be examined before drawing any conclusions about the validity of the capital asset pricing model as translated to an _ex post_ model using the return generating function given by (5).

Chart 1A



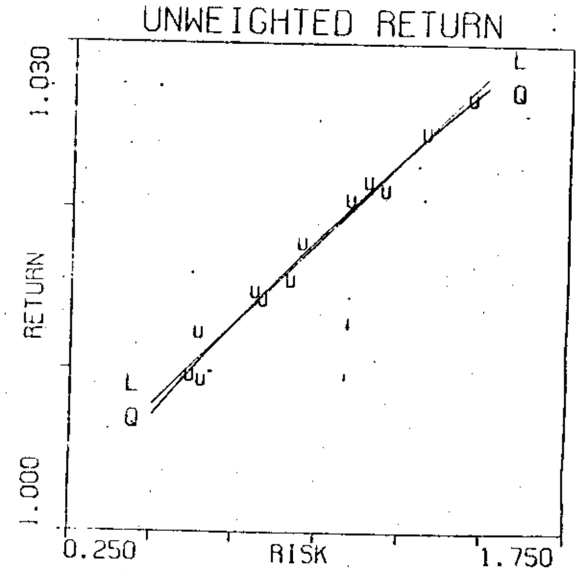
Linear (LL) and Quadratic (QQ) Regressions of Unweighted Returns on Risk for January 1955 to December 1959

Chart 1B



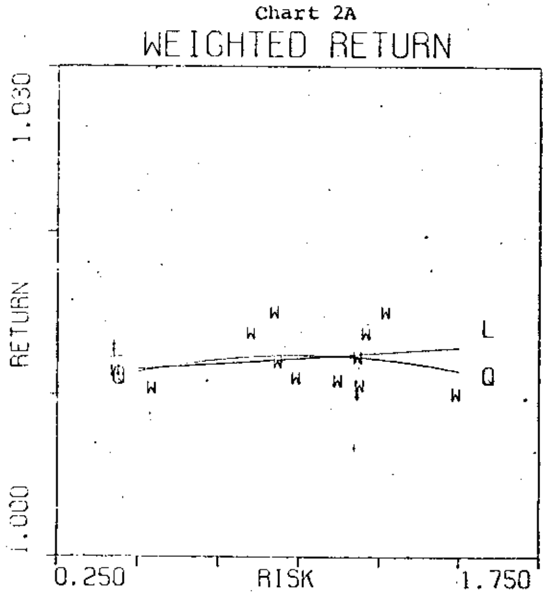
Linear (LL) and Quadratic (QQ) Regressions of Unweighted Returns on Risk for January 1960 to December 1964

Chart 1C



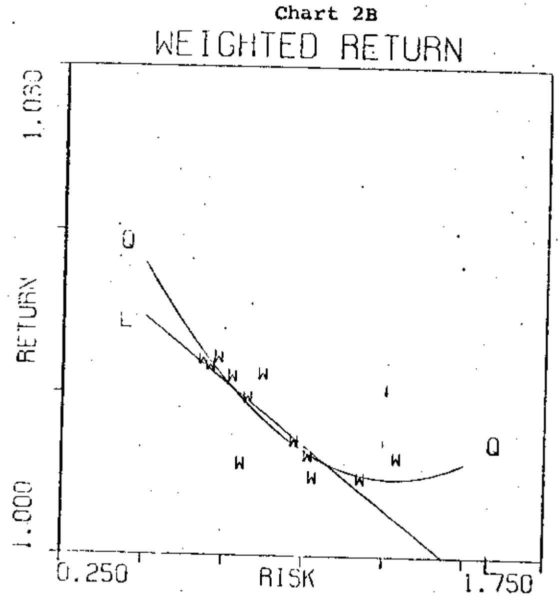
Linear (LL) and Quadratic (QQ) Regressions of Unweighted Returns on Risk for January 1965 to June 1968

Chart 2A



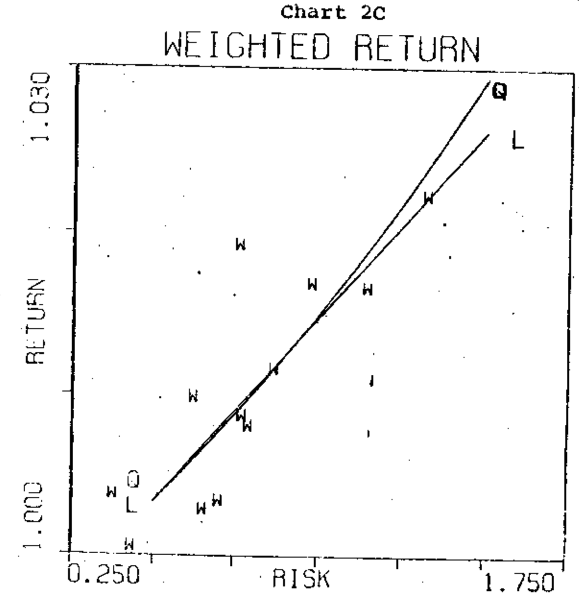
Linear (LL) and Quadratic (QQ) Regressions of Weighted Returns on Risk for January 1955 to December 1959

Chart 2B



Linear (LL) and Quadratic (QQ) Regressions of Weighted Returns on Risk for January 1960 to December 1964

Chart 2C



Linear (LL) and Quadratic (QQ) Regressions of Weighted Returns on Risk for January 1965 to June 1968

If the variance of the zero-beta portfolio is zero,[18] and if the procedures for selecting portfolios have reduced measurement error in beta to negligible proportions, the regressions in Table 1 allow a direct test of the capital asset pricing model. The constant term can be interpreted as a point estimate of the average risk-free rate, while the standard error allows a calculation of a confidence interval. These point estimates are larger than actual risk-free rates during the first two periods, January 1955 through December 1959 and January 1960 through December 1964, and smaller during the last period, January 1965 through June 1968.[19] Further, the standard errors on these estimates confirm that the differences are significant for all the linear regressions.

If the variance of the zero-beta portfolio is positive, the cross-sectional regressions provide only a point estimate of the risk-free rate. Although the estimate is unbiased under the hypothesis that the expected return on a zero-beta asset is the risk-free rate, it is inappropriate to use the usual standard error on the constant to construct a confidence interval. From the previous section, the <u>ex post</u> return on a zero-beta portfolio consisting only of common stocks, R_{ot}, can be approximated by

(9) $$R_{ot} = R_{ft} + \mu_t$$

where μ_t equals the random terms in (7). The existence of a large stock of riskless assets, like government bills, would, as pointed out above, seem to require that $E(R_{ot}) = R_{ft}$.[20]

Substituting (9) into (8) and averaging over t, one obtains for asset or portfolio i

(10) $$R_i = (R_f + \mu) + (R_m - R_f - \mu)\beta_i + \epsilon_i$$

The constant term in the regression of R_i on β_i, therefore, has an expected value of R_f, but the usual confidence interval constructed with the standard error of the constant would be centered around $(R_f + \mu)$. Without knowing the exact realized sample value of μ, one cannot directly test the hypothesis that the expected value of R_o equals R_f.

Recognizing that standard cross-sectional tests do not provide an adequate test of the capital asset pricing model if the zero-beta portfolio has a non-zero variance, Black, Jensen, and Scholes [2] have suggested and implemented a time-series test. This study replicated their tests on the data used in the cross-sectional tests for equally weighted portfolios and, despite the somewhat different way in which the portfolios were formed, arrived at numbers similar to those they obtained. These results are given in Table 2 and indicate if one takes the numbers at face value that at least for the first two periods the expected return on the zero beta portfolio differs significantly from the average risk-free rate.

These time series tests are based upon the following logic: Substituting (9) into (8) yields:

Table 2

Equal-Weighted Time-Series Estimates of Average Monthly

Returns ($\bar{R}_o$) on Zero-Beta Portfolio

Period	R_o	Standard Error of $\bar{R}_o$	t-Value of $\bar{R}_o$ from R_f
1/55 - 12/59	1.0116	.0027	3.5
1/60 - 12/64	1.0126	.0045	2.2
1/65 - 6/68	0.9971	.0049	- 1.3

$$(11) \quad R_{it} = R_{ft} + \beta_i (R_{mt} - R_{ft}) + (1 - \beta_i)\, \mu_t + \epsilon_{it} \quad .$$

Solving for R_{ft}, one obtains

$$(12) \quad R_{ft} = \frac{R_{it} - \beta_i R_{mt}}{1 - \beta_i} + \mu_t + \frac{\epsilon_{it}}{(1 - \beta_i)}$$

An estimate of the risk-free rate, R_{ft}, is therefore given by the first term on the right of the equality. If $E(\mu_t)$ and $E(\epsilon_{it})$ are both equal to zero, the estimate $\hat{R}_{ft}$ is unbiased.

Since $\hat{R}_{ft}$ will have a very large variance when β_i is near one, [2] suggests forming a weighted average of the $\hat{R}_{ft}$'s for each t and then averaging these averages over time to arrive at an overall estimate, $\hat{R}_f$, of the risk-free rate. Formally, $\hat{R}_f$ will be given by

$$(13) \quad \hat{R}_f = \sum_{t=1}^{T} \frac{1}{T} \sum_{i=1}^{N} \frac{(1 - \beta_i)^2}{\left[\sum_{j=1}^{N} (1 - \beta_j)^2\right]} \cdot \frac{R_{it} - \beta_i R_{mt}}{(1 - \beta_i)}$$

where N is the number of portfolios and T the number of time periods.

After $(1 - \beta_i)$ is cancelled, the second summation may be recognized as the least squares coefficient, b_t, in the cross-sectional regression:[21]

$$(14) \quad R_{it} - \beta_i R_{mt} = b_t (1 - \beta_i) + \epsilon_{it}$$

where the constant is suppressed and ϵ_{it} is a disturbance satisfying the usual regression assumptions. Thus, the time-series estimate of [2] can be interpreted as an average of coefficients from cross-sectional regressions of the capital asset pricing model with the constant suppressed. Indeed, by taking the average in (13) first over time and then over the portfolios, one can see that the time-series estimate of [2] can be derived from one cross-sectional regression of the form (14) with R_{it} and R_{mt} replaced by their time-series averages.

If β_i and R_{mt} are measured without error and every risky asset is included, the variance of the time series estimate $\hat{R}_f$, $E(\hat{R}_f - R_f)^2$, will be

$$(15) \quad Var(\hat{R}_f) = \frac{\sigma^2(\mu_t)}{T} + \frac{1}{\left[\sum_{i=1}^{N} (1 - \beta_i)^2\right]} \cdot \frac{\sigma^2(\epsilon_{it})}{T}$$

The first term to the right of the equality sign is the variance of the time series average of μ_t and the second term is the variance due to sampling error. If $\sigma^2(\mu_t)$ were equal to zero, the estimates of R_f provided by the time series test and by the cross-sectional regressions with the constant suppressed would be identical and have the same variance. Thus, the time series tests of [2] reduce to an analysis of a series of cross-sectional regressions of a particular form.[22]

Despite the somewhat different statistical approaches used in testing the capital asset pricing model, our earlier tests [6], the more recent tests of [2], and the tests in this paper do suggest that the estimates of the risk-free rates differ significantly and sometimes by large amounts from actual risk-free rates. The cross-sectional regressions however confirm the assumption of linearity between ex post returns and risk for NYSE stocks. The next section discusses possible implications of these results.

III. Conclusion

The evidence in this paper seems to require a rejection of the capital asset pricing theory as an explanation of the observed returns on all financial assets if the return generating process for common stocks takes the general form given by (5). This theory implies untenable estimates of the rates of return on assets which are risk-free or virtually so.

The failure of this theory to explain returns on different types of financial assets may stem from the assumption of a perfectly functioning short-selling mechanism. Generally in short sales the seller cannot use the proceeds for purchasing other securities. In addition, he has to deposit cash margin equal recently to 65% of the market value of the sales, unless he deposits securities which he owns with a market value roughly three times the cash margin which would otherwise be required. This obviously places a fairly severe wealth constraint on his short sales.

On the surface, the requirement for a perfectly functioning short sales mechanism seems more restrictive than the standard assumption that an individual can borrow or lend at the risk-free rate. Margin accounts allow at least some borrowing at rates slightly above the risk-free lending rate and there are virtually no institutional barriers to the lending mechanism. Yet the capital asset pricing theory may be robust to violations of the short sales assumption if it so happened that each investor's optimal portfolio involved no negative or short holdings. In this case, one could think of an investor's portfolio as consisting of a linear combination of the market portfolio and a zero-beta portfolio. Such a zero-beta portfolio might require short sales if it were actually to be held. However, if in combination with the market portfolio there were no net short positions, no actual short sales need to have taken place. Thus, it is theoretically possible that the short sales assumption may be less restrictive than the usual risk-free rate assumption.

Moreover, the behavior of corporate issuers in supplying securities to the market could correct, at least partially, for the deficiencies in the short-selling mechanism predicated by this theory. If, for example, the return on stocks implies a much higher zero-beta return than the return on high-grade corporate bonds, so that the market for corporate securities is out of equilibrium, then there is no action on the demand side which would correct for this disequilibrium if sufficient short sales of bonds (associated with purchases of stocks) are not possible. With unlimited short sales the disequilibrium should disappear since investors could obtain a higher return for given beta by selling bonds short and using the proceeds to lever lower beta stocks to the level of beta they desire.

Without short sales, the disequilibrium could still be corrected by corporations issuing more bonds and in the process raising bond yields since this would tend to lower their cost of capital. However, for U.S. Government securities, including Treasury bills, there would be no similar adjustment process, though short-sales should be easier to effectuate than in corporate bonds.

The fact that the relationship of average realized returns for NYSE-listed common stocks to their corresponding betas appears very close to linear in each of the three periods analyzed suggests that the capital asset pricing model and its associated short-sales assumptions (and perhaps adjustments on the supply side) may be useful in explaining returns on well-seasoned common stocks. Then the portfolios of holders of these issues could be viewed as linear combinations of the market portfolio and some zero-beta portfolio which would only include such stocks and no other financial assets.

Why, however, should this theory provide reasonably satisfactory explanations for differential returns on NYSE stocks but not on all financial assets? A possible reason is that the market for well-seasoned common stocks is at least partially segmented from the markets for other assets such as bonds. There is some independent but not very strong empirical support for such segmentation.[23] So far as the securities market is concerned the external evidence for segmentation is strongest as between stocks and corporate bonds.

Even if there is segmentation on the demand side by investors in corporate securities, it is not clear why corporate issuers have not largely eliminated this market imperfection by their actions on the supply side. If near zero-beta risky assets such as long term corporate bonds were included in the analysis, the observed risk-return tradeoff over the entire range of risk would certainly have been highly non-linear in all periods.[24] Thus the required rate of return on high-grade bonds appears much lower than that of common stock on a risk-adjusted basis.[25]

These results suggest that even without allowing for the tax advantages of debt financing, the cost of bond financing may have been substantially smaller than the risk-adjusted cost of stock financing and probably smaller than the risk-adjusted cost of internal financing. Considering the big tax advantage of bonds, the question arises why corporations did not place even more reliance on such financing. One answer may be that corporate management in its attempt to avoid the risk of bankruptcy and to preserve its own position has shied away from debt financing, in preference to the retention of earnings, whereas this risk is readily diversified by individual investors. There is some evidence that the historically large risk premium required on stock as compared with bonds has diminished in recent years.

The fact that the differential between the required rates of return on high-grade corporate bonds and on stock on a risk-adjusted basis has persisted for so long, even though it seems to have diminished, is consistent either with segmentation of markets, inadequacies of the return generating model used in this paper, or a deficient short-sales mechanism which for corporate issues is not corrected by the behavior of issuers (perhaps because of a different risk-return relationship between investors and issuers of corporate securities). Of the three possible explanations, the last seems least plausible, since it is not clear why a deficient short-sales mechanism would result in a linear relationship of returns on risk for well-seasoned stocks but not for all assets.[26] While inadequacies of the return generating model for risky assets, rather than the inability of the capital asset pricing theory to explain expected returns on different types of assets, may be responsible for our results, it is difficult to conceive of a plausible generating model which would reconcile these results with this theory. Thus, in the current state of testing of the capital asset theory, the evidence points to segmentation of markets as between stocks and bonds, even though there are few legal restrictions which would have this effect. Until such segmentation vanishes, if it does indeed exist, and until more comprehensive and more satisfactory theories (and return generating models are developed), the best and safest method to formulate the risk-return tradeoff is to estimate it empirically over the class of assets and the period of interest.

FOOTNOTES

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[1] See Friend and Blume [6]. A subsequent paper by Black, Jensen, and Scholes [2] further confirms the inability of the market-line theory to explain differential returns in the stock market. They use our earlier cross-sectional analysis over a longer period of time as well as a new time series analysis to arrive at this conclusion.

[2] If one uses the simple average of the twelve previous monthly changes in the Consumer Price Index as one's assessment of the change in the next month, the variance of the errors from this forecast was .00000339 for the period from January 1955 through June 1968.

[3] Actually, one must prove that β_i in (2) is the β_i of the capital asset pricing model. For a proof of this point, see [8].

[4] The expected value of the rescaled $\tilde{\delta}$ is still zero.

[5] Again, it is being implicitly assumed that the β_i in (5) is the β_i in the capital asset pricing model. Following the logic of [8], this assumption can easily be proved for this generating process.

[6] Cf. Jensen [8] for a description of this approximation.

[7] The x_i^o's will be such that $\Sigma x_i^o \beta_i = 0$ and $\Sigma x_i^o = 1$.

[8] The reason for this convergence to zero is different from the reason given in fn. 7. The x_i^o's result from the solution of a quadratic programming problem and not from market parameters.

Ross [11] shows that $\sum_{i=1}^{N} x_i^o \epsilon_i$ will almost surely approach zero as N increases in such a quadratic programming problem.

[9] If returns are measured in nominal terms, it has sometimes been suggested that risk-free assets are completely different animals from risky assets and that short sales are only permitted in risky assets. This is clearly not true.

[10] See E. Malinvaud [10]. Variations on this technique have been used in [2], [3], [4], and [6].

[11] Since closing prices are not always realizable or investors may not always have had advanced information of a delisting, it would have been preferable to use the first realizable price after delisting, but this would have been prohibitively expensive.

[12] The results with these redistributions which are described below are quite close to those obtained without monthly redistributions other than those reflecting delistings. Further adjusting for commissions similarly does not change the results significantly.

[13] Geometric returns were also calculated for each portfolio and the statistical analysis presented subsequently in this paper was replicated using these returns rather than the arithmetic averages. Although the geometric returns were of course slightly less than the arithmetic averages, there were no substantive differences in the implications of the analysis and so the analysis using geometric returns is not presented.

[14] Formally, the weighted regression follows from a quadratic loss function in which the loss for the i^{th} stock in the linear risk-return relationship is expressed as $W_i(R_i - a - b\beta_i)^2$ rather than $(R_i - a - b\beta_i)^2$ where W_i is the market value weight and a and b are the linear parameters. This is equivalent to assigning a loss to estimation errors proportional to the value of the stock.

[15] The reader is referred to [3] and [4] for theoretical and empirical discussions of this point.

[16] In estimating these regressions, it is implicitly assumed that the process generating these returns is stationary over time. For the short periods used in this paper the assumption appears plausible and further, the empirical evidence of Black, Jensen, and Scholes [2] confirms that the generating process after World War II and through 1966 is reasonably stationary.

[17] The letters U and W in the charts are centered over the

actual point.

[18]The authors wish to thank Fischer Black for several conservations which were very useful in writing the remainder of this section.

[19]The average monthly rates of return on three month government bills were approximately 0.21, 0.25, and 0.37 percent respectively during these periods.

[20]Ross has argued that if the world is not mean-variance, $E(R_{ot})$ might well exceed R_{ft}, and thus that our results might be interpreted as consistent with his theory at least for the first two periods.

[21]In the regression $Y_i = bX_i + \epsilon_i$, the least squares coefficient will be $b = \Sigma X_i Y_i / \Sigma X_i^2$.

[22]Under the usual regression assumptions, Fama and MacBeth [5] have suggested that the return implied by a cross-sectional regression when β is set to zero can be interpreted as the return on a minimum variance zero-beta portfolio constructed from the portfolios used in the regression. The text has derived the explicit cross-sectional regression which is equivalent to the time-series tests of [2]. The test of the capital asset pricing model in [5] which is closest to that of the time-series test of [2] does not suppress the constant and in fact is equivalent to our cross-sectional tests if $\sigma^2(\mu_t)$ equals zero.

[23]For instance, in 1967, households including personal trusts and non-profit institutions owned 83.9 percent of all corporate stocks, 4.5 percent of corporate and foreign bonds, 35.2 percent of state and local obligations, 29.8 percent of U.S. government securities, and 2.7 percent of mortgages. For more detail about the holdings of different groups, the reader is referred to Friend, Blume, and Crockett [7].

[24]This statement is based upon the observation that risk-free investment opportunities would have a beta coefficient of zero and upon an analysis of 20 year AA corporate bonds held one quarter. Regressing the quarterly market rate of return for these bonds on the Standard and Poor's 500 adjusted for dividends over the period 1960 through 1970 yields a beta coefficient of

0.126 (with a t-value of 2.57) which is only moderately larger than zero. However, the beta coefficient was insignificantly different from zero in the early 1960's. The data used in these calculations were provided by the Portfolio Evaluation Service of Merrill Lynch.

[25] See Friend, Blume and Crockett [7] for a long-run perspective, as well as the previous analysis in this paper. The reader should note that the 1965-68 period covered in this paper may provide an exception to the general rule, but it may simply reflect the inadequacies of the return generating function used to relate _ex post_ to _ex ante_ magnitudes in that period.

[26] Another possible explanation of the observed phenomenon is that the beta coefficient only measures a part of what investors mean by risk, but there is evidence that the stated risk objectives of mutual funds are related to the beta coefficients for their portfolios. (Cf. Friend, Blume, and Crockett [7].)

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